Dynamical Mass Gaps In Gauge Theory From Holographic Gravity

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New understanding of holographic descriptions of mesons

A stab at predicting the spectra of arbitrary AF gauge theories

Sofia October 2015

QCD

An SU(3) gauge theory of quark color

$$D^\mu = \partial^\mu + igC^{a\mu}T^a$$

$$\mathcal{L} = -\frac{1}{4}G^{\mu\nu}G_{\mu\nu} + i\bar{\psi}_L \not{D}\psi_L + i\bar{\psi}_R \not{D}\psi_R + m\bar{\psi}_L\psi_R$$

$$\psi \rightarrow e^{ig\theta^a(x)T^a}\psi$$

$$C^{a\mu} \rightarrow C^{a\mu} - \partial^\mu\theta^a(x)$$

Quarks come in 3 colours

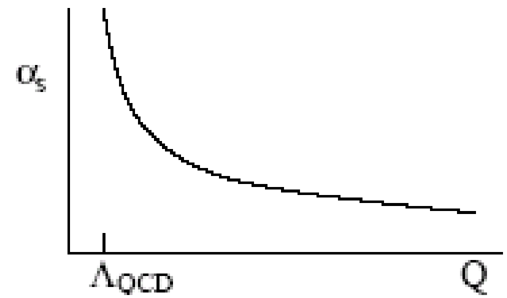
8 gluons mediate the force

$$G^{\mu\nu} = \partial^\mu C^{a\nu}T^a - \partial^\nu C^{a\mu}T^a + g[C^{a\mu}T^a, C^{b\nu}T^b]$$

Strong Dynamics

$$\beta = -\frac{11}{3}N_c \; + \; \frac{2C(R)}{3}N_f^{\rm chiral} \; (+\frac{C(R)}{6}N_{\rm scalar}^{\rm real})$$

The force is asymptotically free (Wilczek, Gross, Politzer)



Confinement:

Quarks can not be liberated from hadrons.

Hadronization in even electron positron collisions is very messy and uncomputable from first principles… as for heavy ion collisions….

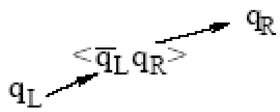
Strong Dynamics

Chiral Symmetry Breaking:

Quark masses $\ll \Lambda_{QCD}$ - so effectively massless

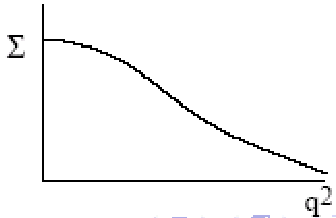
Left handed and right handed spinning quarks become distinct when they travel at the speed of light - quark number is preserved in each sector (chiral symmetry)

Strong interactions make it energetically favourable to fill space with pairs



A dynamical quark mass is generated (1/3 the proton mass)

The pseudo-Goldstone bosons of the symmetry breaking are the pions

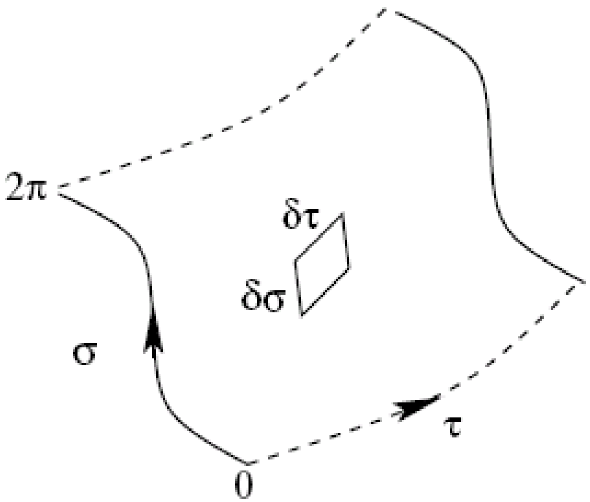


$$m_u, m_d, m_s \ll \Lambda_{QCD}$$

$$SU(3)_L \otimes SU(3)_R$$

$$SU(3)_V$$

String Theory



$$\delta s = \delta\sigma\delta\tau\sin\theta$$
$$= \sqrt{\delta\sigma^2\delta\tau^2 - (\delta\sigma.\delta\tau)^2}$$
$$= \delta\sigma\delta\tau\sqrt{\left(\frac{dX^\mu}{d\sigma}\right)^2\left(\frac{dX^\nu}{d\tau}\right)^2 - \left(\frac{dX^\mu}{d\sigma}\frac{dX_\mu}{d\tau}\right)^2}$$

Action = area of the world sheet

$$= T\int d\sigma^2\sqrt{\det\left(G_{\mu\nu}\frac{dX^\mu}{d\sigma^a}\frac{dX^\mu}{d\sigma^b}\right)}$$

$T = 1/2\pi\alpha'$ is string tension

$$\frac{\partial p^\mu_\tau}{\partial\tau} + \frac{\partial p^\mu_\sigma}{\partial\sigma} = 0$$

$$p^\mu_\sigma = \frac{\partial L}{\partial X'^\mu} = 0$$

$$\sigma = 0, 2\pi$$

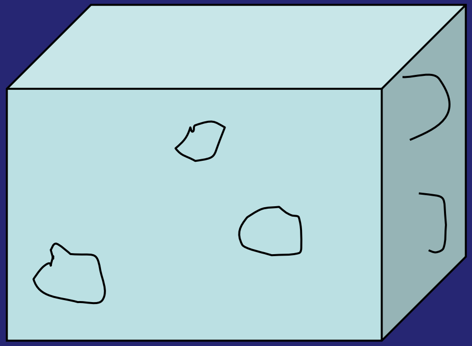
Waves with no energy flux off the end of a string…

Correctly reproduces Regge trajectories $M^2 \sim J$

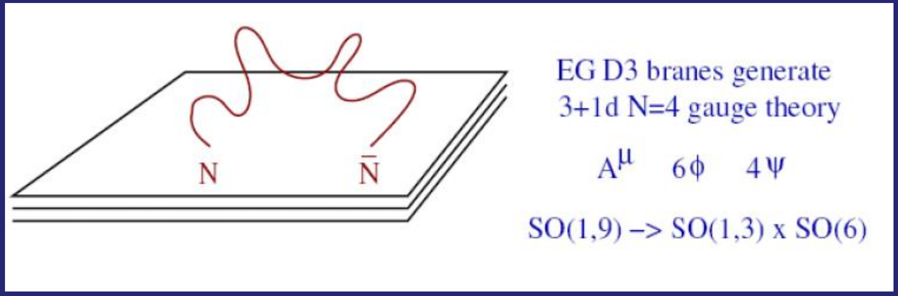
AdS/CFT Correspondence History

Maldacena, Witten

T -> infinity



Open strings can be tied to defect D-branes, dimensionally reducing the field theory



The simplest case is 3+1d N=4 SYM

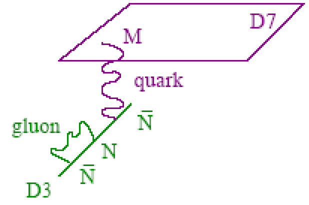
The closed string sector looks initially redundant but has turned out to provide an alternative description of the field theory via a weak-strong coupling duality.

Important but baroque string constructions

- Lie close to N=4 SYM
- With quarks
- Mild susy breaking
- Can see confinement, chiral symmetry breaking
- Predict the meson spectrum
- We can't do QCD (wrong particle content, running coupling, no asymptotic freedom) but have ingredients for new phenomenological modelling…

Adding Quarks

Bertolini, DiVecchia…; Polchinski, Grana; Karch, Katz…



Quarks can be introduced via D7 branes in AdS

The brane set up is

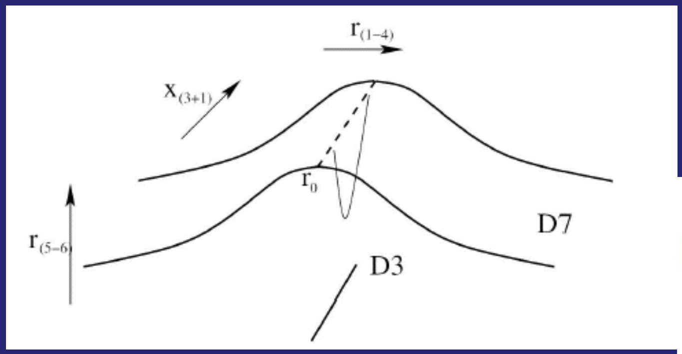
	0	1	2	3	4	5	6	7	8	9
D3	—	—	—	—	•	•	•	•	•	•
D7	—	—	—	—	—	—	—	—	•	•

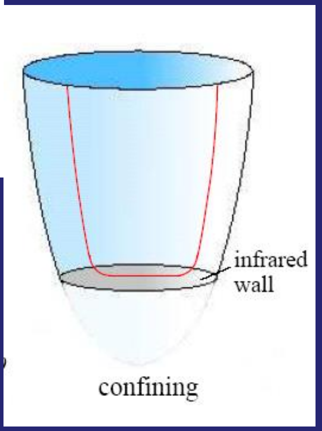
fills radial dirn of AdS, S^3 embedded in S^5 (directions 4–7); $U(1)_A$ (directions 8–9)

We will treat D7 as a probe - quenching in the gauge theory.

