ЗА НОБЕЛОВАТА НАГРАДА ПО ФИЗИКА 2022: КВАНТОВО СПЛИТАНЕ И НЕРАВЕНСТВА НА БЕЛ

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ИЯИЯЕ - БАН

София, 1 декември 2022 г.

Част II.
НЕРАВЕНСТВАТА НА БЕЛ
И НАРУШАВАНЕТО ИМ В ЕКСПЕРИМЕНТИ
СЪС СПЛЕТЕНИ ФОТОНИ

- Скрити параметри и неравенства на Bell-CHSH
- Теоретичната възможност за нарушаване на неравенствата на Бел в квантовата механика
- Експерименти със сплетени фотони. Запушване на "пробойните" (loopholes). Приложения

Scientific Background on the Nobel Prize in Physics 2022

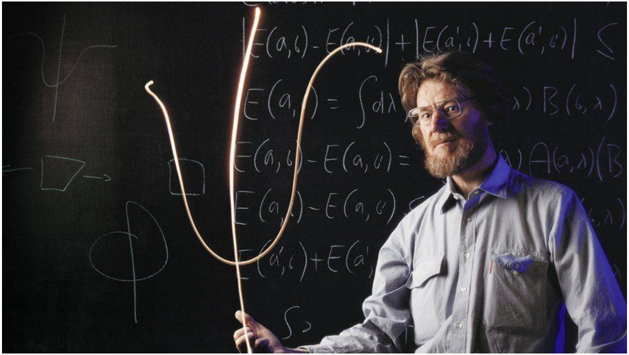
FOR EXPERIMENTS WITH ENTANGLED PHOTONS ESTABLISHING THE VIOLATION OF THE BELL INEQUALITIES AND PIONEERING QUANTUM INFORMATION SCIENCE

Nobel Committee Physics
PRESS RELEASE 2022

Alain Aspect, John Clauser and Anton Zeilinger have each conducted groundbreaking experiments using entangled quantum states, where two particles behave like a single unit even when they are separated. Their results have cleared the way for new technology based upon quantum information.

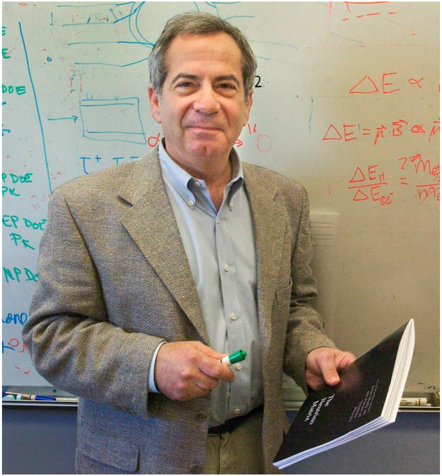
David Bohm (1917 Wilkes-Barre PA, US - 1992 London)

John Stewart Bell (1928 Belfast - 1990 Geneva)



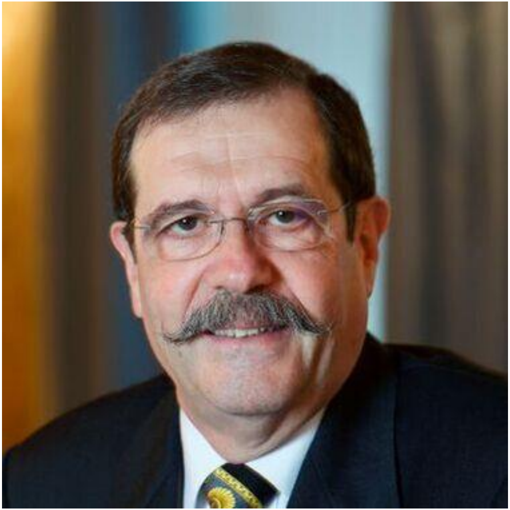
Abner Shimony
(1928 Columbus OH, US - 2015 New Haven CT, US)

Stuart Jay Freedman (1944 Hollywood CA, US - 2012 Santa Fe NM, US)

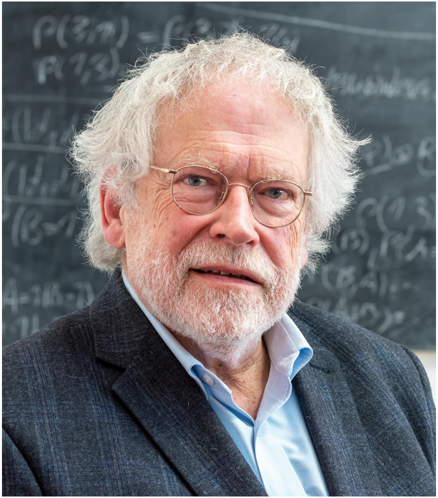


John Francis Clauser - 1942 Pasadena CA, US

Alain Aspect - 1947 Agen, Lot-et-Garonne, France



Anton Zeilinger - 1945 Ried im Innkreis, Austria



Скрити параметри

и неравенства на Bell-CHSH

The Bell(-CHSH) inequalities

J.S. Bell, Physics/Physique/Fizika 1 (1964) 195.

