8-th International Workshop on Complex structures and vector fields Sofia, 21-26 August 2006

Abstracts

August 18, 2006

AUGUST 21 Differential Geometry

8:00 – 9:15	REGISTRATION
9:15 – 9:45	OPENING (Dodunekov, J. Stamenov, S. Dimiev, V. Gerdjikov)
9:45 – 10:30	**Yasuo Matsushita** Almost Kähler-Einstein structures on 8-dimensional Walker manifolds

10:30 – 10:50 Coffee break

10:50 - 11:35	**Peter Popivanov** On the hypoellipticity of complex valued vector fields
11:35 - 12:15	**Toshiaki Adachi** Killing helices on a complex projective space and canonical magnetic fields on geodesic spheres

Lunch time

15:30 - 16:10	**Vestislav Apostolov** Generalized Kähler manifolds, commuting complex structures and split tangent bundles
16:10 - 16:50	**Rossen Dandoloff** Anholonomy of a moving space curve and the Schrdinger equation

16:50 - 17:10 Coffee break

17:10 - 17:50	**Hideya Hashimoto** On almost Hermitian structures of 6-dimensional submanifolds in the octonions
17:50 - 18:30	**Norio Ejiri** Complex submanifolds and Lagrangian submanifolds associated with minimal surfaces in tori

AUGUST 22 Mathematical Physics and PDE

9:00 - 9:45	**Julian Lawrinowicz** Clusters and fractals in mathematical physics
9:45 - 10:30	**Vladimir Georgiev** Nonexistence of nonzero resonances for Schrdinger operators with singular perturbation

10:30 10:50 Coffee break

10:50 - 11:30	**Vladimir Gerdjikov** Hamiltonian aspects of soliton equations with deep reductions
11:30 - 12:00	**Tihomir Valchev** Bäcklund transformations and Riemann-Hilbert problem for N-wave equations with additional symmetries

Lunch time

15:30 - 16:10	**Georgi Grahovski** The Caudrey-Beals-Coifman systems and the gauge group action
16:10 - 16:50	**Nikolay Kostov** The Manakov model as two moving interacting curves

16:50 - 17:10 Coffee break

17.10 - 17.50	**Stoil Donev** Integrability, curvature and description of photon-like objects
17:50 - 18:30	**Tadashi Sugiyama** Characterization of totally umgilic immersions by curves of order 2

AUGUST 23 Differential Geometry

9:00 - 9:45	**Kouei Sekigawa** Some critical almost Kähler structures
9:45 - 10:25	**Sadahiro Maeda** Characterization of parallel isometric immersions of space forms into space forms in the class of isotropic immersions

10:25 - 10:45 Coffee break

10:45 - 11:15	**Milen Hristov** On the real hypersurfaces of Hermitian manifolds
11:15 - 11:45	**Galya Nakova** On some non-integrable almost contact manifolds with Norden metric of dimension 5
11:45 - 12:25	**Bozhidar Iliev** Normal frames and linear transports along paths in line bundles. Applications to classical electrodynamics

Lunch time

15:30 - 16:00	**Velichka Milousheva** On the geometric structure of hypersurfaces of conullity two in Euclidean space
16:00 - 16:40	**Mancho Manev** On quasi-Kähler manifolds with Norden metric

16:40 - 17:00 Coffee break

17:00 - 17:30	**Dobrinka Gribacheva** Lie groups as four-dimensional Riemannian product manifolds
17:30 - 18:00	**Marta Teofilova** Lie groups as several complex manifolds with Norden metric

Conference dinner

AUGUST 24 EXCURSION

AUGUST 25 Complex Analysis

9:00 - 9:40	**Vladimir Balan** KCC and linear stability for the Parkinson tremor model
9:40 - 10:20	**Stancho Dimiev** Bi-complex analytic pseudo-Euclidean geometry and applications

10:20 - 10:40 Coffee break

10:40 - 11:10	**Nikolai Nikolov** On the definition of Kobayashi-Buseman pseudometric
11:10 - 11:40	**Petar Stoev** Differential forms of bi-complex variables
11:40 - 12:10	**Petya Furlinska** Polynomials of bi-complex variables

Lunch time

15:30 - 16:00	**Branimir Kiradjiev** Quadratic forms on the bi-complex algebras
16:00 - 16:30	**Lilya Apostolova** Real analyticity of the hyper-Kähler almost Kähler manifolds

16:30 - 16:50 Coffee break

16:50 - 17:20	**Assen Kyuldjiev** Symmetries of the Manev problem and its real Hamiltonian form
17:20 - 18:00	**Rossen Ivanov** Camassa-Holm equation as a geodesic flow of the right invariant metric

AUGUST 26 Complex Analysis and Applied Mathematics

9:00 - 9:40	**Hiroshi Matsuzoe** Information geometry and affine differential geometry
9:40 - 10:20	**Vesna Manova** Distributional boundary values of the functions in the Djarbjashian classes

10:20 - 10:40 Coffee break

10:40 - 11:20	**Mihail Konstantinov** Effects of finite arithmetic – myths and realities
11:20 - 11:50	**Angel Ivanov, George Venkov** Global existence of the solution to the Hartree equation
11:50 - 12:20	**Ilona Zasada** Phase transitions in binary alloy thin films
12:20 - 12:50	**Marin Marinov** Homogenization Of Nonlinear Parabolic Operators Of High Order

Lunch time

15:30 - 16:00	**Vanya Markova** Application of Unscented and Extended Kalman Filtering for Estimating Quaternion Motion
16:00 - 16:30	**Peace Nwaobilor** Non-oscillating trajectories of vector fields and Zariskis local uniformization
16:30 - 17:00	**Clement Ijeh** Centroidal Voronoi tessellation based algorithms for vector fields visualization and segmentation

17:00 - 17:20 Coffee break

17:20 - 17:50	**Oladipupo Ipadeola** Anisotropic depth migration on complex-structure land data
17:50 - 18:20	**Isaak Ibenme** Dislocations in complex materials
18:20 - 18:50	**CLOSING**

Killing helices on a complex projective space and canonical magnetic fields on geodesic spheres

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We study the moduli space of helices of low orders which are generated by some Killing vector fields on a complex projective space. We call such helices Killing. For a real space form the moduli space, the set of congruence classes, of helices has a "building structure", but for a complex projective space the moduli space of all Killing helices has a quite different structure.