Minimize D7 world volume with DBI action

$$S_{D7} = -T_7 \int d\xi^8 \sqrt{P[G_{ab}]}, \qquad P[G_{ab}] = G_{MN} \frac{dx^M}{d\xi^a} \frac{dx^N}{d\xi^b}$$





Ingredient 1: Extra 5th dimension

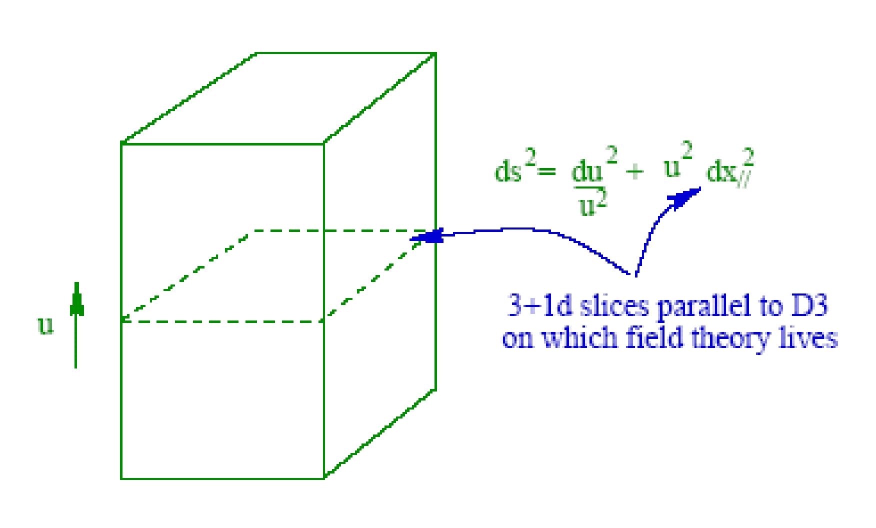
dilatations

$\int d^4x(\partial\varphi)^2$ invariant to $x \to e^{\alpha}x, \ \ \varphi \to e^{-\alpha}\varphi$

$ds^2 = u^2 dx_{//}^2 + \frac{du^2}{u^2}$ invariant to

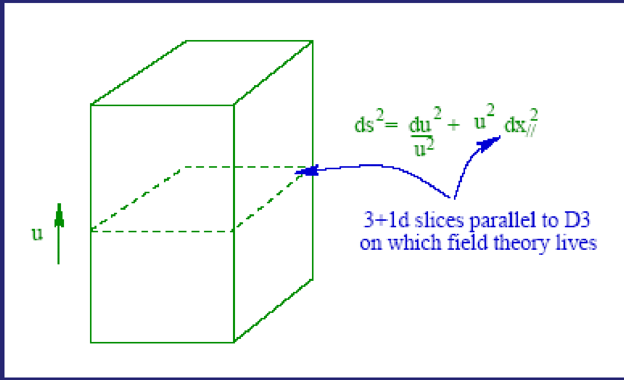
$x \to e^{\alpha}x, \ \ u \to e^{-\alpha}u$

The radial direction is a field theory energy scale.



Different u is looking at gauge theory at different RG scale…..

Ingredient 2: Bulk Fields are O & J



$$S = \int d^4x dr \, \sqrt{-g}(g^{mn}\partial_m\phi\partial_n\phi + M^2\phi^2)$$

$$\mathrm{EoM}: \; \partial_u[u^5(\partial_u\phi)] - u^3M^2\phi = 0$$

$$\phi = \frac{a}{u^\Delta} + \frac{b}{u^{(4-\Delta)}}$$

$$M^2 = \Delta(4-\Delta)$$

Fields in the bulk are gauge invariant objects that run with RG scale eg operators and sources…

a & b are the source and the operator…

the dimensions match perturbative regimes (an accident of susy at strong coupling)

Breitenlohmer-Freedman Bound

A scalar in AdS is stable until msq < - 4 ie Δ < 2

Cooking Up Holographic QCD

Conformal invariance is only broken by log running so AdS space is a good start…

$$ds^2 = u^2 dx_{3+1}^2 + \frac{du^2}{u^2}$$

TABLE I: Operators/fields of the model

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	p	Δ	$(m_5)^2$
$\bar{q}_L \gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R \gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

We pick fields to describe the most important operators in QCD and set their masses by the naïve dimension.

We must though include the dynamics that leads to a mass gap…

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left\{\frac{11}{3}N_c - \frac{2}{3}N_f\right\} - \frac{g^5}{(4\pi)^4}\left\{\frac{34}{3}N_c^2 - \frac{N_f}{N_c}\left[\frac{13}{3}N_c^2 - 1\right]\right\} + \cdots$$

Using the 't Hooft coupling, and setting $\frac{N_f}{N_c} \to x$ we obtain

$$\lambda \equiv g^2 N_c \quad , \quad \dot{\lambda} = -b_0\lambda^2 + b_1\lambda^3 + \mathcal{O}(\lambda^4)$$

with

$$b_0 = \frac{2}{3}\frac{(11-2x)}{(4\pi)^2} \quad , \quad \frac{b_1}{b_0^2} = -\frac{3}{2}\frac{(34-13x)}{(11-2x)^2}$$

We could include a field for the log running coupling… but it talks to our operators through the running anomalous dimension of qq…

$m \, \bar{q}q$

$$\gamma_m^{(1)} = \mu \frac{d \ln m_q}{d\mu} = \frac{3(N_c^2-1)}{4N_c\pi}\alpha$$

$$m^2 = \Delta(\Delta - 4)$$

$$\Delta m^2 = -2\gamma$$

When the BF bound is violated X will develop a vev and generate the gap

Dynamic AdS/QCD

Timo Alho, NE, KimmoTuominen
1307.4896

$$S = \int d^4x \, d\rho \, \mathrm{Tr} \, \rho^3 \left[\frac{1}{\rho^2 + |X|^2} |DX|^2 + \frac{\Delta m^2}{\rho^2} |X|^2 + \frac{1}{2\kappa^2} (F_V^2 + F_A^2) \right]$$

$$X = L(\rho) \, e^{2i\pi^a T^a} .$$

$$ds^2 = \frac{d\rho^2}{(\rho^2 + |X|^2)} + (\rho^2 + |X|^2) dx^2 ,$$

D7 probe action in AdS expanded to quadratic order

X is now a dynamical field whose solution will determine the condensate as a function of m

We use the top-down IR boundary condition on mass-shell: $X'(\rho = X) = 0$

X enters into the AdS metric to cut off the radial scale at the value of m or the condensate – no hard wall

The gauge DYNAMICS is input through Δm

$$m^2 = \Delta(\Delta - 4)$$

$$\Delta m^2 = -2\gamma = -\frac{3(N_c^2 - 1)}{2N_c \pi} \alpha$$

The only free parameters are Nc, Nf, m, Λ

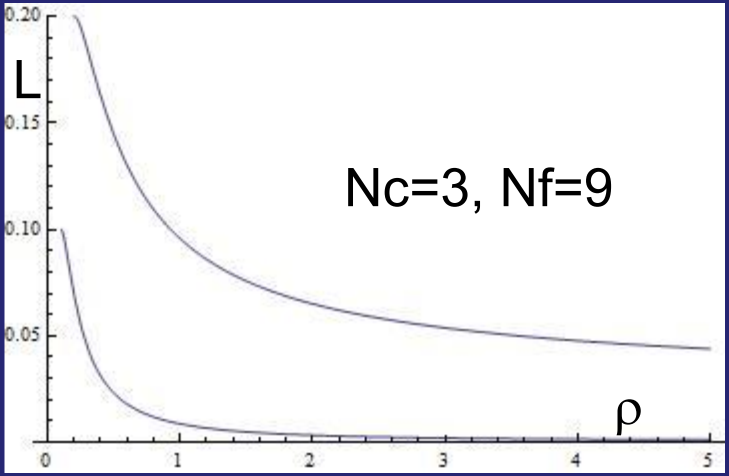
Formation of the Chiral Condensate

We solve for the vacuum configuration of L

$$\partial_\rho[\rho^3\partial_\rho L] - \rho\Delta m^2 L = 0\,.$$

Shoot out with

L'(ρ=L) = 0



Read off m and qq in the UV…

Now solve for meson masses by looking at linearized fluctuations about this vacuum…

