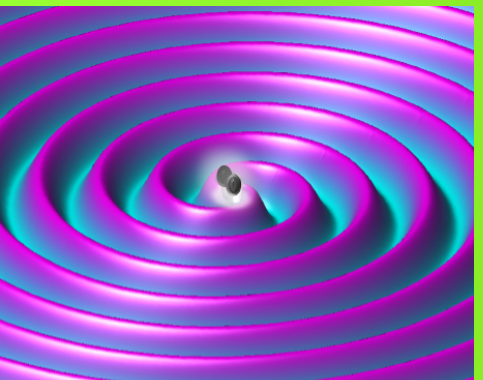
Гравитационните вълни – дългият път до откритието

Деница Стайкова

План на презентацията:

1. Откритието
2. Сигналът
3. Какво са гравитационните вълни?
4. Как работи детектора?
5. Как се смятат вълновите шаблони
5.1. Пост-нютоново приближение
5.2. Числена гравитация
5.3. Теория на пертурбациите
6. Астрофизичните резултати и бъдещето



Открити!

- Събитието: GW150914 от 15.09.2015
- Амплитуда на вълните: $h \sim 10^{-21}$
- Произход: Сливане на 2 черни дупки с маса (29^{+4}_{-4} $M_\odot$ и 36^{+5}_{-4} $M_\odot$) до трета с маса 62 $M_\odot$
- Продължителност на сливането: 0.2s
- Излъчена енергия: 3 $M_\odot$, пикова светимост: 200 $M_\odot$/s
- По-ярки от максималната светимост на видимата Вселена в този момент!
- Разстояние до обекта: 410 Mpc
- Статията във PRL свалена 229 000 пъти за 24ч, още 12 съпътстващи статии

Selected for a Viewpoint in *Physics*

PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.*[*]

(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 21 January 2016; published 11 February 2016)

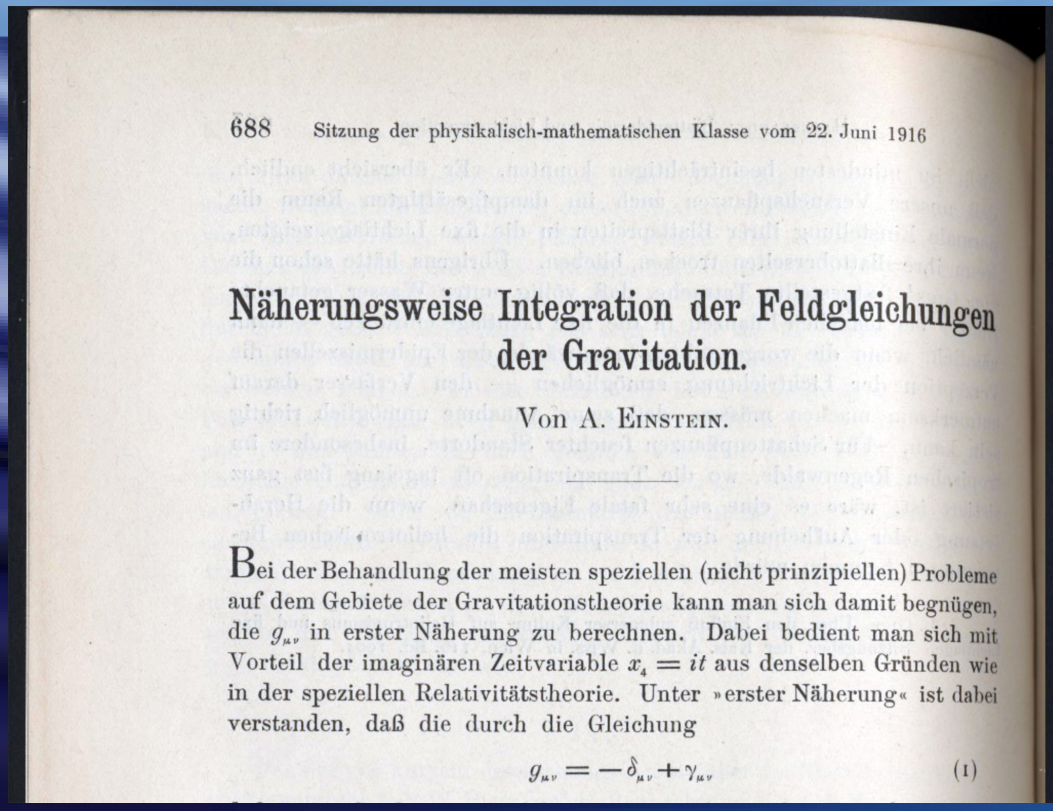
On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4}M_\odot$ and $29^{+4}_{-4}M_\odot$, and the final black hole mass is $62^{+4}_{-4}M_\odot$, with $3.0^{+0.5}_{-0.5}M_\odot c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

DOI: 10.1103/PhysRevLett.116.061102

Малко история

- Предречени от Айщайн през 1916г
- Първо доказателство през 1974г – двойна с-ма пулсари на Тейлър и Хълс: PSR B1913+16
- Първи предложения за лазерен интерферометър Герценштейн и Пустовойт в Русия и Уебер и Уайс в САЩ
- 1972г. Уайс идентифицира фундам. източници на шум
- Първи прототипи в лаборатории в САЩ, Германия и Шотландия
- 1980г. 40 метров и 1.5 метров прототип в САЩ (NSF)
- 1990 одобрява строежа на два 4 километрови интерферометъра – в Ханфорд, Вашингтон и Ливингстън, Луизиана

- 3 фази на LIGO: начална (2000-2005), подобрена (2009-2010) и напреднала (2015-)
- 75 институции от 15 нации по цял свят, ~1000 учени
- Над 1.1 млрд. долар инвестиции
- Над 70 статии публикувани от колаборацията едва последните 13 с положителен резултат!
- 74 групи за допълнително ЕМ проследяване
- 25 екипа са участвали в проследяването на GW150914 със сателити и наземни телескопи
- 19 порядъка по честоти от ЕМ спектъра!
- Невероятен успех на екипната работа в огромен мащаб!



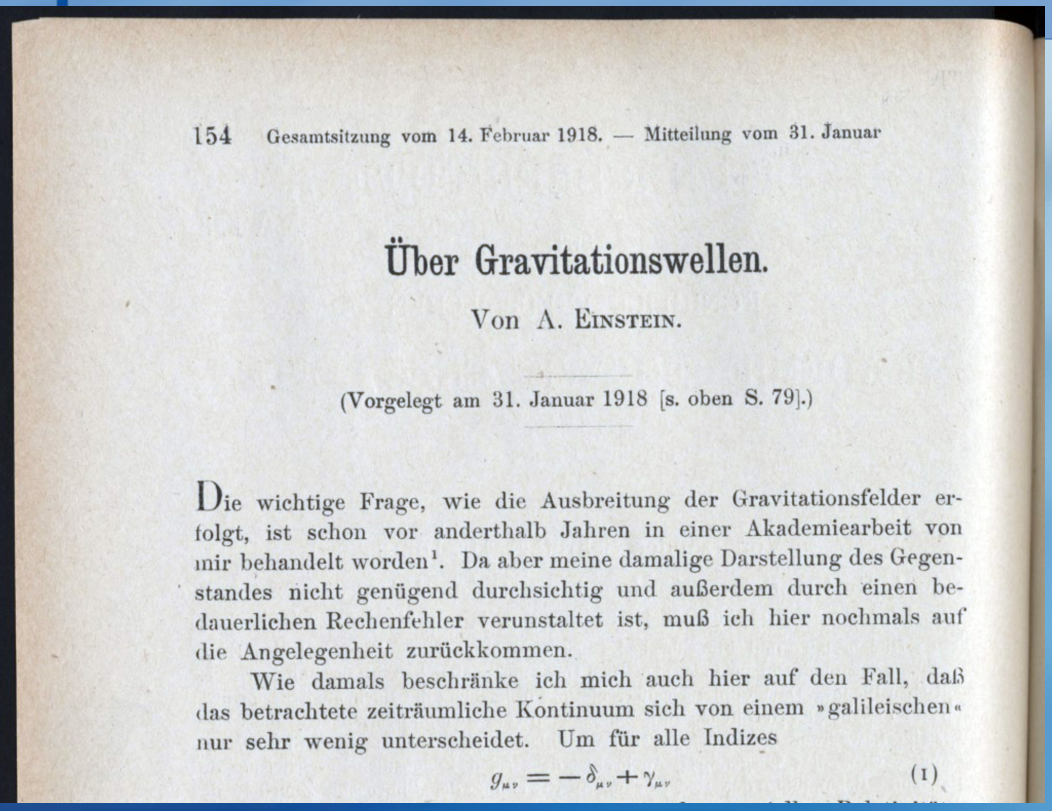
688 Sitzung der physikalisch-mathematischen Klasse vom 22. Juni 1916

Näherungsweise Integration der Feldgleichungen der Gravitation.

Von A. Einstein.

Bei der Behandlung der meisten speziellen (nicht prinzipiellen) Probleme auf dem Gebiete der Gravitationstheorie kann man sich damit begnügen, die $g_{\mu\nu}$ in erster Näherung zu berechnen. Dabei bedient man sich mit Vorteil der imaginären Zeitvariable $x_4 = it$ aus denselben Gründen wie in der speziellen Relativitätstheorie. Unter »erster Näherung« ist dabei verstanden, daß die durch die Gleichung

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \qquad (1)$$



154 Gesamtsitzung vom 14. Februar 1918. — Mitteilung vom 31. Januar

Über Gravitationswellen.

Von A. Einstein.

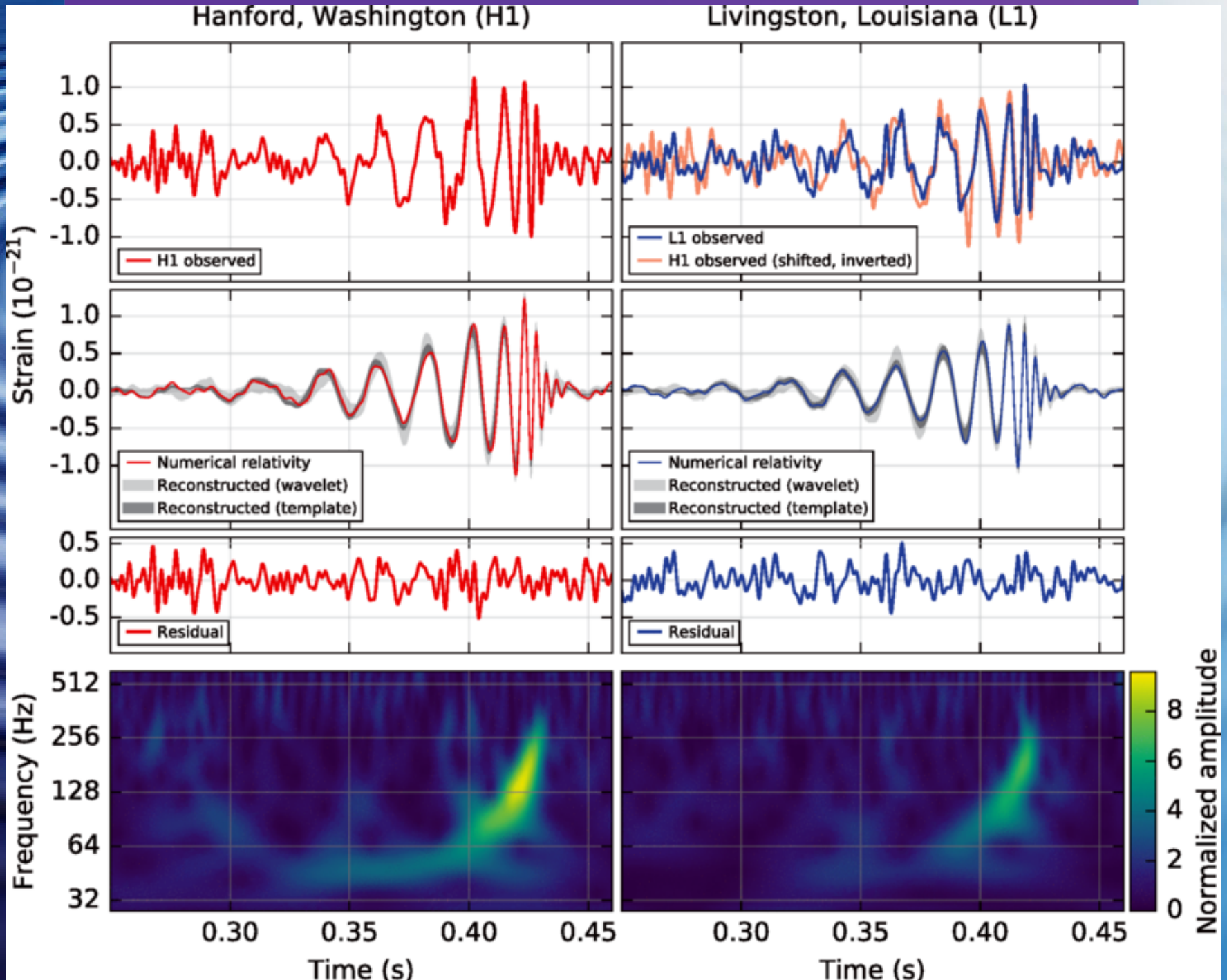
(Vorgelegt am 31. Januar 1918 [s. oben S. 79].)

Die wichtige Frage, wie die Ausbreitung der Gravitationsfelder erfolgt, ist schon vor anderthalb Jahren in einer Akademiearbeit von mir behandelt worden[1]. Da aber meine damalige Darstellung des Gegenstandes nicht genügend durchsichtig und außerdem durch einen bedauerlichen Rechenfehler verunstaltet ist, muß ich hier nochmals auf die Angelegenheit zurückkommen.

Wie damals beschränke ich mich auch hier auf den Fall, daß das betrachtete zeiträumliche Kontinuum sich von einem »galileischen« nur sehr wenig unterscheidet. Um für alle Indizes

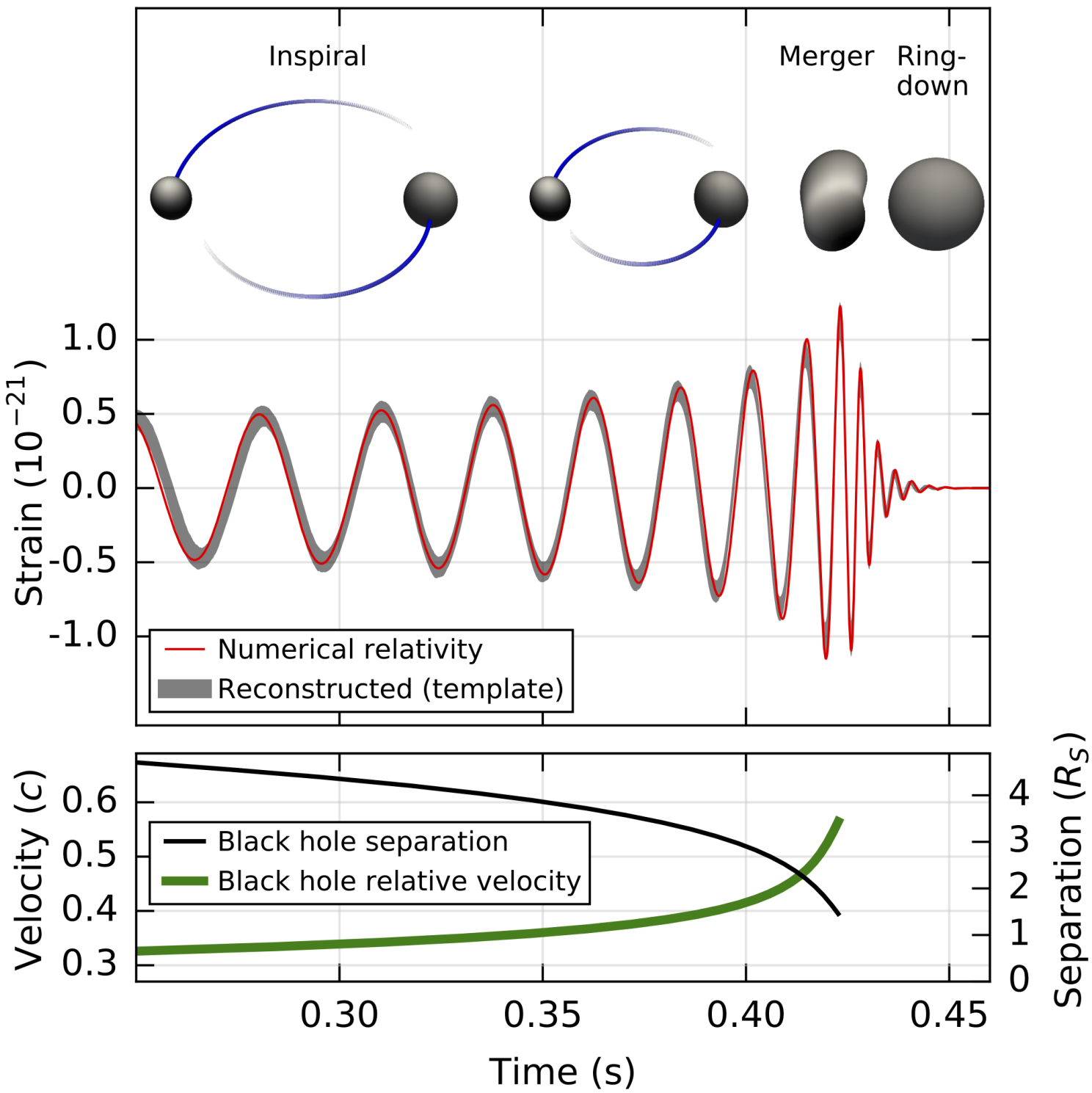
$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \qquad (1)$$

Сигналът:


Hanford, Washington (H1)
Livingston, Louisiana (L1)
Strain (10^{-21})
H1 observed
L1 observed
H1 observed (shifted, inverted)
Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)
Residual
Frequency (Hz)
512
256
128
64
32
Normalized amplitude
Time (s)
0.30
0.35
0.40
0.45

