Приложение 1

Detailed Description of the Project and the Research Plan

Title: Quantum Structure and Geometric Nature of Fundamental Forces

1. Current Status of Scientific Research in Bulgaria in the Topical Scientific Area of the Project

The senior members of the proposed project team are leading internationally recognized experts in Bulgaria in the field of modern theoretical and mathematical physics as well as in various areas of modern mathematics with applications in physics. They have contributed in a significant way to the advance in several world-wide actively developing scientific trends such as Conformal Field Theory (CFT) in two and higher dimension, studying the non-perturbative properties of string theory of fundamental particle interactions at ultra-high energies - Anti-de-Sitter/Conformal-Field-Theory (AdS/CFT) duality and integrability structures, the deeper mathematical structure of string theory, the impact of string theory in cosmology and astrophysics, the role of conformal symmetry in condensed matter physics, algebraic aspects and geometric structure of integrable dynamical systems.

Members of the project team have achieved significant results in various specific areas such as: the general structure of two- and higher-dimensional conformal field theory; study of fractional level Wess-Zumino-Novikov-Witten models in two-dimensional conformal field theory; the representation theory of four-dimensional superconformal symmetry, in particular, the classification of all unitary irreducible representations with positivive energy; the representation theory of N=2 super-Virasoro algebras, in particular, the construction of the characters of all unitary irreducible representations; quantum groups and their representations, in particular, canonical procedure for the q-deformations of the semisimple real algebras; deeper understanding of the Hopf-type algebra actions arising in two-dimensional conformal field theory as generalized internal symmetries; the affine coset construction of the edge state's effective theory in the Zk parafermion quantum Hall states; lattice gauge theories; the systematic formulation and detailed study of additional non-isospectral symmetries of integrable hierarchies of soliton-type equations, including their supersymmetric generalizations and exact soliton-like solutions for the pertinent (super-)tau-functions.

More recently there has been an active work on the following related topics: axiomatics and models of CFT with global conformal invariance in higher dimensions; analyzing validity of AdS/CFT duality beyond the simplest supergravity approximation and the relation with exactly solvable (integrable) lower-dimensional systems; realization of the AdS/CFT correspondence as intertwining symmetry between the AdS and the CFT pictures; classification of all unitary irreducible representations with positivive energy of six-dimensional superconformal symmetry; conformal field theory computation of mesoscopic persistent currents in the Zk parafermion quantum Hall states; formulation and study of a new class of string and *Dp-brane* theories without *ad hoc* dimensionfull scales, where the pertinent string/brane tensions arise as additional dynamical degrees of freedom, as well as a new class of (Weyl-conformally) invariant lightlike branes and their role in black hole physics and cosmology.

2. Relevance of the Scientific Research Topic of the Project in Bulgaria and Europe

Modern string theory is a unifying theory of all fundamental forces in Nature at ultra-high energies of Planck's scale. For the first time in the history of physics string theory offers adequate self-consistent quantization of gravity, in particular, the understanding of the intrinsic nature of such basic cosmological objects like quantum black holes and wormholes in the Universe and, among other things, it reveals the possibility that our own Universe might turn out to be just one "copy" of many more parallel universes embedded in a higher-dimensional space-time. String theory offers a unified framework for a qualitative and quantitative description of the fundamental physical properties and phenomena in the world of elementary particles, among others – the phenomenology of the fundamental particles interactions and, in particular, quark confinement in quantum chromodynamics. Furthermore, string theory has an enormous symmetry group which allows for a natural incorporation of the currently experimentally verified Standard Model (SM) of electromagnetic, weak and strong nuclear forces. In the past few years it has been realized that string theory is intimately connected, and can provide answers to fundamental problems, also in other primary areas of physics – in cosmology (evolution of the early Universe, the cosmological constant problem), relativistic nuclear physics (relativistic heavy ions collisions), condensed matter physics (Hall effect, new nonstandard mechanisms of superconductivity).

On the other hand quantum field theory models, whose formulation within the standard perturbation theory with respect to the coupling constant(s) has been completed long ago, have yielded so far some of the most precise predictions confirming experiments in high-energy physics. In spite of this success, until these days we are still lacking non-perturbative models of interacting quantum fields in four space-time dimensions and this situation is considered as one of the primary challenges in modern mathematical physics.

The proposed project aims to get further insights into how matter behaves at very short distances, and what is the structure of the universe at times around and shortly after the big bang. In recent years, remarkable progress has been made in understanding the evolution of the universe and about the interactions of the elementary particles at short distances, respectively ,at very high energies. Both in particle physics and also in cosmology, physics is described by well established theoretical models, the Standard Model (SM) of elementary particles and the Concordance Model of standard cosmology. Being experimentally confirmed to a very high level of accuracy, both theories however suffer serious theoretical problems, whose lack of understanding is a clear signal of new physics beyond the standard descriptions. More specifically, the hierarchy problem in the SM of particle physics and also the existence of massive neutrinos are clear indications of new physics beyond the SM mass scales of about 100 GeV. Complementarily, but most likely intimately related to the problems in particle physics, there are the big puzzles in standard cosmology, such as the dark matter and the dark energy problems. Moreover, in standard cosmology there is strong evidence that the universe started with the Big Bang, a classical singularity in space-time, which is however theoretically problematic and poorly understood. Finally, there is the grand problem of formulating a unified quantum theory that combines gravity and gauge interactions at very short distances, respectively, time scales. Clearly, one hopes to obtain from a quantum gravity theory further insight into the quantum nature of space-time, its quantum emergence at the Big Bang and also about the related problem of the quantum nature of black hole singularities.

The investigation of the topological quantum computers based on the two-dimensional conformal field theory, Ising and Z_N parafermion models appears to be one of the most significant applications of the fundamental concepts and methods in string theory as well as of the beautiful mathematical theory of quantum groups and braid-group statistics to concrete physical models of strongly correlated electron systems in two-dimensional space. The expected experimental confirmation of the existence of non-Abelian exchange statistics would open new horizons for application of the fundamental research to the development of the quantum computers, nano technologies and quantum information theory in general.

