Representations of Birman–Murakami–Wenzl algebra and reflection equation.

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Plan

1. **Introduction.**
   - Definition of Birman–Murakami–Wenzl (BMW) algebra.
   - A-type Hecke and BMW algebras
   - Baxterized elements for BMW algebra.
   - Jucys–Murphy elements for BMW algebra.
2. **Idempotents for BMW algebra**
   - Up-down Young tableaux.
   - Inductive formula for idempotents in BMW algebras.
   - Reflection equation and evaluation homomorphism.
   - Fusion

Introduction.

The family of the BMW-algebras $BMW_n(q,\nu)$ depends on a discrete parameter $n = 1, 2, \dots$ and two continuos parameters, q and ν.
The aim of my report is to present you the construction of the complete set of pairwise orthogonal minimal idempotents for the Birman-Murakami-Wenzl algebras. These idempotents are needed for the construction of the representation theory of the BMW algebras and their affine extensions. By means of the Schur-Weyl duality these idempotents are important also for the construction of finite dim. irreps of quantum groups SO and Sp types.

Different aspects of the representation theory of the BMW algebras and their affine extensions were discussed by J. Birman and H. Wenzl, A. Beliakova, C. Blanchet, R. Leduc, A. Ram, M.Nazarov, A.Molev, O.Ogievetsky, a.o.

The representation theory of $BMW_n(q, \nu)$ is based on the investigation of the commutative subalgebra in $BMW_n(q, \nu)$ which is generated by Jucys–Murphy elements y_i $(i = 1, \ldots, n)$. This subalgebra is analog of the Gelfand-Zetlin subalgebra in enveloping algebra of Lie algebras. The spectrum of Jucys-Murphy elements distinguishes the basis vectors in a representation space and thus imply formulas for the complete set of pairwise orthogonal minimal idempotents. In fact, for generic values of the parameters q, ν the algebra, generated by the Jucys–Murphy elements, is a maximal commutative subalgebra in $BMW_n(q, \nu)$.

The formulas for the complete set of pairwise orthogonal minimal idempotents (for generic values of the parameters q and ν) which are obtained *in terms of the Jucys–Murphy elements are known*.

Here I suggest another formulas for the same set of idempotents *but in the form of ordered product of baxterized elements (R-matrices).* These formulas looks like a fusion of the solutions of *the reflection equation*. So, our way to describe the idempotents is in the spirit of what is often referred to as the *fusion procedure*.

Birman–Murakami–Wenzl (BMW) algebra

1. Braid group $\mathcal{B}_{M+1}$

A braid group $\mathcal{B}_{M+1}$ is generated by invertible elements T_i $(i = 1, \ldots M)$ subject Coxeter relations:

$$\text{Braid}: \quad T_i\, T_{i+1}\, T_i = T_{i+1}\, T_i\, T_{i+1}\ ,$$

$$\text{Locality}: \quad T_i\, T_j = T_j\, T_i \quad (|i-j| > 1)\ ,$$

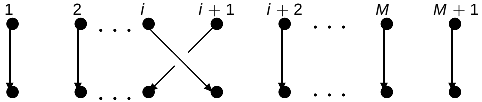
Since we have new elements $(T_i)^k \in \mathcal{B}_{M+1}$ for all $k = 2, 3, 4, \ldots$, it is clear that

$$\text{ord}(\mathcal{B}_{M+1}) = \infty$$

.

Intertwining of two threads

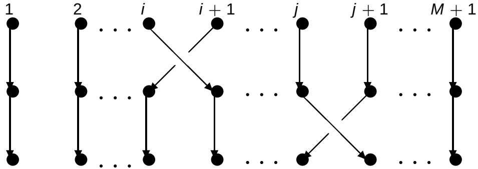
$T_i =$



$=$ braid of $(M+1)$ strands

Locality

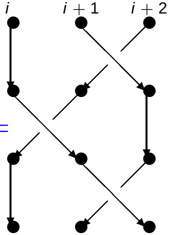
$T_i T_j =$



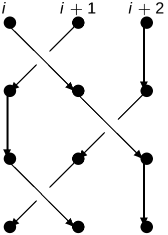
$= T_j T_i$

Braid Relation

$T_{i+1} T_i T_{i+1} =$



$=$

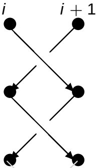


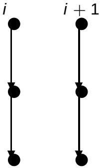
$= T_i T_{i+1} T_i$

Braid relation is one of the Reidemeister moves.