Резултатите в цифри

	EOBNR	IMRPhenom	Overall
Detector-frame total mass $M/\mathrm{M}_\odot$	$70.3^{+5.3}_{-4.8}$	$70.7^{+3.8}_{-4.0}$	$70.5^{+4.6\pm0.9}_{-4.5\pm1.0}$
Detector-frame chirp mass $\mathcal{M}/\mathrm{M}_\odot$	$30.2^{+2.5}_{-1.9}$	$30.5^{+1.7}_{-1.8}$	$30.3^{+2.1\pm0.4}_{-1.9\pm0.4}$
Detector-frame primary mass $m_1/\mathrm{M}_\odot$	$39.4^{+5.5}_{-4.9}$	$38.3^{+5.5}_{-3.5}$	$38.8^{+5.6\pm0.9}_{-4.1\pm0.3}$
Detector-frame secondary mass $m_2/\mathrm{M}_\odot$	$30.9^{+4.8}_{-4.4}$	$32.2^{+3.6}_{-5.0}$	$31.6^{+4.2\pm0.1}_{-4.9\pm0.6}$
Detector-frame final mass $M_\mathrm{f}/\mathrm{M}_\odot$	$67.1^{+4.6}_{-4.4}$	$67.4^{+3.4}_{-3.6}$	$67.3^{+4.1\pm0.8}_{-4.0\pm0.9}$
Source-frame total mass $M^\mathrm{source}/\mathrm{M}_\odot$	$65.0^{+5.0}_{-4.4}$	$64.6^{+4.1}_{-3.5}$	$64.8^{+4.6\pm1.0}_{-3.9\pm0.5}$
Source-frame chirp mass $\mathcal{M}^\mathrm{source}/\mathrm{M}_\odot$	$27.9^{+2.3}_{-1.8}$	$27.9^{+1.8}_{-1.6}$	$27.9^{+2.1\pm0.4}_{-1.7\pm0.2}$
Source-frame primary mass $m_1^\mathrm{source}/\mathrm{M}_\odot$	$36.3^{+5.3}_{-4.5}$	$35.1^{+5.2}_{-3.3}$	$35.7^{+5.4\pm1.1}_{-3.8\pm0.0}$
Source-frame secondary mass $m_2^\mathrm{source}/\mathrm{M}_\odot$	$28.6^{+4.4}_{-4.2}$	$29.5^{+3.3}_{-4.5}$	$29.1^{+3.8\pm0.2}_{-4.4\pm0.5}$
Source-fame final mass $M_\mathrm{f}^\mathrm{source}/\mathrm{M}_\odot$	$62.0^{+4.4}_{-4.0}$	$61.6^{+3.7}_{-3.1}$	$61.8^{+4.2\pm0.9}_{-3.5\pm0.4}$
Mass ratio q	$0.79^{+0.18}_{-0.19}$	$0.84^{+0.14}_{-0.21}$	$0.82^{+0.16\pm0.01}_{-0.21\pm0.03}$
Effective inspiral spin parameter χ_eff	$-0.09^{+0.19}_{-0.17}$	$-0.03^{+0.14}_{-0.15}$	$-0.06^{+0.17\pm0.01}_{-0.18\pm0.07}$
Dimensionless primary spin magnitude a_1	$0.32^{+0.45}_{-0.28}$	$0.31^{+0.51}_{-0.27}$	$0.31^{+0.48\pm0.04}_{-0.28\pm0.01}$
Dimensionless secondary spin magnitude a_2	$0.57^{+0.40}_{-0.51}$	$0.39^{+0.50}_{-0.34}$	$0.46^{+0.48\pm0.07}_{-0.42\pm0.01}$
Final spin a_f	$0.67^{+0.06}_{-0.08}$	$0.67^{+0.05}_{-0.05}$	$0.67^{+0.05\pm0.00}_{-0.07\pm0.03}$
Luminosity distance $D_\mathrm{L}/\mathrm{Mpc}$	390^{+170}_{-180}	440^{+140}_{-180}	$410^{+160\pm20}_{-180\pm40}$
Source redshift z	$0.083^{+0.033}_{-0.036}$	$0.093^{+0.028}_{-0.036}$	$0.088^{+0.031\pm0.004}_{-0.038\pm0.009}$
Upper bound on primary spin magnitude a_1	0.65	0.71	0.69 ± 0.05
Upper bound on secondary spin magnitude a_2	0.93	0.81	0.88 ± 0.10
Lower bound on mass ratio q	0.64	0.67	0.65 ± 0.03
Log Bayes factor $\ln \mathcal{B}_{\mathrm{s/n}}$	288.7 ± 0.2	290.1 ± 0.2	—

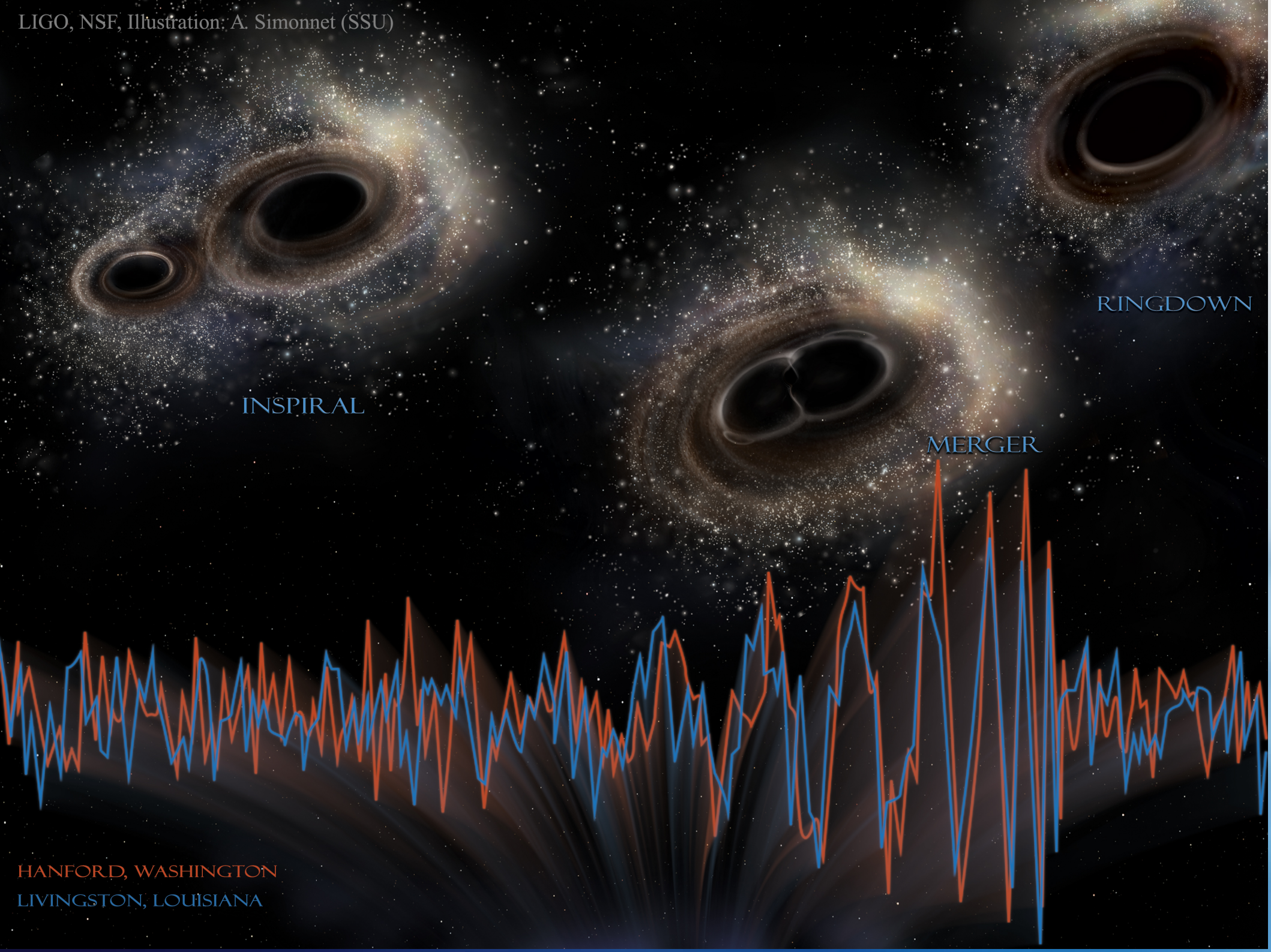


Отклонения от GR определени на <4%

Скорост преди сливането v=0.5c-0.6c
Досега най-бързата двойна пулсарна с-ма J0737-3039: v=2x10^{-3}c

$\dot{P}_{\rm orb} \sim -10^{-14}$–$10^{-12}$

Най-сетне режим на силна гравитация!


LIGO, NSF, Illustration: A. Simonnet (SSU)
INSPIRAL
MERGER
RINGDOWN
HANFORD, WASHINGTON
LIVINGSTON, LOUISIANA

Какво са гравитационните вълни?

Уравненията на Айнщайн:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Или във вакуум:

$$R_{\mu\nu} = 0$$

Линейни пертурбации на плоска метрика:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + \mathcal{O}\left((h_{\mu\nu})^2\right)$$

където

$$|h_{\mu\nu}| \ll 1$$

Получаваме следното линеаризирано уравнение:

$$\partial^\alpha \partial_\nu h_{\mu\alpha} + \partial^\alpha \partial_\mu h_{\nu\alpha} - \partial_\alpha \partial^\alpha h_{\mu\nu} - \partial_\mu \partial_\nu h - \eta_{\mu\nu}(\partial^\alpha \partial^\beta h_{\alpha\beta} - \partial^\alpha \partial_\alpha h) = 16\pi T_{\mu\nu}$$

Въвеждаме следният тензор (без следа):

$$\bar{h}_{\mu\nu} := h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$$

$$-\partial^\alpha \partial_\alpha \bar{h}_{\mu\nu} - \eta_{\mu\nu}\, \partial^\alpha \partial^\beta \bar{h}_{\alpha\beta} + \partial^\alpha \partial_\mu \bar{h}_{\nu\alpha} = 16\pi T_{\mu\nu}\,,$$

Използвайки калибровъчната свобода:

$$\partial_\alpha \bar{h}^{\mu\alpha} = 0\,.$$

Получаваме вълново уравнение:

$$\Box\, \bar{h}_{\mu\nu} = 0\,.$$

С решения:

$$\bar{h}_{\mu\nu} = A_{\mu\nu} \exp(i\kappa_\alpha x^\alpha)$$

Вълновото уравнение:

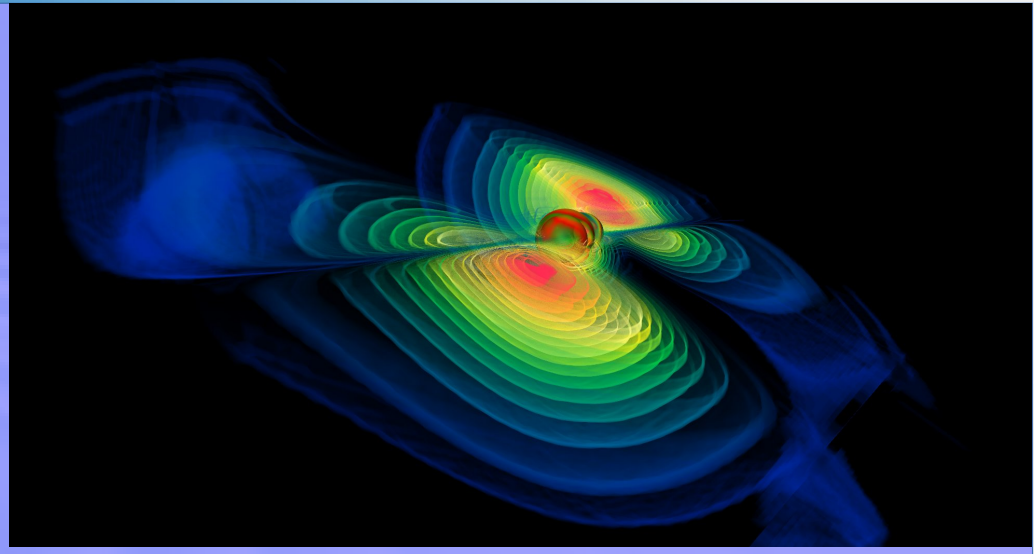
$$\Box \bar{h}^{\alpha\beta} \equiv \left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right) \bar{h}^{\alpha\beta} = 0$$

С решения:

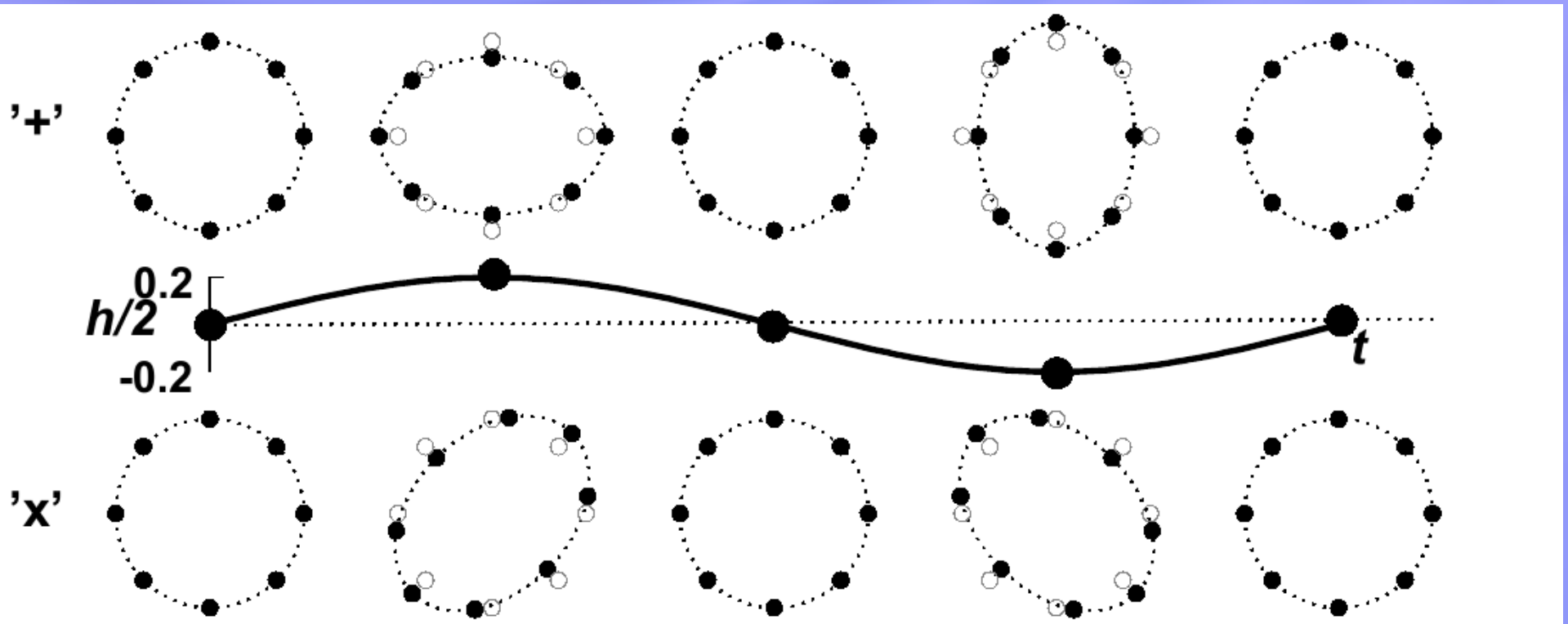
$$\bar{h}_{\alpha\beta} = \mathcal{A}\mathbf{e}_{\alpha\beta} \exp(ik_\gamma x^\gamma),$$

Допълнителни изисквания:

$$\mathbf{e}^{0\alpha} = 0 \Longrightarrow \mathbf{e}^{ij} k_j = 0 \quad (transverse\ wave),$$
$$\mathbf{e}^{i}{}_{i} = 0 (traceless\ wave).$$



Гравитациони вълни: спин (s=2)
Скорост на разпространение: c
Амплитуда 10^(-21)-10^(-18)



Как се отразяват на пространство-времето:

От геодезичното уравнение следва:

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0$$

$$\frac{d^2x^i}{d\tau^2} = -\Gamma^i{}_{00} = -\frac{1}{2}(2h_{i0,0} - h_{00,i}) = 0,$$

Т.е. в тази калибровка, частиците не се движат – От $h_{0a}=0$ → това е коорд. система движеща се със свободно падащата частица.

От уравнението за геодезичното отклонение:

$$\frac{d^2\xi^i}{dt^2} = R^i{}_{0j0}\xi^j.$$

$$R^i{}_{0j0} = \frac{1}{2}h^{TTi}{}_{j,00}$$

$$\frac{d^2\xi^i}{dt^2} = \frac{1}{2}h^{\mathrm{TT}i}{}_{j,00}\xi^j.$$

Гравитационната вълна води до осцилации на разстоянието между 2 близки точки

Вълна движеща се в z-посока, + поляризацията е:

$$\mathbf{e}^{0\alpha} = \mathbf{e}^{z\alpha} = 0 \qquad \mathbf{e}^{xx} = -\mathbf{e}^{yy} \qquad \mathbf{e}^{xy} = 0$$

arXiv: 1005.4735

$$ds^2 = -dt^2 + (1+h_+)dx^2 + (1-h_+)dy^2 + dz^2$$

$$h_+ = \mathcal{A}\mathbf{e}^{xx}\exp[-i\omega(t-z)]$$

Оценка за амплитудата:

$$h = 10^{-21}\left[\frac{E_{gw}}{0.01\ \mathrm{M}_\odot\mathrm{c}^2}\right]^{\frac{1}{2}}\left[\frac{r}{20\ \mathrm{Mpc}}\right]^{-1}\left[\frac{f}{1\ \mathrm{kHz}}\right]^{-1}\left[\frac{\tau}{1\ \mathrm{ms}}\right]^{-\frac{1}{2}}.$$

Какво мерят детекторите:

$$\bar{h}_{\mu\nu}(\mathrm{T.T.})=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times \exp(i\delta) & 0 \\ 0 & h_\times \exp(i\delta) & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \exp[i(\omega t - kZ)]$$

Ju, Blair, Zhao
http://www3.mpifr-bonn.mpg.de/staff/sbritzen/gravwaves

$$(A)^j_K=\begin{bmatrix} \cos\psi\cos\phi-\cos\theta\sin\phi\sin\psi, & -(\sin\psi\cos\phi+\cos\theta\sin\phi\cos\psi), & \sin\theta \\ \cos\psi\sin\phi+\cos\theta\cos\phi\sin\psi, & -\sin\psi\sin\phi+\cos\theta\cos\phi\cos\psi, & -\sin\theta \\ \sin\theta\sin\psi, & \sin\theta\cos\psi, & \cos\theta \end{bmatrix}$$

Най-общо:

$$\delta l/l_0=-\sin 2\Omega\,[(A^x_X A^y_X - A^x_Y A^y_Y)h_+ + (A^x_X A^y_Y + A^x_Y A^y_X)\exp(i\delta)h_\times].$$

Линейно поляризирана вълна: $h_+=h$, $h_x=0$

$$\delta l=\sin 2\Omega\,[\cos 2\phi \sin 2\psi \cos\theta + \tfrac{1}{2}(1+\cos^2\theta)\sin 2\phi \cos 2\psi]\,l_0 h.$$

Кръгово поляризирана вълна: $h_+= h_x=h$, $\delta=\pi/2$

$$\delta l=[\tfrac{1}{2}\sin 2\phi(1+\cos^2\theta)+i\cos 2\phi\cos\theta]\exp(-2i\psi)\,h l_0 \sin(2\Omega).$$

Θ,φ,Ψ – ъгли между коорд. Система на GW и тази на детектора
Schutz and Tinto (1987)
Abadie et al.(2011)

