Towards a geometric approach to the formulation of the Standard Model

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Towards a geometric approach to the formulation of the Standard Model

- Both universal constants c and $\hbar$ have deep geometrical meaning.
- c appears in the metric of the pseudo Euclidean space

$$ds^2 = c^2 dt^2 - (d\vec{x})^2$$

The corresponding 3-dimensional velocity space has Lobachevsky geometry with negative curvature $-1/c^2$

All velocities are bounded from above $v \leq c$

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The Planck constant $\hbar$ is involved explicitly in the commutation relations of the observables and it is the quantum of angular momentum.

It determines also a bound, but on the measurements

$$\Delta x \Delta p \geq \hbar$$

Appearance of universal constants in the description of physical phenomena is a signal that in these cases important role is played by geometric arguments.

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- The present, experimentally confirmed, theory of elementary particles is the **Standard Model**, based on the group:

$$SU_c(3)\otimes SU_L(2)\otimes U(1)$$

Elementary particles do not have composite structure and are described by local fields. The SM depends on finite number of fields of this kind:

-three families of **leptons and quark** fields

-four **vector bosons** $W^{\pm}, Z^{0}, \gamma;$

-an octet of **gluon fields** g

- hypothetical **Higgs boson** field, with different from zero vacuum expectation value

$$\langle 0|H(x)|0\rangle = h_0$$

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It is obvious that, as far as, the "elementary particles" are finite number their masses are bounded from above by a constant M

Our main idea: **mass spectrum of all particles described by local fields should cuts off on a certain mass** M :

$$m \leq M$$

This universal parameter is called **fundamental mass**. In fact **we introduce a new notion of local field intrinsically consistent with the above limitation.**

Objects with masses $m > M$ can not be considered as elementary and to them does not correspond a local field.

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A geometric realization of this idea is if M is considered as the curvature radius of momentum anti de Sitter space:

$$p_0^2 - \vec{p}^{\,2} + p_5^2 = M^2$$

For free particle $p_0^2 - \vec{p}^{\,2} = m^2$ the condition $m \leq M$ is automatically satisfied.

Flat limit: $|p_0|, |\vec{p}| << M,\ p_5 \cong M;\ M \to \infty$

Simultaneously there exists a second surface:

$$p_0^2 - \vec{p}^{\,2} - p_5^2 = -\mathfrak{M}^2$$

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$$p^2 - p_5^2 + M^2 = 0$$

$$m = Msh\mu$$

$$-p_5^2 + M^2 sh^2\mu + M^2 = -p_5^2 + M^2 ch^2\mu =$$

$$-(p_5 - Mch\ \mu)(p_5 + Mch\ \mu)$$

New Klein-Gordon equation:

$$2M(p_5 - Mch\mu)\Psi(p, p_5) = 0$$

$$\Psi(p, p_5) \neq 0, p_5 > 0, \Psi(p, p_5) = 0, p_5 < 0,$$

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- In the flat limit:

$$M \to \infty$$

$$|p_5| = M\kappa = M\sqrt{1+\frac{p^2}{M^2}} \approx M + \frac{p^2}{2M}$$

$$Mch\mu = M\sqrt{1+sh^2\mu} = M\sqrt{1+\frac{m^2}{M^2}} \approx M + \frac{m^2}{2M}$$

$$2M(p_5 - Mch\mu) = 2M(M + \frac{p^2}{2M} - M - \frac{m^2}{2M}) =$$

$$= p^2 - m^2$$ and we get the usual K-G operator

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- There is no difference between the free field theory in curved and flat (pseudo Euclidean) momentum space

$$\Psi_1(p,\kappa) \neq 0, \Psi_2(p,-\kappa) = 0$$

$$(p_5 + Mch\mu)(p_5 - Mch\mu)\Psi_1(p,\kappa) = (p_5^2 - M^2 ch^2\mu)\Psi_1(p,\kappa) =$$

$$(p^2 + M^2 - M^2 - M^2 sh^2\mu)\Psi_1(p,\kappa) = (p^2 - m^2)\Psi_1(p,\kappa) = 0$$

But the equation:

$$2M(p_5 - Mch\mu)\Psi(p, p_5) = 0$$

is non local -

$$p_5 = \sqrt{p^2 + M^2}$$

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- How can we have a local field theory on the surface:

$$p^2 - p_5^2 = -M^2$$

Fourier transform:

$$\varphi(x, x_5) = \int e^{ip_L x^L} \delta(p_L^2 + M^2) \varphi(p, p_5) d^5 p$$

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- We have:

$$\left(-\frac{\partial^2}{\partial x_\mu \partial x^\mu}+\frac{\partial^2}{\partial x_5 \partial x^5}-M^2\right)\varphi(x,x_5)=0$$

All fields (including interacting) should satisfy this fundamental equation!