In my talk, we first point out that the moduli space of circles has a lamination structure. In view of this structure we find that the moduli space of circles is a disjoint union of the moduli space of trajectories for Kähler magnetic fields and the moduli space of other circles. In order to clarify the structure of the moduli space of helices on a complex projective space, we study in the second stage special helices of proper order 4, and construct an embedding of the moduli space of circles other than trajectories for Kähler magnetic fields into the moduli space of helices of proper order 4. At the last stage, we point out some Killing helices of proper order 4 on a complex projective space are extrinsic shapes of trajectories for canonical magnetic fields on its geodesic spheres.

Generalized Kähler manifolds, commuting complex structures, and split tangent bundles.

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I will discuss generalized Kähler manifolds for which the induced complex structures commute, and then relate the existence problem of such manifolds to a conjecture by Beauville concerning Kähler manifolds with splittable holomorphic tangent bundle.

Real analyticity of the hyperkähler almost Kähler manifolds

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Using the techniques in the paper by Kutzschebauch and Loose [1] it is proved that a smooth manifold equipped with two symplectic-homotopic symplectic structures ω_0 and ω_1 admits a real analytic structure such that ω_0 and ω_1 are real analytic ones. Then the result is used to prove that each hyperkähler almost Kähler manifolds with respect to two fundamental forms obtain a real analytic almost Kähler structure C^∞ equivalent to the given hyperkähler almost Kähler structure.

References

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KCC and linear stability for the Parkinson tremor model

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The paper investigates the KCC (structural) stability of the brain-stimulated Parkinson tremor dynamical system. After a brief presentation of the known linear stability results of the model, are determined and studied the KCC-invariants. It is emphasized that the spectral properties of the second KCC-invariant provide practical information for the behavior of the investigated model parameters.

The Finslerian spray and the local nonlinear connection of the second-order differential system associated to the basic SODE are evidentiated, and integrability issues are discussed.

Selected References

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Anholonomy of a moving space curve and the Schrödinger equation as a moving space curve

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The time evolution of a space curve is associated with a geometric phase. This phase arises because of the path dependence of the rotation of the natural Frenet-Serret triad as one moves along the curve. The corresponding non rotational triad undergoes Fermi-Walker parallel transport. The geometric phase, or the Fermi-Walker phase is associated with a second geometric phase, the so called "incompatibility" phase whose presence is linked to the Schrodinger equation. We present a general condition for the evolution of a space curve in terms of geometric phases.

Double-complex analytic pseudo-Euclidean geometry and applications

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Double-complex analyticity can be developed on double-complex algebras, which are complex 2-dimensional vector spaces endowed with multiplication law. The first example of a double-complex algebra (non-commutative) with a rich set of analytic functions was developed by Fueter (1935), for regular quaternion functions $f(q) \in \mathbb{H}$, $q \in \mathbb{H}$, $q = t + xi + yj + zk$, $t,\ x,\ y,\ z \in \mathbb{R}$, via the Cauchy-Riemann approach. If we write $q = v + jw$, $j^2 = -1$, $v,\ w \in \mathbb{C}$ ($v = t + ix$, $w = y - iz$), and $f(q) = g(v, w) + jh(v, w)$, the so called Cauchy-Riemann-Fueter equations hold, namely

$$\frac{\partial g}{\partial \bar{v}} = \frac{\partial h}{\partial \bar{w}}, \qquad \frac{\partial g}{\partial w} = -\frac{\partial h}{\partial v},$$

according to Fueter's notations .

Since there is no other double-complex division algebras than $\mathbb{H}$, we search non - division double-complex algerbras with a rich set of holomorphic functions. Here we stress on the non-division commutative algebra $C(1, j)$ of double-complex numbers $\alpha = z + jw$, $j^2 = i$, z, w, $i \in \mathbb{C}$. It is to remark that $C(1, j)$ is a complexification of the real algebra of even anti-cycles numbers $R(1, j, j^2, j^3)$, $j^4 = -1$.

According to the first author a double-complex function $f(\alpha) = f_0(z, w) + jf_1(z, w)$ is holomorphic iff it satisfy the following Cauchy-Riemann type equations

$$\frac{\partial f_0}{\partial z} = \frac{\partial f_1}{\partial w}, \qquad \frac{\partial f_0}{\partial w} = j\frac{\partial f_1}{\partial z},$$

f_0 called the even part of f and f_1 the odd one. The functions f_0 and f_1 are holomorphic functions of two complex variables. The set of zero divisors $z + jw$ in $C(1, j)$ is an analytic set in $\mathbb{C} \times \mathbb{C}$ satisfying the equation $z^2 - iw^2 = 0$. Some interconnection between holomorphic double-complex functions and holomorphic functions of two complex variables is demonstrated, developed by the second author, P. Stoev and B. Kiradjiev.