On the purely mathematical side, geometry traditionally plays a central role in the mathematical formalism of the fundamental theories in modern physics, first of all – in string theory. The relationships between geometry and mathematical physics received a great impetus from the discovery of the integrability phenomenon. Just to mention few, the Korteweg-de Vries equation works not only in the theory of weakly dispersive nonlinear waves but also in two-dimensional quantum gravity as well as in the geometry of moduli spaces of algebraic curves. Matrix integrals, introduced in the statistics of nuclear energy levels, proved to be connected with quantum gravity, conformal field theory, solutions of soliton equations, moduli spaces, singularity theory, Gromow-Witten invariants. Random matrices and random permutation problems, including certain stochastic processes, have also appeared on the scene in relation with sorting problems in combinatorics, multivariate statistics and growth models.

3. Description of the beneficiary Institution

The Institute for Nuclear Research and Nuclear Energy (INRNE) of the Bulgarian Academy of Sciences (BAS) is founded in 1972. INRNE is the biggest leading complex centre in Bulgaria for scientific research and applications of the nuclear science and technologies. INRNE guarantees a high quality performance of research and innovation activities, addressed to support important national programs, keeping abreast with the modern scientific achievements. With its long-standing experience and active collaboration with leading European and international institutions, INRNE contributes to the progress of the nuclear science.

The main scientific achievements of INRNE are in the following directions:

- Theory of the elementary particles, quantum field theory, string theory, theory of atomic nuclei, soliton interactions and quantum phenomena
- Experimental physics of the elementary particles, nuclear reactions, structure of atomic nuclei, cosmic rays and gamma-astrophysics at ultra high energies
- Neutron interactions and cross sections, physics of the fission
- Reactor physics, nuclear energy and nuclear safety and security
- Dosimetry and radiation safety
- Monitoring and management of the environment, radioecology
- Radiochemistry, high precision analyses of substance, development and production of radioactive preparation
- Nuclear and neutron methods for investigations of substances
- Nuclear instrument design and production.

The Institute's staff of about 350 (1 academician, 3 corresponding members of BAS, 14 professors, 74 associate professors, 78 assistant professors) work in 25 laboratories, 2 scientific experimental facilities and 8 departments providing general support activities.

The annual scientific production of the INRNE consists of about 400 papers published in foreign and Bulgarian refereed journals and in Proceedings of International conferences.

The INRNE has a highly qualified scientific potential, well developed infrastructure, broad international cooperation and longstanding traditions in scientific research and PhD training. Annually INRNE organizes and supports a series of international conferences, meetings and workshops.

The INRNE participates in many projects, e.g. in 5th, 6th and 7th Framework Programs of EC. It has many bilateral contracts with scientific and research organizations from Austria, France, Germany, Greece, Israel, Italy, Russia, Spain, UK, USA, and other countries all over the world.

4. Detailed Description of the Research Plan

4a. Novelty of the Project Objectives. Existing Research on the Project Topics

The quest for a unification of all matter and all forces is in our opinion an important scientific enterprise with many very original scientific concepts, ideas and developments. The Standard Model of elementary particle physics together with Einstein's theory of General Relativity are each in very good agreement with all known experimental phenomena at particle accelerators and in cosmology. Nevertheless the large number of undetermined parameters and the unexplained patterns of particles and forces in the Standard Model call for a unified description which also should include a quantum version of General Relativity. Supergravity theories and their quantum realization in superstring theory are the most promising candidates for this extension of the Standard Model. In particular, they have given birth to many new ideas and concepts for unification, and have proved themselves over many years as abundant sources for very original new developments in theoretical particle physics. On the one hand string theory has triggered an enormous exchange of ideas with modern mathematics in the fields of differential geometry, algebraic geometry and topology; this exchange has even introduced new areas of research in mathematics, perhaps most notably the idea of mirror symmetry. Also the relation between infinite dimensional symmetries in string theory and the mathematicians' work on infinite-dimensional algebras is another good example of the interplay between physics and mathematics in our field. On the other hand, supersymmetric field theories, superstring theory, D-branes and M-theory were also of very high scientific value for new developments in particle physics phenomenology. The possible existence of large extra dimensions, which could be observed at new particle colliders, the investigation of orbifold field theories in higher dimensions (deconstructing gauge theories) and the role of anomalies on branes are excellent examples of the fruitful interplay between string theory and particle physics beyond the Standard Model. Finally, many new developments in quantum field theory were inspired by string theory, for example non-perturbative duality in supersymmetric field theories, non-commutative gauge theories, topological field theories and, last but not least, non-perturbative considerations in gauge theories via dual gravitational models.

Although we have gained a lot of new scientific insights into quantum field theory, superstrings and M-theory over recent years, and have broadened our knowledge of the possible basic structure of our universe, the entire subject is by far not completed: it will continue to be a very active and vital area of research for some time to come. Specifically, not only further exploration of already understood concepts will be necessary in the future, but also the recent appearance of new directions and the persistence of fundamental open problems predict that many years of original and sound scientific development in theoretical and mathematical high energy physics still lie in front of us. The development of new concepts and the invention of new ideas will be necessary in order to make further real progress in the field. Most importantly, a fundamental formulation of M-theory is still lacking: we do not precisely know the elementary degrees of freedom and the huge symmetry group of M-theory – the key to that problem could be a deeper understanding of supermembranes and of infinite dimensional symmetry algebras, which are related to higher spin fields in field theory language. Another very deep and basic problem is the question of vacuum stability and the problem of the apparently huge vacuum degeneracy in superstring theory. Precise predictions, which can really be confronted with experiments, can only be made after we have understood what the ground-state in string theory looks like, by what dynamical mechanism it is selected and what its stability properties are. The construction of brane models and flux compactifications on spaces with very few moduli (i.e. rigid spaces) are likely to open up new solutions in that direction. Concerning the stability of string vacua, we also have to know how string backgrounds develop in time, in particular shortly after the big bang. Ultimately we would like to understand why we so far observe only three macroscopic, large space dimensions in cosmology of accelerator experiments, we wish to know whether future colliders have the chance to discover new extra dimensions. The explanation of the apparently non-zero cosmological constant is a real challenge in string theory or supergravity theories; moreover the question whether the holographic principle extends also to de Sitter like space-times is very interesting from the point of view of counting degrees of freedom of the entire universe. We would also like to extend the investigation of the quantum properties of black holes to more realistic non-BPS black hole solutions, such as those astronomers observe in the centers of galaxies. Finally the interplay between geometry and the investigation of gauge theories via string theory must be pushed forward so that we learn more about non-perturbative physics in realistic gauge theories.

