9-th International Workshop on Complex Structures, Integrability and Vector Fields
Sofia, August 25-29, 2008

Abstracts

August 21, 2008

Organizers:

- University of Niigata, Japan
- University of Pisa, Italy
- Institute for Nuclear Research and Nuclear Energy
  Bulgarian academy of sciences, Bulgaria
- Institute of Mathematics and Informatics
  Bulgarian academy of sciences, Bulgaria

Programm

25.08 IMI

8.30-9.00	Registration		
9.00-9.20	Opening		
	Complex Structures and Differential operators		Complex and Algebraic Structures
9.20-10.00 10.00-10.40	K. Sekigawa P. Popivanov	15.00-15.40 15.40-16.20	I. Ramadanoff J. Lee, J. H. Park, K. Sekigawa
Coffee break		Coffee break	
11.00-11.40 11.40-12.20 12.20-13.00	H. Hashimoto T. Adachi H. Matsuzoe	16.40-17.20 17.20-18.00 18.00-18.40	Y. Christodoulides S. Maeda M. Hristov

26.08 INRNE

	Integrability and Vector Fields		Integrability and Differential Geometry
9.00-9.40 9.40-10.20	N. Kostov, V. Tsiganov R. Ivanov	15.00-15.40 15.40-16.20	V. Gerdjikov, N. Kostov V. M. Vassilev, P. Djondjorov, I. M. Mladenov
Coffee break		Coffee break	
10.40-11.20 11.20-12.00 12.00-12.40	D. Henry T. Valchev G. Grahovski	16.40-17.20 17.20-18.00	M. Kokubu V. Atanasov, R. Dandoloff, R. Balakrishnan

28.08 INRNE

	Differential Geometry and Quantum Mechanics		Differential Geometry and Complex Structures
9.00-9.40 9.40-10.20	D. Trifonov L. Stepien	15.00-15.40 15.40-16.20	T. Koda G. Dimitrov, V. Tsanov
Coffee break		Coffee break	
10.40-11.20 11.20-12.00 12.00-12.40	P. A. Djondjorov, V. M. Vassilev, I. M. Mladenov B. Iliev S. Chun, J. H. Park, K. Sekigawa	16.40-17.20 17.20-18.00	J. Lawrinowicz, M. Novak-Kepcryk, O. Suzuki S. Dimiev, M. S. Marinov, J. Jelev
20.00	**Conference Dinner**		

29.08 IMI

	Complex Structures and Vector fields		Complex Structures and Vector fields
9.00-9.40 M. 9.40-10.20	S. Donev, M. Tashkova M. Manev, M. Teofilova	15.00-15.40 15.40-16.20	Y. Euh, J. H. Park, K. Sekigawa M. Teofilova
Coffee break		Coffee break	
10.40-11.20 11.20-12.00 12.00-12.40	B. Aneva L. Apostolova, S. Dimiev G. Nakova	16.40-17.20 17.20-18.00	A. Petrov Closing

A discrete model of Kähler magnetic fields on a complex space form

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A graph (V, E) is a 1-dimensional CW-complex which consists of a set V of vertices and a set E of edges. Graphs are frequently considered as discrete models of Riemannian manifolds of non-positive curvatures. Paths on a graph correspond to geodesics on a Riemannian manifold.

In the last ten years, the speaker has been studied trajectories for Kähler magnetic fields which are generalization of geodesics on a Kähler manifold. It is natural to consider there should be a discrete model of Kähler manifolds on which we have natural objects corresponding to trajectories for Kähler magnetic fields.

In this talk, the speaker propose to consider graphs whose edges are colored by 2 colors as discrete models of Kähler manifolds. For such a graph, edges are separated into primary edges and auxiliary edges. We consider the adjacency matrix for auxiliary edges to be stochastic. We especially study regular graphs which should be corresponds to complex space forms. We count closed prime paths on such a graph and study the asymptotic behavior of their number with respect to their lengths.