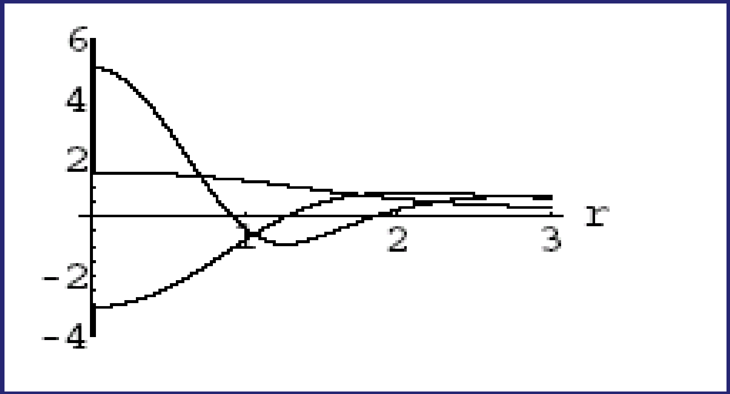
Example Numerics – Meson Masses

The gauge field equation of motion

$$\left[\partial_z\left(\frac{1}{z}\partial_z V^a_\mu(q,z)\right)+\frac{q^2}{z}V^a_\mu(q,z)\right]_\perp=0.$$

$$V\sim V(z)e^{-iq.x}, \qquad q^2=-M^2$$

Numerically shoot from UV… seek solutions with V'(wall)=0…

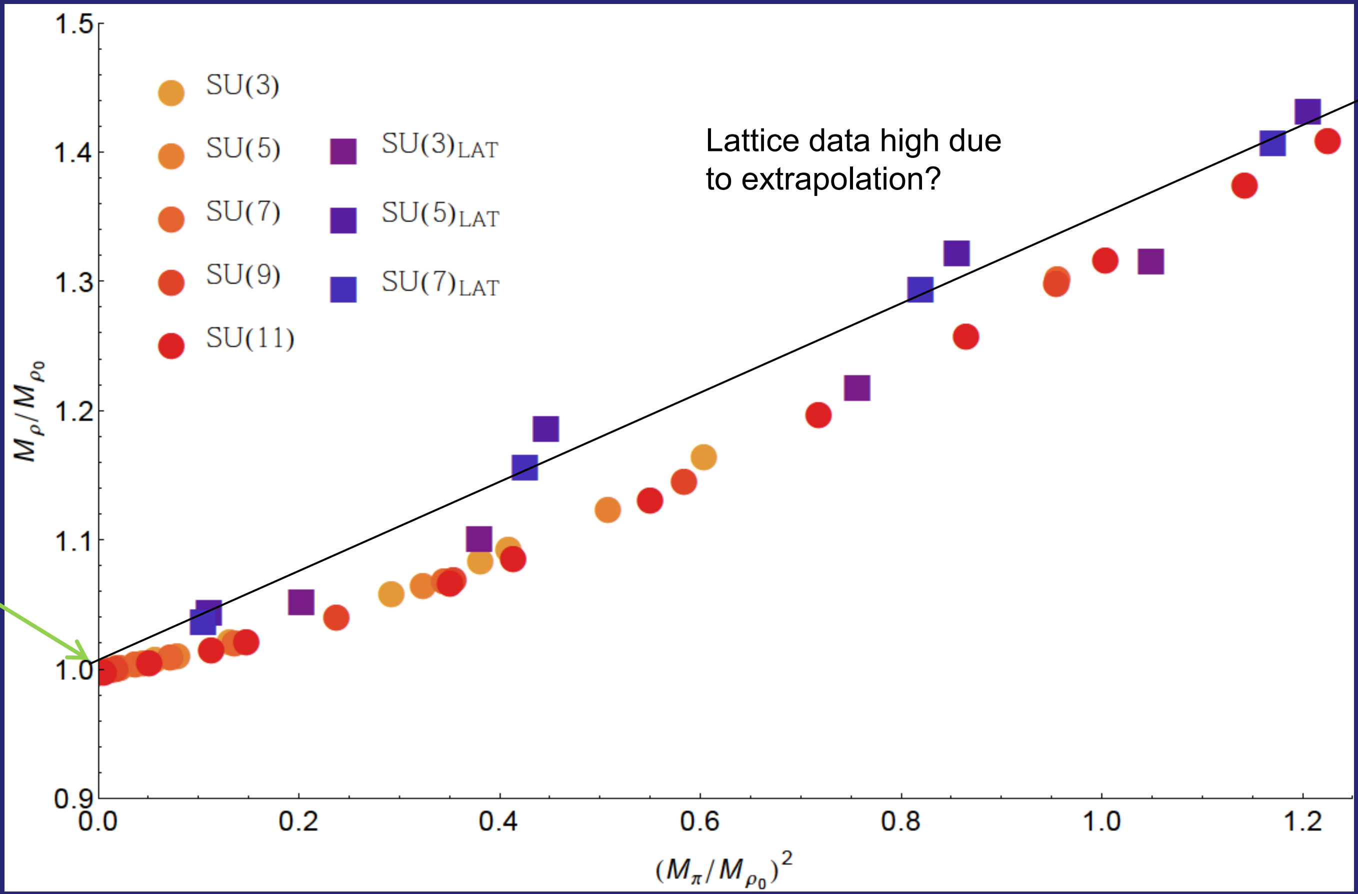


Sub Vs back into the action and integrate over radial coordinate to determine decay constants….

Fix external currents normalization by matching to UV QCD Πvv/ Πaa

SU(Nc) gauge + 3 quarks

NE, Erdmenger & Mark Scott

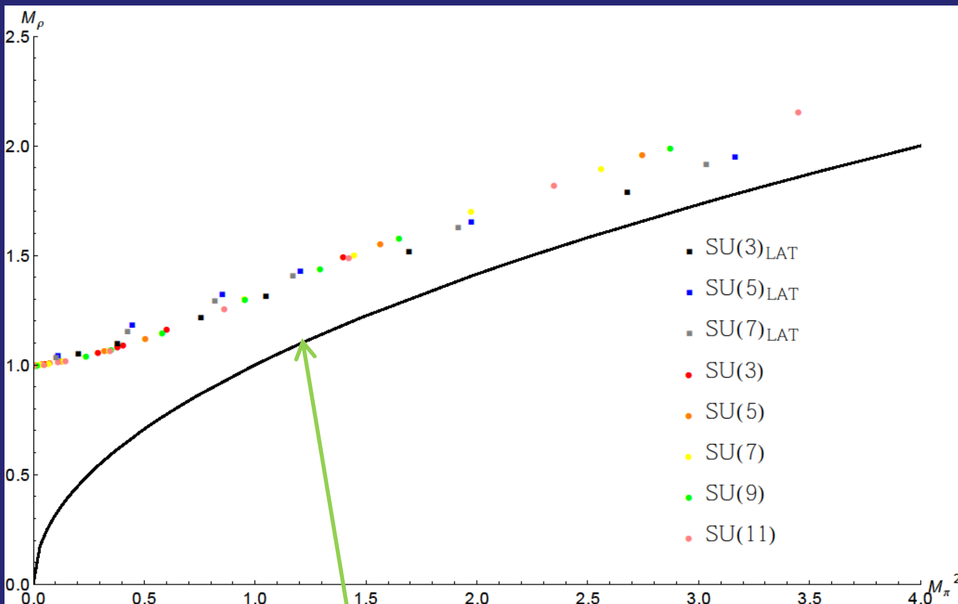


There is very little Nc dependence – basically quenched…

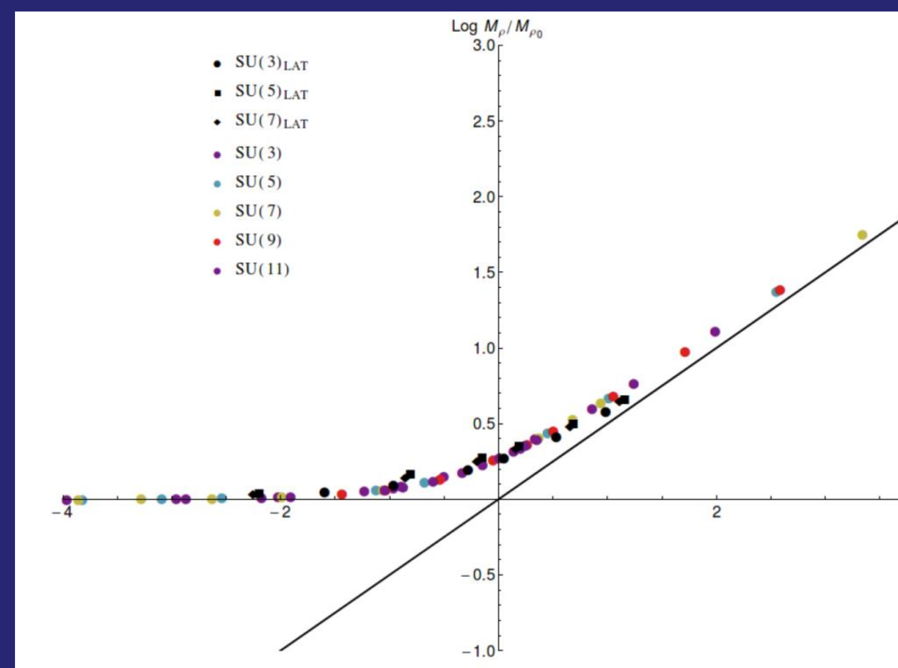
Hence comparison to quenched lattice data (Bali et al… arXiv1304.4437)

All of these models lie within 10% on any point....