Twisting braid

It's impossible to unravel

$T_i^{2+1} =$ 

$\neq$ 

$= \mathbf{1}$

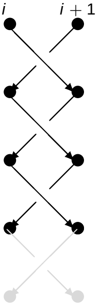
Untwisting braid

$T_i^{-1} =$

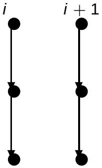
Twisting braid

It's impossible to unravel

$T_i^{2+1} =$



$\neq$



$= \mathbf{1}$

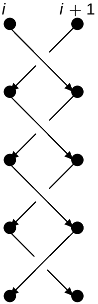
Untwisting braid

$T_i^{-1} =$

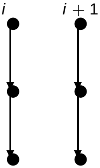
Twisting braid

It's impossible to unravel

$$T_i^{2+1} =$$



$$\neq$$



$$= \mathbf{1}$$

Untwisting braid

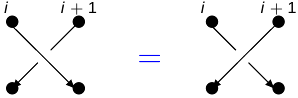
$$T_i^{-1} =$$

2. Finite-dimensional quotients of group algebra of $\mathcal{B}_{M+1}$:

A. Symmetric group S_{M+1}: $T_i^2 = 1 \;\Rightarrow (T_i - 1)(T_i + 1) = 0$,
$\dim(S_{M+1}) = (M+1)!$

Skein relation

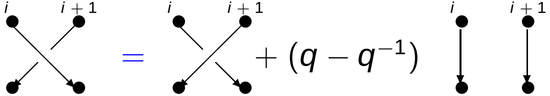
Any braid can be unraveled

$T_i =$  $= T_i^{-1}$

B. Hecke algebra H_{M+1}: $(T_i - q)(T_i + q^{-1}) = 0$, q – parameter.
$\dim(H_{M+1}) = (M+1)$ For $q^2 = 1$ we have $H_{M+1} \to \mathbb{C}[S_{M+1}]$.

Skein relation

Braids are nontrivial

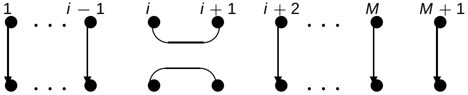


$$T_i = T_i^{-1} + (q - q^{-1})\,\mathbf{1}$$

C. BMW$_{M+1}$ algebra :

$$\begin{cases} \boxed{(T_i - q)(T_i + q^{-1})(T_i - \nu) = 0}, \\ \boxed{\kappa_i \, T_{i+1}^{\pm 1} \, \kappa_i = \nu^{\mp 1} \, \kappa_i} \quad (q, \nu \in \mathbb{C}), \end{cases}$$

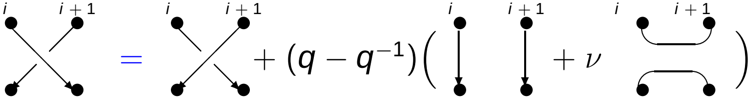
$$\kappa_i := \frac{(q - T_i)(T_i + q^{-1})}{\nu(q - q^{-1})} \quad \Rightarrow \quad \kappa_i T_i = T_i \kappa_i = \nu \kappa_i \quad (i = 1, \ldots, M),$$