Askey-Wilson algebra and related spectral problem

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We consider an interacting lattice system with a flow with boundary Askey-Wislon symmetry. One of the generators is the second order difference operator for the Askey-Wilson polynomials. We use the procedure of algebraic Bethe ansatz to diagonalize the transition matrix of the system.

Double-complex Laplace operator

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The so-called double-complex Laplace operator appears in the function theory over the algebra $C(1,j)$ of double-complex variables $\alpha = z + jw$, with $j^2 = i$, $i \in \mathbb{C}$, $j \notin \mathbb{C}$. It is denoted $\Delta_+ = \partial^2/\partial z^2 + i\partial^2/\partial w^2$. The analogous operator $\Delta_- = \partial^2/\partial z^2 - i\partial^2/\partial w^2$ reduces to the first one, namely $\Delta_- = i\Delta_+^\tau$, where Δ_+^τ coincides with Δ_+ with transposed variables.

We study some typical problems for the considered operator, like

1. eigenvalue and eigenfunction problem, i.e. the problem of the solutions of the equation $\Delta_+ f(z,w) = \lambda f(z,w)$, following the standard method of separation of variables.

2. The initial value problem for the equation $\Delta_+ f(z,w) = 0$ in the whole space.

3. The initial value problem for some compact cartesian products in $\mathbb{C} \times \mathbb{C}$.

A general discussion about the treatment of the notion of the symbol is given. The pure real representation of the equation $\Delta_+(f + ig) = 0$ led to a system of two partial differential equations (PDE) of second order and to a drastic difference with the ordinary ellipticity. However, using the ordinary formulas for the complex operators $\partial/\partial z$ and $\partial/\partial w$ we present Δ_+ in real form as follows

$$\Delta_+ = \frac{1}{4}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)^2 + \frac{i}{4}\left(\frac{\partial}{\partial u} - i\frac{\partial}{\partial v}\right)^2,$$

where $z = x + iy$, $w = u + iv$. The quadratic form

$$B(\xi_1, \xi_2, \xi_3, \xi_4) = \frac{1}{4}(\xi_1 - i\xi_2)^2 + \frac{i}{4}(\xi_3 - i\xi_4)^2,$$

whose matrix

$$\begin{pmatrix} 1 & -i & 0 & 0 \\ -i & -1 & 0 & 0 \\ 0 & 0 & i & 1 \\ 0 & 0 & 1 & -i \end{pmatrix}$$

can be accepted to be a (complex type) symbol for Δ_+.

It is proved that with the help of two appropriate coordinate transformations

$$(z,w) \longrightarrow (x,y,u,v) \longrightarrow (x',y',u',v') \longrightarrow (Z,W)$$

we obtain the following complex presentation in the new variables Z and W

$$\Delta_+ = \frac{\partial}{\partial \overline{Z}} \frac{\partial}{\partial \overline{\overline{W}}}.$$

Geometry-induced quantum potentials and qubits

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A quantum particle confined within a rod feels the bend of the curve that represents the axis of that rod via the induced quantum effective potential. As a result the quantum motion is coupled with the solutions of the classical equations of motion for this rod. The Kirchhoff model of a rod gives the conformation dynamics which allows for curvature based solitary waves. Applying this to the problem of electron transport, this permits us to formalize and quantify the concept of a *conformon* (including a trapped electron wave packet) that has been hypothesized in biology. Another interesting application represents the open tight knot whose curvature profile induces a quantum two-well potential, namely the knot acts as a qubit.

Herglotz functions and spectral theory

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We review the theory of value distribution for boundary values of Herglotz functions, that is functions analytic and with positive imaginary part in the complex upper half-plane, and its applications to the theory of the Weyl-Titchmarsh m-function and the spectral analysis of Sturm-Liouville differential operators. We also present the generalized theory of value distribution for boundary values of Herglotz functions, its connection with compositions of Herglotz functions, and some results about the integral representation of composed Herglotz functions.