Фазата от PN:
Chirp Mass

$$\mathcal{M}=\frac{(m_1 m_2)^{3/5}}{M^{1/5}} \simeq \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3} f^{-11/3}\dot{f}\right]^{3/5}$$

$$M = m_1 + m_2$$

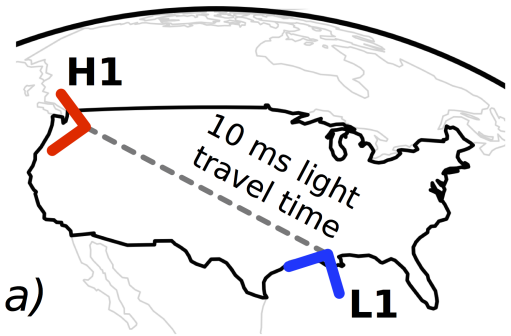
$$q = m_2/m_1 \leq 1,$$

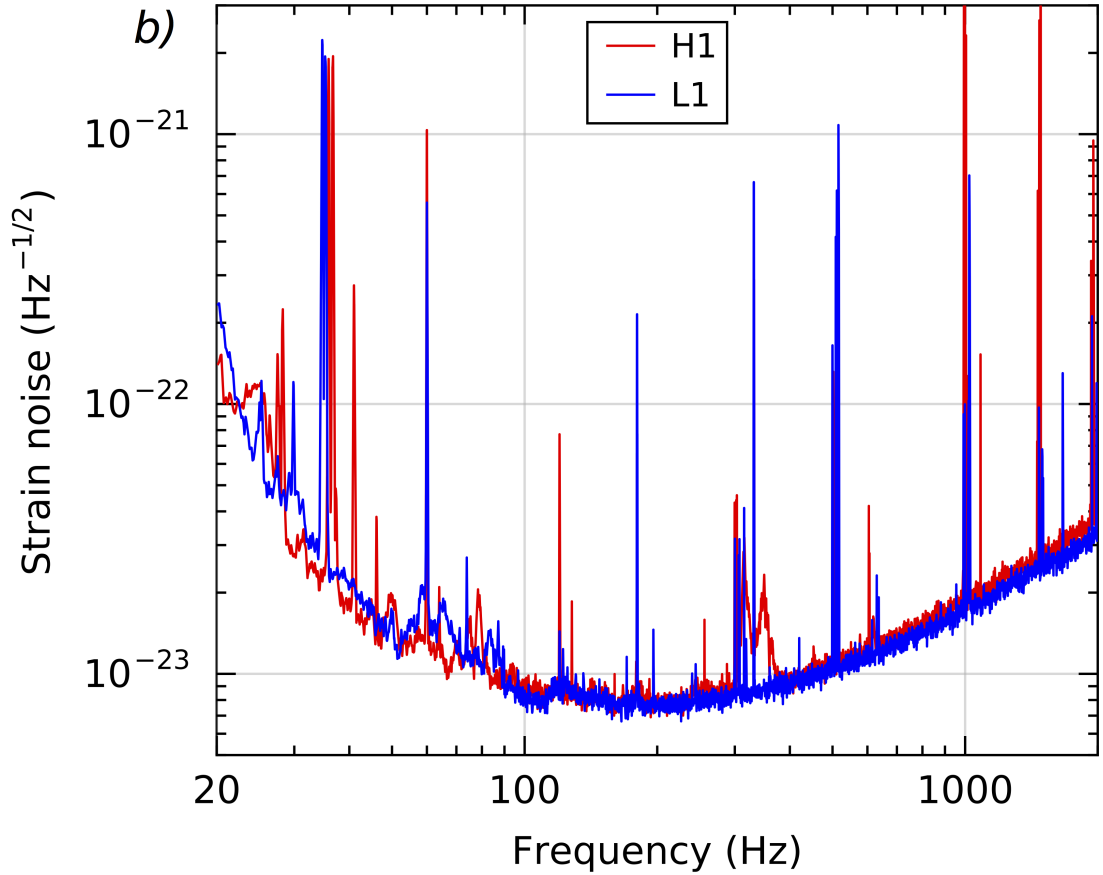
$$h_k = F_k^{(+)} h_+ + F_k^{(\times)} h_\times$$

$$h_+(t) = A_{\mathrm{GW}}(t)\left(1+\cos^2\iota\right)\cos\phi_{\mathrm{GW}}(t)\,,$$

$$h_\times(t) = -2A_{\mathrm{GW}}(t)\cos\iota\sin\phi_{\mathrm{GW}}(t)\,,$$

Детекторът:





Test Mass

L_y = 4 km

Test Mass

Power Recycling

Beam Splitter

L_x = 4 km

Laser Source

20 W

Test Mass

100 kW Circulating Power

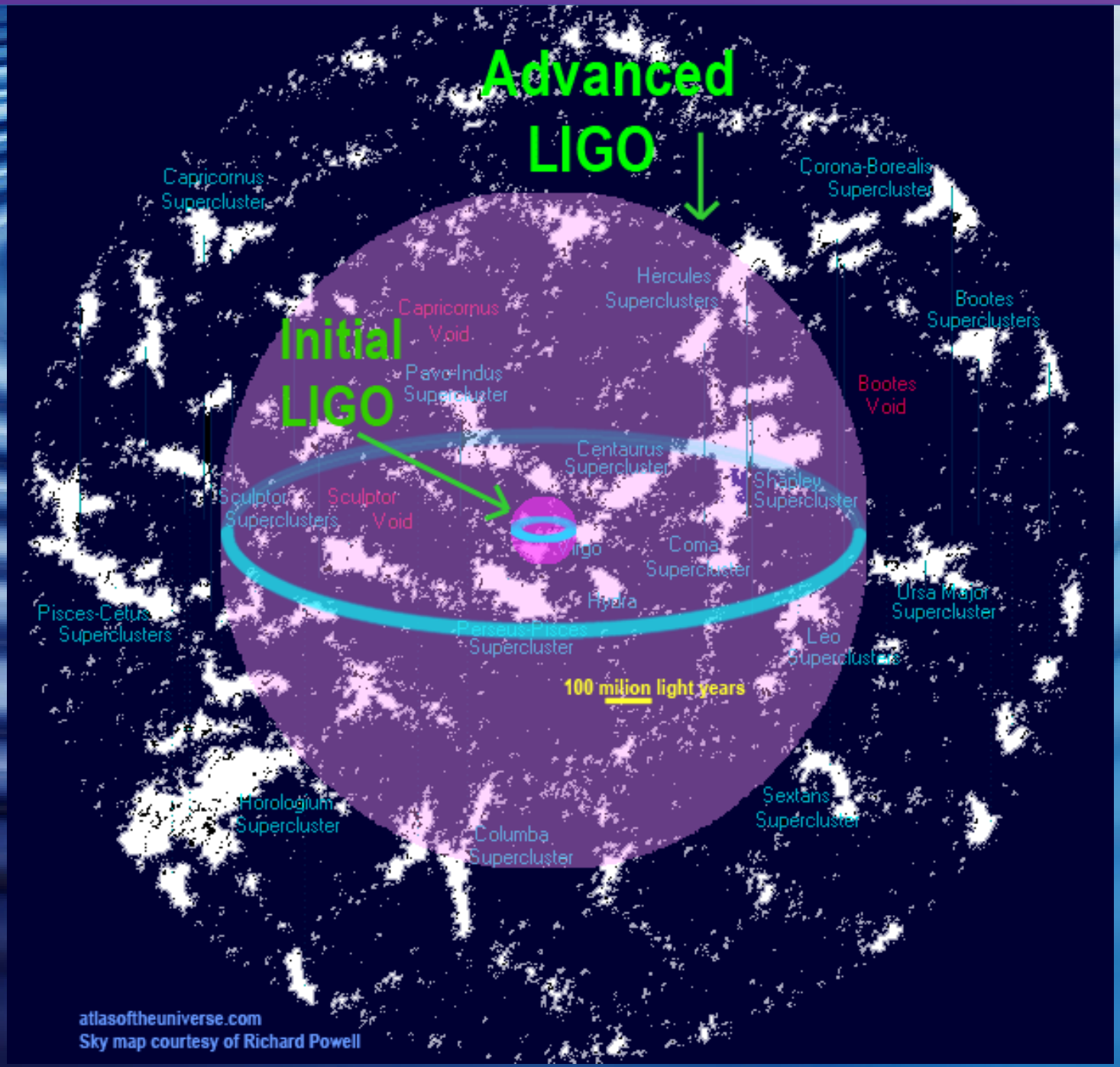
Test Mass

Signal Recycling

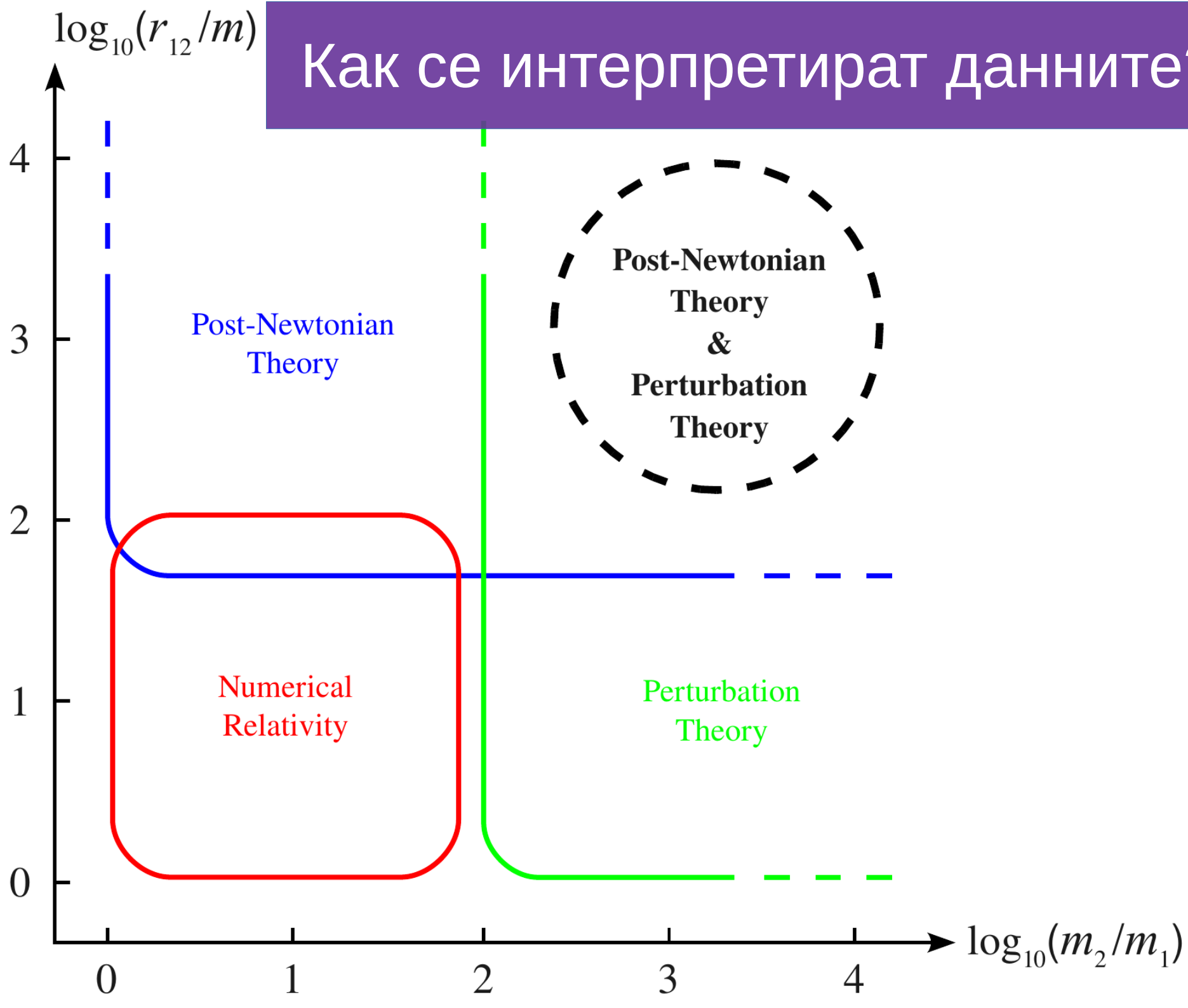
Photodetector

11:20:36
04:20:36

Чувствителност на aLIGO:



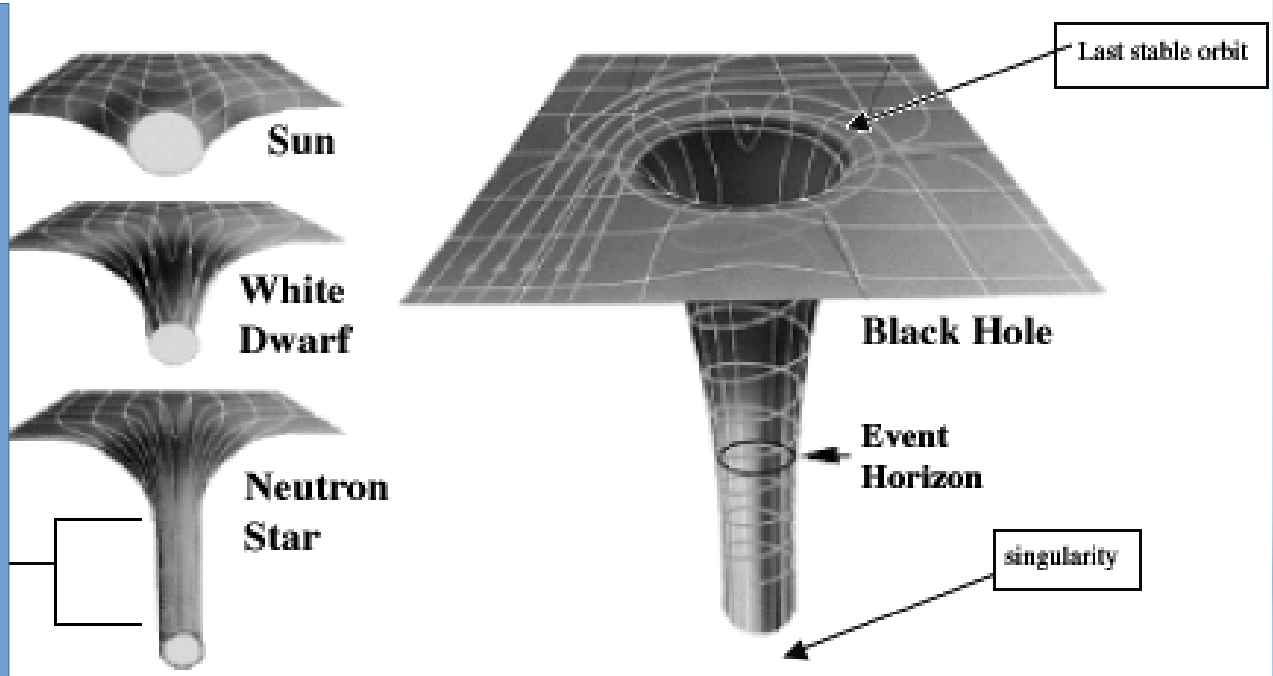
Как се интерпретират данните?



Уравненията на Айнщайн и техните решения:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- 10 силно нелинейни ЧДУ от 2ри ред за метриката $g_{\mu\nu}$
- свързват кривината на пространство-времето $R_{\mu\nu}$ и R с материята и полетато в него $T_{\mu\nu}$.
- 6 независими заради тъжд. на Бианки.
- 4 калибровъчни ст. на свобода: избор на координати
- няма суперпозиция за силни полета



$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma^2}\right) dt^2 - \frac{4Mar}{\Sigma^2} \sin^2\theta\, dt\, d\phi + \frac{\Sigma^2}{\Delta} dr^2 + \Sigma^2 d\theta^2 + \frac{A}{\Sigma^2} \sin^2\theta\, d\phi^2 ,$$

$$\Sigma^2 := r^2 + a^2 \cos^2\theta$$

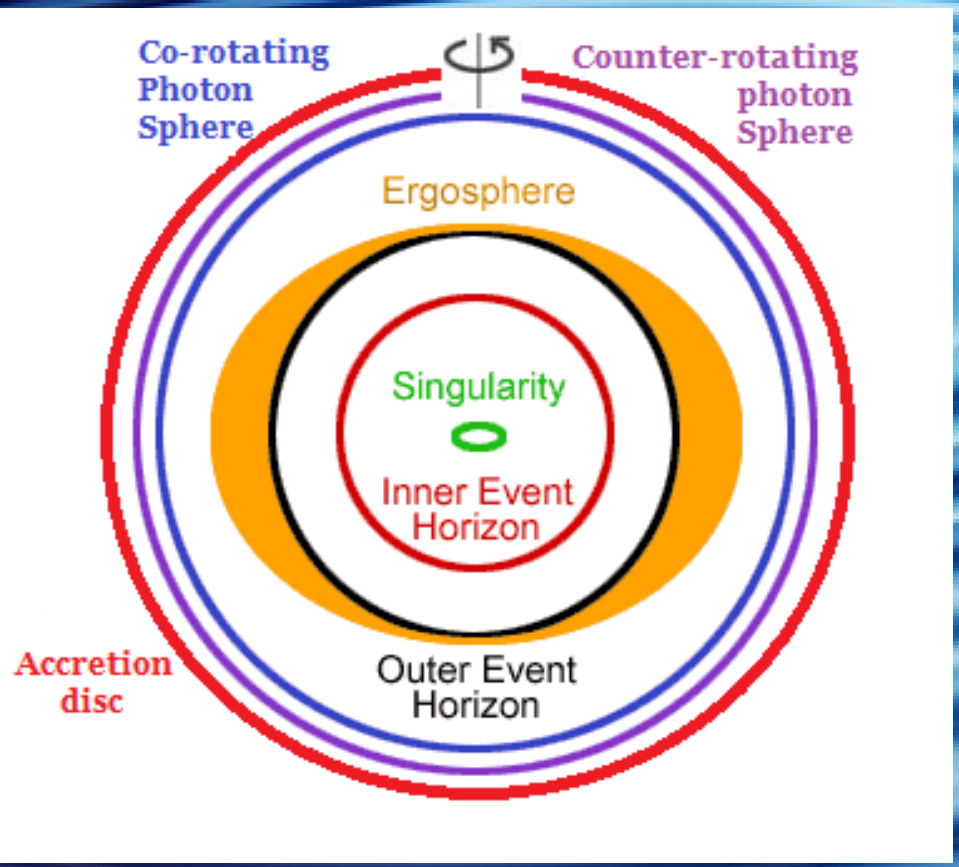
$$\Delta := r^2 - 2Mr + a^2$$

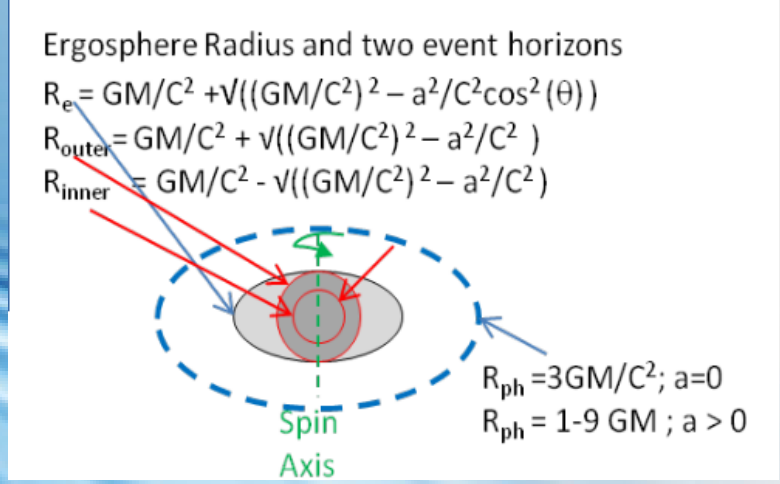
$$A := (r^2 + a^2)^2 - a^2 \Delta \sin^2\theta$$