$\kappa_i =$ 

$$\dim(\mathrm{BMW}_{M+1}) = (2M-1)!! = 1 \cdot 3 \cdots (2M-1)$$

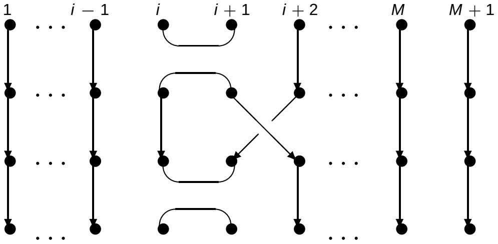
Skein relation

Braids are nontrivial



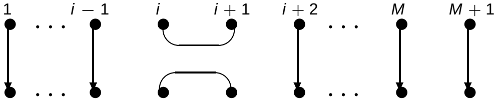
$$T_i = T_i^{-1} + (q - q^{-1}) \, \mathbf{1} + \nu \, (q - q^{-1}) \, \kappa_i$$

$\kappa_i \; T_{i+1} \; \kappa_i =$



$\sim$

$\sim \;\; \kappa_i \;\; =$

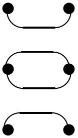
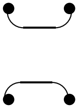


Since the loop is unraveled, these two braids are identical (up to the multiplier ν^{-1}):

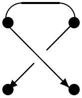
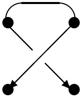
$$\kappa_i \; T_{i+1} \; \kappa_i = \nu^{-1} \; \kappa_i$$

The closure of the braids gives links and knots K.

The trivial link is obtained from relation $\kappa_i^2 = \left(1 - \frac{\nu-\nu^{-1}}{q-q^{-1}}\right)\kappa_i$

 $= \left(1 - \frac{\nu-\nu^{-1}}{q-q^{-1}}\right) \cdot$ 

There are two kinds of tadpoles which appear as pictures for relations $\kappa_i T_i = \nu\kappa_i$ and $\kappa_i T_i^{-1} = \nu^{-1}\kappa_i$

 $= \nu$     $= \nu^{-1}$ 

Louis Kauffman proved that if normalize the polynomial L_K (obtained from K by applying skein relation etc.) via relation

$$F_K = \nu^{w(K)} L_K \ ,$$

where $w(K)$ – twist number of the diagram (knot) K, then F_k turns out to be good invariant polynomial of knots (links). These polynomials are called Kauffman polynomials.

3. Definition and basic relations.

The Birman–Murakami–Wenzl algebra $BMW_n(q,\nu)$ over $\mathbb{C}$ is generated by the invertible elements $T_1,\dots,T_{n-1}$ with the following defining relations

$$T_i\,T_{i+1}\,T_i = T_{i+1}\,T_i\,T_{i+1}\;, \quad T_i\,T_j = T_j\,T_i \quad \text{for } |i-j|>1\;,$$

$$\kappa_i\,T_i = T_i\,\kappa_i = \nu\,\kappa_i\;,$$

$$\kappa_i\,T_{i-1}^{\varepsilon}\,\kappa_i = \nu^{-\varepsilon}\,\kappa_i\;,\quad \kappa_i\,T_{i+1}^{\varepsilon}\,\kappa_i = \nu^{-\varepsilon}\,\kappa_i\;, \tag{1}$$

where $\varepsilon=\pm 1$ and

$$\kappa_i := 1 - \frac{T_i - T_i^{-1}}{q-q^{-1}}\;.$$

Here q and ν are complex parameters.
The quotient of the algebra $BMW_n(q,\nu)$ by the ideal generated by the elements $\kappa_1,\dots,\kappa_n$ is isomorphic to the Hecke algebra $H_n(q)$ (applying $\kappa_i=0$ we obtain Hecke conditions). The classical limit $q\to 1$ of $BMW_n(q,\nu)$ is *the Brauer algebra* $Br_n(\omega)$.

4. Baxterized elements are firstly introduced by V.Jones, and J.Murakami

$$T_i(u,v) := T_i + \frac{q-q^{-1}}{v/u-1} + \frac{q-q^{-1}}{1+\nu^{-1}qv/u}\,\kappa_i \quad \in \quad BMW_n(q,\nu)\;,$$

where u and v – *spectral parameters*. The elements $T_i(u,v) = T_i(\frac{u}{v})$ depend on the ratio $\frac{u}{v}$; later we shall evaluate u and v in terms of contents of boxes of Young diagram; this explains the use of both spectral variables in the notation $T_i(u,v)$. The classical limit $q \to 1$ gives the baxterized operator in the Brauer algebra (Zamolodchikov and Karowski-Weisz R-matrices for so and sp algebras).