Tangent sphere bundles with η-Einstein structure

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A contact metric manifold $(\bar{M}, \eta, \bar{g}, \phi, \xi)$ is called an η-Einstein manifold if the Ricci tensor $\bar{\rho}$ is of the form $\bar{\rho}$ =3D $\alpha\bar{g} + \beta\eta \otimes \eta$, with α and β being smooth functions. We study the geometric properties of the base manifold for the tangent sphere bundle of constant radius r satisfying the η-Einstein condition with the standard contact metric structure. The main theorems are the following :

Theorem 1 *Let M be an $n(\geq 2)$-dimensional Riemannian manifold and T_rM be the tangent sphere bundle of M of radius r equipped with the standard contact metric structure. If T_rM is an η-Einstein manifold, then α and β are constant.*

Theorem 2 *Let M be an $n(\geq 3)$-dimensional locally symmetric space and T_rM be the tangent sphere bundle of M of radius r equipped with the standard contact metric structure. Then T_rM is an η-Einstein manifold if and only if M is a space of constant sectional curvature $\frac{1}{r^2}$ or $\frac{n-2}{r^2}$.*

Cyclic hypercomplex systems

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Let K denotes the field of complex numbers or the field of real numbers. By definition

$$K(j_0, j_1, \ldots, j_n) := \{a_0 j_0 + a_1 j_1 + \cdots + a_n j_n \ : \quad a_k \in K, \quad k = 0, 1, 2, \ldots, n\},$$

$$\dim K(j_0, j_1, \ldots, j_n) = n + 1,$$

is called an $(n+1)$-dimensional hyper-K-system with units j_k. An addition is defined naturally by the coefficients, and the multiplication with help of the units:

$$j_k j_l = c_0^{k_l} j_0 + c_1^{kl} j_1 + \cdots + c_n^{k_1} j_n, \qquad j_l j_k = c_0 lk j_0 + c_1^{lk} j_1 + \cdots + c_n^{lk} j_n.$$

In general we have $j_k j_l \neq j_l j_k$, $k, l = 0, 1, 2, \ldots, n$, i.e. a non-commutative multiplication. We see that $K \subset K(j_0, j_1, j_2, \ldots, j_n)$. Supposing that $j_0 j_0 = j_0$, $j_0 j_k = j_k j_0 = j_k$, we set $j_0 = 1$.

Examples: 1 Real dimension 2: $R(1, j) = \{a_0 + ja_1 \ : a_0, a_1 \mathbb{R}\}$ *(complex numbers); $R(1, j)$, $j^2 = +1$, (bi-real numbers); $R(1, j)$, $j^2 = 0$, (Study numbers).*

Complex dimension 2: $C(1, j)$, $j^2 = i$, $C(1, j) = \{z_0 + jz_1, z_0, z_1 \in \mathbb{C}\}$*; double-complex numbers $\alpha = z + jw$, $z, w \in \mathbb{C}$; a commutative algebra with zero-divisors, denoted DC.*

Real dimension 4: $R(1, i, j, k)$*, $i^2 = j^2 = k^2 = -1$, $ij = k$, $ij \neq ji$, a non-commutative division real algebra (quaternions) denoted by $\mathbb{H}$, $q = x_0 + ix_1 + jx_2 + kx_3$, $x_k \in \mathbb{R}$, $q \in \mathbb{H}$; $R(1, i, j, e)$, $i^2 = j^2 = -1$, $e^2 = +1$, $ij = e$, a commutative non-division real algebra of the bicomplex numbers, denoted by BC.*

Complex dimension 4: $C(1, i, j, k)$*, $i^2 = j^2 = k^2 = -1$, $ij = k$, $ij \neq ji$. This is a complex non-commutative non-division algebra (biquaternions). If $h = z_0 + iz_1 + jz_2 + kz_3$, h can be represented in the form $h = q + ip$, where $q, p \in \mathbb{H}$.*