SU(Nc) gauge + 3 quarks



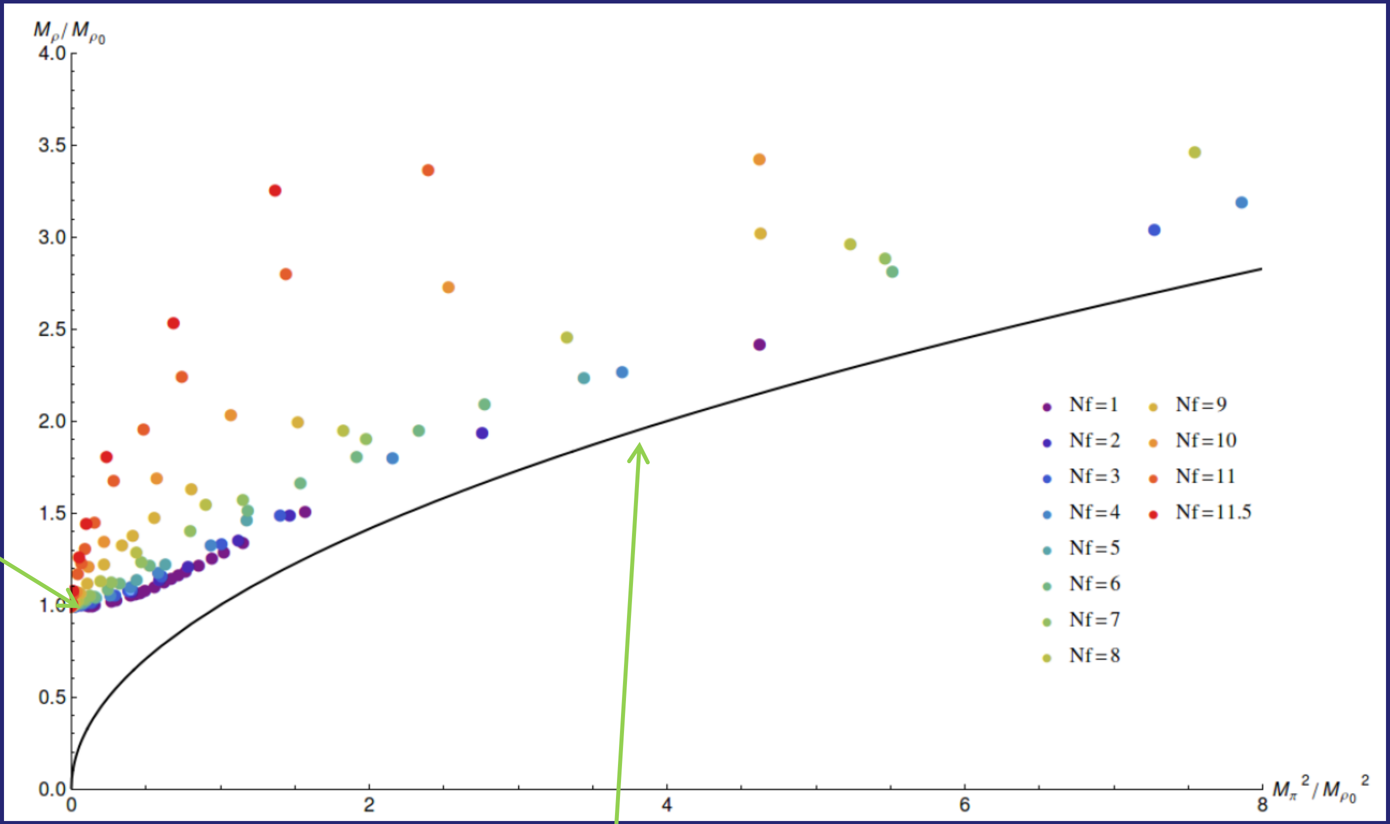
M_rho = M_pi



SU(3) gauge theory + Nf quarks

M_rho = 1
defines Λ

Real QCD
lies here



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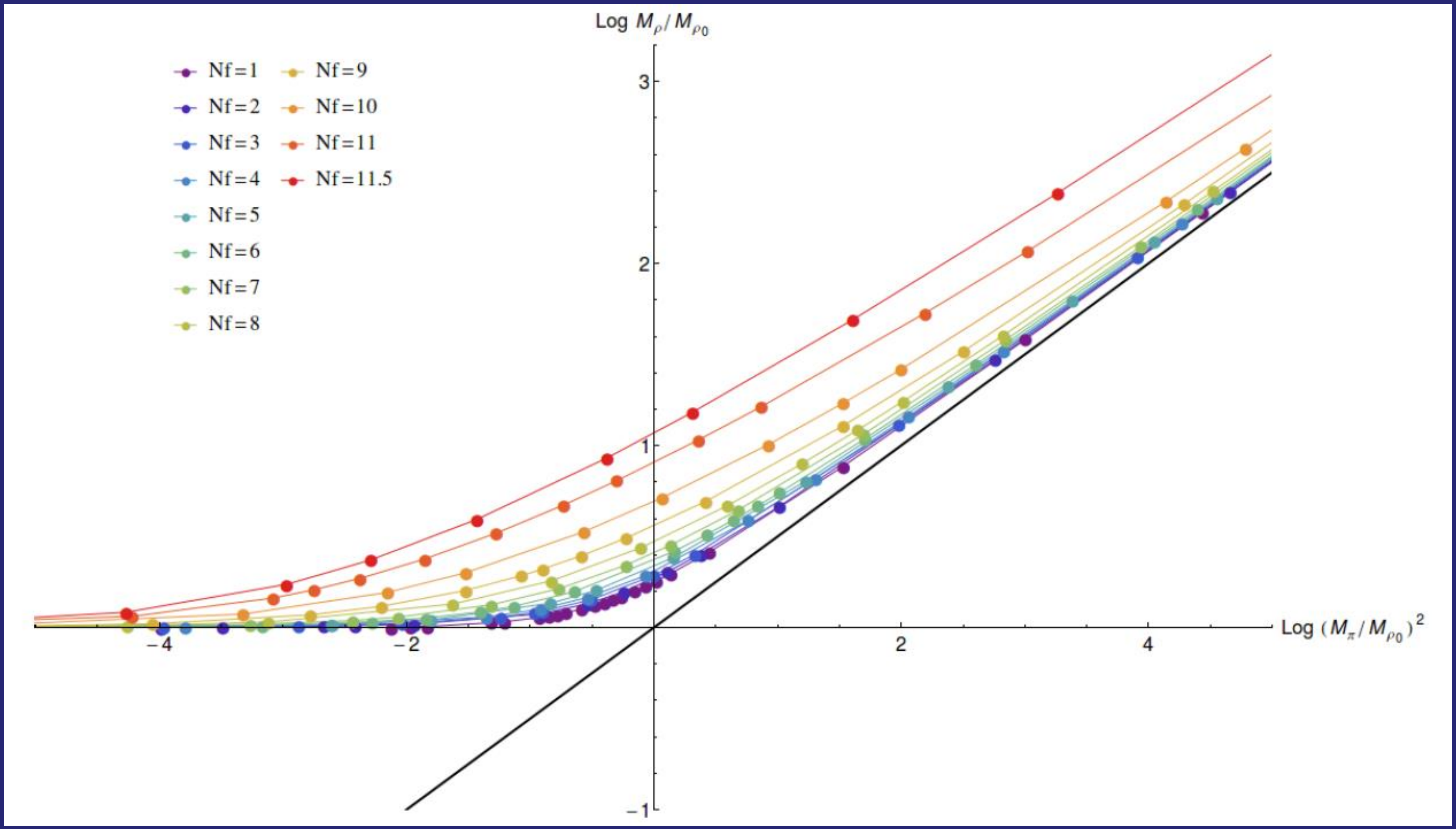
SU(3) gauge theory + Nf quarks



We do see new behaviour as Nf heads towards 12….