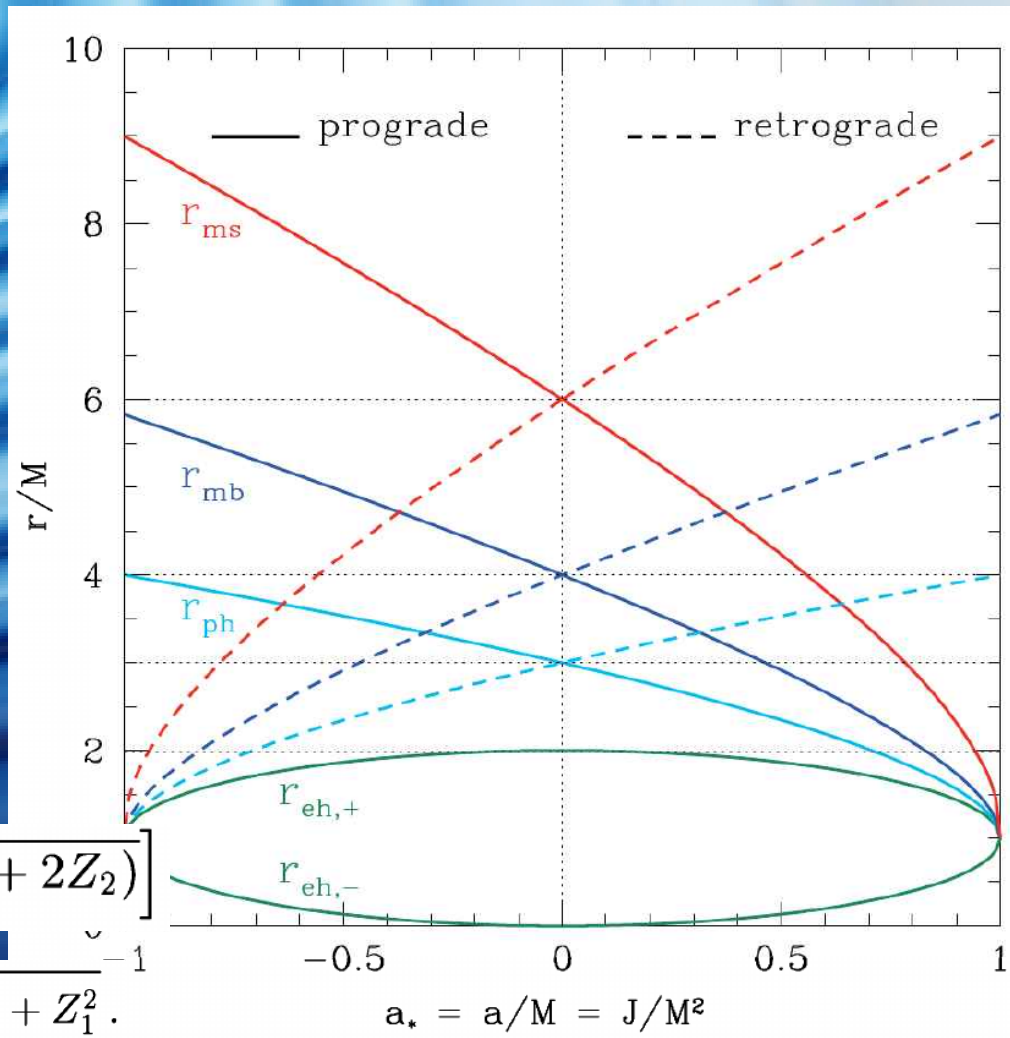
$$r_{\mathrm{s},\pm} = M \pm \sqrt{M^2 - a^2 \cos^2\theta}\,.$$

$$r_{\mathrm{eh},\pm} = M \pm \sqrt{M^2 - a^2}$$

Черната дупка на Кер:







$$r_{\mathrm{mb},\pm} = 2M \mp a + 2M^{1/2}\left(M \mp a\right)^{1/2}$$

$$r_{\mathrm{ms},\pm} = r_{\mathrm{ISCO}} = M\left[3 + Z_2 \mp \sqrt{(3 - Z_1)(3 + Z_1 + 2Z_2)}\right]$$

$$Z_1 := 1 + (1 - a^2)^{1/3}\left[(1 + a)^{1/3} + (1 - a)^{1/3}\right] \quad Z_2 := \sqrt{3a^2 + Z_1^2}\,.$$

Пост-нютоново развитие:

A correction of order $(v/c)^n$ to a Newtonian expression is said to be of $(n/2)$PN order.

ЛЛ псевдо-тензор
$\sim \partial_n{}^2 g^{ab}, \partial_n g^{ab} \partial_m g^{cd}$

Основни променливи:

$$\mathfrak{g}^{\alpha\beta} = \sqrt{-g} g^{\alpha\beta}$$

У-я на Айнщайн
Във формулировка на
Ландау и Лифшиц

$$\partial_{\mu\nu} H^{\alpha\mu\beta\nu} = \frac{16\pi G}{c^4}(-g)\left(T^{\alpha\beta} + t^{\alpha\beta}_{\mathrm{LL}}\right)$$

$$H^{\alpha\mu\beta\nu} = \mathfrak{g}^{\alpha\beta}\mathfrak{g}^{\mu\nu} - \mathfrak{g}^{\alpha\nu}\mathfrak{g}^{\beta\mu}$$

$$t^{\alpha\beta}_{\mathrm{LL}} \sim \partial\mathfrak{g} \cdot \partial\mathfrak{g} = \text{field energy-momentum}$$

$$\partial_\beta\left[(-g)\left(T^{\alpha\beta} + t^{\alpha\beta}_{\mathrm{LL}}\right)\right] = 0$$

Линеаризация:

$$h^{\alpha\beta} = \eta^{\alpha\beta} - \mathfrak{g}^{\alpha\beta}$$

$$\partial_\beta h^{\alpha\beta} = 0 \Leftrightarrow \partial_\beta \tau^{\alpha\beta} = 0$$

$$\Box h^{\alpha\beta} = -\frac{16\pi G}{c^4}\tau^{\alpha\beta}$$

$$\Box = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\tau^{\alpha\beta} = (-g)\left(T^{\alpha\beta} + t^{\alpha\beta}_{\mathrm{LL}} + t^{\alpha\beta}_{\mathrm{H}}\right)$$

$$t^{\alpha\beta}_{\mathrm{H}} \sim \partial h \cdot \partial h + h\partial^2 h$$

Развитие на h^{ab}
по степени на G

$$\Box h^{\alpha\beta}_{n+1} = -\frac{16\pi G}{c^4}\tau^{\alpha\beta}[h_n]$$

$$h^{\alpha\beta}_0 = 0,$$

Living Rev. Relativity, 17, (2014) 2
http://relativity.livingreviews.org/Articles/lrr-2014-2/
Eric Poisson Lecture Notes:
http://www.physics.uoguelph.ca/poisson/research/postN.pdf

$$\boldsymbol{a}_A = -\sum_{B\neq A}\frac{GM_B}{r_{AB}^2}\boldsymbol{n}_{AB} + \frac{1}{c^2}\Bigg(-\sum_{B\neq A}\frac{GM_B}{r_{AB}^2}\Big[v_A^2 - 4(\boldsymbol{v}_A\cdot\boldsymbol{v}_B) + 2v_B^2 - \frac{3}{2}(\boldsymbol{n}_{AB}\cdot\boldsymbol{v}_B)^2 - \frac{5GM_A}{r_{AB}} - \frac{4GM_B}{r_{AB}}\Big]\boldsymbol{n}_{AB} + \sum_{B\neq A}\frac{GM_B}{r_{AB}^2}\Big[\boldsymbol{n}_{AB}\cdot(4\boldsymbol{v}_A - 3\boldsymbol{v}_B)\Big](\boldsymbol{v}_A - \boldsymbol{v}_B) + \sum_{B\neq A}\sum_{C\neq A,B}\frac{G^2M_BM_C}{r_{AB}^2}\Big[\frac{4}{r_{AC}} + \frac{1}{r_{BC}} - \frac{r_{AB}}{2r_{BC}^2}(\boldsymbol{n}_{AB}\cdot\boldsymbol{n}_{BC})\Big]\boldsymbol{n}_{AB} - \frac{7}{2}\sum_{B\neq A}\sum_{C\neq A,B}\frac{G^2M_BM_C}{r_{AB}r_{BC}^2}\boldsymbol{n}_{BC}\Bigg) + O(c^{-4})$$

$$\boldsymbol{r}_{AB} = \boldsymbol{r}_A - \boldsymbol{r}_B,$$
$$r_{AB} = |\boldsymbol{r}_A - \boldsymbol{r}_B|,$$
$$\boldsymbol{n}_{AB} = \boldsymbol{r}_{AB}/r_{AB}$$

1PN у-я за движение

$$h_+ = \frac{2\eta Gm}{c^2R}\beta^2\Big[(1+2\pi\beta^3)H^{[0]} + \beta H^{[1/2]} + \beta^2H^{[1]} + \beta^3H^{[3/2]} + \dots\Big]$$

$$H^{[0]} = -(1+C^2)\cos 2\Psi,$$
$$H^{[1/2]} = -\frac{\Delta}{8}S\Big[(5+C^2)\cos\Psi - 9(1+C^2)\cos 3\Psi\Big]$$
$$H_\times^{[1]} = \frac{1}{3}C\Big[(17-4C^2) - (13-12C^2)\eta\Big]\sin 2\Psi - \frac{8}{3}(1-3\eta)S^2C\sin 4\Psi$$
$$H_\times^{[3/2]} = \frac{\Delta}{96}SC\Big[(63-5C^2) - 2(23-5C^2)\eta\Big]\sin\Psi - \frac{9\Delta}{64}SC\Big[(67-15C^2) - 2(19-15C^2)\eta\Big]\sin 3\Psi + \frac{625\Delta}{192}(1-2\eta)S^3C\sin 5\Psi$$

$$m = M_1 + M_2, \qquad \eta = \frac{M_1M_2}{(M_1+M_2)^2}, \qquad \Delta = \frac{M_1 - M_2}{M_1 + M_2}$$

$$S = \sin\iota, \qquad C = \cos\iota, \qquad \Omega = \text{angular velocity}, \qquad \beta = \left(\frac{Gm\Omega}{c^3}\right)^{1/3} \sim v/c$$

1.5PN гравит. излъчване
На 2на система

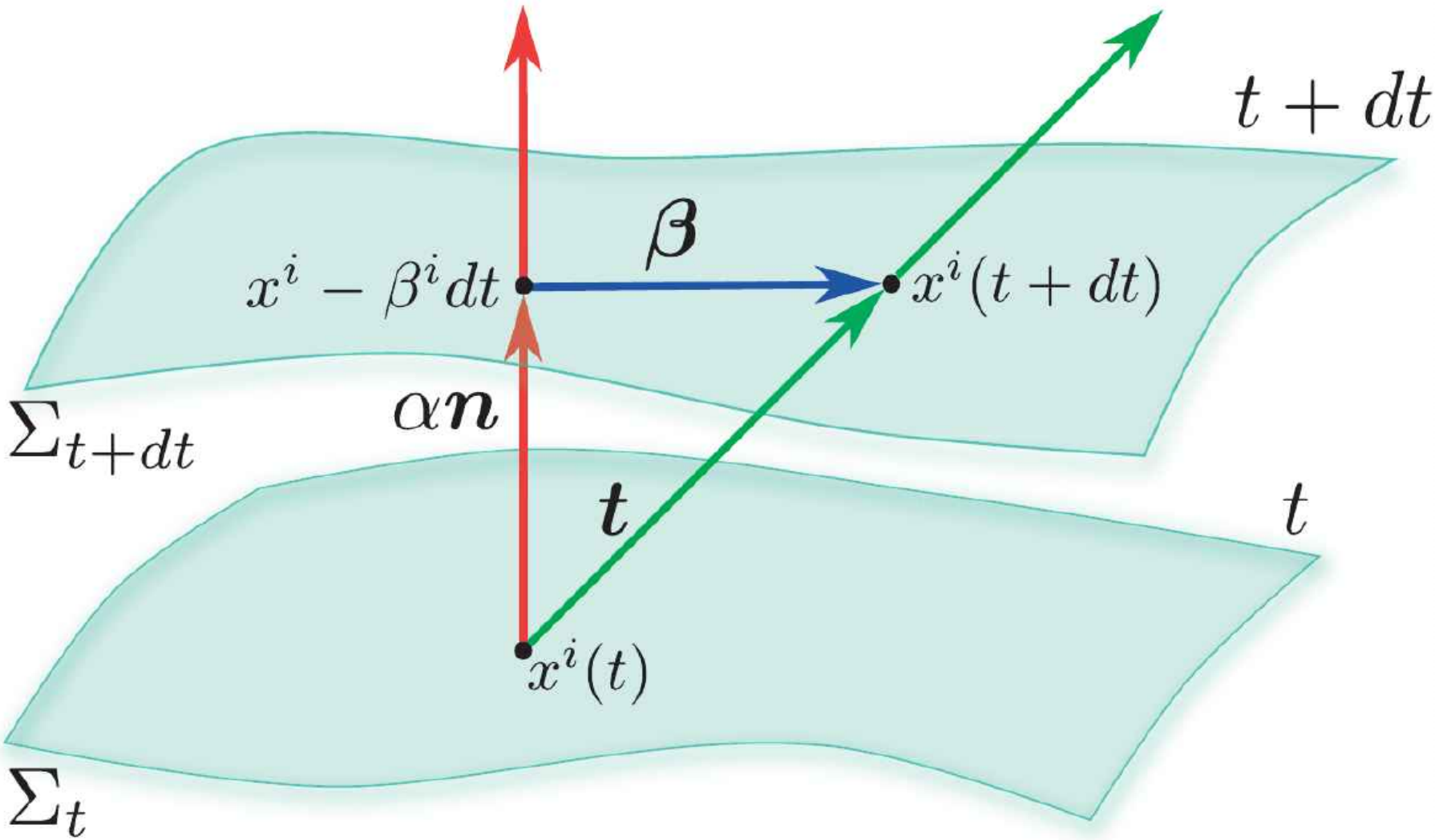
$$\Psi = \Omega\left(t - R/c - \frac{2Gm}{c^3}\ln\frac{4\Omega R}{c}\right)$$

Числена гравитация:

$$x^i_{t+dt} = x^i_t - \beta^i(t, x^j)dt\,.$$

$$\boldsymbol{n}^\mu = \frac{\mathbf{1}}{\boldsymbol{\alpha}}\left(1, -\beta^i\right)$$

$$\beta^\mu = (0, \beta^i)$$



3+1 формализъм:

$$n_\mu = A\Omega_\mu = A\nabla_\mu t\,,$$

$$n^\mu n_\mu = -1,$$

$$n_\mu = (-\alpha, 0, 0, 0)$$

Дефинираме простанствена метрика

$$\gamma_{\mu\nu} := g_{\mu\nu} + n_\mu n_\nu\,,$$

Оператор проектиращ върху хиперповърхността

$$\gamma^{\mu\nu} := g^{\mu\nu} + n^\mu n^\nu$$

Оператор проектиращ върху времето

$$n^\mu = \frac{1}{\alpha}\left(1, -\beta^i\right)$$

Двата проектора са ортогонални

$$\gamma^\mu{}_\nu := g^{\mu\alpha}\gamma_{\alpha\nu} = g^\mu{}_\nu + n^\mu n_\nu$$

Всеки 4-тензор може да се разложи чрез тях

$$N^\mu{}_\nu := -n^\mu n_\nu\,,$$

$$\gamma^\alpha{}_\mu N^\mu{}_\nu = 0\,.$$

Тогава координатното време ще бъде:

$$U^\mu = \gamma^\mu{}_\nu U^\nu + N^\mu{}_\nu U^\nu$$

$$\boldsymbol{t} = \boldsymbol{e}_t = \partial_t := \alpha\boldsymbol{n} + \boldsymbol{\beta}$$

$$\beta^\mu = (0, \beta^i)$$

Тогава 3+1 метриката е :

$$ds^2 = -(\alpha^2 - \beta_i\beta^i)dt^2 + 2\beta_i dx^i dt + \gamma_{ij} dx^i dx^j$$

$$\sqrt{-g} = \alpha\sqrt{\gamma}$$

Lecture notes E. Gourgoulhon
http://luth.obspm.fr/IHP06/lectures/gourgoulhon/lectures_eg.pdf

ADM разлагане:

Дефинираме 3-ков. производна:

$$D_\alpha \gamma^\beta{}_\delta = 0$$

$$D_\nu := \gamma^\mu{}_\nu \nabla_\mu = (\delta^\mu_\nu + n_\nu n^\mu)\nabla_\mu$$

$$D_\alpha T^\beta{}_\delta = \gamma^\mu{}_\alpha \gamma^\beta{}_\nu \gamma^\rho{}_\delta \nabla_\mu T^\nu{}_\rho \,.$$

3-д символи на Кристофел

$$^{(3)}\Gamma^\alpha_{\beta\gamma} = \frac{1}{2}\gamma^{\alpha\delta}\left(\partial_\beta \gamma_{\gamma\delta} + \partial_\gamma \gamma_{\delta\beta} - \partial_\delta \gamma_{\beta\gamma}\right)$$

3д тензор на Риман

$$2D_{[\alpha}D_{\beta]}W_\delta = {}^{(3)}R^\gamma{}_{\delta\beta\alpha}W_\gamma$$

$$^{(3)}R^\gamma{}_{\delta\alpha\beta}n_\gamma = 0$$

$$^{(3)}R^\mu{}_{\nu\kappa\sigma} = \partial_\kappa {}^{(3)}\Gamma^\mu_{\nu\sigma} - \partial_\sigma {}^{(3)}\Gamma^\mu_{\nu\kappa} + {}^{(3)}\Gamma^\mu_{\lambda\kappa}\,{}^{(3)}\Gamma^\lambda_{\nu\sigma} - {}^{(3)}\Gamma^\mu_{\lambda\sigma}\,{}^{(3)}\Gamma^\lambda_{\nu\kappa}\,.$$

3д тензор на Ричи и скаларна кривина

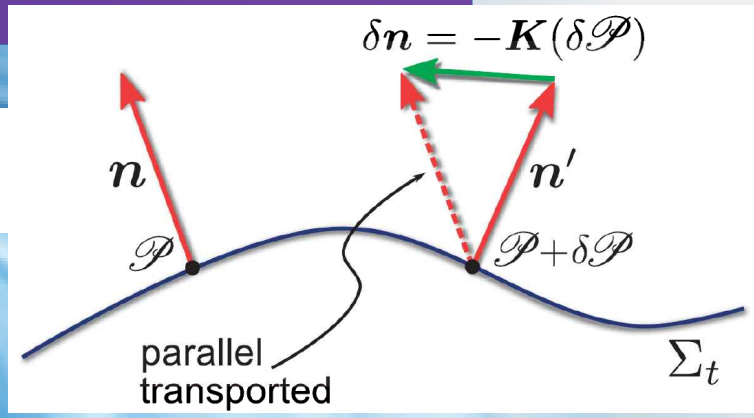
$$^{(3)}R_{\alpha\beta} := {}^{(3)}R^\mu{}_{\alpha\mu\beta}$$

$$^{(3)}R := {}^{(3)}R^\mu{}_\mu$$

Външната кривина:

1-ва дефиниция

$$K_{\mu\nu} := -\gamma^{\lambda}{}_{\mu}\nabla_{\lambda}n_{\nu} = -(\delta^{\lambda}_{\mu} + n^{\lambda}n_{\mu})\nabla_{\lambda}n_{\nu}$$



Ускорение на Ойлеров наблюдател (чисто пространствено)

$$a_{\nu} := n^{\mu}\nabla_{\mu}n_{\nu} \qquad n^{\mu}a_{\mu} = 0 \qquad a_{\nu} = D_{\nu}\ln\alpha$$

2-ра дефиниция

$$K_{\mu\nu} = -\nabla_{\mu}n_{\nu} - n_{\mu}a_{\nu} = -\nabla_{\mu}n_{\nu} - n_{\mu}n^{\gamma}\nabla_{\gamma}n_{\nu}$$

3-та дефиниция

$$K_{ij} = -\frac{1}{2}\mathscr{L}_{\boldsymbol{n}}\gamma_{ij}$$

$$\mathscr{L}_{\boldsymbol{V}}T^{\alpha}{}_{\beta} = V^{\mu}\partial_{\mu}T^{\alpha}{}_{\beta} - T_{\beta}{}^{\mu}\partial_{\mu}V^{\alpha} + T^{\alpha}{}_{\mu}\partial_{\beta}V^{\mu}$$

Ако приложим дефиницията на производна на Ли
Получаваме първото проектирано еволюционно уравнение

$$\mathscr{L}_{\boldsymbol{n}} = \frac{1}{\alpha}\mathscr{L}_{\alpha\boldsymbol{n}} = \frac{1}{\alpha}(\mathscr{L}_{\boldsymbol{t}} - \mathscr{L}_{\boldsymbol{\beta}}) = \frac{1}{\alpha}(\partial_t - \mathscr{L}_{\boldsymbol{\beta}})$$