The idea that gauge fields and strings are related has a long history . The modern view on this relationship derives from the qualitative picture of color confinement in quantum chromodynamics (QCD): the Faraday lines of the chromo-electric field are to be pinched to a thin tube because of the strong interactions and/or non-trivial structure of the vacuum in the Yang-Mills theory. The recent developments have challenged this point of view, and revealed a whole class of gauge theories that are exactly equivalent to a theory of infinitely thin, idealized strings. The strings propagate in the bulk of a higher-dimensional space of which the four-dimensional space-time is the boundary. The string is in some sense holographically projected onto the lower-dimensional screen.

The AdS/CFT correspondence is a concrete realization of the large-N duality, so far unique for the four-dimensional theories. Interestingly, a further simplification occurs when the ‘t Hooft coupling λ becomes large. The string tension is proportional to √λ, which means that at large λ the string behaves classically, since quantum fluctuations of its world-sheet are penalized by large tension. This is a remarkable result: it shows that the strongly-coupled SYM, which is a complicated quantum theory, is equivalently described by classical strings or, in the large-N limit, by classical gravity in the Anti-de-Sitter space. Our understanding of AdS/CFT for wrapped branes is much more rudimentary than for branes at conical singularities. The main obstacles have been rooted in the inability to move away from the near horizon limit, usually only the AdS geometries are known. The lesson from branes on cones is that in order to get a real handle on field theory dynamics–to write down the particle content and superpotential for a dual of a specific AdS solution–the associated Calabi-Yau geometry must be known. The main point of this program is to give a way of associating a special holonomy metric to an AdS metric.

The importance of these issues is rooted to their relation to other important phenomena. Due to its dual nature, the AdS/CFT correspondence can be used to gain insight into the nature of quantum gravity as well as to understand the non- perturbative behavior of gauge fields. The idea that the strongly-coupled regime of any large-N gauge theory is holographically dual to classical gravity in is very attractive for many reasons, but not necessarily true. Nevertheless, a number of semi-phenomenological models were proposed to describe the strongly-coupled regime of QCD using holography.

Another development where the gauge/string duality was instrumental is the holographic description of hard scattering and the precision calculations of scattering amplitudes. It is possible to reformulate many known or expected features of hard scattering in gauge theories in the holographic language. It is also possible to calculate the Yang-Mills scattering amplitudes at strong coupling using classical string theory, and thus establish a link to perturbative calculations of multiparticle amplitudes in QCD and SYM which have been carried out up to rather high orders in perturbation theory. These calculations revealed unexpected structures and symmetries that are likely to persist at the non-perturbative level.

It is very important to study the integrable structures in greater detail and better understand their properties not only in AdS5 × S5 geometry but also in larger class of backgrounds which are potentially related to QCD.

The quest for non-perturbative models of quantum field theory is a task with more than 50 years history and it has become one of the central problems of mathematical physics today. In this regard the vertex algebras approach in CFT in higher space-time dimensions is a new promising method. It is of primary interest to find a connection between globally conformal invariant fields and certain observable quantum fields in gauge theories. Another interesting question is to develop a theory of supersymmetric vertex algebras in higher dimensions, which already exist in the two-dimensional CFT.

Some of the common mathematical themes that underlie and unify the tasks to be addressed by the present project are as follows. The study of Riemannian geometry in the complex setting often yields strong and interesting results that can have an impact both on Riemannian geometry and algebraic geometry. For example Kähler-Einstein metrics and minimal submanifolds in Kähler manifolds are two subjects where the interplay between real methods from partial differential equations and complex geometry yields deep insights.

Symplectic geometry is a part of geometry where "almost-complex" methods already play a large role, and this area forms an integral part of the proposed research. Moreover "moment-map" ideas play a very significant role in other parts of the research programme, for example in the construction of extremal Kähler metrics and quaternionic-Kähler and hyper-Kähler metrics. In addition, infinite-dimensional analogues of the correspondence between symplectic and stable quotients provide a good conceptual framework for the understanding of many phenomena in gauge theory and complex differential geometry. Thus, the aim of exploiting this conceptual framework to the full unifies several of the research tasks.

Twistor methods give a correspondence between holomorphic geometry and low-dimensional conformal geometry, and also allow the use of complex methods in the study of quaternionic geometry, gauge theory and integrable systems, all of which are subjects included in this research proposal.

Although the above described part of the research to be undertaken by the proposed project is in pure mathematics, in fact it is closely related to, and inspired by, developments in supersymmetric quantum field theory and superstring theory which lie in the core of the project. The input from these areas takes the form of conjectures (especially in Donaldson-Yang-Mills theory and in Seiberg-Witten theory) as well as new geometric structures (Frobenius manifolds, special Kähler geometry) which often turn out to be of interest in complex differential geometry. Conversely, mathematical developments in these subjects have feedback in physics. This is well-known for gauge theory, but it also applies to quaternionic geometry and exotic holonomy, which are of increasing interest in string theory via the co called *D*-branes (*Dirichlet*-barnes).

4b. Significance of the Project Objectives. Their Relevance Towards Solution of Important Scientific Problems

Although the Standard Model of particle physics as a quantum field theory is very well understood, our understanding of string theory, brane dynamics and supergravity theories is still very incomplete, despite all the progress made in recent years. In this project we are planning to attack some of the main unsolved issues in supergravity theory and string theory, where we hope to make substantial progress by combining and focusing the various types of expertise and research interests that are present in the participating team members. In particular we will focus on the issue of how to relate the mathematical and geometrical properties of string and supergravity theory to elementary particle physics and also to astroparticle physics. The new collider experiment at LHC (CERN's Large Handron Collider) , which is expected to produce exciting new experimental results like the possible discovery of supersymmetry or other so far unknown elementary particles during the upcoming years, will hopefully also act as a guideline for pinning down some of the geometrical and topological features of superstrings. In this way new experiments may act as a valuable compass in the landscape of superstring and supergravity vacua. We ourselves are not aiming to enter a full fledged phenomenological analysis of new physics. The scientific strength and expertise of the present project team lies in connecting the mathematical structures of superstrings and supergravity to the observables in particle and astroparticle models.