The baxterized elements satisfy Yang-Baxter equation (YBE)

$$T_i(u_2,u_3)T_{i+1}(u_1,u_3)T_i(u_1,u_2) = T_{i+1}(u_1,u_2)T_i(u_1,u_3)T_{i+1}(u_2,u_3)\;,$$

(as nontrivial identity in $BMW_n(q,\nu)$) and unitarity condition

$$T_i(u,v)\,T_i(v,u) = f(u,v)^{-1}\;,$$

where $f(u,v) = \frac{(u-v)^2}{(u-q^2v)(u-q^{-2}v)} = f(v,u)$.

Let

$$Q_i(u,v;c) := T_i\left(\frac{1}{cuv}\right) = T_i + \frac{q-q^{-1}}{cuv-1} + \frac{q-q^{-1}}{1+\nu^{-1}qcuv}\kappa_i \ .$$

We keep three arguments for the later convenience.
It follows from YBE that

$$T_i(u_2,u_3)\, Q_{i+1}(u_1,u_3;c)\, Q_i(u_1,u_2;c) =$$
$$= Q_{i+1}(u_1,u_2;c)\, Q_i(u_1,u_3;c)\, T_{i+1}(u_2,u_3)\ .$$

5. Jucys–Murphy elements of the algebra BMW_n are defined by

$$y_1 = 1 \, , \quad y_{k+1} = T_k \cdot y_k \cdot T_k = T_k \cdots T_2 \, T_1^2 \, T_2 \cdots T_k \, , \\ k = 1, \dots, n-1 \, .$$

The elements $y_1, \dots, y_n$ pairwise commute. The commutative subalgebra, generated by the Jucys–Murphy elements $y_1, \dots, y_n$, of the algebra BMW_n with generic parameters q and ν is maximal.

If we extend BMW_n by the element $y_1 \neq 1$ with defining relations

$$T_1 \, y_1 \, T_1 \, y_1 = y_1 \, T_1 \, y_1 \, T_1 \, , \quad \kappa_j \, y_{j+1} \, y_j = y_j \, y_{j+1} \, \kappa_j = Z_0 \, \kappa_j \\ \kappa_j \, y_j^r \, \kappa_j = Z_r \, \kappa_j \, ,$$

where Z_r – central elements, so we obtain affine extension of the BMW algebra.

Idempotents for BMW algebra. Up-down tableaux.

We depict a partition $\lambda \vdash n$ and $\lambda = (\lambda_1, \lambda_2, \dots)$ by the Young diagram containing λ_1 boxes in the first row, λ_2 boxes in the second row and so on.

Example. Young diagram $\lambda = (4, 3, 1) \vdash 8$ with quantum content

1	q^2	q^4	q^6
q^{-2}	1	q^2	
q^{-4}			

For the box which is in the a-th row and b-th column of λ, the quantum content is defined by $q^{2(b-a)}$.

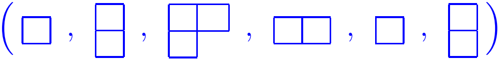
Basis vectors v_{U_n} in the space of the irreducible representation ρ_λ of BMW_n corresponding to λ are indexed by the up-down tableaux of shape λ, that are sequences $U_n = (\Lambda_1, \Lambda_2, \dots, \Lambda_n)$ of Young diagrams Λ_j such that:

1.) $\Lambda_1 = \square$;

2.) Λ_j (for $j = 2, \dots, n$)) differs from Λ_{j-1} by exactly one box (added or removed);

3.) $\Lambda_n = \lambda$.