Крайни уравнения:

$$S_{\mu\nu} := \gamma^{\alpha}{}_{\mu}\,\gamma^{\beta}{}_{\nu} T_{\alpha\beta}\,,$$
$$S_{\mu} := -\gamma^{\alpha}{}_{\mu}\, n^{\beta} T_{\alpha\beta}\,,$$
$$S := S^{\mu}{}_{\mu}\,,$$
$$E := n^{\alpha}\, n^{\beta} T_{\alpha\beta}\,,$$

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + \mathscr{L}_{\boldsymbol{\beta}} \gamma_{ij}$$
$$= -2\alpha K_{ij} + D_i \beta_j + D_j \beta_i$$

6 ЧДУ

Gauss-Codazzi

$$\gamma^{\mu}{}_{\alpha}\,\gamma^{\nu}{}_{\beta}\,\gamma^{\rho}{}_{\delta}\,\gamma^{\sigma}{}_{\lambda}\, R_{\mu\nu\rho\sigma} = {}^{(3)}R_{\alpha\beta\delta\lambda} + K_{\alpha\delta} K_{\beta\lambda} - K_{\alpha\lambda} K_{\beta\delta}\,.$$

Скаларен Gauss:

$$\gamma^{\alpha\mu}\gamma^{\beta\nu} R_{\alpha\beta\mu\nu} = R + 2R_{\mu\nu} n^{\mu} n^{\nu} = {}^{(3)}R + K^2 - K_{\mu\nu} K^{\mu\nu}$$

Codazzi-Mainardi

$$\gamma^{\rho}{}_{\beta}\gamma^{\mu}{}_{\alpha}\gamma^{\nu}{}_{\lambda} n^{\sigma} R_{\rho\mu\nu\sigma} = D_{\alpha} K_{\beta\lambda} - D_{\beta} K_{\alpha\lambda}\,.$$

И от:

$$\gamma^{\alpha}{}_{\mu}\gamma^{\beta}{}_{\nu} n^{\delta} n^{\lambda} R_{\alpha\delta\beta\lambda} = \mathscr{L}_{\boldsymbol{n}} K_{\mu\nu} - \frac{1}{\alpha} D_{\mu} D_{\nu} \alpha + K^{\lambda}{}_{\nu}\, K_{\mu\lambda}$$

6 ЧДУ

$$\partial_t K_{ij} = -\, D_i D_j \alpha + \beta^k \partial_k K_{ij} + K_{ik} \partial_j \beta^k + K_{kj} \partial_i \beta^k$$
$$+ \alpha \left({}^{(3)}R_{ij} + K K_{ij} - 2 K_{ik} K^{k}{}_{j} \right) + 4\pi\alpha \left[\gamma_{ij} \left(S - E \right) - 2 S_{ij} \right)\right]\,,$$

Hamiltonian (energy) Constraint:

$${}^{(3)}R + K^2 - K_{ij} K^{ij} = 16\pi E$$

Momentum Constraint:

$$D_j (K^{ij} - \gamma^{ij} K) = 8\pi S^i$$

Начални Условия 4 у-я

The BSSNOK formulation: Конфромно безследна

$\tilde{\gamma}_{ij} = \phi^2 \gamma_{ij}$, $\tilde{\gamma}^{ij} = \phi^{-2}\gamma^{ij}$ $\tilde{\gamma} := \det(\tilde{\gamma}_{ij}) = 1$

Безследна външна кривина

$$A_{ij} := K_{ij} - \frac{1}{3}\gamma_{ij}K \,, \quad \tilde{A}_{ij} = \phi^2 A_{ij} = \phi^2 (K_{ij} - \frac{1}{3}\gamma_{ij}K) \quad \tilde{A}^{ij} = \phi^{-2}A^{ij}$$

Нови гами: $\tilde{\Gamma}^i := \tilde{\gamma}^{jk}\tilde{\Gamma}^i_{jk} = \tilde{\gamma}^{ij}\tilde{\gamma}^{kl}\partial_l \tilde{\gamma}_{jk}$,

$$\tilde{\Gamma}^i_{jk} = \Gamma^i_{jk} - \frac{1}{3}(\delta^i_j \Gamma^m_{km} + \delta^i_k \Gamma^m_{jm} - \gamma_{jk}\gamma^{il}\Gamma^m_{lm})$$
$$= \Gamma^i_{jk} + 2(\delta^i_j \partial_k \ln\phi + \delta^i_k \partial_j \ln\phi - \gamma_{jk}\gamma^{il}\partial_l \ln\phi) \,,$$

$${}^{(3)}R_{ij} = {}^{(3)}\tilde{R}_{ij} + {}^{(3)}\tilde{R}^{\phi}_{ij},$$

$${}^{(3)}\tilde{R}_{ij} := -\frac{1}{2}\tilde{\gamma}^{lm}\partial_l\partial_m\tilde{\gamma}_{ij} + \tilde{\gamma}_{k(i}\partial_{j)}\tilde{\Gamma}^k + \tilde{\Gamma}^k\tilde{\Gamma}_{(ij)k} + \tilde{\gamma}^{lm}\left(2\tilde{\Gamma}^k_{l(i}\tilde{\Gamma}_{j)km} + \tilde{\Gamma}^k_{im}\tilde{\Gamma}_{kjl}\right) ,$$

$${}^{(3)}\tilde{R}^{\phi}_{ij} := \frac{1}{\phi^2}\left[\phi\left(\tilde{D}_i\tilde{D}_j\phi + \tilde{\gamma}_{ij}\tilde{D}^k\tilde{D}_k\phi\right) - 2\tilde{\gamma}_{ij}\tilde{D}^k\phi\,\tilde{D}_k\phi\right]$$

Крайната BSSNOK система:
/от 12 евол. величини
$\gamma_{ij}, K_{ij},$
До 15
$\tilde{\gamma}_{ij}, \tilde{A}_{ij}, \phi, K, \tilde{\Gamma}^i$

$$\partial_t\tilde{\gamma}_{ij} = -2\alpha\tilde{A}_{ij} + 2\tilde{\gamma}_{k(i}\partial_{j)}\beta^k - \frac{2}{3}\tilde{\gamma}_{ij}\partial_k\beta^k + \beta^k\partial_k\tilde{\gamma}_{ij} \,,$$

$$\partial_t\tilde{A}_{ij} = \phi^2\Big[-D_iD_j\alpha + \alpha\Big({}^{(3)}R_{ij} - 8\pi S_{ij}\Big)\Big]^{\mathrm{TF}} + \beta^k\partial_k\tilde{A}_{ij} + 2\tilde{A}_{k(i}\partial_{j)}\beta^k$$
$$+ \alpha(\tilde{A}_{ij}K - 2\tilde{A}_{ik}\tilde{A}^k{}_j) - \frac{2}{3}\tilde{A}_{ij}\partial_k\beta^k \,,$$

$$\partial_t\phi = \frac{1}{3}\phi\,\alpha K - \frac{1}{3}\phi\,\partial_i\beta^i + \beta^k\partial_k\phi \,,$$

$$\partial_t K = -D_iD^i\alpha + \alpha\Big[\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2 + 4\pi(E+S)\Big] + \beta^i\partial_i K \,,$$

$$\partial_t\tilde{\Gamma}^i = \tilde{\gamma}^{jk}\partial_j\partial_k\beta^i + \frac{1}{3}\tilde{\gamma}^{ik}\partial_k\partial_j\beta^j + \frac{2}{3}\tilde{\Gamma}^i\partial_j\beta^j - \tilde{\Gamma}^j\partial_j\beta^i - 2\tilde{A}^{ij}\partial_j\alpha + \beta^j\partial_j\tilde{\Gamma}^i$$
$$+ 2\alpha\Big(\tilde{\Gamma}^i_{jk}\tilde{A}^{jk} - 3\tilde{A}^{ij}\partial_j\ln\phi - \frac{2}{3}\tilde{\gamma}^{ij}\partial_j K\Big) - 16\pi\alpha\tilde{\gamma}^{ij}S_j \,,$$

Крайни уравнения:

$$S_{\mu\nu} := \gamma^{\alpha}{}_{\mu}\,\gamma^{\beta}{}_{\nu} T_{\alpha\beta}\,,$$
$$S_{\mu} := -\gamma^{\alpha}{}_{\mu}\, n^{\beta} T_{\alpha\beta}\,,$$
$$S := S^{\mu}{}_{\mu}\,,$$
$$E := n^{\alpha}\, n^{\beta} T_{\alpha\beta}\,,$$

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + \mathscr{L}_{\boldsymbol{\beta}} \gamma_{ij}$$
$$= -2\alpha K_{ij} + D_i \beta_j + D_j \beta_i$$

6 ЧДУ

Gauss-Codazzi

$$\gamma^{\mu}{}_{\alpha}\,\gamma^{\nu}{}_{\beta}\,\gamma^{\rho}{}_{\delta}\,\gamma^{\sigma}{}_{\lambda}\, R_{\mu\nu\rho\sigma} = {}^{(3)}R_{\alpha\beta\delta\lambda} + K_{\alpha\delta} K_{\beta\lambda} - K_{\alpha\lambda} K_{\beta\delta}\,.$$

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Codazzi-Mainardi

$$\gamma^{\rho}{}_{\beta}\gamma^{\mu}{}_{\alpha}\gamma^{\nu}{}_{\lambda} n^{\sigma} R_{\rho\mu\nu\sigma} = D_{\alpha} K_{\beta\lambda} - D_{\beta} K_{\alpha\lambda}\,.$$

И от:

$$\gamma^{\alpha}{}_{\mu}\gamma^{\beta}{}_{\nu} n^{\delta} n^{\lambda} R_{\alpha\delta\beta\lambda} = \mathscr{L}_{\boldsymbol{n}} K_{\mu\nu} - \frac{1}{\alpha} D_{\mu} D_{\nu} \alpha + K^{\lambda}{}_{\nu}\, K_{\mu\lambda}$$

6 ЧДУ

$$\partial_t K_{ij} = -\, D_i D_j \alpha + \beta^k \partial_k K_{ij} + K_{ik} \partial_j \beta^k + K_{kj} \partial_i \beta^k$$
$$+ \alpha \left({}^{(3)}R_{ij} + K K_{ij} - 2 K_{ik} K^{k}{}_{j} \right) + 4\pi\alpha \left[\gamma_{ij} \left(S - E \right) - 2 S_{ij} \right)\right]\,,$$

Hamiltonian (energy) Constraint:

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Momentum Constraint:

$$D_j (K^{ij} - \gamma^{ij} K) = 8\pi S^i$$

Начални Условия 4 у-я

Калибровъчни условия: York-Lichnerowicz

$\bar{\gamma}_{ij} := \psi^{-4}\gamma_{ij}$

$\bar{\gamma} := \det(\bar{\gamma}_{ij}) = \psi^{-12}\gamma$

$\bar{A}^{ij} = \psi^{10}A^{ij}$

$\bar{A}_{ij} = \psi^{2}A_{ij}$

12 евол. У-я
Трябват 12 нач. Усл.
4 – ограничения
4 – калибровъчни
Остават 4
От 12 $\{\gamma_{ij}, K_{ij}\}$

Hamiltonian (energy) Constraint:

$$\bar{D}^2\Psi - \frac{\Psi}{8}\bar{R} + \frac{\Psi^{-7}}{8}\bar{A}_{ij}\bar{A}^{ij} - \frac{1}{12}\Psi^5 K^2 = -2\pi\Psi^5\rho$$

$$\bar{D}^2 := \bar{\gamma}^{ij}\bar{D}_i\bar{D}_j$$

Momentum Constraint:

$$\bar{D}_i\bar{A}^{ij} - \frac{2}{3}\Psi^6\bar{\gamma}^{ij}\bar{D}_j K = 8\pi\Psi^{10}S^i$$

Решения:
- вакуум: $S^i=0$
- максимално срязване $K=0 \rightarrow A^{ij}, \Psi$
- времева симетрия $K_{ij}=0$ + плоско пр-во

$$\bar{D}_i\bar{A}^{ij} = 0$$

$$\bar{D}^2\Psi - \frac{\Psi}{8}\bar{R} + \frac{\Psi^{-7}}{8}\bar{A}_{ij}\bar{A}^{ij} = 0$$

$$\bar{D}^2_{flat}\Psi = 0$$

За истински Кер:
http://cds.cern.ch/record/354278/files/9805023.pdf

Линейни → суперпозиция!

ЧД с ъгл. момент S_k

$$\bar{A}^{ij} = \frac{3}{r^3}(\epsilon^{ik}_l S_k l^l l^j + \epsilon^{jk}_l S_k l^l l^i)$$

$$l^i = \frac{x^i}{r}$$

$$r^2 = \Sigma x^{i2}$$

ЧД с лин. импулс P_k

$$\bar{A}^{ij} = \frac{3}{2r^2}(P^i l^j + P^j l^i - (\bar{\gamma}^{ij} - l^i l^j) l_k P^k)$$

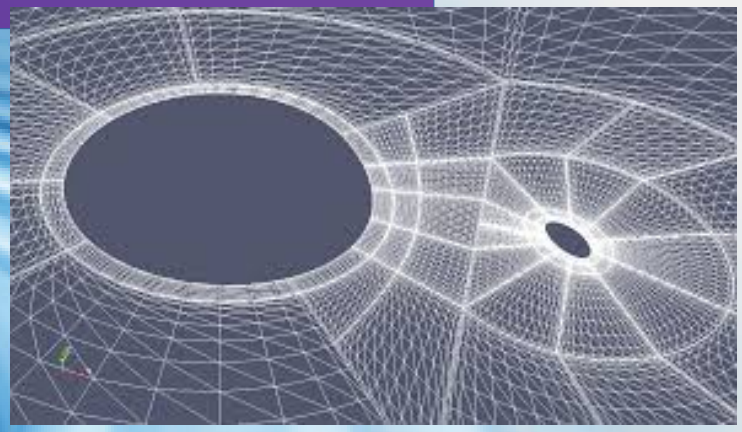
И така:

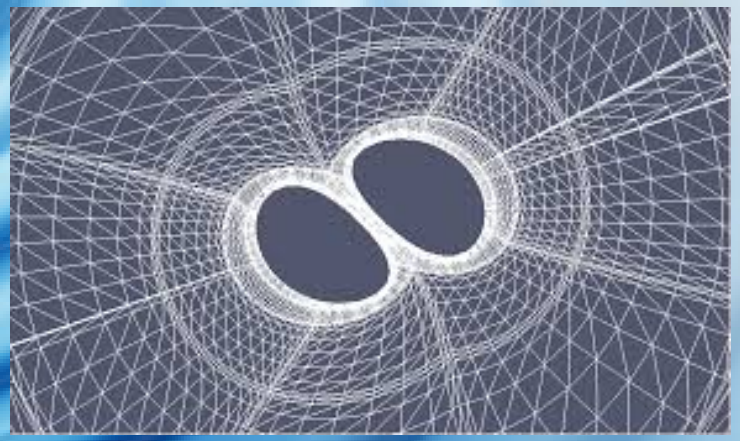
Уравненията на ADM (1960-1970)
Първи опит за симулация: Hanh, Lindquist (1964)
Първа успешна симулация на ЧД: Pretorius (2005)

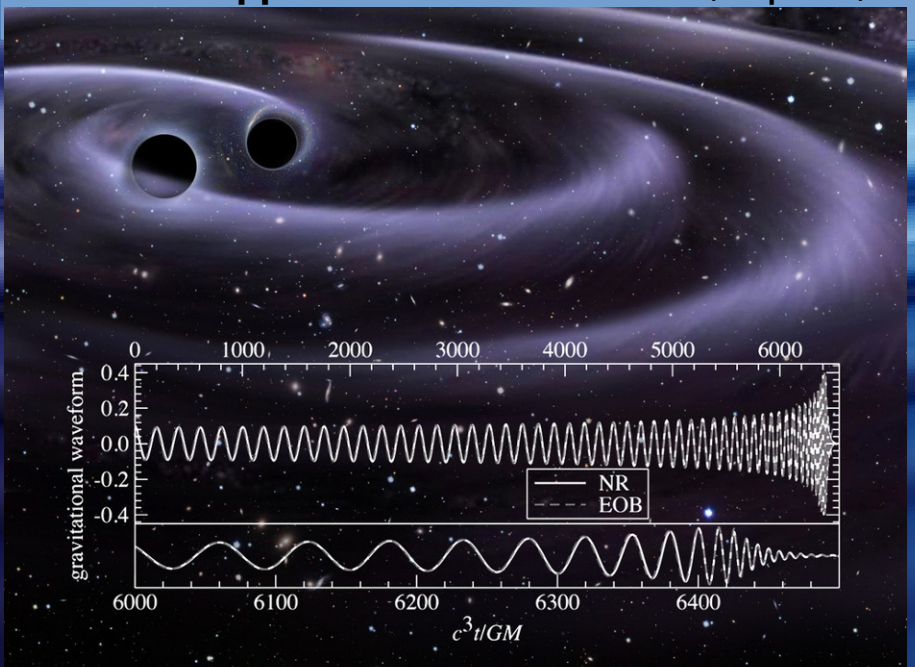
Проблеми:

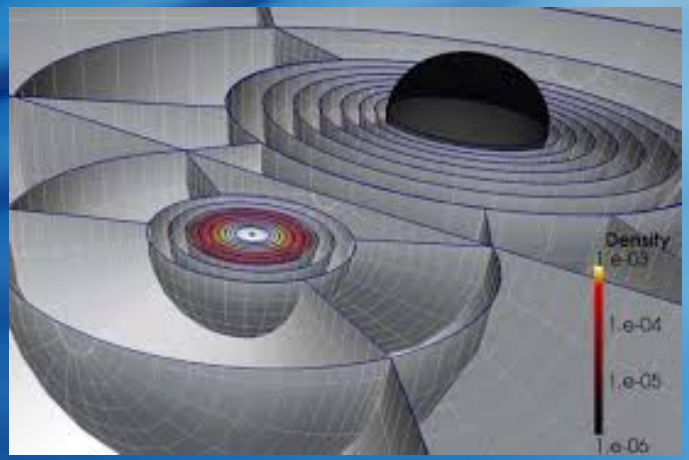
- ADM уравненията не са добре поставени /BSSNOK решава този проблем донякъде/
- Малки грешки се увеличават в процеса на еволюцията
- Черните дупки имат сингулярности - Excision, moving puncture method
- /избор на координати не достигащи сингуляр-ността и спиране на еволюцията до хоризонта/
- Всеки избор на координати е лош.