Our research objectives will span a ***broad range of closely interrelated topics*** which can be subdivided thematically as follows:

(a) "Gauge/Gravity Duality and Integrability in String Theory"

One of the primary aims of the project is to deepen the understanding of the relation between gauge theories and effective string theories in the context of generalized geometric structures, web of dualities and the existing phenomenological data. So called T-duality brings us to the realm of non-geometric backgrounds where we are still lacking of consistent and comprehensive description. Our aims in the T-duality context are related to search for a description of these backgrounds using techniques of doubled geometry. This global picture, which is very useful by itself, will be very helpful in many applications such as in AdS/CFT. The doubled formalism in combination with Berkovits' string on AdS5 × S5 can shed some new light into AdS/CFT since we get a more general geometric picture. The principal tasks within the project will be:

• Study the hidden integrable structures and (supersymmetric) Bethe ansatz solutions of various two-dimensional sigma-models relevant for the AdS/CFT correspondence. One outstanding problem that can be addressed in this context is the relationship between the discrete spin chains and the continuous sigma-models. The AdS/CFT correspondence asserts that the spectrum of the 2d sigma-model with the AdS5 × S5 target superspace is the same as the spectrum of scaling dimensions of local gauge-invariant operators in N=4 SYM. The latter can be found by diagonalizing non-local Hamiltonians of supersymmetric integrable spin chains. As additional mathematical input we plan to use the interplay of three types of representations: positive energy representations, (holomorphic) discrete series representations and finite-dimensional irreps. Another outstanding problem is the study of finite-size effects in the sigma-model on AdS5 × S5 and in the spin chain that describes the spectrum of the planar N = 4 SYM. Perturbative calculation of the finite-size effects is the first logical step towards full-fledged string quantization in AdS5xS5 with periodic boundary conditions. These calculations have been done both at strong coupling in the sigma-model and at weak coupling in the SYM. Very interesting links to the Thermodynamic Bethe Ansatz uncovered recently definitely deserve further investigation.

• Study of string/gauge theory duality for gauge theories with less supersymmetry/other than N=4 SYM (supersymmetric Yang-Mills gauge theory):
The existing asymptotic solution of the AdS/CFT correspondence allows one to calculate the non-perturbative spectrum of states with large quantum numbers, typically with exponential accuracy. Interestingly, some of these states are related to other integrable models, such as the O(6) sigma-model. It is important to study this relationship in more detail, in order to understand how the AdS/CFT correspondence works in details and perhaps to make contact with more general gauge/string duality in other backgrounds.

• Application of the ideas and methods of holographic duality to hard scattering in gauge theories, to the quark-gluon plasma, and possibly to dynamical symmetry and supersymmetry breaking.

• Gravity duals of SYM gauge theories:
We plan to explore these backgrounds (and their T-duals) in pure spinor formulation using doubled geometry; study of the twists and dualities producing backgrounds dual to non-commutative/nonlocal gauge theories

(b) Globally conformal invariant Lie field models. This part of the project aims to develop and study models of local and bilocal Lie fields. There is an old result forbidding the existence of scalar "nonabelian" Lie fields in more than two space-time dimensions. On the other hand, there is no such a result for tensor fields, for which up to now there are no known nontrivial examples. For bilocal Lie fields there are simple models based on Wick products of free fields. Regarding the latter there is a duality between field Lie algebras and gauge groups. The task is to classify the possible dual pairs for both the Fermi and Bose cases. The problem of building local tensor Lie field models has a long history as we have pointed out. One currently developing approach to it is the method of reconstruction of vertex algebras in higher dimensions from models in one dimensions (chiral vertex algebras). In this connection the most important task is to construct models of *W*-algebras, with an extended local conformal symmetry, which allow reconstruction in higher dimensions.

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(c) “Axiomatics and Models of Supersymmetric Vertex Algebras”. We propose to study the incorporation of the supersymmetry within the axiomatic approach to globally conformal invariant quantum field theory.

(d) "Lightlike Branes in the Physics of Black holes, Elementary Particle Physics and Cosmology". This part of the project is devoted to the systematic and detailed study of the dynamics of (Weyl-conformally invariant) lightlike *p*-branes fisrt proposed in the literature by some of the project team members, especially, studying their role in “soldering” along common (event) horizons of different space-time regions with black hole type geometries, as well as their potential applications in the modern “brane-world” scenarios. In particular, we will study lightlike brane-worlds where large extra dimensions could naturally be rendered undetectable (due to the zero eigenvalue of the induced metric on the brane world) provided our Universe is considered as a lightlike brane moving in D > 4 dimensional bulk space-time. The latter is precisely the brane-world scenario obtainable from the consistent unified dynamical (Lagrangian) description of lightlike branes proposed in our earlier works on the subject. Moreover we plan further detailed study of the recently discovered by us “mass inflation” effect triggered by the dynamical tension of lightlike brains which automatically occupy event horizons, in particular, black hole horizons.

(e) “Black Holes in Higher-Dimensional General Relativity”. The black hole type solutions (black rings, black Saturns, concentric and orthogonal black dipole-rings) in higher-dimensional curved space-times demonstrate very interesting properties and features absent in D=4 dimensions. In space-times with D greater than four the black objects are no longer uniquely determined by the conserved asymptotic charges, their horizons possess a much richer topological structure. A lot of research has already been devoted to the black holes in five dimensions where many interesting and important from theoretical point of view exact solutions were found and analyzed. The accumulation of exact solutions naturally raises the question of the classification of the black solutions, which will be one of the main lines of research in the proposed project. The aim is to identify integrable sectors in higher-dimensional Einstein-Maxwell and Einstein-Maxwell-Dilaton gravity and subsequently employ solitonic solution-generating techniques similar to the one used in the context of two-dimensional integrable systems in deriving and analyzing new black hole/black ring solutions.