Cyclic hypercomplex systems

Let us take back the double complex algebra $C(1, j)$, $j^2 = i$. We introduce the following new hypercomplex systems:

$$C(1, j, j^2, j^3), \quad j^4 = i; \qquad C(1, j, j^2, j^3, j^4, j^5, j^6, j^7), \quad j^8 = i, ; \qquad \cdots$$

$$C(1, j, j^2, j^3, \ldots, j^{2n-1}), \quad j^{2n} = i.$$

The elements of $C(1, j, j^2, j^3)$ are denoted $z_0 + z_1 j + z_2 j_2 + z_3 j_3$, $z_k \in \mathbb{C}$, $k = 0, 1, 2, 3$. They are called fourth-complex numbers. Analogically, the elements of $C(1, j, j^2, j^3, j^4, j^6, j^7)$ are called eight complex numbers etc. It is to recall that the analogous real hypercomplex system $R(1, j, j^2, j^3, \ldots, j^{2n-1})$, $j^{2n} = -1$, have been studied in Kazan Geometric School. Their element are called even cyclic numbers. The essential "cyclicsness" is illustrated as follows:

Denoting $j = j_1$ in $C(1, j_1)$, ($j_1^2 = i$, $i \in \mathbb{C}$, $j_1 \notin \mathbb{C}$) and $j = j_2$ in $C(1, j_2, j_2^2, j_2^3)$, $j_2^4 = i$, $i \in \mathbb{C}$, $j_2 \notin C(1, j_1)$. We prove that $C(1, j_2, j_2^2, j_2^3) \simeq C(1, j_1) \oplus j_2 C(1, j_1)$. It is to remark that if we take $C(1, j_1, j_1^2, j_1^3)$ then we obtain $C(1, j_1, j_1^2, j_1^3) \subseteq C(1, j_1)$. The above formulated cyclic property is repeated periodically for higher dimensional hypercomplex systems $C(1, j, j^2, \ldots, j^{2n-1})$.

We develop a matrix representation of some of the above formulated hypersystems, which allow us the develop a matrix function theory for them.

The holomorphic function theory developed up to now for double-complex variables is extended for other hypercomplex systems.

Complex structures related to $SL(3)$

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On some Strain-Curvature relations on Minkowski space-time

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Some interesting relations between appropriately defined strain tensors and corresponding Frobenius curvatures on Minkowski space-time induced by an isotropic closed and co-closed 2-form are established and briefly discussed.

Plane curves associated with integrable dynamical systems of the Frenet-Serret type

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Let the curvature κ of a plane curve Γ be given as a function of the Cartesian coordinates (x, z) of the points the curve is passing through in the Euclidean plane $\mathbb{R}^2$. In other words, we are considering the case when $\kappa = \mathcal{K}(x, z)$ is a known function. Without loss of generality, the curve Γ can be thought of as parameterized by a real parameter t, the co-ordinates of the position vector $\mathbf{x}(t) = (x(t), z(t))$ being determined by the system of equations

$$\ddot{x} + \mathcal{K}(x,z)\dot{z} = 0, \qquad \ddot{z} - \mathcal{K}(x,z)\dot{x} = 0$$

arising from the Frenet-Serret relations. Here the dots denote derivatives with respect to t.

This system can be regarded as a dynamical system of two degrees of freedom determining the motion (trajectories) of a particle of unit mass in which t is playing the role of time.

Here we report three quite interesting cases in which the above system is integrable by quadratures. These cases correspond to the Euler's elastica, its generalization – the so-called Lévy's elastica and the profile curves of the Delaunay's surfaces.

In all of the aforementioned cases, an explicit parametrization of the corresponding plane curves is given.

Nearly Kähler manifolds with vanishing TV Bochner curvature tensor

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We study the local structures of nearly Kähler manifolds with vanishing Bochner curvature tensor as defined by Tricerri and Vanhecke (TV). We show that there does not exist a TV Bochner flat strict nearly Kähler manifold in $2n(\geqq 10)$ dimension and determine the local structures of the manifolds in 6 and 8 dimensions.

