

Квазинормални моди на въртящи се черни дупки

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Атестационен семинар



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План на семинара

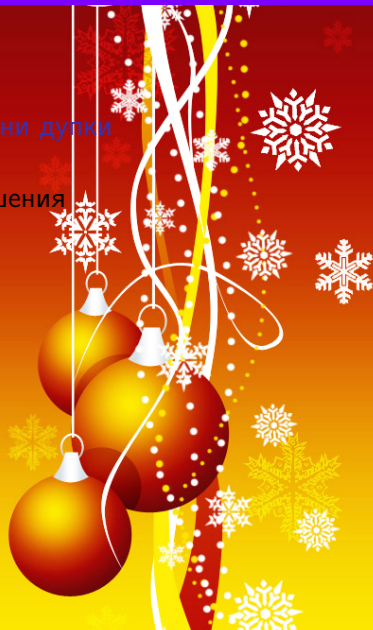
1 Квазинормални моди на въртящи се черни дупки

- Защо квазинормални моди?
- Уравнения на Тьюколски и техните решения

2 Числени резултати

- Двумерен алгоритъм на Мюлер
- Резултатите при липса на въртене
- Резултати за въртяща се черна дупка

3 Публикации



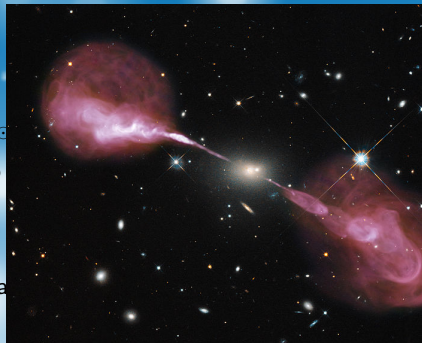
Теоретични срещу наблюдателни черни дупки

“Малко, тъмно и тежко: Но дали е черна дупка?” M.Visser et al.

BHs,GRandStrings 2008:010,2008, arXiv:0902.0346v2

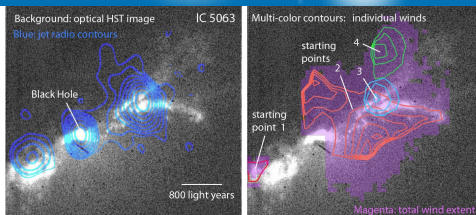
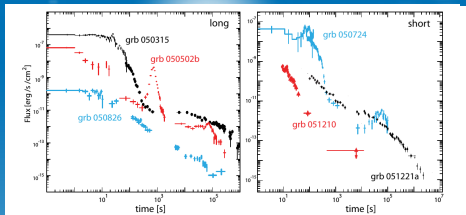
Какво казват наблюдателните данни:

- Най-малката звездна черна дупка: $3.8M_{\odot}$
XTE J1650-500 (типично $M \in (4 - 15)M_{\odot}$)
- Средни по маса черни дупки: $1 \cdot 10^3 - 4 \cdot 10^4 M_{\odot}$
- Супер-масивни черни дупки: $1 \cdot 10^6 - 9 \cdot 10^9 M_{\odot}$
- 2 двойни системи SMBH
- 5 тройни системи
- Липсващи двойни системи неутронна звезда (НЗ) – черна дупка (ЧД)
- Невъзможност към момента да се отдели НЗ от ЧД само по спектъра /случаят GRO J0422+32: от $3.6 M_{\odot}$ (2003) до $2.1 M_{\odot}$ (2012)/



Обекти с огромна разлика в мащабите, които наричаме черни дупки.

Нерешени проблеми?

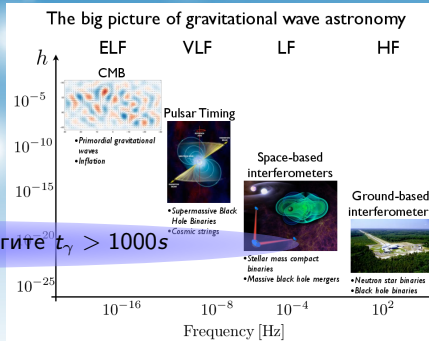


① Липсващи гравитационни вълни от LIGO

② Липсващи гравитационни вълни от Pulsar Timing Arrays /Madison et al. MNRAS, arXiv:1510.08068 [astro-ph.IM]/