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- The action for the free scalar field is:

$$S \approx \int d^5 p \varepsilon(p_5)\delta(p_L^2 + M^2)\varphi(p, p_5)(p_5 - Mch\mu)\varphi(p, p_5) =$$

$$\int \frac{dp^4}{2\kappa}\left[\overline{\varphi_1(p)}(\kappa - Mch\mu)\varphi_1(p) + \overline{\varphi_2(p)}(\kappa + Mch\mu)\varphi_2(p)\right],$$

We introduce new field variables:

$$\varphi(p) = \frac{\varphi_1(p) + \varphi_2(p)}{\kappa}, \chi(p) = \varphi_1(p) - \varphi_2(p)$$

$$\overline{\varphi(p)} = \varphi(-p)$$

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- The action becomes local :

$$S \approx M^2 \int [\kappa^2 \overline{\varphi(p)}\varphi(p) - \overline{\chi(p)}\chi(p) +$$

$$Mch\mu(\overline{\varphi(p)}\chi(p) + \overline{\chi(p)}\varphi(p)]d^4 p$$

$$\overline{\varphi(p)} = \varphi(-p)$$

$$M^2(\kappa^2 - ch^2\mu)\varphi(p) = (p^2 - m^2)\varphi(p) = 0$$

$$\chi(p) = ch\mu\varphi(p)$$

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- Free field Lagrangian for massless neutral scalar fields:

$$\mathcal{L}_0(x) = \frac{1}{2}\left(\frac{\partial\varphi(x)}{\partial x_\mu}\right)^2 + \frac{\mathfrak{M}^2}{2}(\chi(x) - \varphi(x))^2 \equiv \frac{1}{2}\left(\frac{\partial\varphi(x)}{\partial x_\mu}\right)^2 - U_0(\varphi, x)$$

For the total density of the potential energy we have:

$$U(x) = -\frac{\mathfrak{M}^2}{2}\left(\chi(x) - \varphi(x)\right)^2 + \frac{\lambda^2}{4}\left(\varphi^2(x) + \chi^2(x)\right)^2$$

It has a non stable vacuum state.

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- Introducing fields with zero vacuum expectation value:

$$\varphi'(x) = \varphi(x) - \varphi_0 = \varphi(x) - \frac{\mathfrak{M}}{\lambda}$$

$$\chi'(x) = \chi(x) + \chi_0 = \chi(x) + \frac{\mathfrak{M}}{\lambda}$$

We get for the total Lagrangian:

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$$\mathcal{L}(x) = \frac{1}{2}\left(\frac{\partial\varphi'(x)}{\partial x_\mu}\right)^2 - \frac{\mathfrak{M}^2}{2}(\chi'(x) - \varphi'(x) + 2\frac{\mathfrak{M}}{\lambda})^2 +$$
$$+\frac{\lambda^2}{4}\left((\varphi'(x) - 2\frac{\mathfrak{M}}{\lambda})^2 + (\chi'(x) + 2\frac{\mathfrak{M}}{\lambda})^2\right)^2 =$$
$$= \frac{1}{2}\left(\frac{\partial\varphi'(x)}{\partial x_\mu}\right)^2 - \frac{3}{2}\mathfrak{M}^2(\varphi'^2(x) + \chi'^2(x)) + \mathfrak{M}^2\varphi'(x)\chi'(x) + \mathcal{L}_{int}$$

Compare this with anti de Sitter free field Lagrangian:

$$\mathcal{L}(x) = \frac{1}{2}\left(\frac{\partial\varphi(x)}{\partial x_\mu}\right)^2 - \frac{m^2}{2}\varphi(x)^2 - \frac{M^2}{2}(\chi(x) - cos\mu\ \varphi(x))^2 =$$
$$= \frac{1}{2}\left(\frac{\partial\varphi(x)}{\partial x_\mu}\right)^2 - \frac{M^2}{2}(\varphi^2(x) + \chi^2(x)) + M^2\cos\mu\ \varphi(x)\chi(x),$$
$$\cos\mu = \sqrt{1 - \frac{m^2}{M^2}}$$

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- They are identical if:

$$\mathfrak{M}^2 = M^2 cos\mu; \qquad 3\,\mathfrak{M}^2 = M^2.$$

Therefore as a result the field $\varphi(x)$ obtains a mass:

$$m_H = \frac{2\sqrt{2}}{3} M < M.$$

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Important remarks:

1.In our approach all fields (including $\varphi(x)$ and, in a more general case, the Higgs field $H(x)$) before the symmetry breaking may be considered as massless. The point is that here the dimension of mass $[m]$ is generated by the fundamental mass.