(f) "Two-Dimensional Non-Critical String Models". The Liouville gravity in two dimensions is an exactly solvable conformal model which reveals many of the properties of the more complicated closed and open string theories. The renewed interest in this model is caused by the development in the last years of the noncompact c > 25 Virasoro theory (c being the central charge). Both continuum conformal field theory and discrete matrix model methods are being exploited in the study of the model. So far these methods have been applied mainly to quantities in the bulk like the bulk tachyon correlators. The project aims in the extention of this program to the Liouville gravity with matter in the presence of boundaries. Another direction worth exploring is the theory deformed with boundary deformation terms and the study of the corresponding renormalization group flows.

(g) "Two-Dimensional Integrable Models - Application of Quantum Group and Conformal Invariance". This part of the project covers application of conformal and Hopf-type symmetry to various integrable models in two space-time dimensions. Using the global conformal invariance and the related rationality of the theory, some methods originally developed in the two-dimensional framework can be extended to higher dimensions. Other basic research topics of the projects will be:generalized internal symmetries; logarithmic conformal field theory; conformal description of fractional Hall liquids; modular categories and fusion rules.

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(h)"Conformal (Super)Algebras in Various Dimensions". This part of the project is based on work traditional for many of the senior team members for the last 20 years and has direct important applications in string theory. So far we have classified the unitary irreducible representations of conformal supersymmetry in $D = 6$ dimensions, same for the superalgebras $osp(1|2n, R)$, which is relevant for embedding space-time dimensions $D = 9,10,11$ (for $n = 16, 32$). We also made progress in writing character formulae for $D = 4$ conformal supersymmetry, constructed all invariant differential operators and character formulae for $D = 3$ conformal symmetry. Further research within the present project includes the construction of the Minkowskian bulk-to-boundary and boundary-to-bulk intertwining operators in (super-) AdS/CFT correspondence.

(i) "Quantum Groups and Their Representations". This part of the project is similarly based on work traditional in our group for the last 15 years. So far we have worked on non-standard and exotic quantum groups, which are relevant for the integrability aspects of string theories. Another direction is related to the q-deformations of the conformal algebras in various dimensions, also called the AdS quantum groups $SO_q(D,2)$. In particular, we have investigated the q-deformed equations with quantum conformal symmetry, and constructed q-plane wave solutions of these equations. Within the present project we plan to investigate generalized quantum-group internal symmetries and superselection rules in quantum field theory models in lower ($D < 3$) space-time dimensions.Further, we plan to study the relationship among rational conformal field theories, their logarithmic extensions, and appropriate Hopf structures (e.g., finite-dimensional factorizable Hopf algebras).

(j) "Topological Quantum Computation with Non-Abelian Anyons". One of the most spectacular characteristics of the strongly correlated two-dimensional electron systems is the possibility for localized excitations (quasiparticles) which are neither bosons nor fermions to exist. The exchange statistics of these so called anyons, which could only exist in effectively two-dimensional systems, such as the insulator-semiconductor interface like in the quantum Hall effect, gives rise to representations of the braid group instead of the permutation group over the anionic states of matter. The unitary transformation corresponding to the exchanges of such indistinguishable particles, especially of the so called non-Abelian anyons, are not simply phases (as it is for the Abelian anyons) but rather non-trivial matrices that could in principle approximate any unitary transformation and therefore could be used for realization of universal quantum gates and eventually for building quantum computers. The biggest advantage of this approach to quantum computers is due to the topological nature of the braid operations because of which the quantum operations are protected by topology from quantum noise and decoherence. The latter are the most difficult obstacles for the physical implementation of quantum computers and exploiting their unprecedented computational power that could lead to a revolutionary development in almost all spheres of human activity. The topological protection for storing and processing of quantum information offers a unique opportunity for improving the quantum computers hardware instead of correcting the errors, that arise due to the interaction of the qubits with their environment, by sophisticated software algorithms that cost serious overhead in resources.

(k) "Applications of Non-Standard Quantum Statistics to Strongly Correlated Systems". The aim of this part of the project is to explore properties of hardcore bosons and fermions via the previously discovered by one of the team members new type of quantum A-statistics in view of their relevance as good candidates for a description of high-temperature superconductivity.

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(I) "Geometric and Integrability Structures Related to String Theory". In this part of the project the main object to be studied will be quaternionic and quaternionic contact manifolds as well as their analogues. The various notions of quaternionic manifolds arise naturally in the context of supersymmetric nonlinear sigma-models of quantum field theory and in string theory.

An important concept is the so called quaternionic contact structure introduced by Biquard, which arises as a natural fiber bundle at infinity of the hyperbolic space. A special property discovered by Biquard states that the contact 3-form is determined uniquely by the quaternionic structure as well as by the metric on the horizontal bundle. Of utmost importance is the existence of a linear connection which preserves the quaternionic contact structure as well as its Ricci tensor and scalar curvature.

The Riemann and CR-Yamabe problems are fruitful areas of research in geometry and analysis. The main step in solving them is the detailed understanding of the pertinent conformally flat cases. Appropriate models for such cases are given by the corresponding spheres or, equivalently, by the Heisenberg groups with centers of dimensions 0 and 1. This equivalence is realized through Klein transformation, which in the case of the Riemann problem reduces to the ordinary stereographic projection.

4c. Description and Justification of the Novelty of the Scientific Methods

All research topics are intertwined in a very non-trivial way, so that scientists working on one specific theme will also need knowledge and input from the other fields. This will lead to a very positive and dynamic exchange of ideas. Hence, it is in our opinion that the proposed project is scientifically very original since it combines the complementary types of expertise of several leading local research groups in theoretical and mathematical particle physics and mathematics in a very unique way. In particular we have combined team members that are known experts in the fields of superstrings, supersymmetric field theories, general relativity and cosmology, integrable systems, conformal field theory, representations of infinite-dimensional algebras, differential and algebraic geometry, and many other areas of theoretical physics and mathematics. The team composition is such that input from mathematical physics could be used to make contact with expected experimental results, like those from CERN's LHC or with future astrophysical measurements. Therefore, we view ourselves as a research team with a very high degree of multidisciplinarity, especially as regards the very close relation between modern mathematics and theoretical particle physics.