A geometric point of view can be developed introducing a pseudo-scalar product on $C(1, j)$, namely $\langle z + jw, u + jv\rangle := zu - iwv$, and $\langle z + jw, z + jw\rangle = z^2 - iw^2$. Then the isotropic cone of $C(1, j)$ is defined by just the same equation as for the set of zero-divisors. In real coordinates ($z = x + iy$, $w = \xi + i\eta$) the isotropic cone coincides with the intersection the following two real 3-dimensional surfaces in $\mathbb{R}^4$:

$$x^2 - y^2 + 2\xi\eta = 0, \qquad \xi^2 - \eta^2 + 2xy = 0.$$

Introducing appropriates real parameters s and t, one proves that the mentioned isotropic cone is represented by a family of couples of central curves of hyperbolic type parametrized by the points (s, t) such that $st = -1$. The obtained geometry can be considered as an analogous of pseudoeclidean geometry on $\mathbb{C}$ defined by the pseudo-scalar product $\langle x+iy, \xi+i\eta\rangle := x\xi - y\eta$, and $\langle x+iy, x+iy\rangle = x^2 - y^2$. In our case we take the natural neutral metric defined by $|\alpha|^2 = |z|^2 - |w|^2$, $\alpha = z + iw$, and we consider the inverse image of the so called "absolute plane" $O|z||w|$.

The third author will present applications in mechanics.

Integrability, curvature and description of photon-like objects

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This paper aims to present and realize a general idea for description of finite physical objects of photon-like nature making use of the mathematical concepts and structures connected with the Frobenius integrability/nonintegrability. We consider some distribution Δ_o of vector fields on a manifold, then separate some *integrable* subdistribution $\Delta \subset \Delta_o$, representing the integrity of the object considered. The curvatures of all non-integrable subdistributions of Δ are interpreted as generators of processes of internal energy-momentum exchange, i.e. of the internal dynamics of the object. The curvatures of distributions including vector fields from Δ_o and Δ are interpreted as generators of interaction of the physical object with the outside world. A basic example of photon-like objects is considered in detail.

Complex submanifolds and Lagrangian submanifolds associated with minimal surfaces in tori

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We consider some energy function withe parameter on the Siegel upper half spce H_γ and the restriction to complex manifolods in H_γ. We prove that the catastrophe set of the function on complex submanifold are analytic set by using another complex structure different from the standard complex structure of the product of complex spaces and parameter spaces. Moreover the irreducible components containg a nondegenerate critical point give Lagrangian cones in complex Euclidean space (the cotangent bundle of parameter spaces). As example, we have analytic sets associated with minimal surfaces in flat tori and the irreducible component containg minimal surfaces with only Killing Jacobi fields gives a Lagrangian cone.

Polynomials of double-complex variables

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The structure of zero-sets of the polynomials of double-complex variables is studied in some particular cases.

Nonexistence of nonzero resonances for Schrödinger operators with singular perturbation

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The existence of non - trivial solutions to the equation

$$-\Delta u + \mathbf{i} a_j(x) \frac{\partial u}{\partial x_j} + b(x) u = \lambda u, \quad x \in \mathbf{R}^n, \quad \lambda > 0, \tag{1}$$

is a typical obstacle to establish dispersive properties of the time evolution group associated with the perturbed Laplace operator

$$-\Delta + \mathbf{i}a_j(x)\frac{\partial}{\partial x_j} + b(x). \tag{2}$$

The main result of the work establishes that there are no resonances embedded in the positive real line for the family of operators (2) under suitable short - range assumptions at infinity and possible local singularities of the coefficients $a_j(x)$ and $b(x)$.

Hamiltonian aspects of soliton equations with deep reductions

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By soliton equations we mean nonlinear evolution equations (NLEE) in two-dimensional space-time allowing Lax represenation $[L(\lambda), M(\lambda)] = 0$. More specifically we consdier the NLEE related to the Lax operators of the form:

$$L(\lambda)\psi(x,t,\lambda) \equiv i\frac{d\psi}{dx} + (q(x,t) - \lambda J)\,\psi(x,t,\lambda) = 0,$$

where $q(x,t) = [J, Q(x,t)]$. Here $Q(x,t)$ is a generic element of the simple Lie algebra $\mathfrak{g}$ vanishing fast enough for $x \to \pm\infty$ and J is a constant element of the Cartan subalgebra $\mathfrak{h} \subset \mathfrak{g}$.

The soliton equations integrable by applying the inverse scattering method (ISM) [1] to $L(\lambda)$ possess a hierarchy of Hamiltonian structures, i.e. for each such equation one can construct a sequence of Hamiltonians $H^{(k)}$ and symplectic forms $\Omega^{(k)}$, $k = 1, 2, \ldots$, such that each pair $(H^{(k)}, \Omega^{(k)})$ generates the same NLEE for all k. The phase space $\mathcal{M}_J \equiv \{q(x,t)\}$ can be viewed as the co-adjoint orbit of a certain Kac-Moody algebra passing through λJ.

Important classes of NLEE with physical applications are the N-wave equations [1] and the multicomponent nonlinear Schrödinger equations (MNLS) [2]. They are naturally related to the homogeneous and symmetric spaces respectively. We study the reductions of these NLEE following [3]. The reductions allow us to impose on $q(x,t)$ a number of algebraic constraints compatible with the dynamics. Thus we are able to derive new soliton equations with fewer coefficient functions.

The effects of the reductions on the phase space $\mathcal{M}_J$ and on the hierarchies of the Hamiltonian structures are outlined [4]. In doing this we are using two important tools: a) the equivalence of the isnverse scattering problem for $L(\lambda)$ to a Riemann-Hilbert problem and b) the generalized Fourier transforms [5] that allow us to linearize the NLEE.