On multicomponent evolution equations on symmetric spaces with constant boundary conditions

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We analyze the algebraic aspects of solving the multicomponent nonlinear Schrödinger (MNLS) equations [1, 2, 3, 4] and the multicomponent mKdV [5] equations related to the symmetric spaces. This analysis includes:

1. Spectral properties of the MNLS equations under the nonvanishing (constant) boundary conditions.
2. Construction of new equations of MNLS and MmKdV type imposing additional reductions
3. The involutivity of their integrals of motion is proved using the method of the classical R-matrix.
4. Explicit description of their hierarchies of Hamiltonian structures;

Our explicit examples are related to the BD.I, C.I and D.III-types symmetric spaces.

References

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On Nonlinear Schrödinger Equations on Symmetric Spaces and their Gauge Equivalent

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The fundamental properties of the multi-component nonlinear Schrödinger (MNLS) equations on symmetric spaces and their gauge equivalent Heisenberg ferromagnetic type equations are reviewed.

This includes: the properties of the fundamental analytical solutions for these models, the corresponding minimal set of scattering data, the description of the class of nonlinear evolutionary equations, solvable by the inverse scattering method and the recursion operator, related to such systems, the hierarchies of Hamiltonian structures.

The results are illustrated on the example of MNLS model related to $so(5, \mathbb{C})$ algebra, describing $\mathcal{F} = 1$ spinor Bose-Einstein condensate, and its gauge equivalent multi-component Heisenberg ferromagnetic type model.

Deformations of almost complex structures of hypersurfaces of purely imaginary octonions

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Persistence of solutions for some integrable shallow water equations

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We examine the persistence of decay properties for a family of dispersive nonlinear partial differential equations. We show that certain decay properties of the initial data persist for as long as the solution exists. On the other hand, for a subset of the family certain decay rates are possible only for the trivial solution. For example, the only solution that remains with compact support for any further time is the trivial solution.

Some geometric properties and objects related to Bézier curves

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Classical Bézier curves (the parabolic case) with the natural barycentric description are considered. The explicit expressions for curvature, lenght etc. are given. It is introduced the notion trajectory of Bézier curve and by means of the isotomic conjugation the corresponding conic surfaces in the space are obtained.

Heisenberg relations in the general case

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The Heisenberg relations are derived in a quite general setting when the field transformations are induced by three representations of a given group. They are considered also in the fibre bundle approach. The results are illustrated in a case of transformations induced by the Poincare group.

Integrable models for shallow water waves

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The motion of inviscid fluid is described by Euler's equations. In the case of shallow water, one can consider a perturbative asymptotic expansion of Euler's equations to a certain order of smallness of the scale parameters. The so obtained asymptotic equations can be matched to certain integrable equations. The best known example in this regard is the KdV equation. The main aim of the talk is to present an overview of some recent results, concerning the use of integrable equations, like a two-component generalization of the Camassa-Holm equation, Kaup - Boussinesq system and mKdV in modeling the motion of shallow water waves.

On a variation of the *-scalar curvature

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We consider the space of almost Hermitian structures (J, g) on a differentiable manifold M. We discuss critical points of the functional $\int_M (\lambda\tau + \mu\tau^*) dM$ and their stability, where τ and τ^* denotes the scalar curvature and the $*$-scalar curvature of (M, J, g).

On differential geometric aspect of flat fronts in hyperbolic 3-space

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Lax pair for restricted multiple three wave interaction system, quasiperiodic solutions and bi-hamiltonian structure

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We study restricted multiple three wave interaction system (MTWIS) by the inverse scattering method. We develop the algebraic approach in terms of classical r-matrix and give an interpretation of the Poisson brackets as linear r-matrix algebra. We also discuss bi-Hamiltonian structure of MTWIS. The solutions are expressed in terms of known Baker-Akhiezer functions. In particular, for $n = 1$ the solutions are expressed in terms of Weierstrass functions.

Four-dimensional TV Bochner flat almost Hermitian manifolds

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We study curvature properties of almost Hermitian manifolds with vanishing Bochner curvature tensor as defined by Tricerri and Vanhecke and characterize those manifolds.