③ Загадката на GRB – $E_{iso} \sim 10^{53} \text{ erg}$, $t \sim \text{sec}$, $t_{\text{flares}} \sim 10^5 \text{ s}$ живот на центр. двигател за ултрадългите $t_{\gamma} > 1000 \text{ s}$ /APJ, 778:54, 2013, ApJ 766:30, 2013/

④ Образуване на струи, M_{min}^{BH} , магн. полета – нужно $B \sim 10^{15} \text{ G}$

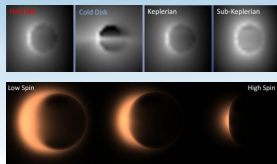
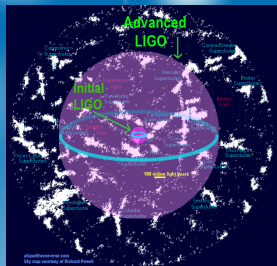


Проекти посветени на изучаването на ЧД:

- Advanced LIGO (09.2015)
- LISA Pathfinder (изстреляна на 3.12.2015!)
- Astrosat (изстрелян 28.09.2015, visible, UV, X-ray)
- Event Horizon Telescope (EHT, VLBI)
- BlackHoleCam project (финансиран от ERC, VLBI)
- Както и Chandra (X-ray), Spitzer (infrared), Fermi (gamma), Swift (transient), NuSTAR (X-ray), Hubble, Kepler etc.

Как да видим черна дупка?

- радио-телескопи /планирана разделителна способност за SgrA* - $4r_{Sh}/$
- гравитационни вълни
- multimessenger approach



Звъненето на черните дупки /Teukolsky (1972)/

Линейна пертурбация на метриката на Кер за
се описва от Уравненията на Тьюколски.
Ъглово уравнение (TAE):

$$\Psi = e^{i(\omega t + m\phi)} S(\theta) R(r)$$

$$\left((1-u^2) S_{lm,u} \right)_{,u} + \left((a\omega u)^2 + 2a\omega s u + s E_{lm} - s^2 - \frac{(m+su)^2}{1-u^2} \right) S_{lm} = 0, \quad (1)$$

и Радиално уравнение (TRE):

$$\frac{d^2 R_{\omega,E,m}}{dr^2} + (1+s) \left(\frac{1}{r-r_+} + \frac{1}{r-r_-} \right) \frac{dR_{\omega,E,m}}{dr} + \left(\frac{K^2}{(r-r_+)(r-r_-)} - \right. \\ \left. is \left(\frac{1}{r-r_+} + \frac{1}{r-r_-} \right) K - \lambda - 4is\omega r \right) \frac{R_{\omega,E,m}}{(r-r_+)(r-r_-)} = 0 \quad (2)$$

където $\Delta = r^2 - 2Mr + a^2 = (r-r_-)(r-r_+)$, $K = -\omega(r^2 + a^2) - ma$,
 $\lambda = E - s(s+1) + a^2\omega^2 + 2am\omega$ и $u = \cos(\theta)$.

За ЕМ пертурбации: $s = -1$. Двата хоризонта са: $r_{\pm} = M \pm \sqrt{M^2 - a^2}$.

Решения в термини на конфлуентни функции на Хойн

Решението на ТАЕ:

$$S_{1,2}(\theta) = e^{\alpha_{1,2} z_{1,2}} z_{1,2}^{\beta_{1,2}/2} z_{2,1}^{\gamma_{1,2}/2} \text{HeunC}(\alpha_{1,2}, \beta_{1,2}, \gamma_{1,2}, \delta_{1,2}, \eta_{1,2}, z_{1,2}) \quad (3)$$

където $z_1 = \cos(\theta/2)^2$, $z_2 = \sin(\theta/2)^2$, а параметрите са:

За случаят $m = 0$:

$$\alpha_1 = -\alpha_2 = 4a\omega,$$

$$\beta_1 = \beta_2 = 1,$$

$$\gamma_1 = -\gamma_2 = -1,$$

$$\delta_1 = -\delta_2 = 4a\omega,$$

$$\eta_1(\omega) = \eta_2(-\omega) = 1/2 - E - 2a\omega - a^2\omega^2$$

За случаят $m = 1$:

$$\alpha_1 = \alpha_2 = -4a\omega,$$

$$\beta_1 = \gamma_2 = 2,$$

$$\gamma_1 = \beta_2 = 0,$$

$$\delta_1 = -\delta_2 = 4a\omega,$$

$$\eta_1(\omega) = \eta_2(-\omega) = 1 - E - 2a\omega - a^2\omega^2$$

Където $\text{HeunC}(\alpha, \beta, \gamma, \delta, \eta, z)$ е решението на:

$$\frac{d^2}{dz^2} H(z) + \left(\alpha + \frac{\beta + 1}{z} + \frac{\gamma + 1}{z - 1} \right) \frac{d}{dz} H(z) + \left(\frac{\mu}{z} + \frac{\nu}{z - 1} \right) H(z) = 0$$