The Conformal Window

SU(Nc) gauge theory with Nf fundamental quarks

Nf=11/2 Nc

No AF

CFT

Nf = 4 Nc

χ SB

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left\{\frac{11}{3}N_c - \frac{2}{3}N_f\right\} - \frac{g^5}{(4\pi)^4}\left\{\frac{34}{3}N_c^2 - \frac{N_f}{N_c}\left[\frac{13}{3}N_c^2 - 1\right]\right\} + \cdots$$

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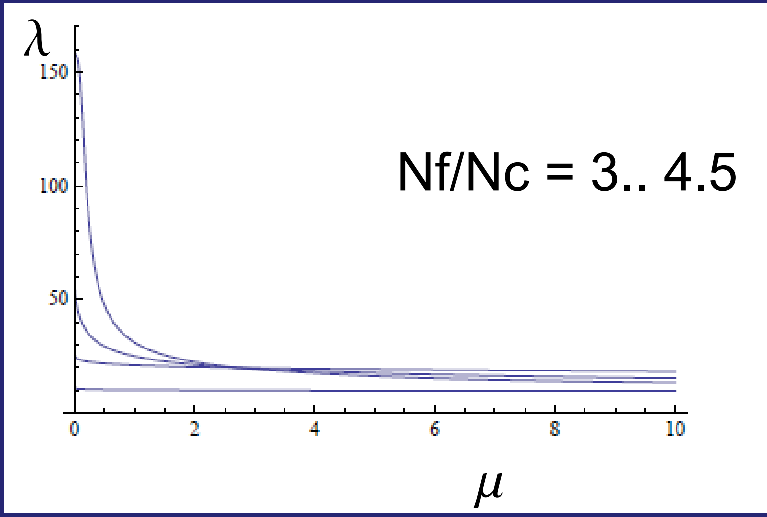


If critical γ = 1…. Nf/Nc ~ 4

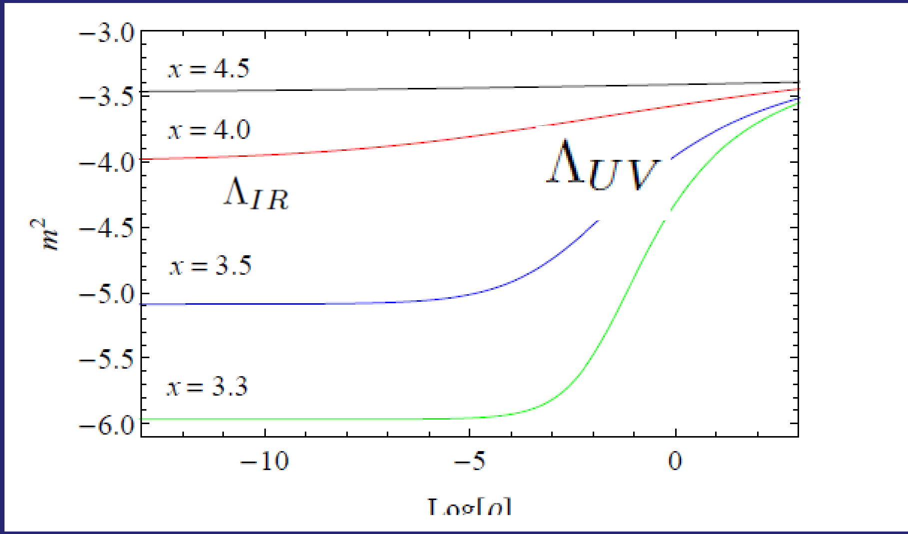
Yamawaki,Appelquist, Terning, Sannino,…

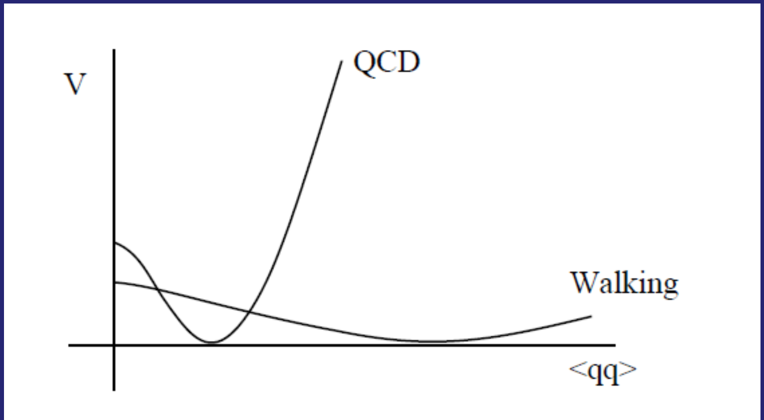
Walking Dynamics Holdom

Just above the CW regime theories have an enhanced UV quark condensate

$$\langle \bar{q}q \rangle_{UV} \sim \Lambda_{UV} \langle \bar{q}q \rangle_{IR} \sim \Lambda_{UV} \Lambda_{IR}^2$$

$$f_\pi \sim \Lambda_{IR}$$

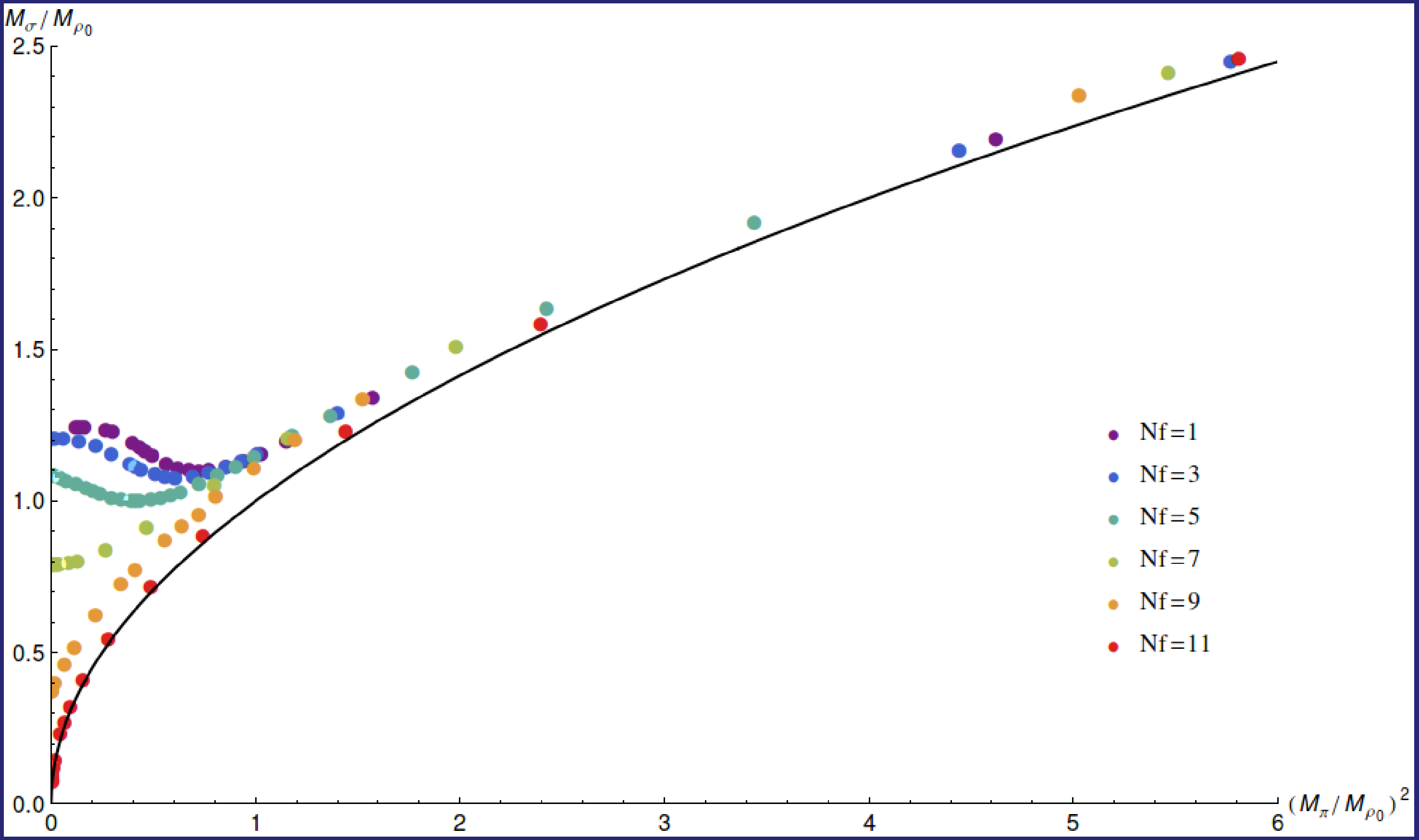




- Is the sigma particle light – a techni-dilaton?
- Is the higgs such a technicolor state?