[3]Supported by the Korea Research Foundation Grant funded by the Korean Government (MOEHRD) KRF-2007-531-C00008.

Mathematical outlook of fractals and chaos related to simple orthorhombic Ising-Onsager-Zhang lattices

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The paper is inspired by a spectral decomposition and fractal eigenvectors for a class of piecewise linear maps due to S.Tasaki et al (1994) and by an ad hoc explicit derivation of the Heisenberg uncertainity relation based on a Peano-Hilbert planar curve, due to M. S. El Nashic (1994). It is also based by an elegant generalization by Z.-D. Zhang (2007) of the exact solution by L.Onsager (1944) to the problem of description of the E. Ising lattices (1925). This generalization involves, in particular, opening the knots by rotation in a higher dimentional space and studying important commutators in the algebra involved. The investigations of Onsager and Zhang, involving quaternionic matrices of order being a power of two, can be reformulated with the use of the quaternionic sequence of P. Jordan algebras implied by the fundamental paper of P. Jordan, J.von Neumann, and E. Wigner (1934). It is closely related to W. Heisenberg approach to quantum theories, as summarized by him in essay dedicated to N. Borh on the occasion of his seventieth birthday (1955). We show that the Jordan structures are closely related to some types of fractals, in particular fractals of the algebraic structure. Our study includes fractal renormalization and the renormalized Dirac operator, Meromorphic Schauder basis and hyperfunctions on fractal boundaries, and a final discussion.

A characterization of Clifford minimal hypersurfaces in terms of their geodesics

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We characterize Clifford minimal hypersurfaces $S^r(nc/r) \times S^{n-r}(nc/(n-r))$ with $1 \leqq r \leqq n-1$ in a sphere $S^{n+1}(c)$ of constant sectional curvature c by observing their geodesics from this ambient sphere.

On the curvature properties of real time-like hypersurfaces of Kähler manifolds with Norden metric

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A type of almost contact hypersurfaces with Norden metric of Kähler Norden manifolds is considered. The curvature tensor and the special sectional curvatures are characterized. The canonical connection on such manifolds is studied and the form of the corresponding Kähler curvature tensor is obtained. Some curvature properties of the manifolds belonging to the widest main class of the considered type of hypersurfaces are given.

Some generalizations of statistical manifolds

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A statistical manifold is a Riemannian manifold with a torsion-free affine connection which satisfies a kind of symmetric condition. Such a manifold is naturally induced from affine hypersurface theory. On the other hand, it is known that affine connections in quantum information geometry have non-zero torsion tensors. In this talk, we introduce some generalizations of statistical manifolds. We then discuss geometry of such statistical manifolds

Some submanifolds of almost contact manifolds with Norden metric

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In this paper we study submanifolds of almost contact manifolds with Norden metric of codimension two with totally real normal space. An example of such submanifold as a Lie subgroup is constructed.

Ten reasons for pursuing multi-commutative quantum theories

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A consistent presentation of the multi-commutativity quantum project (with further references) can be found in **arxiv.org/abs/quant-ph/9711004**.

Here we focus on the motivation of the project, highlighting how it changes the quantum language and takes advantage of its newly gained flexibility. We stress its ultimate goal to overcome the conceptual limitations of the standard quantum theory and any other model that tends to identify quantum physics with non-commutative formalisms.

Hypoellipticity, solvability, subellipticity and solutions with stationary singularities of pseudodifferential operators with symplectic characteristics

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This talk deals with the local properties of pseudodifferential operators with double characteristics. More precisely, we consider the problems of local (non)solvability, hypoellipticity, with loss of regularity larger than one, subellipticity and existence of solutions with prescribed singularities. At the end we discuss the link between global analytic hypoellipticity of complex valued non-singular, real analytic vector fields defined on a compact, connected, orientable, 2 dimensional, real analytic manifold M and the topological structure of M.