Example : $U_6 =$



The images of the Jucys–Murphy elements y_j are diagonal in the chosen basis,

$$\rho_\lambda(y_j) \cdot v_{U_n} = c_j(U_n)\, v_{U_n} \ .$$

Here v_{U_n} is the basis vector associated to the up-down tableau U_n of shape λ, operator $\rho_\lambda(y_j)$ is the image of $y_j \in BMW_n$ in the irrep ρ_λ and the eigenvalue $c_j(U_n)$ is the quantum content of the box added/removed at the step j of $U_n = (\dots, \Lambda_{j-1}, \Lambda_j, \dots)$.

I.e., for the box in the a-th row and b-th column the eigenvalue is defined by $c_j(U_n) = q^{2(b-a)}$ if this box was added to the diagram Λ_{j-1} to obtain the diagram Λ_j and $c_j(U_n) = \nu^2 q^{-2(b-a)}$, if this box was removed from the diagram Λ_{j-1} to obtain the diagram Λ_j. Note that $c_1 = 1$. The up-down tableau U_n is uniquely determined by its content sequence $(c_1(U_n), \ldots, c_n(U_n)) = (y_1, y_2, \ldots, y_n)$. Example:

1	q^2	q^4	q^6
q^{-2}	1	q^2	
q^{-4}			

$\rho_\lambda(y_9) \sim c_9(U_n) =$ $\nu^2 q^{-2}$ (remove) or q^8 (add)

1	q^2	q^4	q^6
q^{-2}	1		
q^{-4}			

1	q^2	q^4	q^6	q^8
q^{-2}	1	q^2		
q^{-4}				

Example of the standard Young tableaux with content

$$U_8 = \begin{array}{|c|c|c|c|} \hline {}^{1}\ 1 & {}^{2}\ q^2 & {}^{4}\ q^4 & {}^{6}\ q^6 \\ \hline {}^{3}\ q^{-2} & {}^{5}\ 1 & {}^{8}\ q^2 & \\ \cline{1-3} {}^{7}\ q^{-4} & & & \\ \cline{1-1} \end{array}$$

For this up-down Young tableaux (were there are no removing boxes) the content sequence is

$$(c_1(U_n), \ldots, c_8(U_n)) = (1, q^2, q^{-2}, q^4, 1, q^6, q^{-4}, q^2) \quad .$$

The numbers in the l.h.s. are eigenvalues of $(y_1, y_2, \ldots, y_n)$.
Above example of the up-down tableau U_6:

$$(c_1(U_6), \ldots, c_6(U_6)) = (1, q^{-2}, q^2, \nu^2 q^2, \nu^2 q^{-2}, q^{-2}) \quad .$$

Inductive formula for idempotents of BMW algebras.

Define a family of $\overline{BMW}_n$-valued rational functions $\mathcal{Y}_j(u_1, \dots, u_j)$, where $j = 1, 2, \dots, n$, by inductive procedure. The first function is

$$\mathcal{Y}_1(u) := \frac{c u\, y_1 - 1}{u - y_1} \;\; , \; ,$$

(for non-affine case we fix $y_1 = 1$, $c = -q^{-1}\nu^{-1}$) and

$$\mathcal{Y}_j(u_1, \dots, u_j) :=$$

$$= Q_{j-1}(u_{j-1}, u_j; c)\, \mathcal{Y}_{j-1}(u_1, \dots, u_{j-2}, u_j)\, T_{j-1}(u_j, u_{j-1})^{-1} =$$

$$= Q_{j-1}(u_{j-1}, u_j; c) \dots Q_1(u_1, u_j; c)\; \mathcal{Y}_1(u_j) \cdot T_1^{-1}(u_j, u_1) \dots T_{j-1}(u_j, u_{j-1})^{-1} \; ,$$

where $j = 1, 2, 3, \dots, n$.