Кодове: Einstein Toolkit, SpeC, LORENE, BAM









И след 10 000 CPU/часа на шаблон

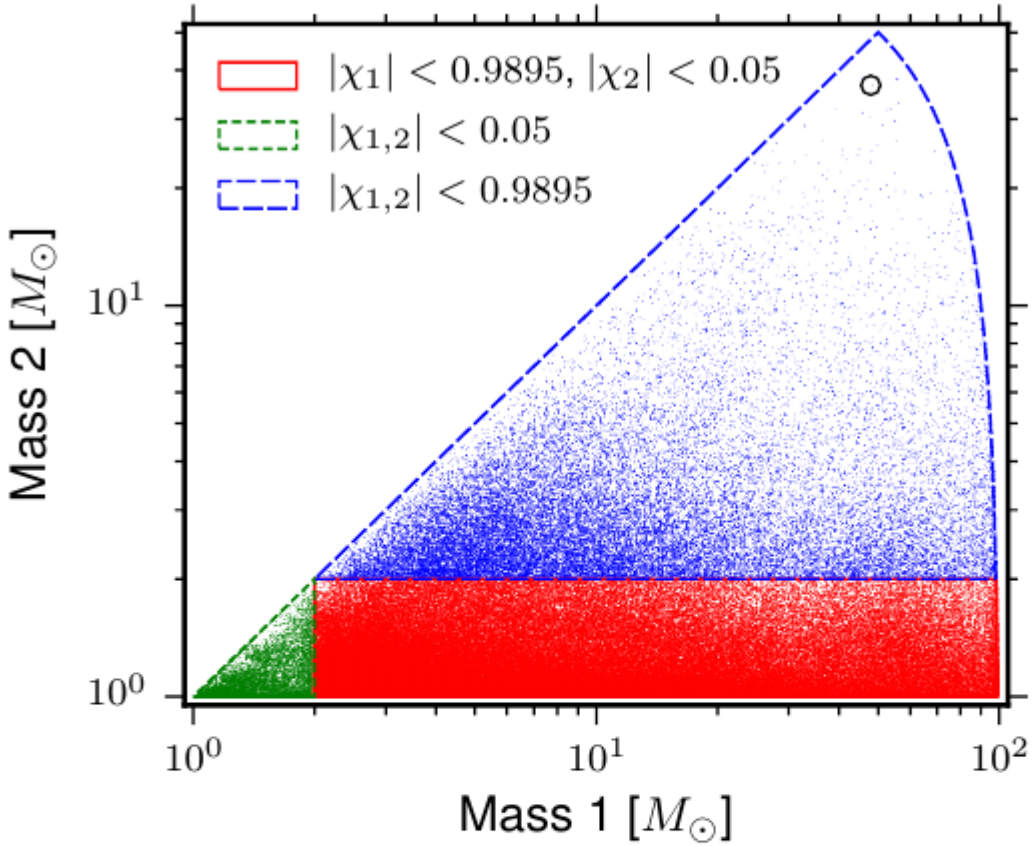


FIG. 1. The four-dimensional search parameter space covered by the template bank shown projected into the component-mass plane, using the convention $m_1 > m_2$. The lines bound mass regions with different limits on the dimensionless aligned-spin parameters χ_1 and χ_2. Each point indicates the position of a template in the bank. The circle highlights the template that best matches GW150914. This does not coincide with the best-fit parameters due to the discrete nature of the template bank.

250 000 шаблона, получени чрез премащабиране
ArXiv:1602.03839

$$\eta = (m_1 m_2)/(m_1 + m_2)^2$$

$$\mathcal{M} = (m_1 m_2)^{3/5}/(m_1 + m_2)^{1/5}$$

$$\chi_{1,2} = c\mathbf{S}_{1,2}/Gm_{1,2}^2$$

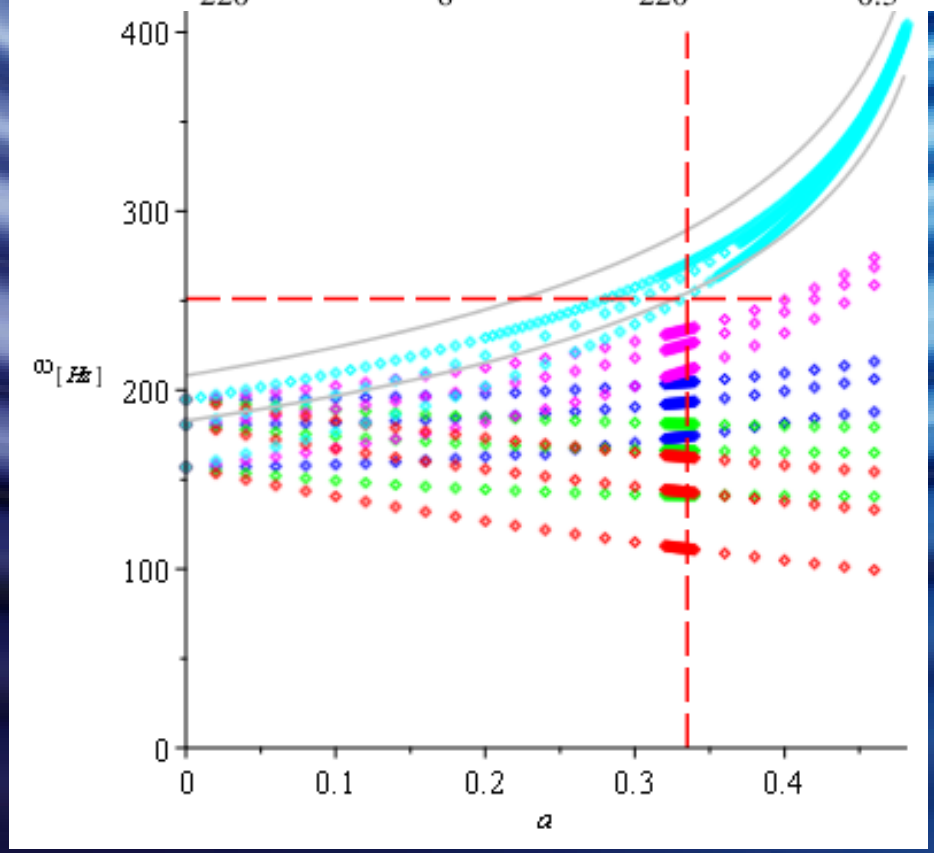
Квази-нормални моди:

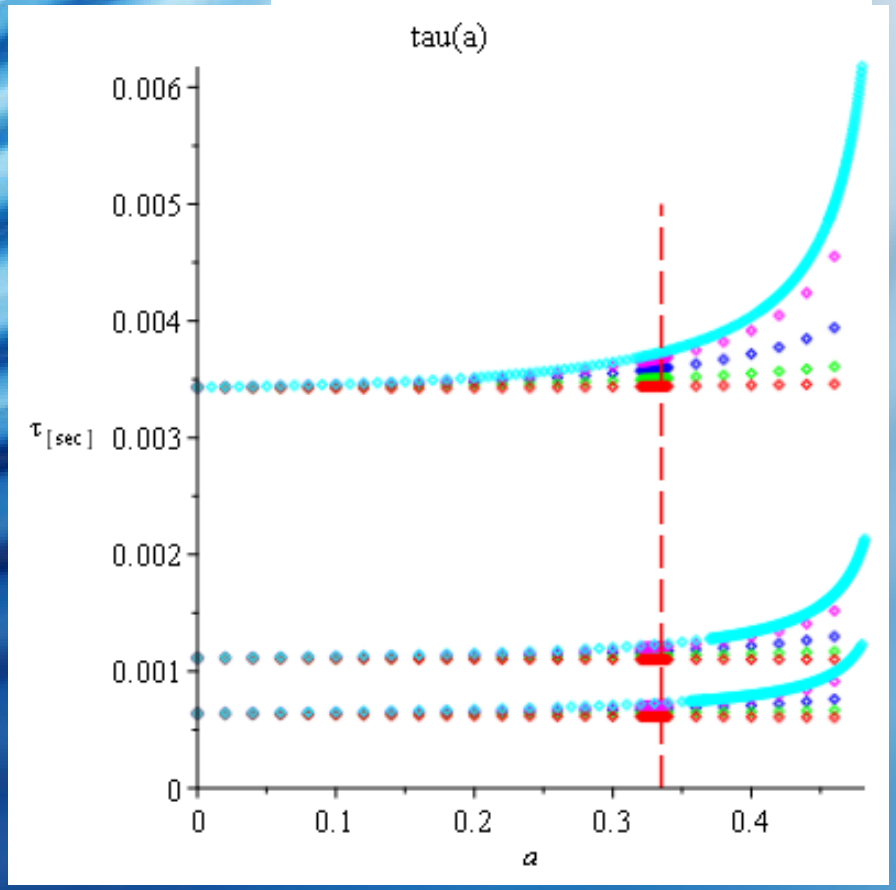
$$h_+ - i\,h_\times = \sum_{\ell m} \frac{\sqrt{(\ell-1)\ell(\ell+1)(\ell+2)}}{2r}\left(\Psi^{(\mathrm{even})}_{\ell m} - i\,\Psi^{(\mathrm{odd})}_{\ell m}\right) {}_{-2}Y_{\ell m}\,,$$

$$\Psi^{(\mathrm{even/odd})}_{\ell m} = A_{\ell m}\left[1 + \frac{1}{2}\left(\frac{\ell(\ell+1)}{2\omega_{\ell m} r}\right)^2 + \mathcal{O}(1/r^4)\right]\exp(-i\omega_{\ell m}(t-r^*))\exp(i\,\delta\phi_{\ell m}) + \mathcal{O}(1/r^2)\,,$$

$$\sin\delta\phi_{\ell m} = \frac{\ell(\ell+1)}{2\omega_{\ell m} r} + \mathcal{O}(1/r^2)\,.$$

We find $f^{\mathrm{QNM}}_{220} = 251^{+8}_{-8}$ Hz and $\tau^{\mathrm{QNM}}_{220} = 4.0^{+0.3}_{-0.3}$ ms





Астрофизични резултати:

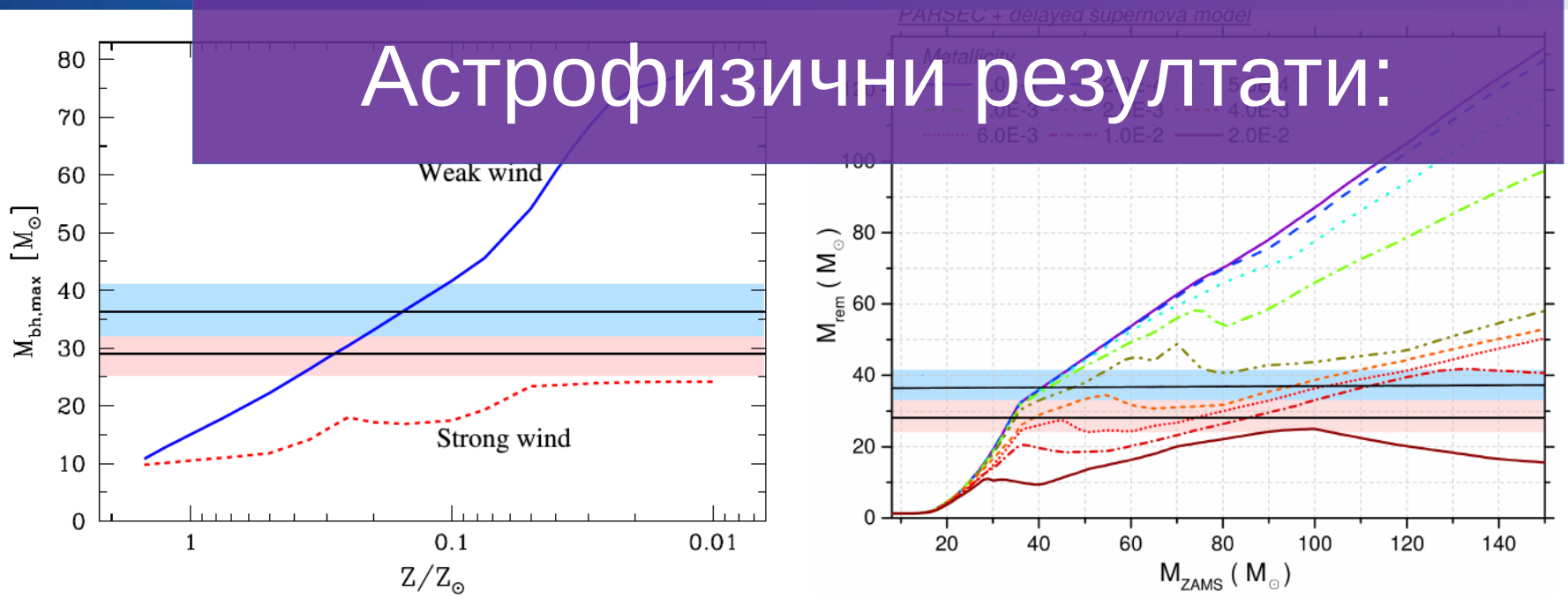


Figure 1. Left: dependence of maximum BH mass on metallicity Z, with $Z_\odot = 0.02$ for the old (strong) and new (weak) massive star winds (Figure 3 from Belczynski et al. 2010a). Right: compact-remnant mass as a function of zero-age main-sequence (ZAMS; i.e., initial) progenitor mass for a set of different (absolute) metallicity values (Figure 6 from Spera et al. 2015). The

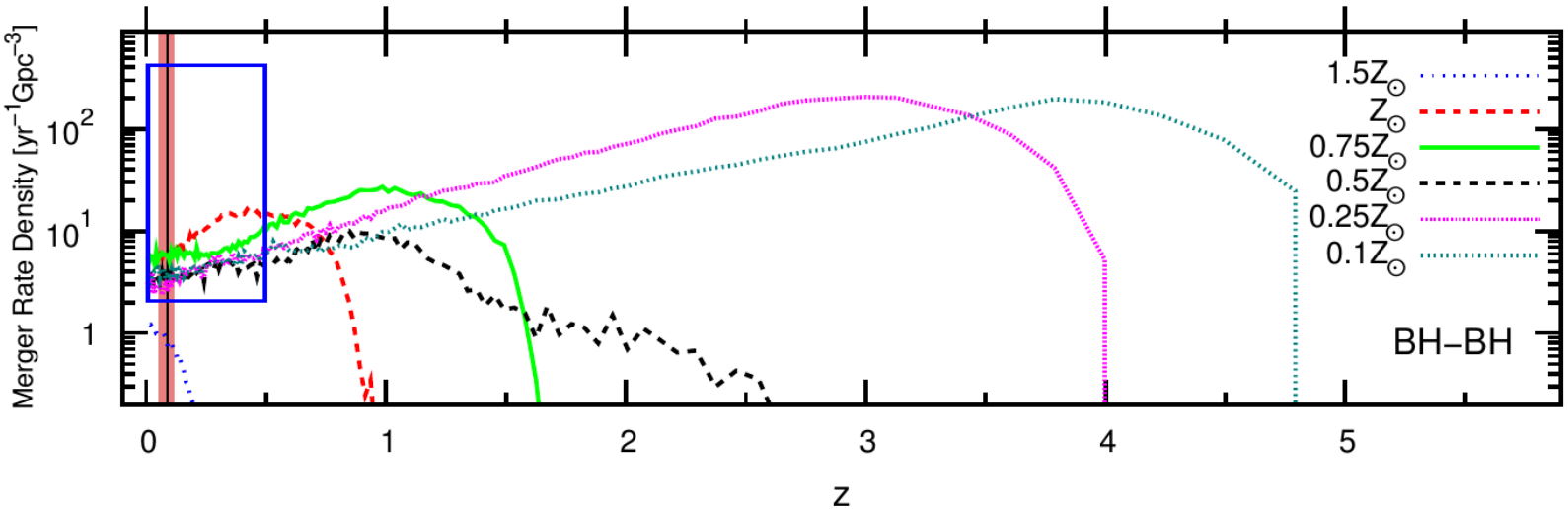
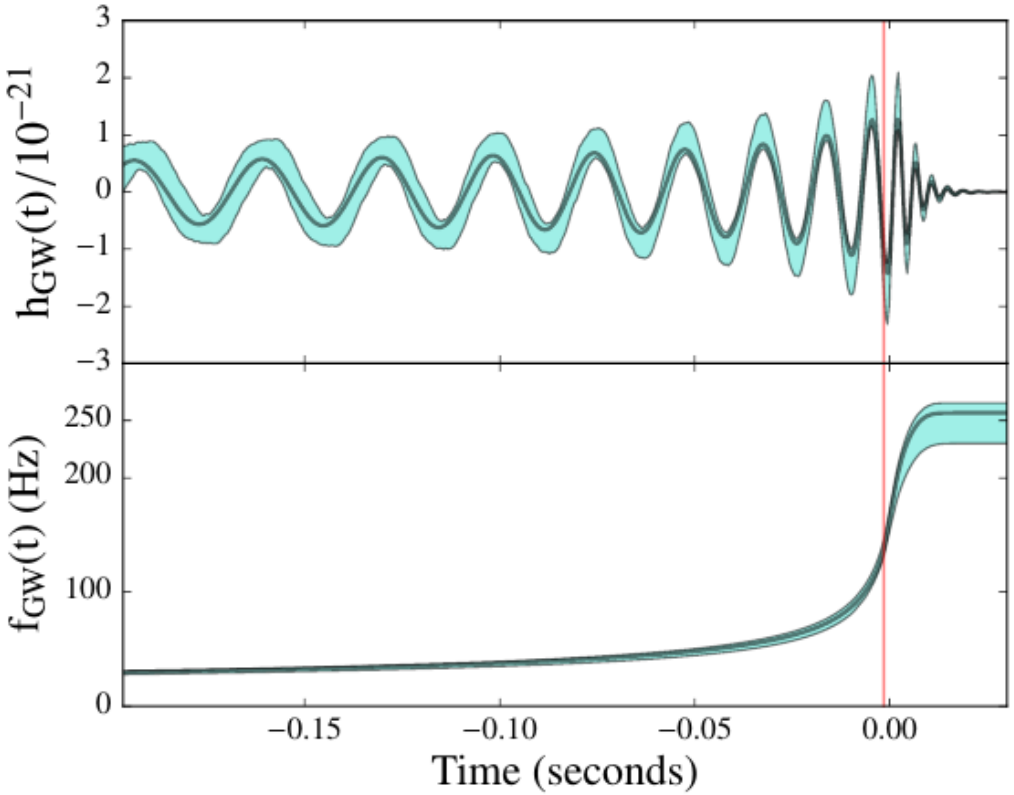


Figure 2. Predictions of BBH merger rate in the comoving frame ($Gpc^{-3}\,yr^{-1}$) from isolated binary evolution as a function of redshift for different metallicity values (adopted from Figure 4 in Dominik et al. 2013). At a given redshift, the total merger rate is the sum over metallicity. The redshift range of GW150914 is indicated by the vertical band; the range of the BBH rate estimates and the redshift out to which a system like GW150914 could have been detected in this observing period are indicated by an open blue rectangular box.