String theory and quantum field theory are an arena of huge crossover of methods from almost all areas of modern mathematics: functional analysis; differential geometry and topology; group representation theory, including representations of infinite-dimensional Lie algebras; abstract algebra and number theory. This in turn creates a fruitful and extremely stimulating feedback towards mathematics itself .

Apart from the purely mathematical methods enumerated above, we will employ in the proposed research project numerous most advanced methods of modern theoretical and mathematical physics such as: methods of superspace geometry and supersymmetric quantum field theories; functional integration techniqies; methods inherent in the theory of continuum integrable systems – Lax ("zero-curvature") representation of integrable nonlinear soliton(-like) equations, solitonic solution-generating techniques; methods of discrete integrable systems on a lattice; generalization of Polyakov formalism for world-volume p-brane actions, originally introduced in earlier works of the team members, which is based on additional (p-1)-rank antisymmetric world-volume gauge fields and alternative non-Riemannian volume form (integration measure); exploring the representation theory (including that of indecomposable representations, in the case of non-semisimple representation categories) of quadratic and Hopf algebras, corresponding to the chiral (vertex) algebras of the pertinent conformal field theory models. Further, methods from the theory of nonstandard quantum statistics, pioneered by some of the team members, will similarly be substantially used.

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Specifically concerning the methodology in topological quantum computation with non-abelian anyons, one of the most promising prototype of a topologically protected quantum computer could be realized by the Pfaffian state which is believed to describe the observed quantum Hall state with filling factor 5/2. For its description (in terms of a two-dimensional effective conformal field theory) our team has a substantial contribution. Among the most important previously obtained results to be employed in the current project we should mention the explicit construction of the representations of the braid group for Pfaffian states containing $2n+2$ Ising anyons which could be used to realize n qubits, the investigation of the possible gate set and the analysis of the computational power of the Ising quantum computer. Another promising model of a topological quantum computer, known as the Fibonacci topological quantum computer, could be realized with the non-Abelian anyons which are believed to exist in the quantum Hall state at filling factor 12/5, that could be described in terms of a two-dimensional conformal field theory with parafermion symmetry.

In the context of the project research on integrability structures and their relations to string theory, the mathematical team members will employ methods from representation theory with highest weights in Fock spaces to describe phase spaces of particle systems governed by trigonometric Calogero-Moser Hamiltonians. Painleve equations will be studied using methods of differential Galois theory. Also we will be employing methods similar to those of Lee-Parker and Lee-Jerison in the solution of Yamabe problem.

4d. Expected Results and Impact

The present research project is centered around string theory as a consistent theory of quantum gravity. One of the main emphases will be to put superstrings and supergravity into the context of particle physics and cosmology. Hence the expected results of the proposed project will contribute to address and seek for plausible solutions of some of the core questions on the structure, origin and future of our Universe.

In the context of our research on the very broad topic of gauge/gravity duality and integrability in string theory we expect results which will contribute to the confirmation of the AdS/CFT duality hypothesis of Maldacena and, let us not forget, that AdS/CFT correspondence is the fundamental milestone of modern non-perturbative string theory. The impact of such confirmation can hardly be overestimated. It will be of enormous importance, as it will lead to a revolutionary breakthrough in understanding how the string theory as the fundamental theory of all forces in Nature selects its various ground states, in particular, the symmetries of the latter and the patterns of subsequent dynamical symmetry breakings, and, correspondingly, the pertinent spectra of fundamental particles, and how all this will ultimately explain the presently observed world of elementary particles and their fundamental interactions at the presently available collision energies in modern particle accelerators.

The rigorous systematic construction of non-perturbative truly interacting models of quantum fields is a long awaited ultimate culmination of modern quantum field theory, therefore, all successful steps in this direction, which we plan to undertake, will undoubtedly create an enthusiastic response by the scientific community in mathematical physics. Specifically we envisage to obtain explicit classification of bilocal Lie fields as well as to construct non-trivial local Lie field theory models.

Discussion of our new types of brane-world scenarios, where “our Universe” is a lightlike brane from the point of view of the embedding higher-dimensional space instead of the customary (massive) Nambu-Goto-type brane, is expected to shed new light on the intrinsic dynamics of the brane-world itself with respect to the extra space dimensions (the latter has never been properly taken into account so far in the context of the standard brane-world scenarios). In particular, we expect to achieve via lightlike brane-world scenarios a natural explanation for the practical unobservability of the extra space dimensions from the point of view of standard observers confined on “our Universe”.

Thanks to the occupation of event horizons by the lightlike branes with co-dimension one, proposed in our research project, we will obtain black hole geometries ***without singularities***. Another new physically interesting effect we can achieve with our lightlike branes (depending on the specific geometries in the space-time regions below and above the horizon occupied by the lightlike brane) is the creation of an effective potential “well” around the horizon trapping the infalling matter.

The study of the non-critical two-dimensional string theory with $c<1$ matter will be extended to manifolds with boundaries. The impact of the boundary deformations for the boundary tachyon correlators and the equations satisfied by them will be understood in detail.

On the topic of quantum group symmetries of two-dimensional integrable models we expect to find explicitly and explore the properties of the (presumably, finite dimensional) algebraic objects governing the fusion rules in low dimensional QFT models (the analogues of the gauge groups in $D \geq 4$).

The expected results in our investigation of the topological quantum computers built on the Ising and parafermion models will appear among the most significant applications of the quantum field theory, the mathematical theory of quantum groups and braid statistics to concrete physical models of strongly correlated electron systems in effective two-dimensional space. The interest towards these topological quantum computers has been recently increasing especially after the series of experiments performed in world leading laboratories such as Bell Labs, Microsoft Station Q, AT&T , etc., which confirmed the existence of the topologically ordered Hall states at filling factor 5/2 (as well as of that at 12/5) and the fractional electric charge (1/4 of the electron charge) of the elementary quasiparticles at 5/2. This state appears to be the most stable quantum Hall state in the second Landau level and almost the only one that could be routinely observed in ultrahigh-mobility semiconductor samples at very low temperature. Thus, our expected theoretical results together with the expected experimental verification of the non-Abelian exchange statistics in the aforementioned quantum Hall state would open new horizons for applications of quantum field theory, quantum groups as well as for the development of quantum computers, nanotechnologies and quantum information theory in general.