4. Fusion function. For an up-down tableau $U_n = (\Lambda_1, \ldots, \Lambda_n)$ of length n denote $c_j = c_j(U_n)$ and let c be a parameter. Define, for $n = 1, 2, \ldots$, the *fusion* function Ψ_n by

$$\Psi_n(u_1, \ldots, u_n) := \left(\prod_{k=1}^{n} \frac{u_k - c_k}{c u_k c_k - 1} \right) \mathcal{Y}_1(u_1) \mathcal{Y}_2(u_1, u_2) \ldots \mathcal{Y}_n(u_1, \ldots, u_n) \, .$$

We denote by $E_{U_n} \in BMW_n$ the primitive idempotent, corresponding to the up-down tableau U_n, that is the projector onto the 1-dimensional subspace spanned by eigenvector v_{U_n}:

$$E_{U_n} \cdot E_{U'_n} = \delta_{U_n, U'_n} \; E_{U_n} \quad , \qquad \sum_{E_{U_n}} E_{U_n} = 1 \, .$$

The following theorem is the main result.

Theorem. *The idempotent E_{U_n} is equal to the subsequent evaluation of Ψ_n at the points $u_i = c_i$ (content of up-down tableau U_n),*

$$E_{U_n} = \; \Psi_n(u_1, u_2, \ldots, u_n)|_{u_1 = c_1} \cdots \Big|_{u_n = c_n}$$

Proof. By induction on n.

Example. We shall illustrate the Theorem by using simplest example of the algebra BMW_2. The algebra BMW_2 is generated by T_1 which has the projector decomposition $T_1 = qS - q^{-1}A + \nu\Pi$. One has

$$S = E_{(1,q^2)} = \frac{1}{q+q^{-1}}\left(T_1 + q^{-1} + \frac{q-q^{-1}}{1-q\nu^{-1}}\kappa_1\right) ,$$

$$A = E_{(1,q^{-2})} = -\frac{1}{q+q^{-1}}\left(T_1 - q + \frac{q-q^{-1}}{1+q^{-1}\nu^{-1}}\kappa_1\right) ,$$

$$\Pi = E_{(1,\nu^2)} = \frac{\nu(q-q^{-1})}{(q^{-1}+\nu)(q-\nu)}\kappa_1 \; .$$

The elements S, A (symmetrizer and antisymmetrizer) and element Π form a complete system of pairwise orthogonal primitive idempotents of BMW_2.

$$S \sim \left(\square\, ,\, \begin{smallmatrix}\square\\ \square\end{smallmatrix}\right), \quad A \sim \left(\square\, ,\, \square\square\right) , \quad \Pi \sim \left(\square\, ,\, \emptyset\right)$$

Hecke algebra. The fusion functions $\Psi_n(u_1, \dots, u_n)$ for the BMW algebra depend on c. So in the Hecke algebra quotient we obtain a one parametric family of fusion functions (for the Hecke algebras), given by $\Psi_n(u_1, \dots, u_n)$, where we have to substitute

$$T_i(u_1, u_2) = T_i + \frac{q - q^{-1}}{u_2/u_1 - 1}, \quad Q_i(u_1, u_2; c) = T_i + \frac{q - q^{-1}}{cu_1 u_2 - 1}.$$

Explicitly the one-parametric family of the fusion functions is

$$\widetilde{\Psi}_n(u_1, u_2, \dots, u_n) := \left(\prod_{k=1}^{n} \frac{u_k - c_k}{cu_k c_k - 1} \frac{cu_k - 1}{u_k - 1} \right) \cdot \\ \cdot \widetilde{Y}_1(u_1) \widetilde{Y}_2(u_1, u_2) \dots \widetilde{Y}_n(u_1, \dots, u_n),$$

where

$$\widetilde{Y}_j(u_1, \dots, u_j) := \overleftarrow{\prod_{j>k\geqslant 1}} \left(T_k + \frac{q - q^{-1}}{cu_k u_j - 1} \right) \overrightarrow{\prod_{1\leqslant k<j}} \left(T_k + \frac{q - q^{-1}}{u_j/u_k - 1} \right).$$

At $c = 0$ we have $Q_i(u_1, u_2; 0) = T_i^{-1}$ and it gives the fusion function proposed by API, A.Molev, and A.Oskin.

Reflection equation (RE).