We conslude with the class of soliton equations containing the affine Toda field theory models [3, 6]. They are related to $L(\lambda)$ with $\mathbb{Z}_h$ reduction where h is the Coxeter number of $\mathfrak{g}$. We derive for them the so-called symplectic basis which allows us to obtain explicitly their action-angle variables [7]. Finally we briefly comment on the limiting case $h \to \infty$.

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The Caudrey-Beals-Coifman Systems and the Gauge Group Action

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The fundamental properties of the generalized Zakharov–Shabat systems with complex-valued Cartan elements and the systems studied by Caudrey, Beals and Coifman (CBC systems) [1, 2] and their gauge equivalent are considered. This class contains such physically important equations like the N-wave equations and their gauge equivalent [3], the complex Toda chain, etc.

In the present talk the following topics will be discussed: the properties of their fundamental analytical solutions (FAS) associated to the corresponding Lax operator; the minimal set of scattering data [4]; the description of the class of nonlinear evolutionary equations solvable by the inverse scattering method and the recursion operator, related to such systems; the hierarchies of Hamiltonian structures.

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Lie groups as four-dimensional Riemannian product manifolds.

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The basic class of the integrable Riemmanian almost product manifolds with Norden metric is considered. Three examples of 4-dimensional Riemmanian product manifolds are constructed by means of Lie groups and Lie algebras. The form of the curvature tensor for each of the examples is obtained.

On almost Hermitian structures of 6-dimensional submanifolds in the octonions

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On the real hypersurfaces of Hermitian manifolds

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There are sixteen classes of real hypersurfaces of Kähler manifold, generated by the four basic classes W_1, W_2, W_4 and W_6 in the classification scheme for almost contact metric manifolds [1]. In [2] these classes are described in terms of the hypersurface second fundamental form. In the partial case when the ambient manifold is complex space form the identities for the Riemannian and for the Hermitian-like curvatures are introduced [3, 5]. The paper deals with the classification problem for the real hypersurfaces of Hermitian manifolds by using the ideas and results in [4, 6].

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Dislocations in Complex Materials

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Deformation of metals and alloys by dislocations gliding between well-separated slip planes is a well-understood process, but most crystal structures do not possess such simple geometric arrangements. Examples are the Laves phases, the most common class of intermetallic compounds and exist with ordered cubic, hexagonal, and rhombohedral structures. These compounds are usually brittle at low temperatures, and transformation from one structure to another is slow. On the basis of geometric and energetic considerations, a dislocation-based mechanism consisting of two shears in different directions on adjacent atomic planes has been used to explain both deformation and phase transformations in this class of materials. We report direct observations made by Z-contrast atomic resolution microscopy of stacking faults and dislocation cores in the Laves phase Cr2Hf. These results show that this complex dislocation scheme does indeed operate in this material. Knowledge gained of the dislocation core structure will enable improved understanding of deformation mechanisms and phase transformation kinetics in this and other complex structures.

Centroidal Voronoi Tessellation Based Algorithms for Vector Fields Visualization and Segmentation

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A new method for the simplification and the visualization of vector fields is presented based on the notion of Centroidal Voronoi tessellations (CVT's). A CVT is a special Voronoi tessellation for which the generators of the Voronoi regions in the tessellation are also the centers of mass (or means) with respect to a prescribed density. A distance function in both the spatial and vector spaces is introduced to measure the similarity of the spatially distributed vector fields. Based on such a distance, vector fields are naturally clustered and their simplified representations are obtained. Our method combines simple geometric intuitions with the rigorously established optimality properties of the CVTs. It is simple to describe, easy to understand and implement. Numerical examples are also provided to illustrate the effectiveness and competitiveness of the CVT-based vector simplification and visualization methodology.

Normal frames and linear transports along paths in line bundles. Applications to classical electrodynamics

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The definitions and some basic properties of the linear transports along paths in vector bundles and the normal frames for them are recalled. The formalism is specified on line bundles and applied to a geometrical description of the classical electrodynamics. The inertial frames for this theory are discussed.

Anisotropic Depth Migration on Complex-Structure Land Data

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Anisotropic depth migration (ADM) in thrust-belt environments has become more commonplace over the past four years. As our understanding of the problems with seismic imaging of structures below dipping clastic strata increases, so does our ability to build anisotropic velocity models and correct for imaging and position errors cause by seismic anisotropy. The theoretical background and numerical modelling of seismic propagation effects of waves through media that exhibits tilted transverse isotropy (TTI) illustrate problems with anisotropy. Results of ADM on three complex-structure datasets illustrate the effectiveness of the solution.

Global existence of the solution to the Hartree equation.

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We consider nonlinear Schrodinger equation with nonlocal interaction and prove that if the solution has finite $H^{1/2}-$ norm at $t = 0$, then it exists globally in time. Moreover, its $H^{1/2}-$ norm remains bounded for all times.

Camassa-Holm equation as a geodesic flow of the H^1 right-invariant metric

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The fundamental role played by the Lie groups in mechanics, and especially by the dual space of the Lie algebra of the group and the coadjoint action will be illustrated through the Camassa-Holm equation (CH). In 1998 Misiolek observed that the CH is a geodesic flow equation on the group of diffeomorphisms, preserving the so-called H^1 metric. This example is analogous to the situations in hydrodynamics, where the infinite-dimensional groups of diffeomorphisms preserve the volume element of the domain of fluid flow and to the theory of rigid body with a fixed point, where the Hamiltonian is actually a left-invariant metric on $SO(3)$. The momentum map and some explicit parametrizations of the diffeomorphism group, related to recently obtained solutions for the CH equation will be presented.