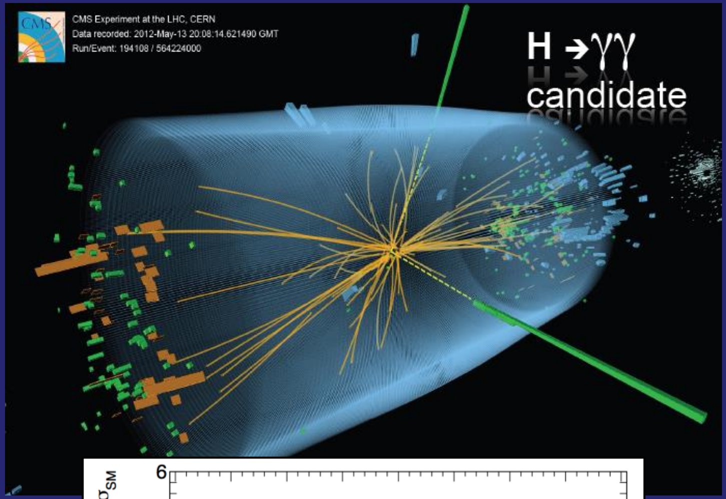
SU(3) gauge theory + Nf quarks

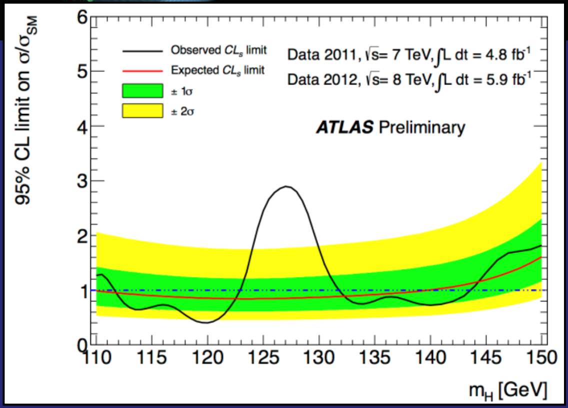
The QCD point is not right for the f0(500) but about right for the f0(980) – is the f0(500) odd eg a molecule ???



We indeed see a light sigma relative to the rho…

The Higgs Connection





Many of us remain perturbed by the discovery of the higgs boson. We used to explain why there were no fundamental scalars by arguing that quantum corrections drag scalar masses to infinity…

The "natural" explanation is that all known scalars are composites of quarks bound by QCD interactions…

Technicolour posits that the higgs is the sigma meson of a new "Technicolour" theory whose chiral symmetry breaking breaks the chiral weak force SU(2)…

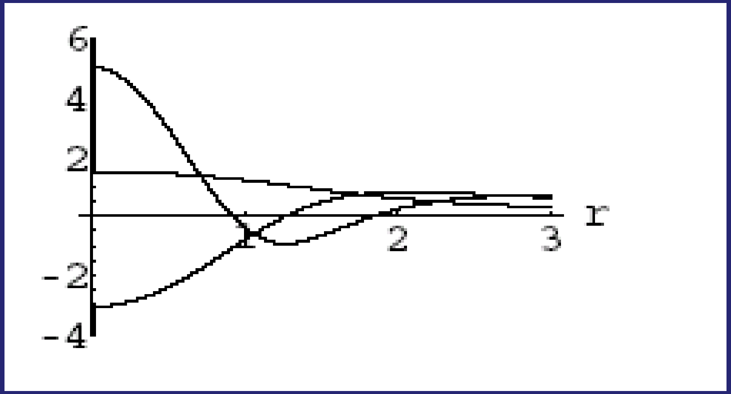
These walking theories do realize the fine tuning (eg Nf -> 12) that might do the trick…

Additional fine tunings seem necessary though

S parameter
H -> $\gamma\gamma$

Regge Behaviour in Excited States

The holographic method works well for broad brush responses to changes in the rate of running… but there are some things it gets wrong…



Generally radially excited states masses grow as

M ~ n

One can also naively describe higher spin states associated with

$$\bar{q}\gamma^{\mu}\partial^{\nu}q,\ \bar{q}\gamma^{\mu}\partial^{\nu}\partial^{\lambda}q,..$$

Again

M ~ J

Phenomenology suggests

$M^2 \sim J$

$M^2 \sim n$

$$S \sim \int d^4x \; d\rho \; \frac{\rho^{1+2s} e^{-\Phi}}{(\rho^2+L^2)^{1-s}} (\partial^\mu A^{\nu\lambda\ldots})^2$$

$$\partial_\rho = \frac{1}{\rho^2+L^2} \partial_z$$

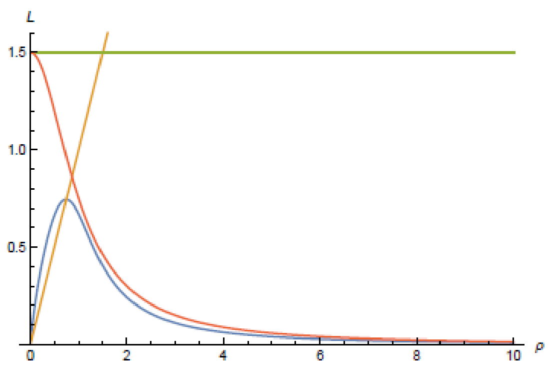
$$\psi = aV$$

$$\partial_\rho \left[\rho^{1+2s} e^{-\Phi} \partial_\rho V \right] + \frac{\rho^{1+2s} e^{-\Phi}}{(\rho^2+L^2)^2} M_n^2 V = 0$$

$$-\psi'' + U\psi = M_n^2 \psi$$

$$\begin{aligned} U &= -\frac{(\rho^2+L^2)^2}{a\rho^{1+2s}e^{-\Phi}} \partial_\rho \left[\rho^{1+2s} e^{-\Phi} \partial_\rho a \right] \\ &= -(\rho^2+L^2)^2 \left[\frac{(1+2s)}{\rho} \left(\frac{1}{a} \partial_\rho a \right) - \partial_\rho \Phi \left(\frac{1}{a} \partial_\rho a \right) + \frac{1}{a} \partial_\rho^2 a \right] \end{aligned}$$

$$L = \frac{1}{1+\rho^2}$$



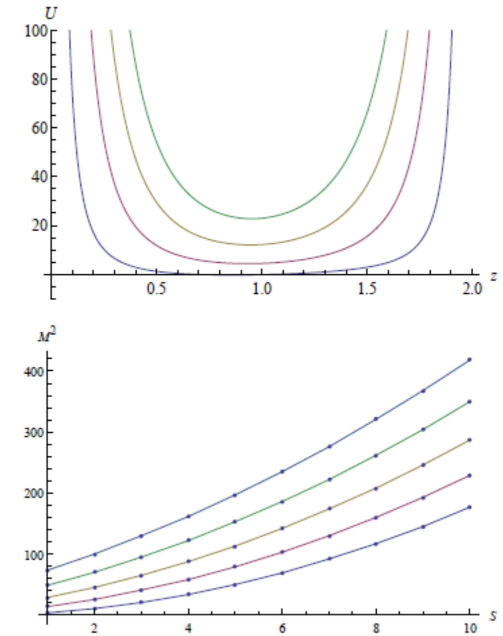


Figure 2: The Schroedinger wells for the Dynamically Generated Mass model for $s = 1, 2, 3, 4$ and a plot of the Mass trajectories vs spin, s, for the excitation numbers $n = 1, 2, 3, 4, 5$.

$$\partial_\rho = \frac{1}{\rho^2 + L^2}\partial_z$$

The square well emerges because at small ρ the constant L dominates and…

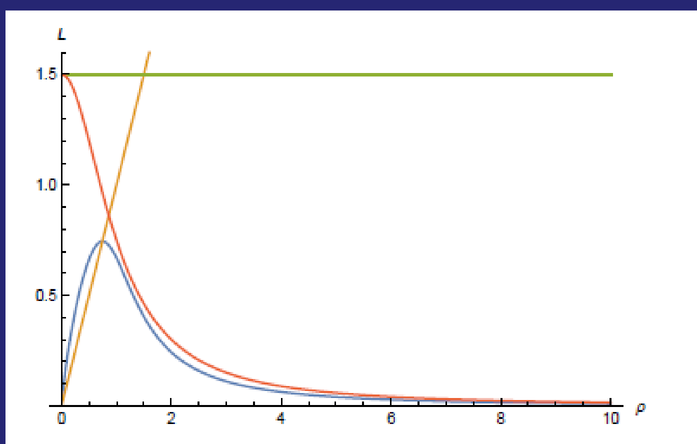
$$z = \arctan[\rho]$$

$0 < \rho < \infty$ maps to $0 < z < \pi/2$.

One can create "softwall" models where L -> 0 in the IR eg

$$L = \frac{z^{1/2}\rho^{1/2}}{\sqrt{1 + z\rho^5}}$$

Gives M ^{2} ~ n at fixed s….