ArXiv: 1602.03846
Модели на двойни с-ми ЧД:
- изолирана двойна с-ма
- двойна система от плътни звездни купове



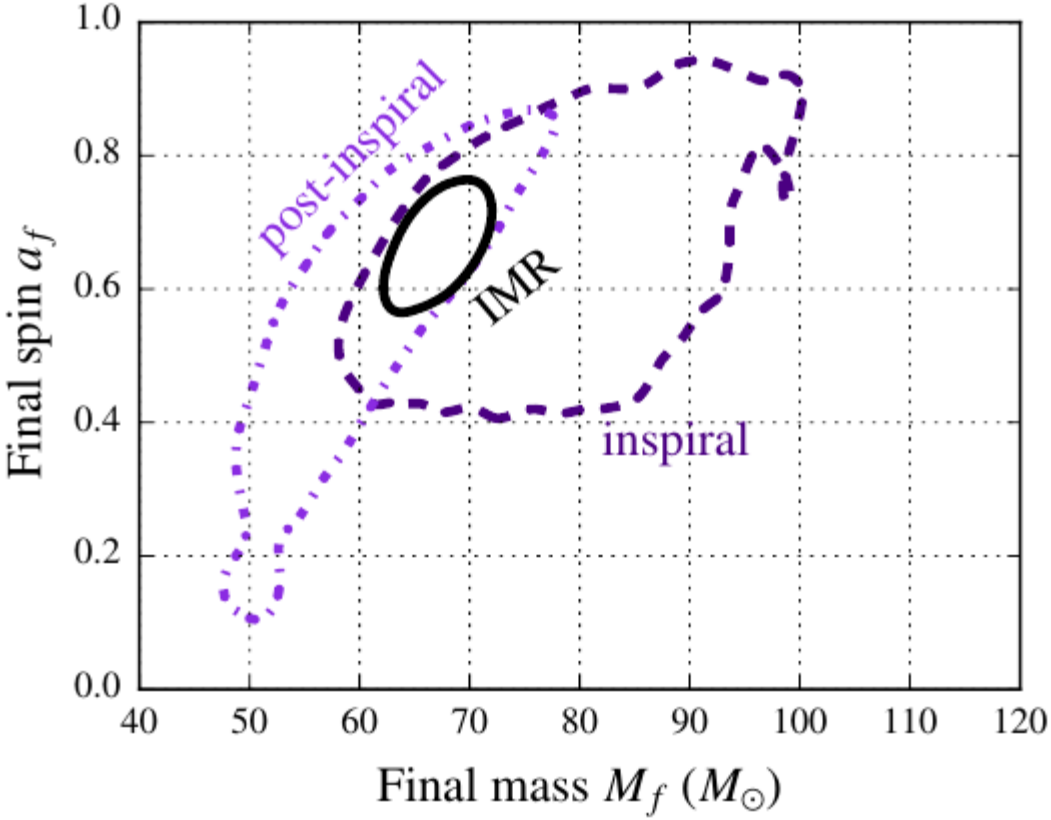
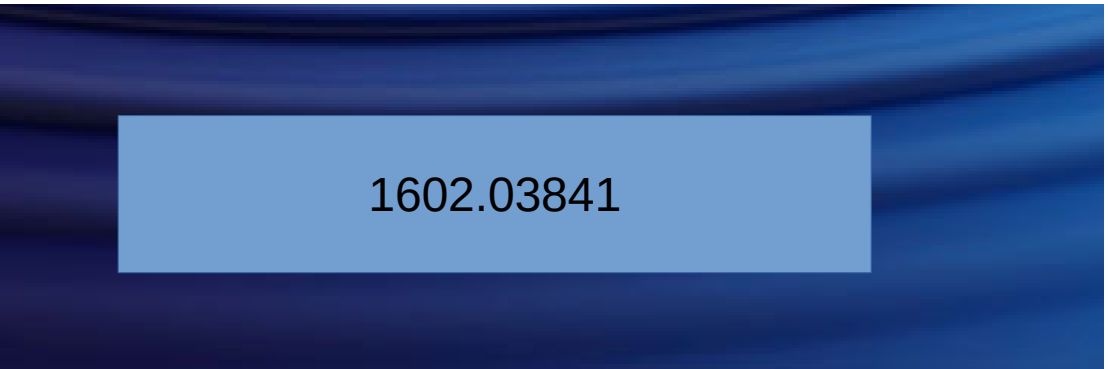
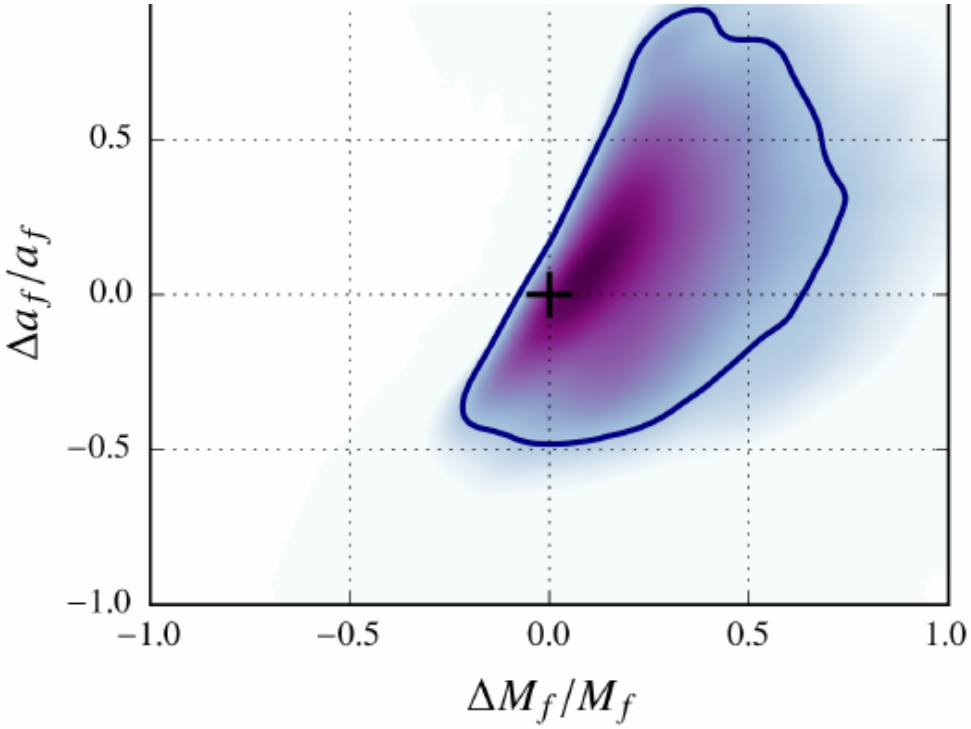
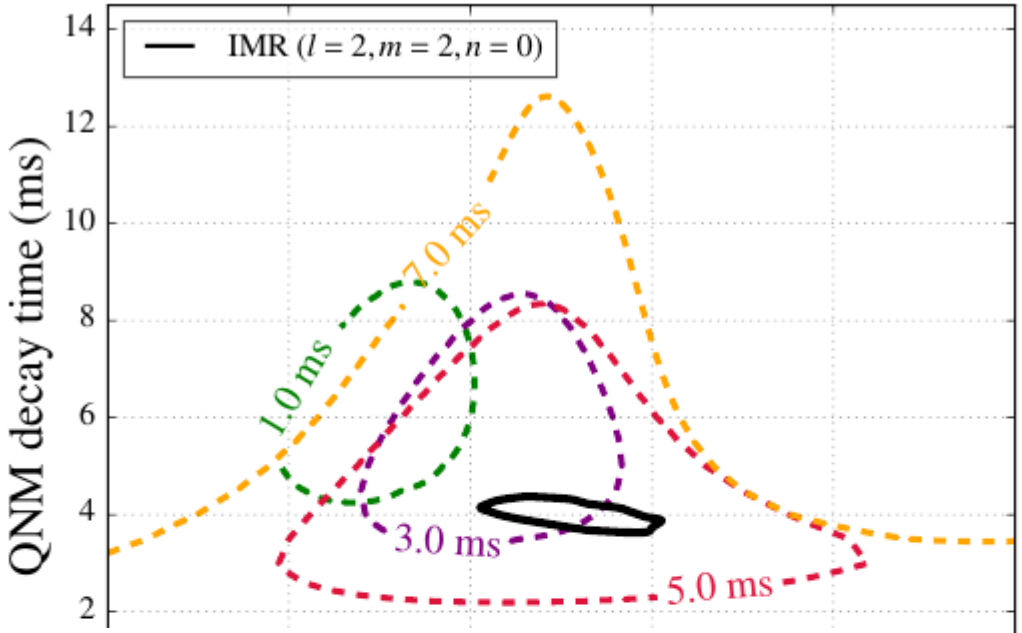
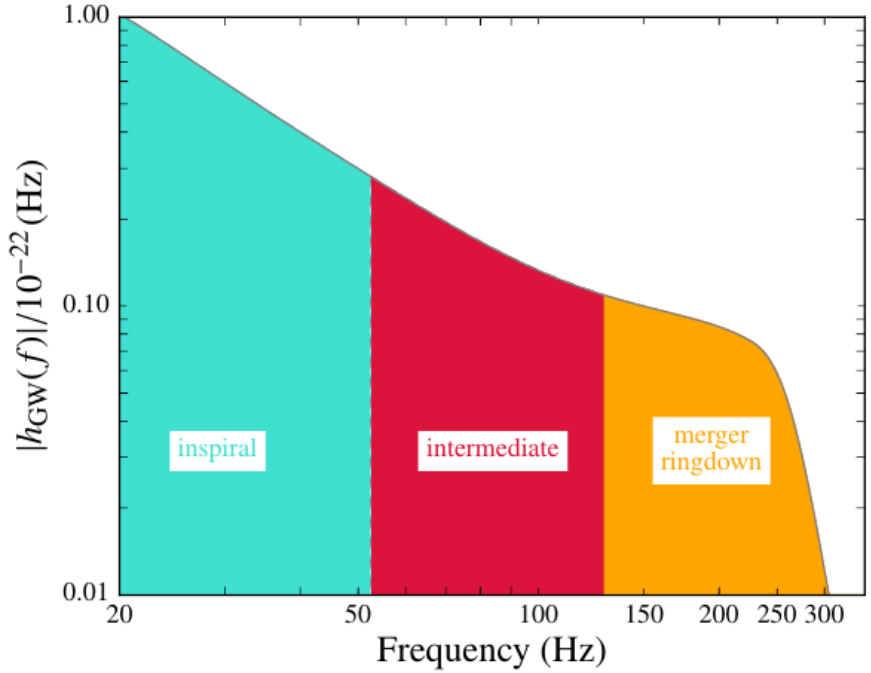


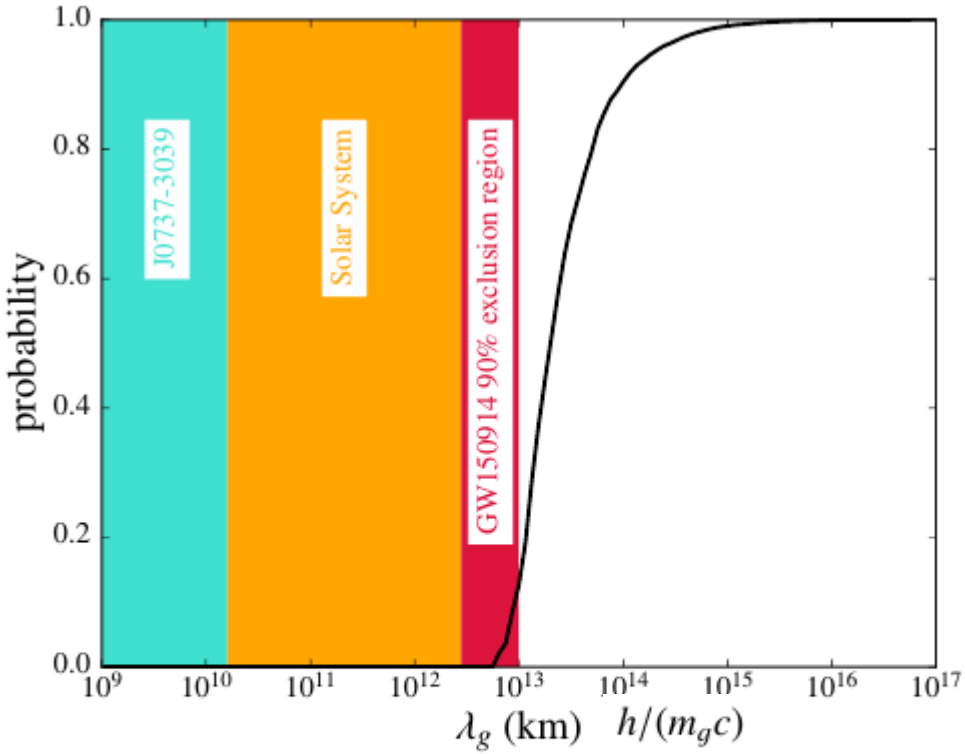
FIG. 2. 90% credible regions for the waveform (upper panel) and GW frequency (lower panel) of GW150914 versus time as estimated by the LALInference analysis [3]. The solid lines in each panel indicate the most probable waveform from GW150914 [3] and its GW frequency. We mark with a vertical line $f_{GW}^{\text{end insp}} = 132$ Hz, which is used in the IMR consistency test to delineate the boundary between the inspiral and post-inspiral parts.











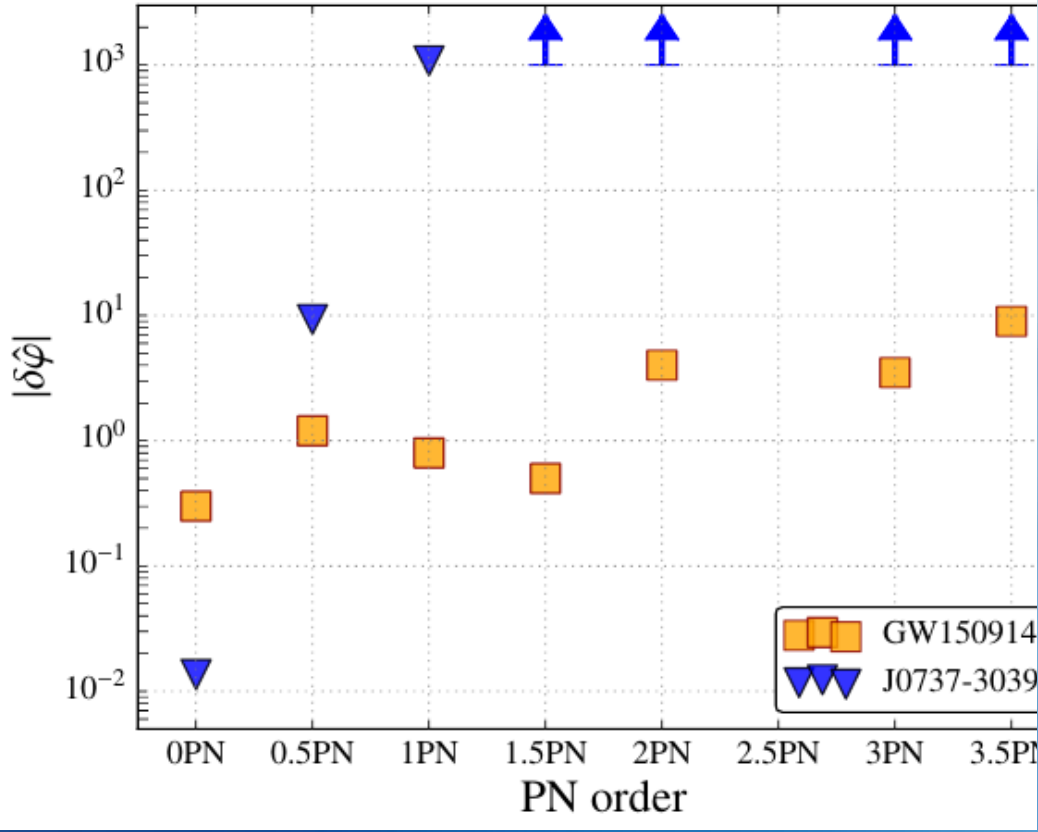
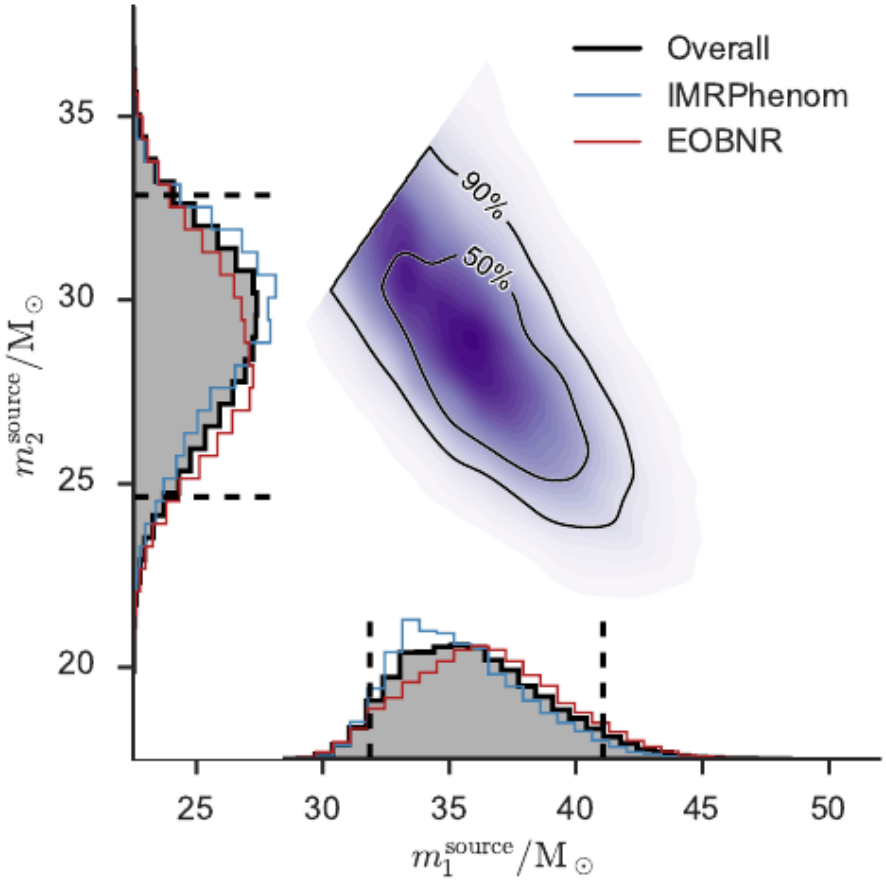
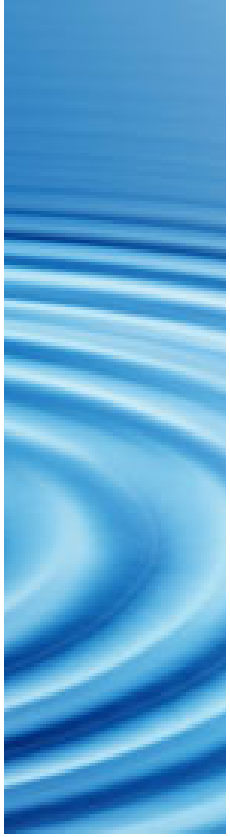


FIG. 8. Cumulative posterior probability distribution for λ_g (black curve) and exclusion regions for the graviton Compton wavelength





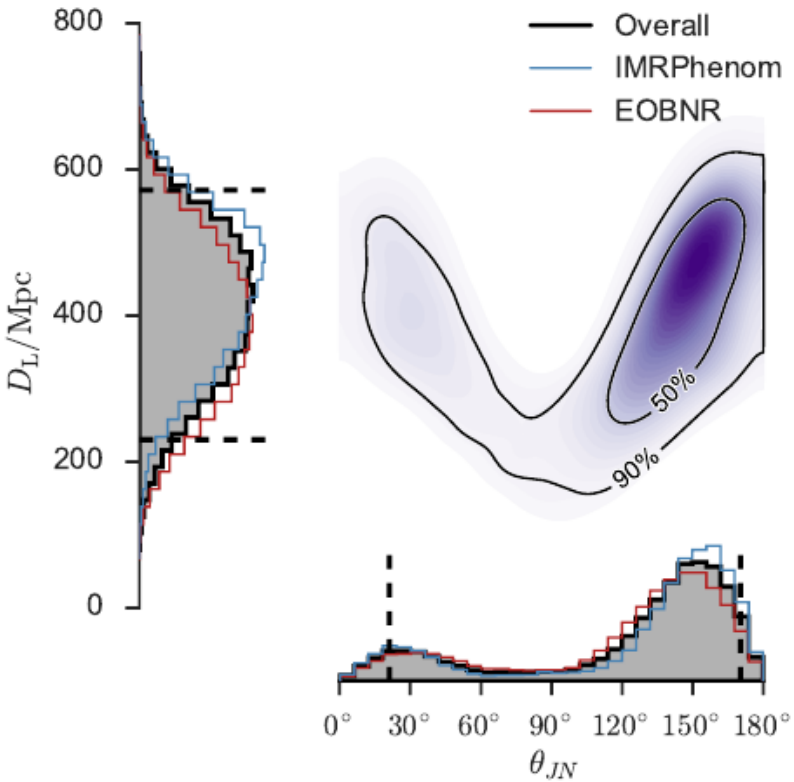
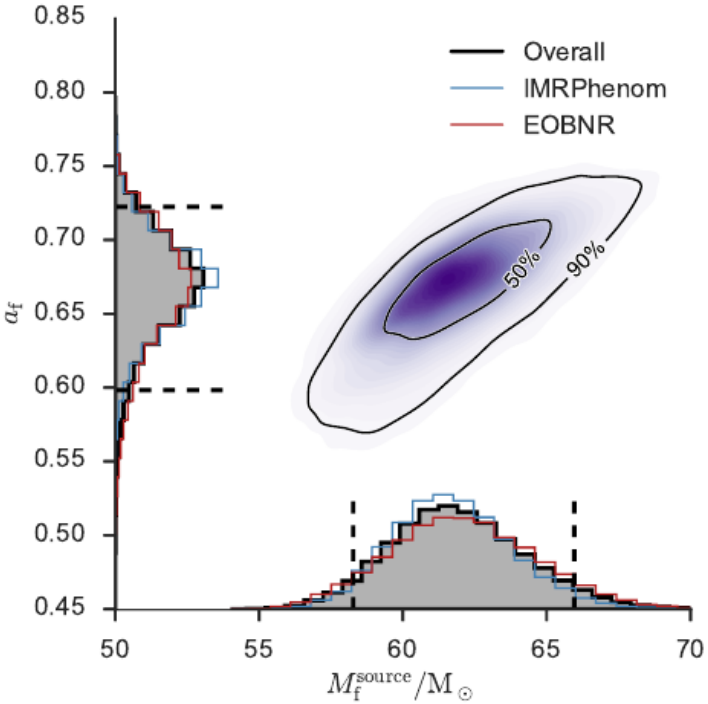