Quadratic forms on the double-complex algebras

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Quadratic forms over double-complex algebras appeared naturally. For simplicity here we consider such forms of two double-complex variables $\alpha, \beta \in C(1,j)$, i.e. $\alpha = z + jw$, z, w, u, $v \in \mathbb{C}$, $j^2 = i$, $i \in \mathbb{C}$.

In general a quadratic form $Q((\alpha,\beta))$ over $C(1,j) \times C(1,j)$ is defined by a bi-linear symetric functional $B(x,y)$ with complex coefficients (defined over the underlying vector space with base $(1,1)$, $(1,j)$, $(j,1)$, (j,j), such that $Q = Q((\alpha,\beta)) = B((\alpha,\beta),(\alpha,\beta))$. In terms of z, w, u, v we have $Q = a(z+jw)^2 + 2b(z+jw)(u+jv) + c(u+jv)^2$, a, b, $c \in \mathbb{C}$. So $Q = a(z^2 + iw^2 + 2jzw) + 2b(zu + iwv + j(zv + wu)) + c(u^2 + iv^2 + 2juv) = a(z^2 + iw^2) + 2b(zu + iwv) + c(u^2 + iv^2) + j(2azw + b(zv + wu) + 2cuv)$. In fact we obtain two quadratic form over $\mathbb{C}^4$. The classic theorem of Lagrange is established in Q-orthogonal basises.

The quadratic form Q is a double-complex holomorphic form with respect to the double-complex variable $z + jw$ iff it satisfies the Cauchy-Riemann type equations introduced by S. Dimiev for $C(1,j)$:

$$\frac{\partial Q_0}{\partial z} = \frac{\partial Q_1}{\partial w} \qquad \frac{\partial Q_0}{\partial w} = j\frac{\partial Q_1}{\partial z},$$

where $Q_0 = a(z^2+iw^2)+2b(zu+iwv)+c(u^2+iv^2)$, and $Q_1 = 2azw+2b(zv+wu)+2cuv$. So the equalities $2az + 2bu = 2az + 2bu$ and $2iaw + i2bv = i(2aw + 2bv)$ show that $Q = Q_0 + jQ_1$ is always holomorphic.

Examples: the quadratic form $a(z + jw)^2$ is holomorphic over $C(1,j)$, $a \in \mathbb{C}$. As $\partial(az^2 + aiw^2)/\partial z = \partial(2azw)/\partial w$ (or $2az = 2az$) and $\partial(az^2 + aiw^2)/\partial w = i\partial(2azw)/\partial z$ (or $2aiw = i2aw$). The same is true for $c(u + iv)^2$. But the $(C \times C)$-holomorphic form $Q = z^2 - iw^2$ ($Q_0 = z^2 - iw^2$, $Q_1 = 0$) (which is important for the definition of the isotropic cone of $C(1,j)$) is not $C(1,j)$-holomorphic.

Another negative example is given by the quadratic form $(\alpha^*)^2 = (z - jw)^2$. It is not double-complex holomorphic as it does not satisfy the above mentioned Cauchy-Riemann system. The same is true for the quadratic forms of the type $a\alpha\alpha^* + b\beta\beta^*$ etc.

Effects of finite arithmetic - myths and realities.

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In this paper we analyze the main effects of using finite arithmetic (FA) in the solution of computational problems. As a particular model of FA we use the standard floating-point binary arithmetic satisfying the IEEE Standard. In particular we describe the influence of the three main groups of constants which govern the effects in using FA:

1. The rounding unit and the range of FA.
2. The relative condition number of the (regular) computational problem.
3. The constants in the definition of numerical stability of the algorithm used to solve the problem.

We give a simple formula which shows how these constants define an upper bound on the relative accuracy of the solution of a regular computational problem in FA.

Next we discuss some myths and misconceptions which are wide spread among users of computer systems for mathematical computations.

Thus the aim of the paper is to help lecturers in informatics and mathematics as well as students and young specialists in science and engineering to avoid bad computational practices.

Symmetries of the Manev Problem and its Real Hamiltonian Form

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The Manev model and its real form dynamics are known to possess Ermanno-Bernoulli type invariants similar to the Laplace-Runge-Lenz vector of the Kepler model. Using these additional invariants, we demonstrate here that both Manev model and its real Hamiltonian form have exactly the same $\mathfrak{so}(3)$ or $\mathfrak{so}(2,1)$ symmetry algebra in addition to the angular momentum algebra. Thus Kepler and Manev models are shown to have identical symmetry algebras and hence sharing more features than previously expected.

The Manakov model as two moving interacting curves

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The two time-dependent Schrödinger equations in a potential $V(s,u)$, u denoting time, can be interpreted geometrically as a moving interacting curves whose Fermi-Walker phase density is given by $-(\partial V/\partial s)$. The Manakov model appears as two moving interacting curves using extended da Rios system and two Hasimoto transformations.

Clusters and fractals in mathematical physics

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It is well known that starting with real structure, the Cayley–Dickson process gives complex, quaternionic, and octonionic (Cayley) structures related to the Adolf Hurwitz composition formula for dimensions $p = 2, 4$ and 8, respectively, but the procedure fails for $p = 16$ in the sense that the composition formula involves no more a triple of quadratic forms of the same dimension; the other two dimensions are $n = 2^7$. Instead, Ławrynowicz and Suzuki (2001) have considered graded fractal bundles of the flower type related to complex and Pauli structures and, in relation to the iteration process $p \to p+2 \to p+4 \to \ldots$, they have constructed 2^4-dimensional "bipetals" for $p = 9$ and 2^7-dimensional "bisepals" for $p = 13$. The objects constructed appear to have an interesting property of periodicity related to the gradating function on the fractal diagonal interpreted as the "pistil" and a family of pairs of segments parallel to the diagonal and equidistant from it, interpreted as the "stamens". As a consequence of an effective, explicit determination of the periods and expressing them in terms of complex and quaternionic structures, the following applications are outlined: fractals vs. biholomorphic invariants, importance of fractals when studying the entropy-depending structures, hyperholomorphy and harmonicity, and applications to nonlinear parabolic equations. The lecture is based on a research joint with Stefano Marchiafava (Roma) and Małgorzata Nowak-Kępczyk (Radom, Poland).