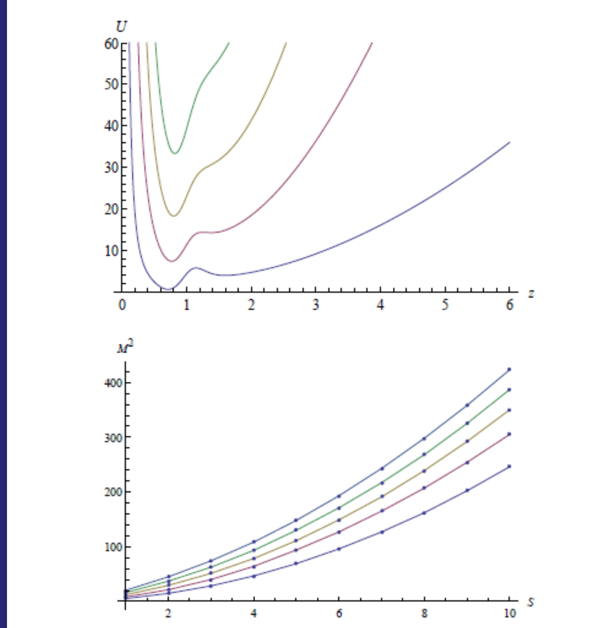
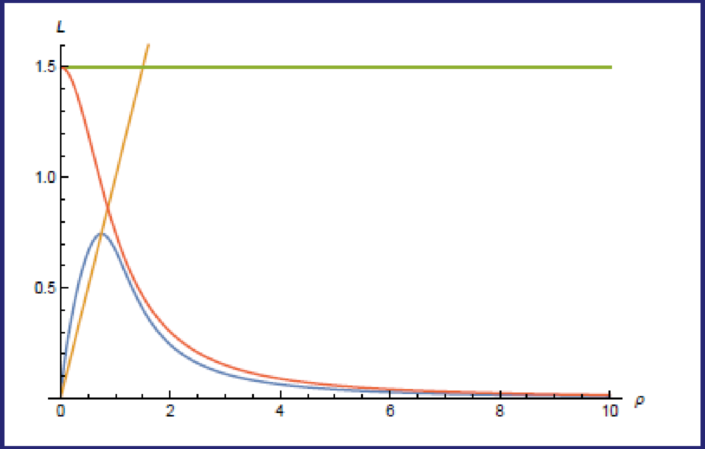
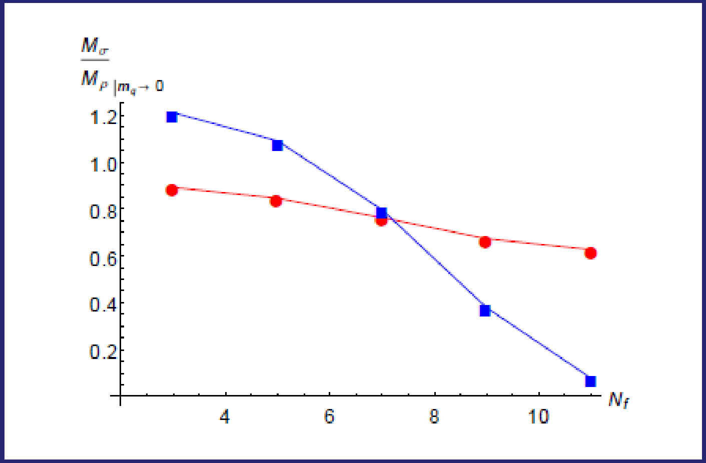


Figure 3: The Schroedinger wells for the softwall model with L given by (33) for $s = 1, 2, 3, 4$ and a plot of the Mass trajectories vs spin, s, for the excitation numbers $n = 1, 2, 3, 4, 5$.

But this means the mesonic physics is sensitive to scales far below the quark mass…

Softwalls also remove the dilaton like nature of the sigma/higgs





Kiritsis & Jarvinen embrace softwalls and deny a light higgs… I think softwalls are wrong and the light higgs is possible…

Regge trajectories are most likely achieved by a transition from point like supergravity fields to honest strings in AdS (which is expected at N < infinity)

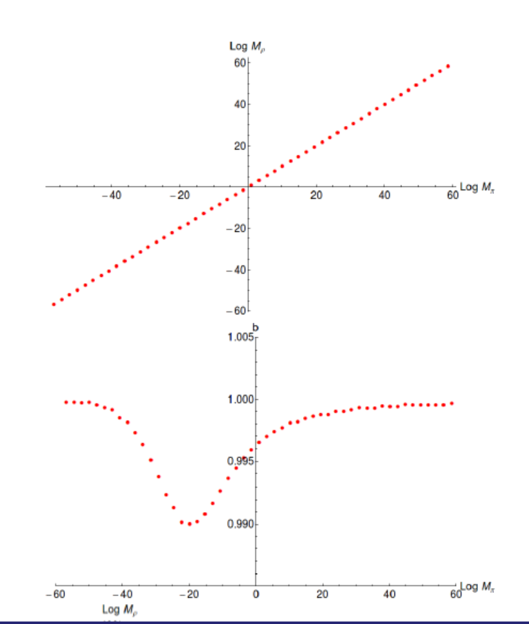
- see Misha Shifman & Cobi Sonnenschien's work….

Extra Games

Theories in the Conformal Window

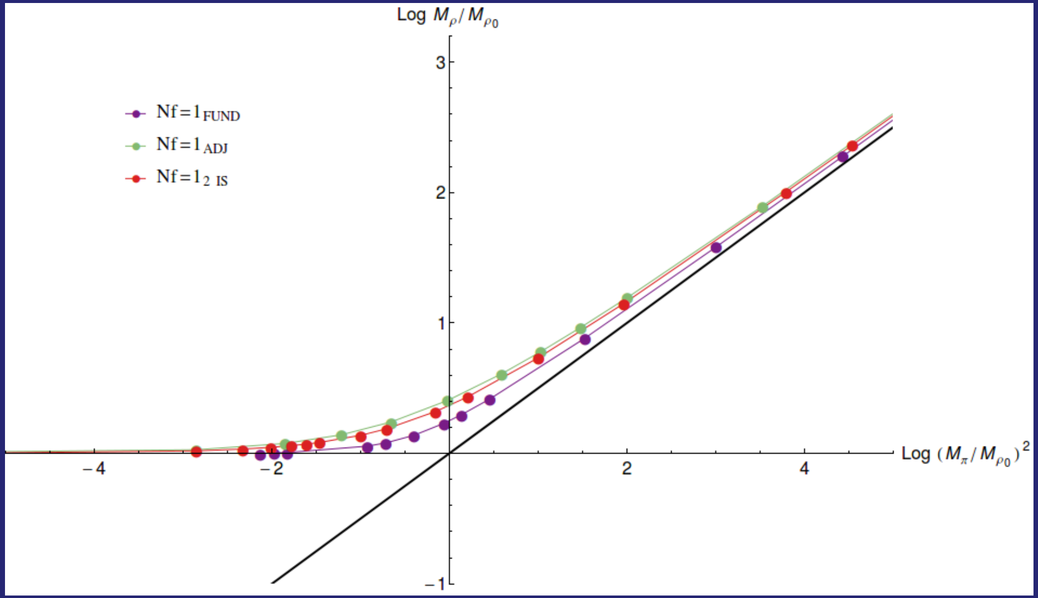
eg SU(3) gauge + 13 flavours scaling behaviours

$$M_\rho \propto M_\pi^b$$



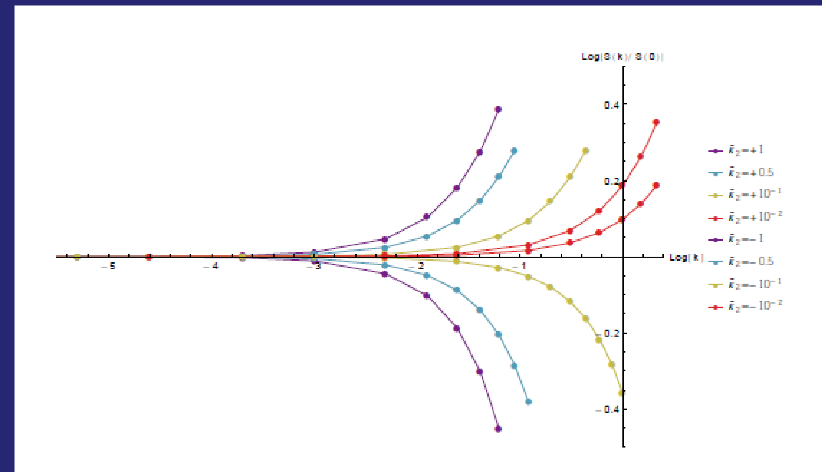
Pick your theory…

eg SU(3) gauge +



Can Dynamically Generated Higher Dimension Operators Trigger Lorentz Symmetry Breaking?