Hardcore quantum particles (bosons and fermions), to be studied in the present project within a novel mathematical framework – that of nonstandard quantum statistics, offer new interesting alternatives for an adequate description of high-temperature superconductivity.

We envisage the following principal results on the topic of geometric and integrable structures related to string theory . We will study in detail the conformal geometry of the quaternionic contact manifolds through the Biquard connection. We will find particular solutions of the quaternionic contact problem of Yamabe on the quaternionic sphere. We will seek for necessary and sufficient conditions for vanishing of the torsoin of Biquard connection. We will obtain conformal deformations which map the standard flat torsionless contact structure on the quaternionic Heisenberg group into a contact structure with traceless part od the horizontal Ricci tensor of Biquard connection.

We will introduce phi-para-holomorphic sectional curvature on the para-Sasakian manifolds and will study the geometry of para-Sasakian manifolds with a constant phi-para-holomorphic sectional curvature (para-Sasakian space forms). We will provide classification of para-Sasakian space forms together with their changes under the action of D-homothetic transformations. We will provide an answer to the problem about the existence of flat (with respect to Levi-Civita connection) para-contact manifolds. We will provide a proof that the contact (para-contact) Bochner tensor in the case of CR-manifolds (respectively, para-CR-manifolds) and in the case of Saskian (respectively, para-Sasakian) manifolds coincides with the Chern-Moser tensor (respectively, the analog of Chern-Moser tensor in the para-contact case obtained by Ivanov-Vasilev-Zamkovoy).

4e. Impact to the training and career of young researchers

Quantum field theories, supergravity theories and superstring theory, geometry of integrable systems are subjects which have gone through various important developments during recent years. It is the aim of our project to provide to our young researchers a stimulating and active research and training environment. Compared to localized institute training, our multi-partner project offers:

1) A well-thought global research program that enhances the research capabilities of the individual institutions. This is highly important in a field that changes constantly and fast.
2) Several training events tailored to the needs of young researchers – regularly organized local seminars, opportunities for attendance of other training events (schools and workshops), in particular, international training events organized by some of the senior project team members, will enable the young researchers to present their work and, therefore, enhance their visibility to the scientific community in the field.
3) The project team encourages contacts with other institutions and/or project teams. This will result in short or longer time exchanges of young researchers that is very beneficial for them. In a fast changing and extremely international field, this aspect is crucial for the global training of young researchers.
4) The multi-partner project team provides the opportunity for young researchers to obtain extra career related skills that are typically unavailable locally in any separate institution.

4f. Impact to the longer term collaborations of the participating teams

The multi-partner project team contains subteams that have already developed close and friendly collaborative linkages among them. The coherence of the project team is driven by this cohesion developed already by many of its members. The present proposed project will offer an outstanding opportunity for continuation and enhancement of this collaborative structure, to develop stronger ties both in research and training, and eventually to turn our proposed project into one of the tightest and most successful consortia in Bulgarian science.

4g. Potential of Knowledge Transfer and Applicability of the Results

As a primary channel of knowledge transfer the research project team will provide 12-month or 24-month training programs for several young researchers (Ph.D. students or early postdocs) - both for members of the project team and outside young researchers. The trained young researchers will be able to attend several regular seminars organized by us on a weekly basis. The first one is a general seminar devoted to reports (sometimes, talks given by foreign visitors) on modern developments in quantum field theory (including string theory) and mathematics relevant to theoretical physics. Another seminar is a specialized one on Lie symmetries and applications in physics. Furthermore, members of the project team are delivering on a regular basis various graduate lecture courses on advanced topics of theoretical physics such as conformal symmetries, quantum groups, etc. organized in collaboration with the Physics Department of Sofia University.

Another important channel of knowledge transfer are the international scientific events. Our project team has an extensive previous experience in organizing numerous international conferences, workshops and schools in quantum field theory, mathematical physics, group theoretical methods including quantum groups, superstrings and supergravity, integrable systems (the latest will be the forthcoming 2008 annual workshop of the FP6 European Research Training Network "Forces-Universe" in Varna). Furthermore, we will be organizing two related training events - the 8-th and 9-th International Workshops "Lie Symmetry and Its Applications in Physics" in Varna (summer 2009 and summer 2011, respectively).

Let us now try to explain the importance, significance and applicability of the expected results in less technical terms immersing our project, its merits and envisaged impact within the general realm of modern theoretical and mathematical physics. Indeed, while our field itself consists of pure theoretical research, it is not without practical importance as it produces novel ideas and insights which trigger the development and construction of important experimental facilities. This in turn has not only major industrial and financial implications but it leads to the development of novel technologies, which find important applications in domains unconnected to particle physics (e.g. medical imaging, IT applications such as the GRID, novel electronics, etc). Sometimes, these developments are unforeseen but turn out to have a revolutionary impact on society – let us only mention the **World Wide Web**, which started as support project for particle physics at CERN and subsequently **changed the face of modern civilization**!

5. Persistence and Steadiness of the Research Activities after Completion of the Project

The present project has far fetched ambitions in delivering substantial contributions to the advance of modern string theory as the basic archetypal theory in modern physics encompassing the description of all fundamental forces in Nature in an elegant unified way. We can easily envision a profound commitment of the scientific efforts of the project team members to pursue the above mentioned scientific challenges ***far beyond*** the expiration of the 3-years project term.

Let us substantiate our pledges by recalling the following obvious facts. Mastery and control over new more powerful energy sources (in view of their inevitable depletion on Earth) and the feasibility of interplanetary transportation and communications in deep space (as the only way out of the chilling option of collapse of human civilization due to exhaustion of our own planet's resources) are absolutely unthinkable without most profound knowledge of the structure and the fundamental forces of matter both at very small distances (of order 10^{-33} cm), as well as at colossal galactic scales. And these are precisely the cardinal problems of modern theory of elementary particles at (ultra-)high energies for many decays ahead in the future where the merits of the present project most naturally fit. The principal long-term efforts of all researchers in this branch of science world-wide, including the present project team members, will be directed towards several closely interrelated priority directions many of which are at the core of present proposed project:

(1) Deeper understanding of the mechanisms of unification of the fundamental forces in Nature (nuclear strong and weak inetarctions, electromagnetic interactions and gravity), in particular:
   - (a) Quantum gravity and nature of the Vacuum;
   - (b) Quantum nature of the space-time singularities in the physics of black holes, "wormholes", the global topology of the Universe;
   - (c) Existence and nature of extra space-time dimensions;
   - (d) Feasibility of the modern "brane-world" cosmological scenarios.