Characterization of parallel isometric immersions of space forms into space forms in the class of isotropic immersions

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We characterize parallel isometric immersions of space forms into space forms in the class of isotropic submanifolds M's under conditions that the mean curvature vector of M is parallel and the sectional curvature K of M^n satisfies some inequality.

On quasi-Kähler manifolds with Norden metric

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The basic class of the non-integrable almost complex manifolds with Norden metric is considered. Its curvature properties are studied. A 4-parametric family of 4-dimensional quasi-Kähler manifolds with Norden metric is constructed by means of a Lie group with a non-Abelian almost complex structure. This family is characterized geometrically. Necessary and sufficient conditions for the special case of isotropic Kähler manifolds are given.

Distributional boundary values of the functions in the Djarbjashian classes

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In the paper by M.Djarbashian [1] the author introduced, on the unit disk $\Delta = \{z,\ |z| < 1\}$, the Djarbjashian spaces denoted by $H^p(\alpha)$, $-1 < \alpha < \infty$, $0 < p < \infty$, i.e.

$$f \in H^p(\alpha) \leftrightarrow \frac{1+\alpha}{\pi} \int_0^1 \int_0^{2\pi} (1-r^2)^\alpha \left|f(re^{i\theta})\right|^p rdr\, d\theta < \infty,$$

with the norm

$$||f||_p^\alpha = \left(\frac{1+\alpha}{\alpha} \int_\Delta (1-|z|^2)^\alpha |f(z)^p| dx\, dy\right)^{1/p}, \qquad z = x+iy.$$

In our paper we found the distributional boundary values of the functions in the Djarbjashian spaces.

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Homogenization Of Nonlinear Parabolic Operators Of High Order

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In this work the problem of homogenization of nonlinear parabolic operators of the type

$$P_\varepsilon(u) = \frac{\partial u}{\partial t} - \sum_{|\alpha|\le m} (-1)^{|\alpha|} D^\alpha a_\varepsilon(\varepsilon^{-\mu} t, \varepsilon^{-r} x, \partial u)$$

is solved, where the functions $a(\tau, y, \zeta)$ are periodic in (τ, y). Depending on the relation between μ and r, the behavior of $P_\varepsilon(u)$ when $\varepsilon \to 0$ is examined. A homogenization operator $P(u)$ is found and the corresponding formulas for calculation of the effective coefficients are proved.

Application of Unscented and Extended Kalman Filtering for Estimating Quaternion Motion

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The unscented Kalman filter is a superior alternative to the extended Kalman filter for a variety of estimation and control problems. However, its effectiveness for improving direction of motion tracking for virtual reality applications in the presence of noisy data has been unexplored. In this paper, we present an empirical study comparing the performance of unscented and extended Kalman filtering for improving model for forecasting. Specifically, we examine wind orientation motion, represented with quaternions, which are critical for correct viewing perspectives in virtual reality. Our experimental results and analysis indicate that unscented Kalman filtering performs equivalently with extended Kalman filtering. However, the additional computational overhead of the unscented Kalman filter and quasi-linear nature of the quaternion dynamics lead to the conclusion that the extended Kalman filter is a better choice for estimating quaternion motion in virtual reality applications.

Almost Kähler-Einstein structures on 8-dimensional Walker manifolds ρ. Counterexamples to the Goldberg conjecture of neutral version

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There is a famous conjecture, by Goldberg [6], which states that the almost complex structure of a compact almost Kähler-Einstein Riemannian manifold is integrable. Sekigawa proved that the Goldberg conjecture is true if the scalar curvature is non-negative [11], [12]. Nurowski and Przanowski [10] have shown that the conjecture is false when one omits the assumption of compactness and one of us (Haze) has demonstrated the same for the neutral-signature analogue of Goldberg's conjecture by constructing a neutral Ricci-flat metric on an open subset of $\mathbb{R}^4$ which is almost-Kähler with respect to a non-integrable almost complex structure. (The assertion in [10] that Sekigawa has established Goldberg's conjecture in four dimensions is erroneous, [13].)

In [8], Haze's metric is shown to be an instance of a Walker metric. By a Walker manifold, we mean a pseudo-Riemannian n-manifold which admits a field of parallel null r-planes, with $r \leq \dfrac{n}{2}$. The corresponding metric is called a Walker metric [14]. When $n = 2k$ and $r = k$, such a metric is of neutral signature. Walker metrics have proven useful in various circumstances, e.g., in [5]. I have recently begun a study of neutral Walker metrics in four dimensions [7], [8]. See also joint works with Davidov and Muškarov, and with my colleague in Spain as follows: [1], [2], [3] and [4].

By considering 8-dimensional Walker manifolds, we are able to exhibit almost Kähler-Einstein neutral structures which are not Kähler, initially on $\mathbb{R}^8$ and then on an 8-torus. Thus, the neutral-signature version of Goldberg's conjecture fails. Whether our result has any bearing on the original conjecture of Goldberg for Riemannian manifolds remains unclear, although our example, being Ricci flat, indicates that Sekigawa's result [11],

[12] fails for neutral signature. Our almost Kähler-Einstein neutral structure on an 8-dimensional torus is the first counterexample for a *compact manifold.*

The counterexamples will be published in Monatshefte für Mathematik [9].