2. In contrast to the standard approach we did not need to introduce a tachion. In certain sense the role of tachion mass is played by the quantity $\mathfrak{M}$, which is the curvature radius of de Sitter p - space.

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- Dirac fields:

$$(-\frac{\partial^2}{\partial x_\mu \partial x^\mu}+\frac{\partial^2}{\partial x_5 \partial x^5}-M^2)\Psi(x,x_5)=0$$

Back to momentum space:

$$(p^2-p_5^2+M^2)\Psi(p,p_5)=0$$

Introduce Dirac matrices:

$$\Gamma^L=(\gamma^\lambda, i\gamma^5); \Gamma^L\Gamma^K+\Gamma^K\Gamma^L=2g^{KL}$$

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- Then:

$$(p^2 - p_5^2 + M^2)\Psi(p, p_5) =$$

$$(iM + p_L\Gamma^L)(iM - p_L\Gamma^L)\Psi(p, p_5) = 0$$

The operators:

$$\Pi_R = \frac{iM + p_L\Gamma^L}{2iM}, \Pi_L = \frac{iM - p_L\Gamma^L}{2iM}$$

are projection operators.

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$$\Pi_R + \Pi_L = 1$$

$$(\Pi_R)^2 = \Pi_R, (\Pi_L)^2 = \Pi_L$$

$$\Pi_R \Pi_L = \Pi_L \Pi_R = 0$$

Moreover in the flat limit $|p_0|, |\vec{p}| << M, p_5 \cong M$

$$\Pi_R \to \frac{1+\gamma^5}{2}; \Pi_L \to \frac{1-\gamma^5}{2}$$

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- New natural definition of “chirality”

$$\Psi(p, p_5) = \Pi_R \Psi(p, p_5) + \Pi_L \Psi(p, p_5) =$$
$$\Psi_R(p, p_5) + \Psi_L(p, p_5)$$

The fundamental “Pentagon” equation in the case of Dirac fields splits to equations for the “right” and “left” fields and this is intrinsic property for the chosen geometry.

Along this lines a generalization of the Standard Model is under consideration!

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- Equations of motion for abelian vector field

$$(p_5 - M)A_\lambda(p, p_5) = p_\lambda A_5(p, p_5)$$

$$(p_5 + M)A_5(p, p_5) = p\;.A\,(p, p_5)$$

$A_5(p, p_5)$ is a gauge degree of freedom .

Gauge group:

$$A_L(x, x_5) \rightarrow A_L(x, x_5) - e^{-iMx_5}\frac{\partial}{\partial x_L}(e^{iMx_5}\lambda(x, x_5))$$

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- Covariant derivative:

$$D_L = \frac{\partial}{\partial x^L} + ieA_L(x, x_5)e^{-iMx_5}$$

$$\mathcal{L}(x) = \Big[-\tfrac{1}{4}\overline{F_{KL}}(x, x_5)F^{KL}(x, x_5) + \\ +2\left|\frac{\partial(e^{-i\mathfrak{M}x_5}A_L(x,x_5))}{\partial x_L} - 2i\mathfrak{M}e^{-i\mathfrak{M}x_5}A_5(x, x_5)\right|^2 \Big]_{x_5=0}$$

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$$\mathcal{L}(x) = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x)$$

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- Equation of motion for the Dirac field

$$\left[p_\mu \gamma^\mu - i\gamma^5 (p_5 - M) - 2Msh\frac{\mu}{2} \right] \Psi(p, p_5) = 0$$

If you multiply by:

$$\left[p_\mu \gamma^\mu - i\gamma^5 (p_5 - M) + 2Msh(\frac{\mu}{2}) \right]$$

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we easily get:

$$2M(p_5 - Mch\mu)\Psi(p, p_5) = 0$$

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Conclusion

A nontrivial generalization of the standard model is in progress, in which the principle of existence of a bound on the mass spectrum of the elementary constituents of matter is advanced in a natural geometric way.