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Monogenic, hypermonogenic and holomorphic Cliffordian functions

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The aim of this talk is to make an overview on three generalizations to higher dimensions of the functions theory of a complex variable. The first one concerns the so-called monogenic functions which were introduced by F. Brackx, R. Delanghe and F. Sommen, the second is the theory of hyper-monogenic functions developed by H. Leutwiler, and the last one studies the theory of holomorphic Cliffordian functions due to G. Laville and I. Ramadanoff. The basic notions in this three theories will be given. Their links and differences will also be commented. The largest part of the mentioned results concerning the holomorphic Cliffordian functions was obtained thanks to joint works with Guy Laville in the Laboratoire de Mathematiques Nicolas Oresme, CNRS UMR 6139, Departement de Mathematiques, Universite de Caen Basse-Normandie.

On some classes of exact solutions of eikonal equation

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As it is known, eikonal equation has some interesting applications in physics: it describes propagation of waves in Minkowski space and it has among others applications in quantum mechanics, too. It is possible, by applying certain decomposition method, to obtain some new wide classes of exact solutions of this equation. These classes and their properties will be presented.

Almost complex connections on almost complex manifolds with Norden metric

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A four-parametric family of linear connections preserving the almost complex structure is defined on an almost complex manifold with Norden metric. Necessary and sufficient conditions

for these connections to be natural are obtained. A two-parametric family of complex connections is studied on a conformal Kähler manifold with Norden metric. The curvature tensors of these connections are proved to coincide on such a manifold.

Pseudo-Hermitian fermion and boson coherent states

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The pseudo-Hermitian fermion and boson annihilation and number operators are diagonalized following where possible the patterns of the standard fermion and boson cases. In both pseudo-fermion and pseudo-boson cases the eigenstates of the pseudo-Hermitian annihilation operators are shown to form bi-overcomplete sets. Explicit constructions are provided on the example of a two-level pseudo-Hermitian system, studied in the physical literature. It is shown that the constructed pseudo-fermionic coherent states exhibit temporal stability.

Integrable dynamical systems of the Frenet-Serret type

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Let the curvature κ of a plane curve Γ be given as a function of the Cartesian co-ordinates (x, z) of the points the curve is passing through in the Euclidean plane $\mathbb{R}^2$, i.e, $\kappa = \mathcal{K}(x, z)$ is a known function. Let the curve Γ be parametrized by a real parameter t. Then, the co-ordinates of the position vector $\mathbf{x}(t) = (x(t), z(t))$ are determined by the system of equations

$$\ddot{x} + \mathcal{K}(x, z)\dot{y} = 0, \qquad \ddot{z} - \mathcal{K}(x, z)\dot{x} = 0 \tag{1}$$

arising from the Frenet-Serret relations. Here the dots denote derivatives with respect to t.

This system can be regarded as a dynamical system of two degrees of freedom determining the motion (trajectories) of a particle of unit mass, t playing the role of time.

Here, we are studying the integrability of system (1), which, in contrast to the case when the intrinsic equation of the curve Γ is known, is not obvious.

For that purpose, first we show that system (1) admits an exact variational formulation. Then we explore the variational symmetries of the system with respect to local Lie groups of point transformations of the dependent variables x, z. As a result, a set of sufficient conditions are obtained which ensure that (1) possess two different constants of motion. Finally, it is proved that any dynamical system of the form (1), which has the latter property, is integrable by quadratures. Moreover, in each such case we can present an explicit parameterization of the corresponding trajectory curves.

Tangent sphere bundles of constant radii

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In the present talk, we shall show that the Riemannian geometry and the contact metric geometry with respect to the standard con- tact metric structures on the tangent sphere bundle of a Riemannian manifold with constant radii are essentially reduced to the respective ones on the unit tangent sphere bundles. Further, we shall provide several applications of the result [3], and mention the geometry of the tangent sphere bundles of Riemannian manifolds with variable radii in connection with the case of constant radii.

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New integrable equations of MKdV type

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We present new examples of multicomponent MKdV type equations associated with symmetric spaces of BDI series. These are integrable equations obtained by using the method of reduction group. Their 1-soliton solutions have been derived as well by applying dressing procedure with a proper dressing factor.

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