The most general solution of the "usual" reflection equation

$$L_j(u)\cdot T_j\left(\frac{1}{cu\,v}\right)\cdot L_j(v)\cdot T_j\left(\frac{u}{v}\right)=T_j\left(\frac{u}{v}\right)\cdot L_j(v)\cdot T_j\left(\frac{1}{cu\,v}\right)\cdot L_j(u)$$

in the BMW algebra is given by the rational function in y_j (API and O.Ogievetsky)

$$L_j(u)=\frac{cu\,y_j-1}{u-y_j}\ ,\qquad c=-q^{-1}\nu^{-1}\ ,$$

and we have $L_1(u)\equiv\mathcal{Y}_1(u)$ (see fusion function). This formula gives the possibility to formulate the evaluation homomorphisms for the quantum universal enveloping algebras which described by "usual" reflection equations (for the classical counterpart it was done by API, A.Molev, O.Ogievetsky). Indeed, in the R-matrix representation of $\overline{BMW}_n$ the affine element $y_1\in\overline{BMW}_n$ is represented as the matrix of generators of quantum algebras of SO and Sp types.

Fusion solution of the RE. Note that elements $\mathcal{Y}_j$ satisfy the RE

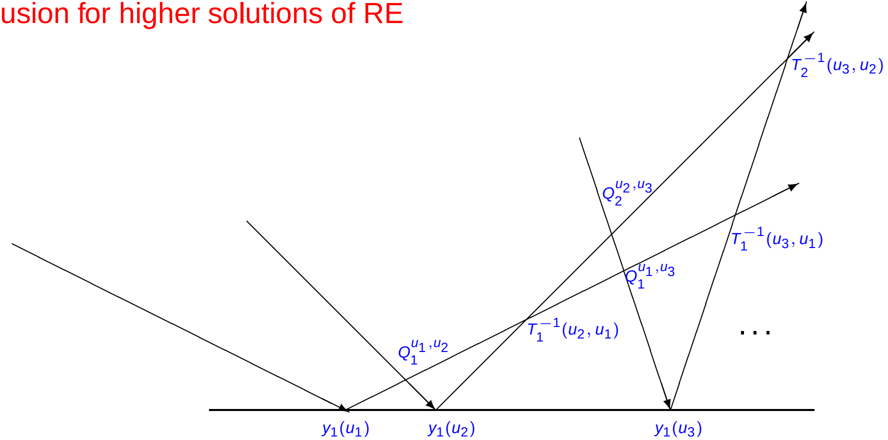
$$\mathcal{Y}_j(u_1,\dots,u_{j-1},u)\cdot T_j\left(\frac{1}{cu\,v}\right)\cdot\mathcal{Y}_j(u_1,\dots,u_{j-1},v)\cdot T_j\left(\frac{u}{v}\right)$$
$$=T_j\left(\frac{u}{v}\right)\cdot\mathcal{Y}_j(u_1,\dots,u_{j-1},v)\cdot T_j\left(\frac{1}{cu\,v}\right)\cdot\mathcal{Y}_j(u_1,\dots,u_{j-1},u)\,.$$

This is shown by induction on j. Thus, the fusion function

$$\Psi_n(u_1,\dots,u_n):=\left(\prod_{k=1}^{n}\frac{u_k-c_k}{cu_kc_k-1}\right)\mathcal{Y}_1(u_1)\cdot\mathcal{Y}_2(u_1,u_2)\cdots\mathcal{Y}_n(u_1,\dots,u_n)\,.$$

is exactly the fusion solution of the reflection equation which describes the reflection of n particles from the boundary.

Fusion for higher solutions of RE



$$\mathcal{Y}_1(u_1) \cdot \mathcal{Y}_2(u_1, u_2) \cdot \mathcal{Y}_3(u_1, u_2, u_3) =$$
$$= y_1(u_1) \cdot (Q_1^{u_1,u_2}\, y_1(u_2)\, T_1^{-1}(u_2, u_1)) \cdot$$
$$\cdot (Q_2^{u_2,u_3} Q_1^{u_1,u_3}\, y_1(u_3)\, T_1^{-1}(u_3, u_1) T_2^{-1}(u_3, u_2))$$

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