FIG. 1. Posterior PDFs for the source-frame component masses $m_1^{\rm source}$ and $m_2^{\rm source}$, where $m_2^{\rm source} \leq m_1^{\rm source}$. In the 1-dimensional marginalised distributions we show the Overall (solid black), IMRPhenom (blue) and EOBNR (red) PDFs; the dashed vertical lines mark the 90% credible interval for the Overall PDF. The 2-dimensional plot shows the contours of the 50% and 90% credible regions plotted over a colour-coded posterior density function.



ce luminosity distance $D_{\rm L}$ and
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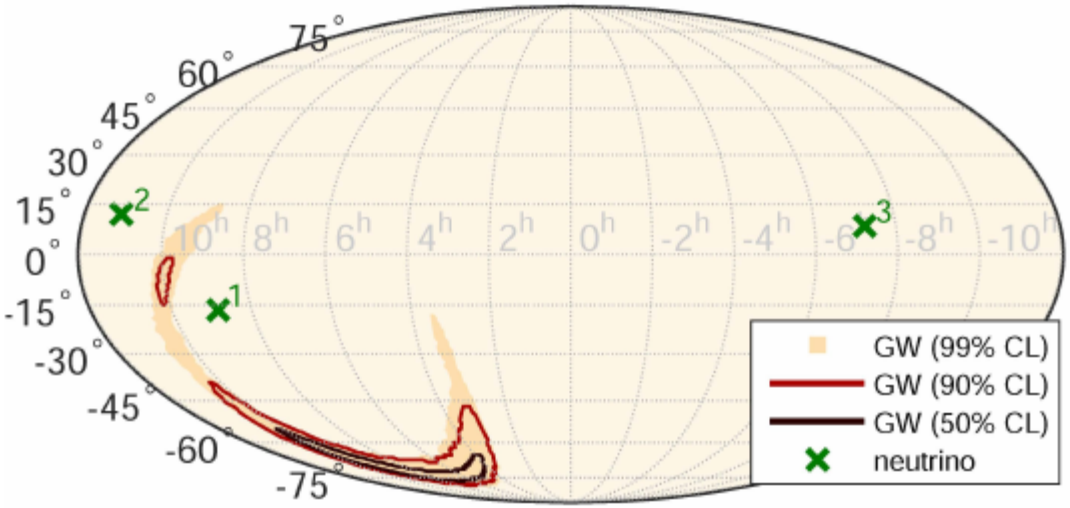

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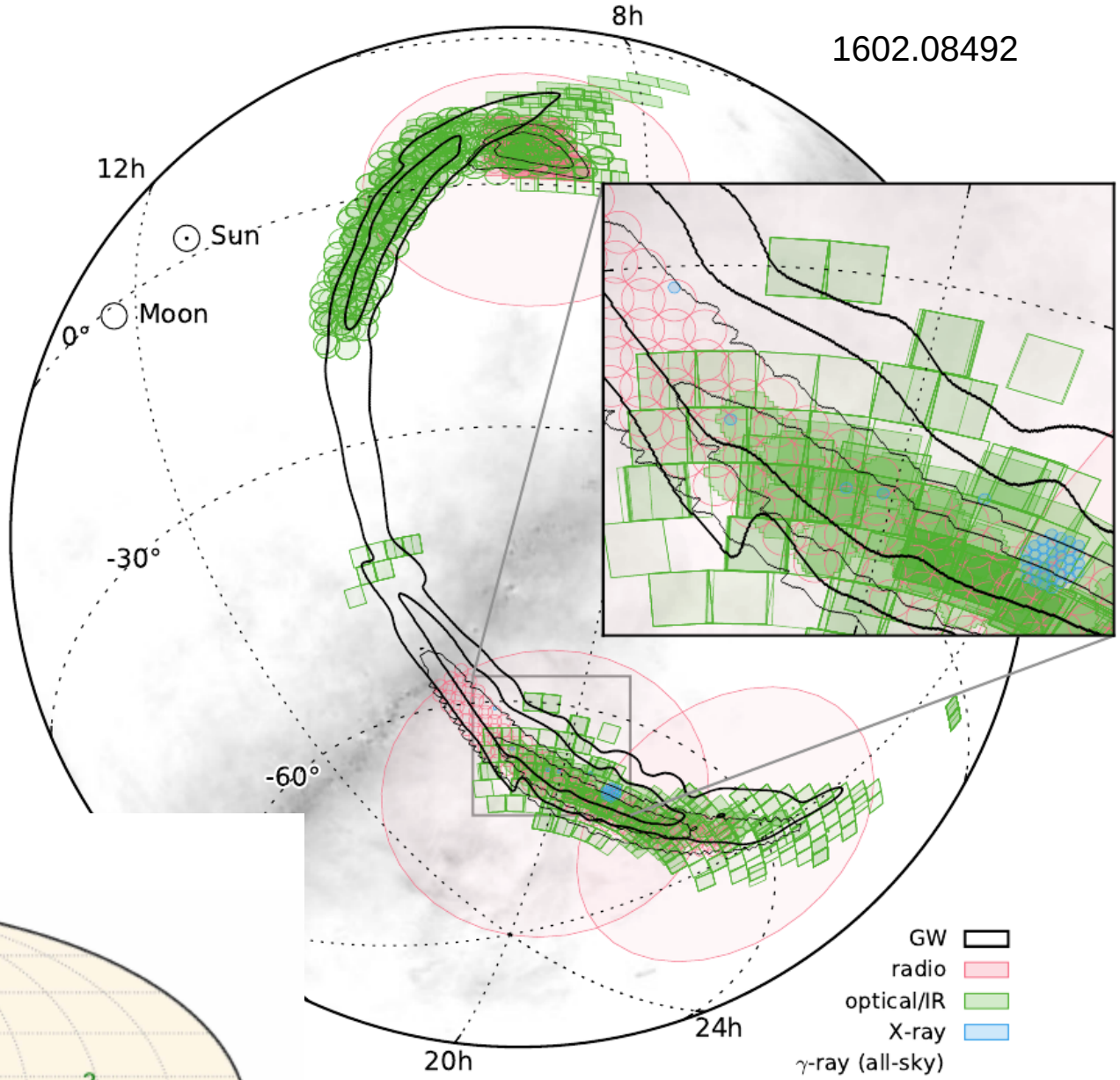
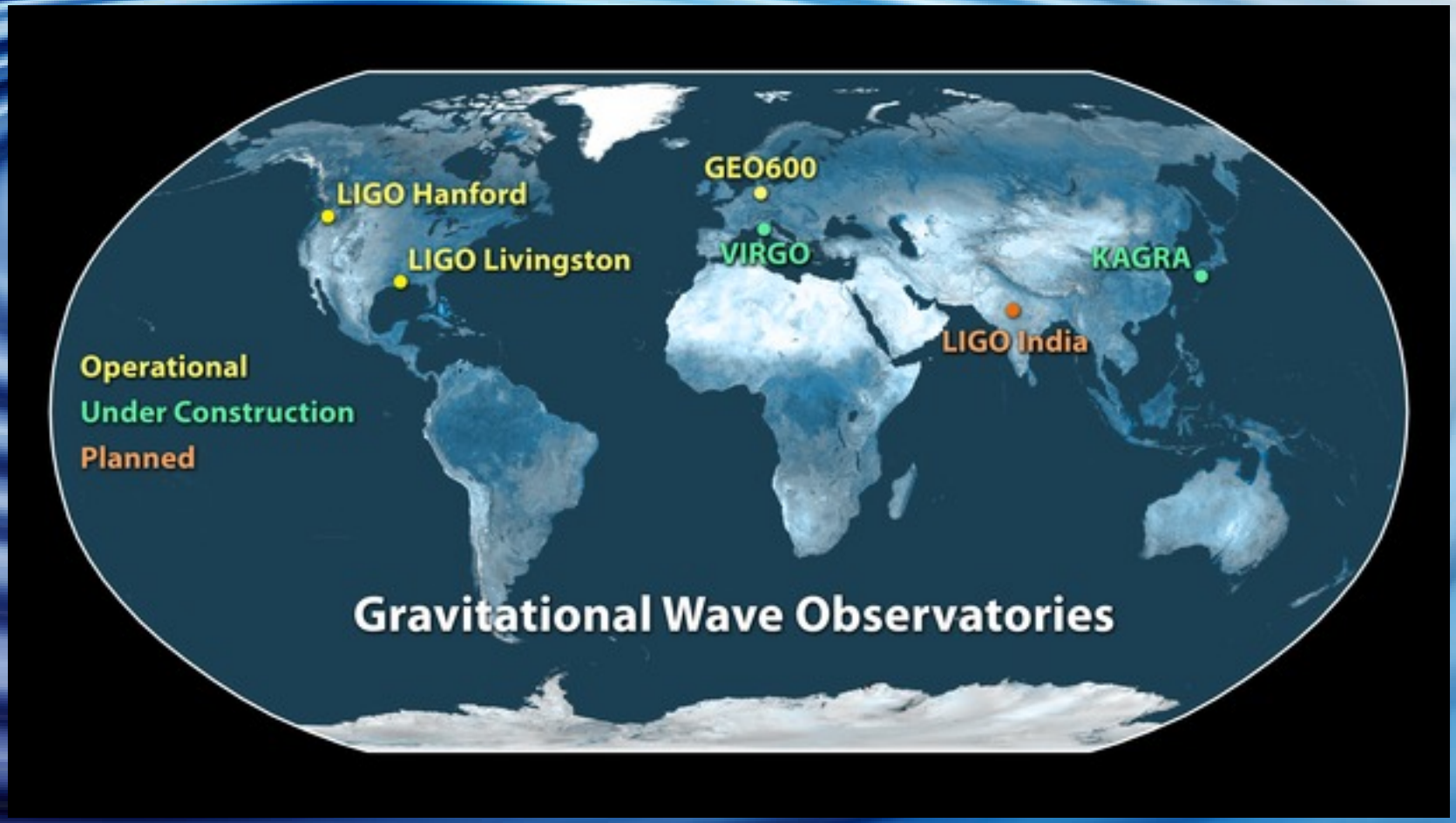
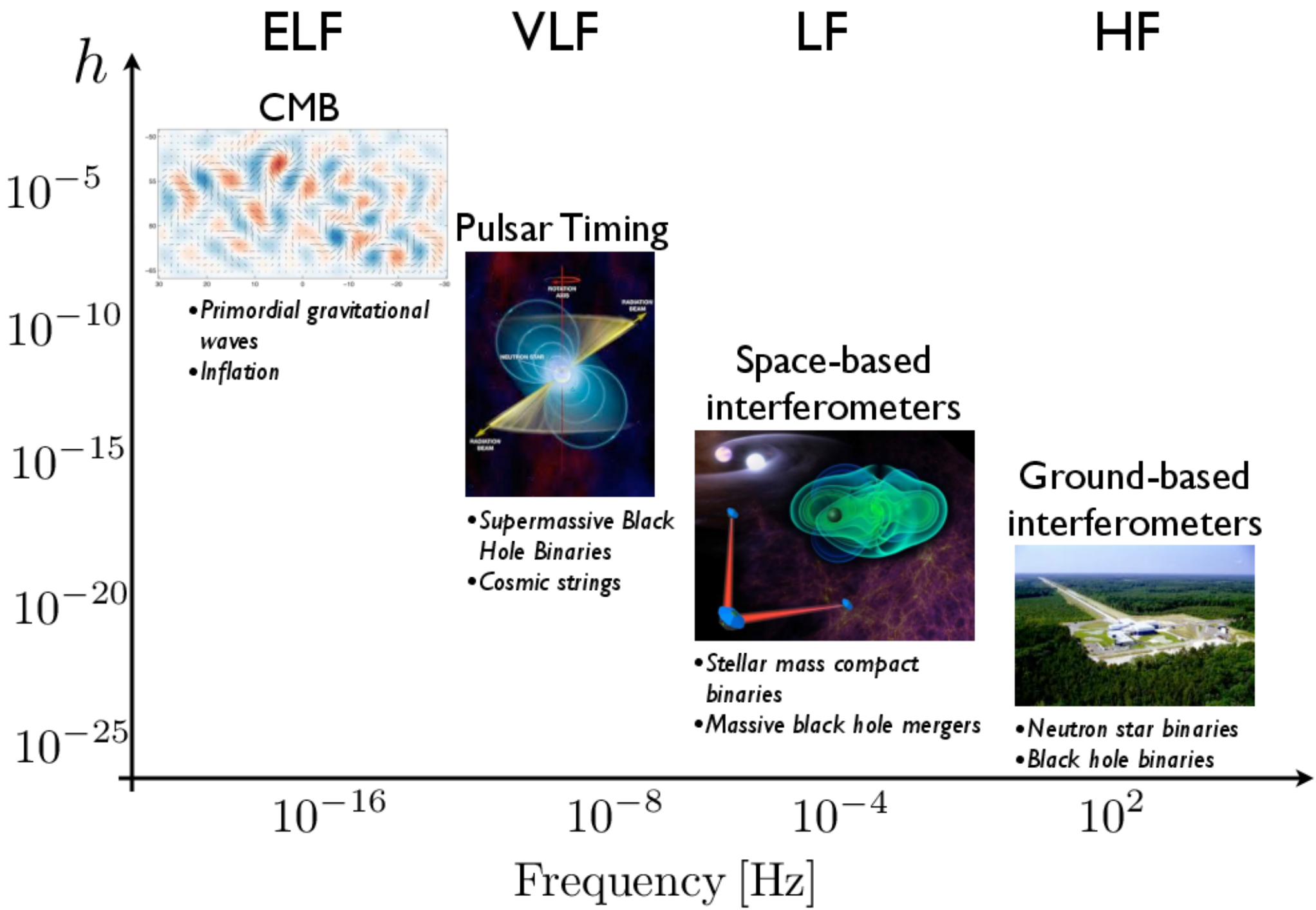
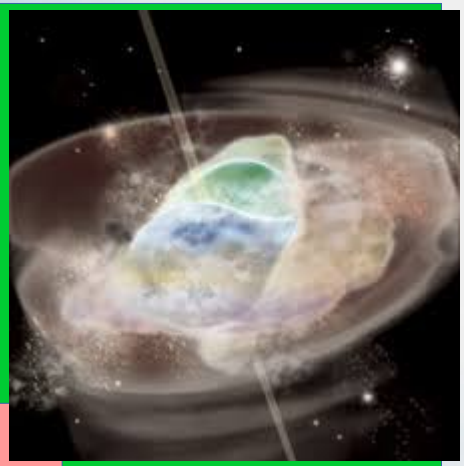


Преспективите пред гравитационната астрономия

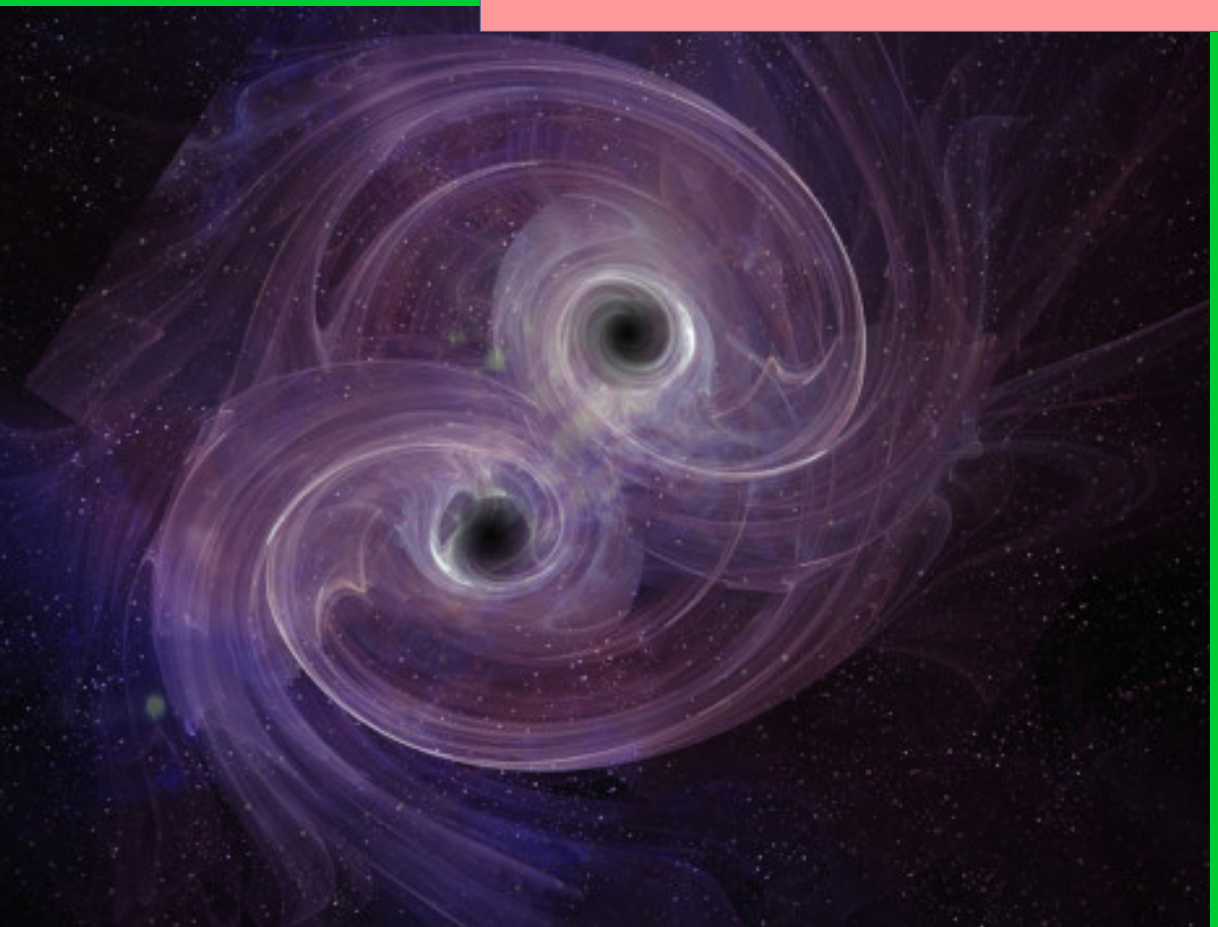


The big picture of gravitational wave astronomy





Благодаря за вниманието!



This is just the beginning,
we've discovered gravitational waves.
It's been a very long road,
but this is just the beginning.
It's the first of many to come...
now that we know binary black holes
are out there,
we'll begin listening to the Universe."
Gabriela Gonazalez, LIGO