и $\delta = \mu + \nu - \alpha(\beta + \gamma + 2)/2, \eta = \alpha(\beta + 1)/2 - \mu - (\beta + \gamma + \beta\gamma)/2$

Решенията на TRE:

$$\begin{aligned}
 R(r) &= C_1 R_1(r) + C_2 R_2(r), \text{ за} \\
 R_1(r) &= e^{\frac{\alpha z}{2}} (r-r_+)^{\frac{\beta+1}{2}} (r-r_-)^{\frac{\gamma+1}{2}} \text{HeunC}(\alpha, \beta, \gamma, \delta, \eta, z) \\
 R_2(r) &= e^{\frac{\alpha z}{2}} (r-r_+)^{\frac{-\beta+1}{2}} (r-r_-)^{\frac{\gamma+1}{2}} \text{HeunC}(\alpha, -\beta, \gamma, \delta, \eta, z),
 \end{aligned} \tag{4}$$

където $z = -\frac{r-r_+}{r_+-r_-}$, а параметрите са:

$$\begin{aligned}
 \alpha &= -2i(r_+ - r_-)\omega, \beta = -\frac{2i(\omega(a^2 + r_+^2) + am)}{r_+ - r_-} - 1, \gamma = \frac{2i(\omega(a^2 + r_-^2) + am)}{r_+ - r_-} - 1, \\
 \delta &= -2i(r_+ - r_-)\omega(1 - i(r_- + r_+)\omega), \\
 \eta &= \frac{1}{2} \frac{1}{(r_+ - r_-)^2} \left[4\omega^2 r_+^4 + 4(i\omega - 2\omega^2 r_-) r_+^3 + (1 - 4a\omega m - 2\omega^2 a^2 - 2E) \times \right. \\
 &\quad \left. (r_+^2 + r_-^2) + 4 \left(i\omega r_- - 2i\omega r_+ + E - \omega^2 a^2 - \frac{1}{2} \right) r_- r_+ - 4a^2 (m + \omega a)^2 \right].
 \end{aligned}$$

За TAE изискваме регулярност на сферата. Т.е. Вронскианът на 2 решения $S_1(\theta)$ and $S_2(\theta)$, да е $W[S_1(\theta), S_2(\theta)] = 0$, или:

$$W[S_1, S_2] = \frac{\text{HeunC}'(\alpha_1, \beta_1, \gamma_1, \delta_1, \eta_1, (\cos(\pi/6))^2)}{\text{HeunC}(\alpha_1, \beta_1, \gamma_1, \delta_1, \eta_1, (\cos(\pi/6))^2)} + \frac{\text{HeunC}'(\alpha_2, \beta_2, \gamma_2, \delta_2, \eta_2, (\sin(\pi/6))^2)}{\text{HeunC}(\alpha_2, \beta_2, \gamma_2, \delta_2, \eta_2, (\sin(\pi/6))^2)} + p = 0 \quad (5)$$

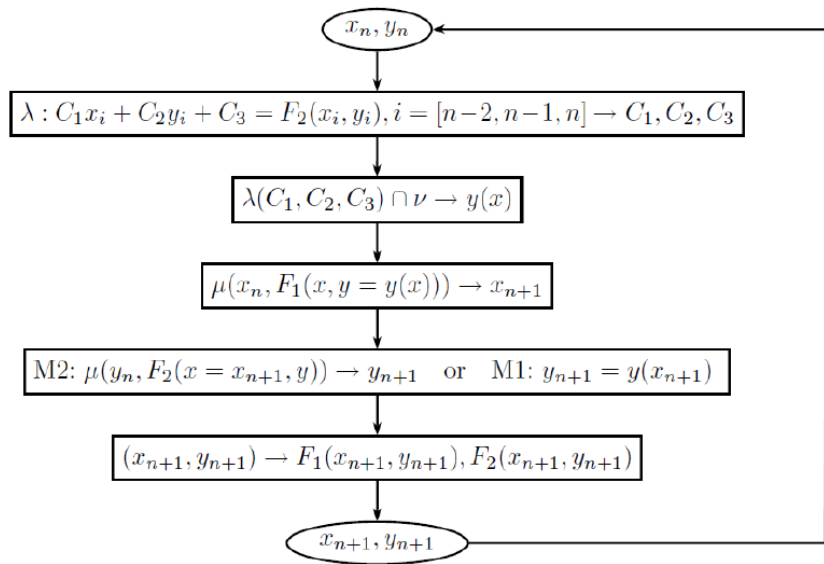
За TRE:

– (black hole boundary conditions) BHBC: R_2 е валидно за $\Re(\omega) \notin (-\frac{ma}{2Mr_+}, 0)$ и $\sin(\arg(\omega) + \arg(r)) < 0$ (DSD).

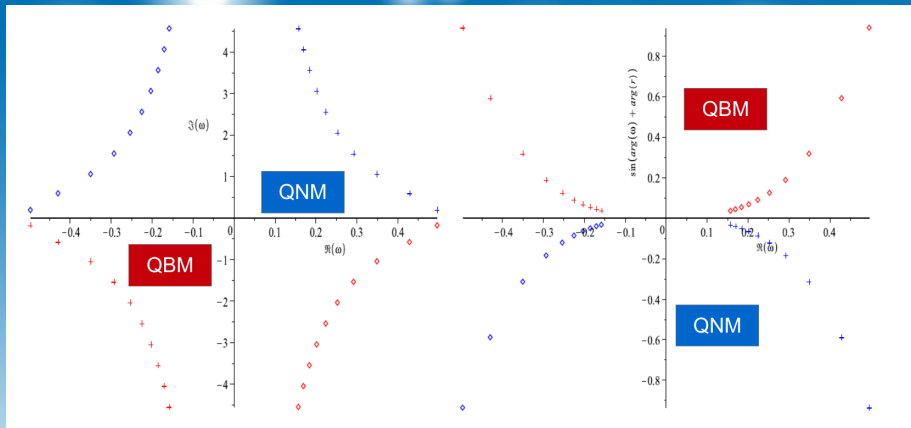
– (quasibound boundary conditions) QBBC: R_1 е валидно за $\Re(\omega) \notin (-\frac{ma}{2Mr_+}, 0)$ и $\sin(\arg(\omega) + \arg(r)) > 0$.

TTM моди липсват в ЕМ случаят, ϵ -методът в най-обща форма е $r = |r|e^{i\arg(r)}$.