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Information geometry and affine differential geometry

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Affine differential geometry studies hypersurfaces immersed into an affine space, and information geometry studies geometric structures of sets of probability distributions. A key fact is that duality of affine connections arises naturally, then these geometries have common geometric ideas.

Let us focus to elucidate the relation between Bayesian statistics and affine differential geometry. Bayesian statistics is a statistical theory which uses previous knowledge for inference, and the knowledge is expressed by a prior probability distribution. Hence existence of prior distributions and their choice are important problems. From the viewpoint of differential geometry, prior distributions can be regarded as volume elements on statistical manifolds, and equiaffine structures work essentially for the existence of natural prior distributions.

In this talk, we give a geometric interpretation of Bayesian statistics. Moreover, we will give sufficient conditions for a statistical submanifold to have an equiaffine structure, as a result for differential geometry.

On the Geometric Structure of Hypersurfaces of Conullity Two in Euclidean Space

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We introduce the notion of a semi-developable surface of codimension two in Euclidean space as a generalization of the notion developable surface of codimension two and give a characterization of the developable and semi-developable surfaces of codimension two in terms of their second fundamental forms. We prove that each hypersurface of conullity two in Euclidean space is locally a foliation of developable or semi-developable surfaces of codimension two.

We give a geometric description of the class $\mathfrak{K}_o$ of hypersurfaces of conullity two with involutive geometric two-dimensional distributions proving that the integral surfaces of these distributions are surfaces with flat normal connection, which are not developable and conversely, that any two-dimensional surface with flat normal connection, which is not developable, generates a hypersurface of conullity two from the class $\mathfrak{K}_o$. In this way the hypersurfaces of conullity two from the class $\mathfrak{K}_o$ are in one-to-one correspondence with the two-dimensional surfaces with flat normal connection, which are not developable.

We characterize the hypersurfaces of conullity two also as envelopes of two-parameter families of hyperplanes, proving that a hypersurface in Euclidean space is locally a hypersurface of conullity two if and only if it is the envelope of a two-parameter family of hyperplanes. This geometric characterization allows us to obtain a parametrization of each hypersurface of conullity two by a pair of a unit vector function $l(u,v)$ and a scalar function $r(u,v)$. We obtain a characterization in terms of a system of partial differential equations for the geometric functions $l(u,v)$ and $r(u,v)$ of two main classes of hypersurfaces of conullity two: the class of minimal hypersurfaces of conullity two and the class of umbilical hypersurfaces of conullity two.

On some non-integrable almost contact manifolds with Norden metric of dimension 5

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A Lie group as a 5-dimensional almost contact manifold with Norden metric is considered. By using the complexification of the real Lie algebra this manifold is constructed. The obtained manifold is non-integrable and belongs to one of the basic classes. Curvature properties of the constructed manifold are given.

On the definition of the Kobayashi-Buseman pseudometric

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We prove that the $(2n-1)$-th Kobayashi pseudometric of any domain $D \subset \mathbb{C}^n$ coincides with the Kobayashi–Buseman pseudometric of D, and that $2n-1$ is the optimal number, in general.

Non-oscillating trajectories of vector fields and Zariski's local uniformization.

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A transcendent non-oscillating trajectory of an analytic germ of real vector field induces a structure of Hardy field for the meromorphic functions. It has a natural valuation associated to it. The study of this valuation allows to get a reduction of singularities of the vector field following the strict transform of the trajectory. More generally, for a holomorphic complex vector field, the above results can be generalized for a given valuation of the field of the meromorphic functions. We obtain in this way a local uniformization in the sense of Zariski, that should be globalized in dimension three, following the classical results of Zariski. The key of these results is a construction (due to J. Cano and Grigoriev-Singer) based on the Newton Polygon of a differential operator, that assures finiteness results on the valuation allowing the local uniformization

On the hypoellipticity of complex valued vector fields

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This talk deals with the (micro)local C^∞ and Gevrey hypoellipticity and subellipticity of some systems of complex valued vector fields. In the case of second-order overdetermined systems of PDE generated by the same vector fields we prove C^∞ hypoellipticity results which do not depend on the lower order terms. The sharp loss of regularity in the Sobolev spaces in the latter case is equal to 1, while in the first one it runs from 1/2 to 1.

Some critical almost Kähler structures

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Let M be a compact orientable manifold of dimension m. We denote by $\mathscr{M}(M)$ the set of all Riemannian metrics on M and $\mathscr{N}(M)$ the set of all Riemannian metrics of the same volume element. It is well-known that a Riemannian metric $g \in \mathscr{N}(M)$ is a critical point of the functional $\mathscr{A}$ on $\mathscr{N}(M)$ defined by

$$\mathscr{A}(g) = \int_M \tau \, dv_g$$

if and only if g is an Einstein metric, where τ is the scalar curvature of g and dv_g is the volume element of the Riemannian metric g.

Now, let M be a compact manifold of dimension $m = 2n$ admitting an almost complex structure. We denote by $\mathscr{AH}(M)$ the set of all almost Hermitian structures and $\mathscr{AH}(M, \Omega)$ the set of all almost Hermitian structures with the same Kähler form Ω. An almost Hermitian manifold $M = (M, J, g)$ with the closed Kähler form Ω ($d\Omega = 0$) is called an almost Kähler manifold. Let $M = (M, J, g)$ be a compact almost Kähler manifold and Ω the corresponding Kähler form. Then, we may note that any almost Hermitian structure $(J, g) \in \mathscr{AH}(M, \Omega)$ is an almost Kähler structure on M. In this case, we denote $\mathscr{AH}(M, \Omega)$ by $\mathscr{AK}(M, \Omega)$. In [1], Blair and Ianus studied critical points of the functional $\mathscr{F}$ on $\mathscr{AK}(M, \Omega)$ defined by

$$\mathscr{F}(J, g) = \int_M (\tau^* - \tau) \, dv_g,$$

where τ^* is the $*$-scalar curvature of (M, J, g), and proved that (J, g) is a critical point of $\mathscr{F}$ on $\mathscr{AK}(M, \Omega)$ if and only if the Ricci tensor ρ is J-invariant.