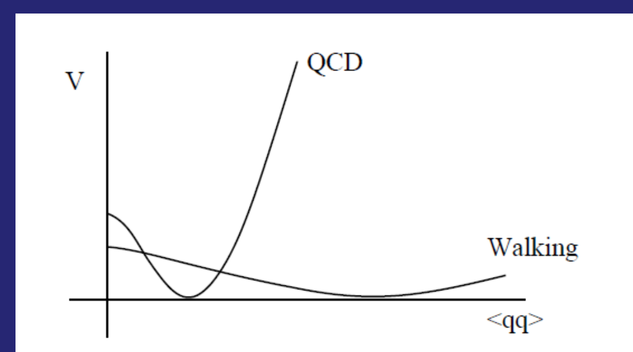
$$\Delta\mathcal{L} = -\frac{\kappa_0}{\Lambda_{UV}^2}|f|^2\partial^\mu\phi^*\partial_\mu\phi$$



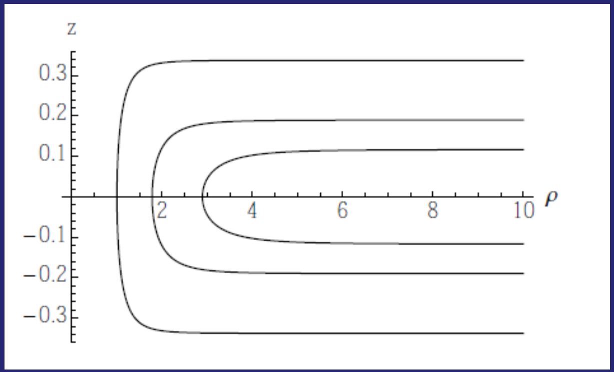
Instability to striped phases?

New contributions to Dark Energy in R^2 gravity…

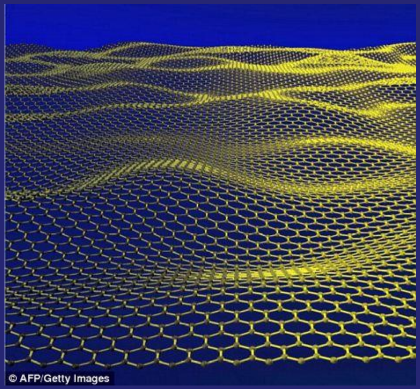
Walking Theories are Inflationary



Lower Dimensionally Systems may teach us about graphene like structures…



Bilayer condensates
…



Conclusions

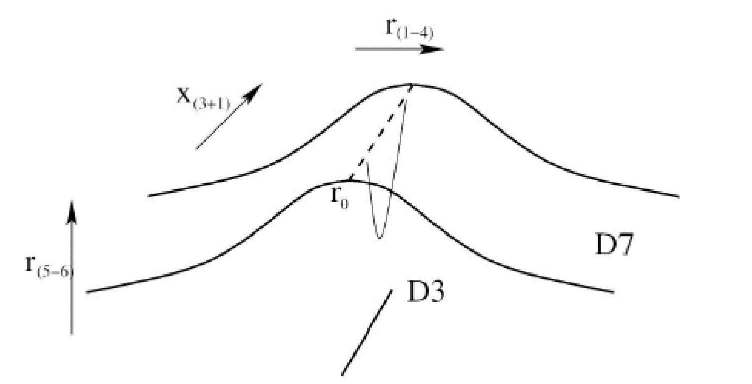
Holographic models of QCD continue to improve… running of γ is crucial…one gets a good description of the lowest lying spectra at better than 10% and you can see generic behaviours with Nc Nf easily...

Holography is a remarkably simple method to get a ball park answer for behaviour… but it still can’t be systematically improved…

On going work: NJL interactions… T μ phase structure… pomeron physics… enlarging to the full QCD spectra…

D3/ Probe D7 Model

Alvares, NE, Kim, 1204.2474



λ

$$S_{D7} = -T \int d^4x d\rho\, \rho^3 e^{\phi} \sqrt{1 + (\partial_\rho L)^2}$$

$$S = \int \mathrm{d}\rho \lambda(r) \rho^3 \sqrt{1 + L'^2}$$

We expand for small L

$$S = \int \mathrm{d}\rho \left(\frac{1}{2} \lambda(r) \Big|_{L=0} \rho^3 L'^2 + \rho^3 \frac{d\lambda}{dL^2}\Big|_{L=0} L^2 \right)$$

we can now make a coordinate transformation

$$\lambda(\rho)\rho^3 \frac{d}{d\rho} = \tilde{\rho}^3 \frac{d}{d\tilde{\rho}}, \qquad \tilde{\rho} = \sqrt{\frac{1}{2} \frac{1}{\int_\rho^\infty \frac{d\rho}{\lambda \rho^3}}}$$

$$L = \tilde{\rho}\phi$$

$$S = \int \mathrm{d}\tilde{\rho} \frac{1}{2} \left(\tilde{\rho}^5 \phi'^2 - 3\tilde{\rho}^3 \phi^2 \right) + \int \mathrm{d}\tilde{\rho} \frac{1}{2} \lambda \frac{\rho^5}{\tilde{\rho}} \frac{d\lambda}{d\rho} \phi^2$$

This is the action of a scalar in AdS with a mass squared of -3 + ρ dependent correction from the gradient of λ

For example if we try to (very naively) input the two loop QCD running of the coupling…

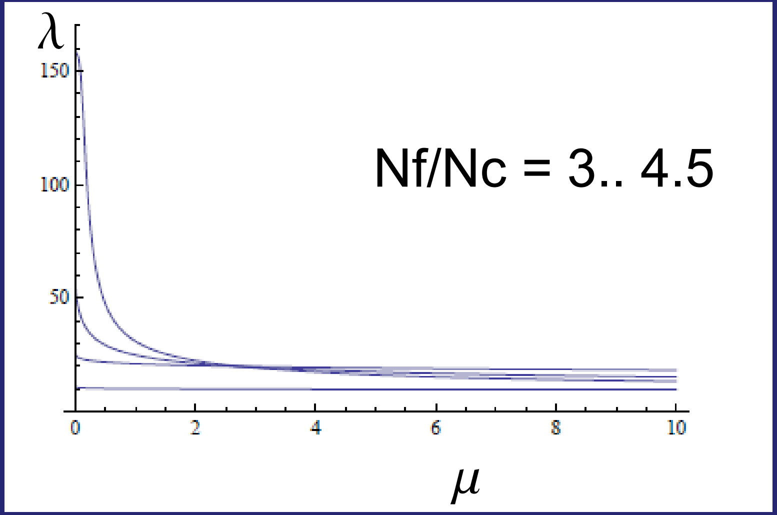
$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left\{\frac{11}{3}N_c - \frac{2}{3}N_f\right\} - \frac{g^5}{(4\pi)^4}\left\{\frac{34}{3}N_c^2 - \frac{N_f}{N_c}\left[\frac{13}{3}N_c^2 - 1\right]\right\} + \cdots$$

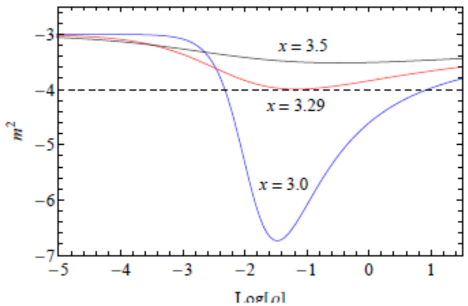
Using the 't Hooft coupling, and setting $\frac{N_f}{N_c} \to x$ we obtain

$$\lambda \equiv g^2 N_c \quad , \quad \dot{\lambda} = -b_0\lambda^2 + b_1\lambda^3 + \mathcal{O}(\lambda^4)$$

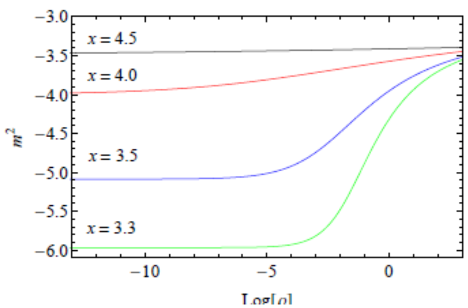
with

$$b_0 = \frac{2}{3}\frac{(11-2x)}{(4\pi)^2} \quad , \quad \frac{b_1}{b_0^2} = -\frac{3}{2}\frac{(34-13x)}{(11-2x)^2}$$





(a)The model with the QCD running imposed in section III ($x = 3.5, 3.29, 3.0$).



(b)The model of section IVa where the QCD anomalous dimension is imposed in the IR ($x = 4.5, 4, 3.5, 3.3$).

We output these running masses… to be compared with the perturbative expectation below…

$$\gamma_m^{(1)} = \mu\frac{d\ln m_q}{d\mu} = \frac{3(N_c^2-1)}{4N_c\pi}\alpha$$

$$m^2 = \Delta(\Delta - 4)$$

Jarvinen & Kiritsis Holographic Model

arXiv:1112.1261 [hep-ph]

- 5d supergravity $ds^2 = e^A dr^2 + dx_4^2$
- λ scalar to represent running coupling
- V(λ, A) – impose preferred IR and UV behaviour

Nf/Nc ~ 4

- ϕ scalar to represent $\bar{q}q$ condensate
- V(ϕ, λ, A) – to determine if $\bar{q}q$ condenses

Glueballs

The model concentrates on the quark states… we're not trying to describe the running of the coupling or Tr F^2… talks later today will try to include that extra scalar correctly… however for us roughly…

Find the dynamical IR quark mass… below that scale run the coupling as pure YM.. Find the IR pole… multiply by 8 and that's the glueball mass!

SU(3)
Nf=3