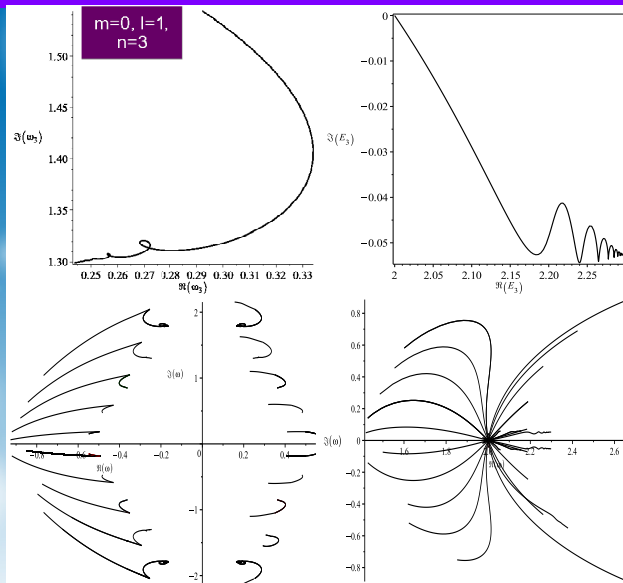


Спектърът при $a=0$ /D.S. and Fiziev (2015)/



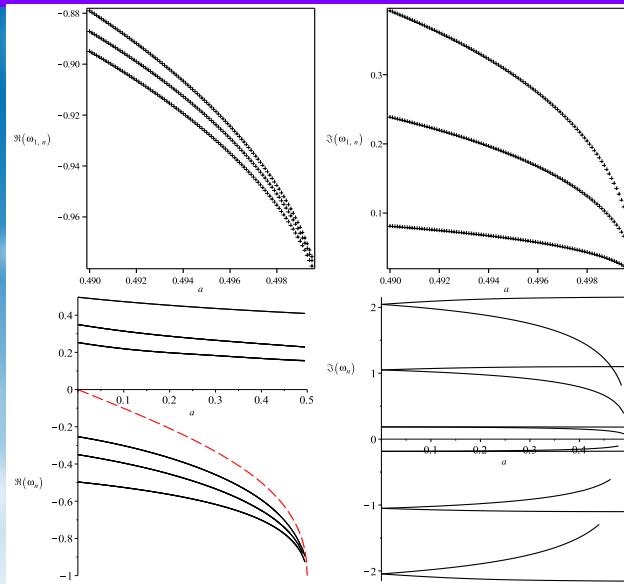
Фигура: а) QNM и QBM моди за $m=0, l=1$ б) Граничното условие за тях: $\sin(\arg(\omega) + \arg(r))=0$

Спектърът при $a \in [0, M]$ /Fiziev and D.S. (2015)/



Фигура: $\omega_{m,n}(a)$ и $E_{m,n}(a)$ за $a = [0, M)$, $m = 0, 1$, $l = 1$ $n = 0..4$

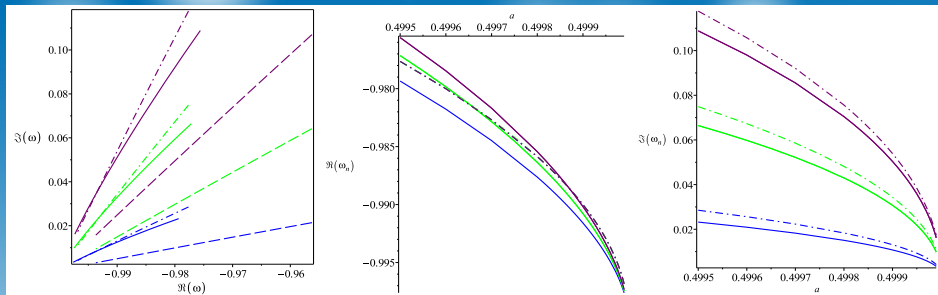
Спектърът при $a \rightarrow M$ /D.S. and Fiziev (2015)/



Фигура: $\Re(\omega_{1,n})(a)$ и $\Im(\omega_{1,n})(a)$ за $a = [0.49, 0.4995]$ за моди $n = 0, 1, 2,$

Аналитично приближение на спектъра при $a \rightarrow M$ /D.S. and

Fiziev (2015)/



Фигура: $\omega^1(a)$ (пунктир), $\omega^2(a)$ (точки-пунктир), и нашите числени резултати за ω_n for $n=0..2$, $m=1$, $l=1$

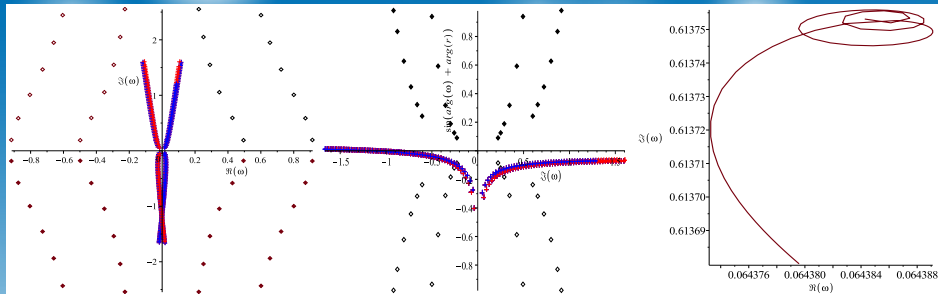
Ако $\Omega = \frac{a}{r_+^2 - a^2}$, $r_{\pm} = M \pm \sqrt{M^2 - a^2}$, $A = 4\pi(r_+ + a^2)$ и $T_{BH} = \frac{r_+ - r_-}{A}$.

Hod (2008) предлага 2 аналитични формули за ω близо до екстремалния режим:

$\omega^1 = m\Omega - i2\pi T_{BH}(n + 1/2)$ и $\omega^2 = m\Omega - i2\pi T_{BH}(n + 1/2 + i\delta)$.

Нашите числени резултати се описват по-добре с ω^2 с $\delta = -1/2 + i/6$, а за $m = 2$ с $\delta = -1/2$. Отклонението от теоретичната формула е $< 5\%$ for $a \rightarrow M$.