We denote by $\mathscr{AK}(M, [\Omega])$ the set of all almost Kähler structures on M with the same Kähler class $[\Omega]$ in the de Rham cohomology group. In [3], [4], Koda studied critical points of the functional $\mathscr{F}_{\lambda,\mu}$ on $\mathscr{AH}(M, \Omega)$ and $\mathscr{AK}(M, [\Omega])$ defined by

$$\mathscr{F}_{\lambda,\mu}(J, g) = \int_M (\lambda\tau + \mu\tau^*) \, dv_g, \qquad (\lambda, \mu) \in \mathbb{R}^2 \backslash (0, 0).$$

On one hand, it is known that the following integral formula $(*)$ holds on a compact almost Kähler manifold $M = (M, J, g)$ ([6]):

$$\int_M \left(f_1 - \frac{1}{2} f_2 + f_3 - 2 f_4 \right) dv_g = 0, \tag{$*$}$$

where f_1, f_2, f_3, f_4 are certain functions on M, which will be introduced in the talk. The above integral formula $(*)$ plays an important role in the arguments on the Goldberg conjecture.

In this talk, we shall discuss the first variation of the above integral formula $(*)$ on $\mathscr{AK}(M, [\Omega])$ and introduce the corresponding Euler-Lagrange type equations. Further, we shall also discuss another first variational problem on $\mathscr{AK}(M, [\Omega])$ for a 4-dimensional compact almost Kähler manifold $M = (M, J, g)$ which is based on the Wu's theorem.

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Differential forms of double-complex variables

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Differential 1-forms over the double-complex algebra $C(1,j)$ are considered

$$\omega = \phi(\alpha)d\alpha + \psi(\alpha)d\alpha^*$$

where $\alpha \in C(1,j)$, $\alpha = z + jw$, $\alpha^* = z - jw$, $j^2 = i$, $i \in \mathbb{C}$, z, $w \in \mathbb{C}$ ($\mathbb{C}$ is the field of complex numbers). By definition

$$\frac{\partial}{\partial\alpha} := \frac{1}{2}\left(\frac{\partial}{\partial z} - ji\frac{\partial}{\partial w}\right), \qquad \frac{\partial}{\partial\alpha^*} := \frac{1}{2}\left(\frac{\partial}{\partial z} + ji\frac{\partial}{\partial w}\right),$$

It is proved that $df = \partial f/\partial\alpha d\alpha + \partial f/\partial\alpha^* d\alpha^*$, or shortly $df = \partial f + \partial^* f$. In the case $\partial f/\partial\alpha^* = 0$, the function $f(\alpha)$ is a holomorphic function of the double-complex variable α. We study the equation $\partial\partial^* f = 0$ of the complex harmonic functions.

Characterization of Totally Umbilic Immersions by Curves of Order 2

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In my talk we study submanifolds by use of extrinsic shapes of some curves having points of order 2. Here a smooth curve γ parameterized by its arclength is said to be of order 2 at $\gamma(t_0)$ if

i) $\nabla_{\dot\gamma}\dot\gamma(t_0) \neq 0$,

ii) $\nabla_{\dot\gamma}\Big(\frac{1}{\|\nabla_{\dot\gamma}\dot\gamma\|}\nabla_{\dot\gamma}\dot\gamma\Big)(t_0)$ is parallel to $\dot\gamma(t_0)$.

We give a condition that they are totally umbilic, which gives an extension of Nomizu-Yano's result on a characterization of extrinsic sphere. We also study Kähler and quaternionic Kähler submanifolds from this point of view and characterize totally geodesic submanifolds in these classes.

Lie groups as several complex manifolds with Norden metric

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The basic classes of the integrable almost complex manifolds with Norden metric are considered. An example of a 4-dimensional conformal Kähler manifold with Norden metric and a 4-dimensional special complex manifold with Norden metric are constructed by means of Lie groups and Lie algebras. Both manifolds are characterized geometrically. The form of the curvature tensor for one of the examples is obtained. The conditions for these manifolds to be isotropic Kählerian are given.

Bäcklund transformations and Riemann-Hilbert problem for N-wave equations with additional symmetries

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We obtain soliton solutions of N-wave equations with reductions imposed on them. The method we make use of is a special type of Bäcklund transformation, namely the dressing method. There is an equivalence between the inverse scattering problem and Riemann-Hilbert problem. This way we construct singular solutions for the Riemann-Hilbert problem associated with the N-wave equation.

Phase transitions in binary alloy thin films

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The theory, of order-disorder phenomena in binary alloy thin films of AB3 type is considered in the context of order parameters of different physical nature. The main idea of the present description was applied at first by Valenta and Sukiennicki in the case of permalloy films for their order-disorder characterisation and consists in the model of pair interactions between nearest neighbour atoms belonging to the sublattices in the form of the monoatomic layers parallel to the surface. Such description is based on the thermodynamic approach to small particles when they should be treated as thermodynamically inhomogeneous systems. The thin film is considered as the thermodynamically inhomogeneous system constructed of the homogeneous subsystems in the form of monoatomic layers. Different order parameters, as lattice long- and short-range order, magnetic order and crystallinity parameters, are introduced and considered in the context of their mutual interdependence.

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