(2) String-theory-inspired qualitatively new physics of elementary particles at energy/distance scales beyond the scales of the currently accepted Standard model of particle interactions and the standard "concordance" model in cosmology, in particular:
   - (a) Explaining hierarchy of mass scales in Nature;
   - (b) Massive neutrinos;
   - (c) Quantum chromodynamics – "confinement" of quarks and gluons, quark-gluon plasma;
   - (d) "Dark" matter and "dark" energy problem in modern cosmology.

(3) Applications of string theory in nuclear physics and condensed matter physics (relativistic heavy ion collisions, Hall effect, new non-standard mechanisms of superconductivity).

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No doubt, the above strategic visions will provide a natural environment both for continuation of the fruitful collaboration among the project team members, as well as for building up new collaborations and join efforts with other teams on national and international level in attacking the cardinal challenges of physics as we have already proved to be capable of through our previous experience.

6. Plan for Dissemination of the Results

The research in the present project will be carried out within a ***wide international collaboration*** with scientists from several leading world-renown institutions in various countries around the globe, which will provide an excellent natural environment for a ***broad dissemination*** of the project results. The current (partial) list of our foreign collaborators includes:

(1) *Austria* – Erwin Schroedinger Institute for Mathematical Physics (ESI), Institute on High Energy Physics and University of Vienna;
(2) *Belgium* – University of Ghent;
(3) *Brazil* – Institute for Theoretical Physics (Sao Paulo);
(4) *France* – C.E.A. Saclay (including “Rila” project of CNRS “Random geometry, quantum gravity and conformal theories”), Université de Paris-Sud (Orsay), Ecole Polytechnique, (Palaiseau), L.A.P.P. Annecy, Université Paul Sabatier (Toulouse); Université Henri Poincaré (Nancy);
(5) *Germany* – Institute for Theoretical Physics (University of Göttingen), Technical University of Clausthal; Max-Planck Institut für Mathematik, Leipzig; Institute for Mathematical Physics (Technical University of Braunschweig)
(6) *Hungary* – K.F.K.I. (Budapest);
(7) *Israel* – Ben-Gurion University (Beer-Sheva);
(8) *Italy* – I.C.T.P. and S.I.S.S.A. (Trieste), University of Trieste; Rome University “Tor Vergata”;
(9) *Russia* – J.I.N.R. (Dubna);
(10) *Switzerland* - Theory Group of C.E.R.N., Geneva;
(11) *United Kingdom* – Imperial and Kings Colleges of University of London, University of York, University of Northumbria at Newcastle;
(12) *United States of America* – University of Delaware (Newark); Michigan University (Ann Arbor); Pennsylvania State University (Abington); North Carolina State University (Raleigh).

Several members of the project team are currently participating in several large Research Training Networks financed by the European Commision and in other European multipartner projects:

(a) ***“Constituents, Fundamental Forces and Symmetries of the Universe”*** – FP6 Marie Curie Actions, Research Training Network, Project **MRTN-CT-2004-005104** (homepage: http://www.theorie.physik.uni-muenchen.de/~luest/forcesuniverse.html)**;**

(b) ***“European Network in Geometry, Mathematical Physics and Applications”*** **(ENIGMA)** - FP6 Marie Curie Actions, Research Training Network, Project **MRTN-CT-2004-565** (homepage: http://enigma.sissa.it);

(c) ***“European Differential Geometry Endeavour”*** **(EDGE Network)** (homepage: http://edge.imada.sdu.dk/welcome.php3);

(d) ***“Quantum systems related to noncommutative geometries, their symmetries and evolution equations***” - 3-node network (Clausthal-Leipzig-Sofia Cooperation), financed by the Alexander von Humboldt Foundation (homepage: http://www.humboldt-foundation.de);

as well as in the recently expired:

(e) ***“Integrable Models and Applications: From Strings to Condensed Matter”*** (**EUCLID)** - FP5 European Network, Project **HPRN-CT-2002-00325,** (homepage: http://www-users.york.ac.uk/~ec9/fp5data.html).

Other outstanding opportunities for dissemination of project achievements is to present them at prestgious international scientific events (conferences, workshops and schools) and it is here where the financial support from the project funds will prove of crucial importance. Also our own project team has an extensive previous experience in organizing numerous international conferences, workshops and schools in quantum field theory, mathematical physics, group theoretical methods including quantum groups, superstrings and supergravity, integrable systems etc, which always have enjoyed an overwhelming and very high level international attendance.

7. Management Structure of the Project

The proposed network will have a very transparent and efficient management and organizational structure, which already functioned very well in the previous research projects with participation of the current team members. This structure will ensure a very smooth and efficient administration of the present project.

Specifically the management structure is planned as follows:

a) Coordinator – Prof. Ivan Todorov: He represents the multi-partnet project as a whole to the outside and is of course the relevant primary contact person.The coordinator supervises the general functioning of the project.The coordinator collects all relevant financial informations, and he is responsible for the financial and scientific reports to the Fund.

b) Deputy coordinator – Prof. Emil Horozov: He will assist the coordinator in all relevant issues and will also function as a contact person for the team members from the Faculty of Mathematics and Informatics of Sofia University.

c) Prof. Radoslav Rashkov will act as a contact person for the team members from the Faculty of Physics of Sofia University.

d) Prof. Emil Nissimov will function as person responsible for the liason between the team members from all three institutions.

Management know-how and experience of the team has been successfully tested on numerous occasions, especially regarding the organizing of international conferences, workshops and schools in quantum field theory, mathematical physics, group theoretical methods, superstrings and supergravity, integrable systems.

Distribution of all prospective project funds will be regulated by a special agreement among the project team members from the various participating institutions officially sanctioned and signed by the pertinent institution's administrations.