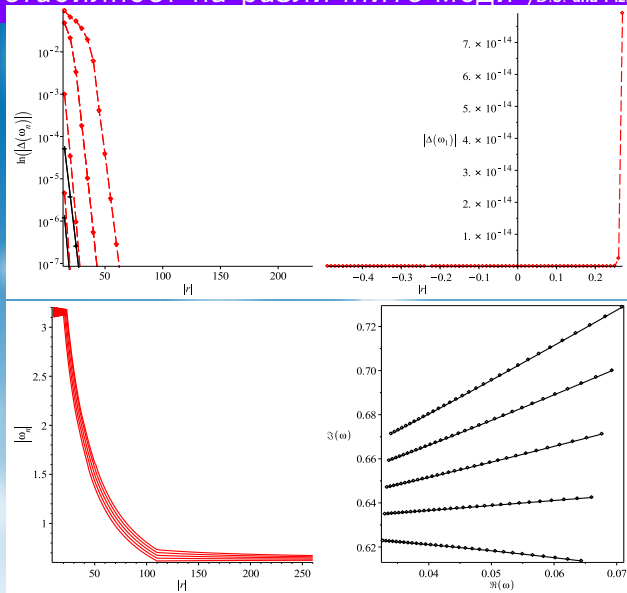
Spurious modes /D.S. and Fiziev (2015)/



Фигура: (а) Нефизични моди (кръстчета) с КНМ и КБМ (ромбчета) за $a = 0$, $m = 0, l = 1, 2$. (б) Граничното условие за тях $\sin(\arg(\omega) + \arg(r))$ (с) модата $n = 3$ за $a \in [0, M)$

Допълнителните нефизични моди бяха изследвани числено с епсилон-метода, за да се намери критерий за тяхното отсяване.

Числена стабилност на различните МОДИ /D.S. and Fiziev (2015)/



Фигура: Горе: QNM Долу: Spurious

- ❶ "Numerical stability of the electromagnetic quasinormal and quasibound modes of Kerr black holes" Denitsa R. Staicova , Plamen P. Fiziev, Bulg. Astr. J., 23 (2015), issn: 1313-2709, (arXiv:1511.09081)
- ❷ "New results for electromagnetic quasinormal and quasibound modes of Kerr black holes " Denitsa R. Staicova, Plamen P. Fiziev Astrophysics and Space Science, June 2015, 358:10, (arXiv:1412.4111), SJR=0.760, H=50
- ❸ Solving systems of transcendental equations involving the Heun functions, P. Fiziev, D. Staicova arXiv:1201.0017,), Am. J. of Comp. Math. Vol. 02 : 02, pp.95 (2012), Google based impact factor=0.51, H=8
- ❹ New results for electromagnetic quasinormal modes of black holes, D. Staicova, P. Fiziev, arXiv:1112.0310, internal report
- ❺ Application of the confluent Heun functions for finding the quasinormal modes of nonrotating black holes, P. Fiziev, D. Staicova, Phys. Rev. D 84, 127502 (2011), SJR=2.041, H=253
- ❻ Two-dimensional generalization of the Muller root-finding algorithm and its applications, P. Fiziev, D. R. Staicova, arXiv:1005.5375, Internal Report, SU (2011)

- 7 The Spectrum of Electromagnetic Jets from Kerr Black Holes and Naked Singularities in the Teukolsky Perturbation Theory, D. Staicova, P. Fiziev, Astrophys Space Sci (2011) 332: 385-401 , SJR=0.760, H=50
- 8 Toward a New Model of the Central Engine of GRB, P. Fiziev, D. Staicova, Bulgarian Astronomical Journal, 11, pp. 13-21, 2009
- 9 A new model of the Central Engine of GRB and the Cosmic Jets, P. Fiziev, D. Staicova, Bulgarian Astronomical Journal, 11, pp. 3-11, 2009

Публикувани доклади на конференции:

- 10 P. Fiziev, and D. Staicova, "Towards New Paradigms: Proceeding of the Spanish Relativity Meeting 2011", Ed. by I.B. Jimenez, J.S.R. Cembrano s, A. Dobado, et. Al, Book Series: AIP Conference Proceedings, Vol 1458, pp. 395-398, (2012) , SJR=0.152, H=53

Цитати:

Забелязани цитати:

- (9): 1 независим цитат (2015)
- (7): 1 независим цитат (2011)
- (6): 3 независими цитата (2011, 2012, 2013)
- (4): 7 независими цитати (2012, 2013а,б, 2014, 2015а,б,в):
- (5): 9 независими цитати (2012, 2013а,б,в,г), 2014(а,б), 2015(а,б):
- (10): 2 независими цитати (2013а,б)

Общо: 23 независими цитати

От тях цитати през 2013: 9

От тях цитати през 2014: 3

От тях цитати през 2015: 6

Конференции с доклади: 2013-2015:

IberiCOS 2015 Xth Iberian Cosmology Meeting /доклад: Minimal Dilatonic Gravity from cosmology to compact massive objects/

XI. International Workshop Lie Theory and Its Applications In Physics /19. 06.2015 доклад: The Heun functions and their applications in astrophysics applications in astrophysics/

10-та научна конференция на Съюза на астрономите в България Българичик, 2 – 5 юли 2015 г./с доклад Minimal Dilatonic Gravity from cosmology to compact massive objects/

Командировки:

15.09.2014-15.10.2014 – Университетът в Аликанте, Испания

20.10.2014-21.11.2014 – Университетът Гьоте във Франкфурт, Германия /по COST Action: MP1304/

Училища /в рамките на Cost Action NewCompStar/:

“The many faces of compact stars”: Barcelona (Spain), September 22 – 26, 2014

“Dense matter in compact stars: Experimental and observational signatures, 21.-25.09.15 , Bucharest, Romania

Семинари и други

Семинари:

„The Heun functions and their applications in astrophysics.“ (Семинар в ИЯИЯЕ, БАН, 19.12.2013)

„Compact stars in minimal dilatonic gravity“. (Семинар в Astro-Coffee, ИТР, Frankfurt, Germany, 18.11.2014)

„Compact static stars in Minimal Dilatonic Gravity“. (Семинар в ИЯИЯЕ, БАН, 12.2014)

„Minimal Dilatonic Gravity from cosmology to compact massive stars“ (Семинар в Института по Астрономия, БАН, 12.03.2015)

Статии в годишника на ИЯИЯЕ:

Electromagnetic Spectra of Rotating And Non-Rotating Black Holes, Denitsa Staicova, Plamen Fiziev, 2013, p. 86

Minimal dilatonic gravity in compact relativistic stars, Denitsa Staicova, Plamen Fiziev, 2014

Постери:

Постер за 43 годишнината на ИЯИЯЕ (43 years 1972 – 2015 INRNE),

“Electromagnetic quasi-normal modes of rotating black holes”, Denitsa Staicova and Plamen Fiziev

Това е!

Благодаря за вниманието!



References and credits I



Teukolsky S.A.

Rotating Black Holes: Separable Wave Equations for Gravitational and Electromagnetic Perturbations
Phys.Rev.Lett 29, 1114-1118 (1972)



Fiziev P.P. (2006)

Exact Solutions of Regge-Wheeler Equation and Quasi-Normal Modes of Compact Objects
CQG 23, 2447-2468 (2006), arXiv:0509123 [gr-qc]



Fiziev P.P. (2009)

Classes of exact solutions to the Teukolsky master equation
PRD 80 , 124001 (2009), arXiv:0906.5108 [gr-qc]



Fiziev P.P. (2010a)

Classes of exact solutions to the Teukolsky master equation
CQG 27, 135001 (2010a), arXiv:0908.4234v4 [gr-qc]



Fiziev P.P. (2010b)

Novel relations and new properties of confluent Heun's functions and their derivatives of arbitrary order
J. Phys. A: Math. Theor. 43, 035203, arXiv:0904.0245 [math-ph]



Hubble+VLA, LIGO, EHT, Gehrels et. al (2009), National and Kapodistrian University of Athens, NanoGrav

Цитати:

(10): 1 независим цитат (2015)

BC da Cunha, F Novaes, „Kerr scattering coefficients via isomonodromy“, Journal of High Energy Physics, 2015

(8): 1 независим цитат (2011)

D. Pugliese, H. Quevedo, and R. Ruffini, “Equatorial circular motion in Kerr spacetime”, Phys. Rev. D 84, 044030 (2011)

(7): 3 независими цитата (2011, 2012, 2013)

M. Cadoni, P. Pani, “Holography of charged dilatonic black branes at finite temperature” - JHEP 1104:049, (2011)

Pani, P. "Applications of black hole perturbation theory."The European Physical Journal Plus 127.6 : 1-52, (2012)

Neville I. Robinson, Andrew J. Love, Hidden channels of groundwater flow in Tóthian drainage basins, Advances in Water Resources 62:71–78 · December (2013)

(4):7 независими цитати (2012, 2013a,б, 2014, 2015a,б,в):

M.A. Dariescu, C. Dariescu, “Approximative Analytic Study of Fermions in Magnetar's Crust: Ultra-relativistic Plane Waves, Heun and Mathieu Solutions and Beyond”, Astrophysics and Space Science, 2012 – Springer, 10.1007/s10509-012-1101-y. 341, 1. 2, pp.429-435 (2012)

M.A. Dariescu et al. , „Analytic Study of Fermions in Graphene; Heun Functions and Beyond“, Romanian Journal of Physics, 58, 7-8 (2013)

Цитати:

- C. Dariescu, M-A Dariescu, C. Cretu, The Quantum Treatment of the 5D-WarpedFriedmann–Robertson–Walker Universe in SchrödingerPicture, Int J Theor Phys (2013) 52:1345–1353, DOI 10.1007/s10773-012-1450-x (2013)
- Motsepe Kganya A, Shatalov Michael. Y and Joubert Stephan V, „Application Of Heun Functions To Vibratory Rotor Gyroscope, Their Numerical Computation And Accuracy Estimation“, Applied Mathematics and Computation, 239, 47-55, (2014)
- A.E. Sitnitsky, „Exact solution of Smoluchowski's equation for reorientational motion in Maier-Saupe potential“, Physica A: Statistical Mechanics and its Applications 419, 373-384, (2015)
- Piotr Bizoń, Michał Kahl , „Wave maps on a wormhole“, arXiv:1412.5371 [gr-qc], Physical Review D 91, 065003 (2015)
- AE Sitnitsky, Probability distribution function for reorientations in Maier-Saupe potential, arXiv:1509.03439
- (6): 9 независими цитати (2012, 2013a,6,в,г), 2014(а,6), 2015(а,6).
- M.A. Dariescu, C. Dariescu, "Approximative Analytic Study of Fermions in Magnetar's Crust: Ultra-relativistic Plane Waves, Heun and Mathieu Solutions and Beyond", Astrophysics and Space Science– Springer, 10.1007/s10509-012-1101-y. 341, I. 2, pp 429-435 (2012)
- M. Hortascu "Heun functions and their uses in physics Mathematical Physics. Proceedings of the 13th Regional Conference, Antalya; pp23-39, World Scientific (2013)

Цитати:

- H. Hassanabadi, S. Zarrinkamar, A. A. Rajabi "Could we treat the Regge-Wheeler equation in an easier way?" The European Physical Journal Plus, June, 128:61, (2013)
- H. R. Christiansen, M. S. Cunha "Solutions to position-dependent mass quantum mechanics for a new class of hyperbolic potentials", J.Math.Phys. 54, 122108, (2013)
- M. S. Cunha, H. R. Christiansen "Analytic results in the position-dependent mass Schrodinger problem", Commun. Theor. Phys. vol. 60, No. 6, pp. 642-650, (2013)
- V. Balan, A.M. Manukyan, E.M. Ovsiyuk, V.M. Red'kov, O.V. Veko, . Confluent Heun functions and the Coulomb problem for spin 1/2 particle in Minkowski space, arXiv:1410.8300 [math-ph], (2014)
- H. R. Christiansen, M. S. Cunha , Energy eigenfunctions for position-dependent mass particles in a new class of molecular hamiltonians, J. Math. Phys. 55, 092102 (2014)
- C. Dariescu, M.A. Dariescu, Quantum Analysis of $k=-1$ Robertson–Walker Universe, Foundations of Physics, 2015 – Springer (2015), 10.1007/s10701-015-9922-5
- N. Saad, On the solvability of confluent Heun equation and associated orthogonal polynomials, arXiv:1509.00090 (2015)
- (11): 2 независими цитати:
- M. Hortascu "Heun functions and their uses in physics Mathematical Physics: Proceedings of the 13th Regional Conference, Antalya; pp23-39, World Scientific (2013)
- P.Pani, „Advanced Methods in Black-Hole Perturbation Theory" International Journal of Modern Physics A, Vol. 28 (2013) 1340018 (2013)