J.F. Clauser, M.A. Horne, A. Shimony, R.A. Holt, Phys. Rev. Lett. 26 (1969) 880.

E.P. Wigner: Am. J. Phys. 38 (1970) 1005.

J.S. Bell, Introduction to the hidden-variable question, in: Foundations of Quantum Mechanics (1971) 171-81 (*with the derivation Bell-CHSH inequality reproduced below*)

A. Aspect, Bell's Theorem: The naive view of an experimentalist, in: Quantum [Un]speakables (2002) 119-153; quant-ph/0402001

Bell 1966

J.S.Bell, On the problem of hidden variables in quantum mechanics, Rev. Mod. Phys. 38:3 (1966) 447-452

- reconsiders and objects to von Neumann's "proof on the mathematical impossibility of hidden variables in quantum theory"; also, to Jauch and Piron, and Gleason's treatments of the problem

CHSH 1969

. . . There is an extensive literature purporting to prove the inconsistency of hidden-variable theories with the statistical predictions of quantum mechanics. These proofs *, though mathematically valid, rest upon physically unrealistic postulates. **Bell succeeded in replacing these postulates by a physically reasonable condition of locality. He showed that in a Gedankenexperiment of Bohm (a variant of that of EPR) no local hidden-variable theory can reproduce all of the statistical predictions of quantum mechanics.** This result is somewhat ironical in view of Einstein's convictions that quantum mechanical predictions concerning spatially separated systems are incompatible with his conditions for locality unless hidden variables exist. ...

Wigner 1970

It has often been suggested that the stochastic nature of quantum mechanical measurement is not the result of the failure of determinism. Rather, it is suggested, our inability to predict the outcomes of quantum mechanical measurements is due to the lack of of knowledge of the values which some **'hidden parameters'** are assuming. The values of these hidden parameters (the exact nature of which remains unspecified) do uniquely determine the behavior of the system they describe... However, the values of these hidden variables cannot be obtained directly. The quantum mechanical state vectors correspond to statistical distributions of these variables, not to definite values of them...

Bell inequality essentials (for EPR-Bohm)

$A\,,\ B$ – both taking values 1 or - 1 , $\langle\, A\, B\,\rangle_{exp} =$

$= \frac{1}{N}\,(N_{++} - N_{+-} - N_{-+} + N_{--})\,,\ \ N = N_{++} + N_{+-} + N_{-+} + N_{--}$

$A_1\,,\ A_2\,;\ B_1\,,\ B_2$ – all taking values 1 or - 1

$2^4 = 16$ different values

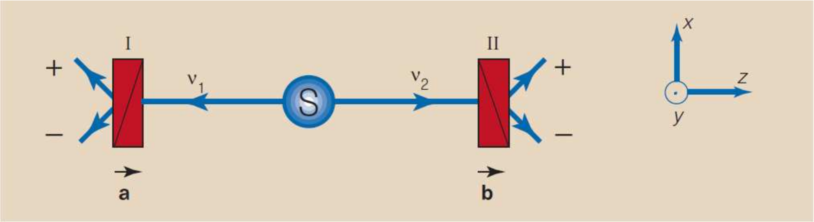
four independent quadratic combinations

$A_i B_j\ :\quad A_1 B_1\,,\ A_1 B_2\,,\ A_2 B_1\,,\ A_2 B_2$

$S := A_1 B_1 - A_1 B_2 + A_2 B_1 + A_2 B_2 =$

$= A_1\,(B_1 - B_2) + A_2\,(B_1 + B_2) = \pm 2$

$\Rightarrow\ |\langle\, S\,\rangle| \leq 2$



EPR-Bohm type experiment with photons (A. Aspect): The two entangled photons travelling in opposite directions away from a source are analyzed (by Alice and Bob) by linear polarizers in orientations a and b, respectively. One measures the probabilities of joint detections in the output channels at various orientations of the polarizers

SETTING (assuming hidden variables (HV))

- a pair of linearly polarized photons emitted by a source in opposite directions, characterized by some supplementary parameters (HV) λ
- two analyzers, A in orientation **a** and B in orientation **b** which may depend on some additional parameters λ'
- the polarizations **a** and **b** are represented by two unit vectors in a plane orthogonal to their common line of propagation
- the possible outcomes of the polarization measurement $A(\mathbf{a}, \lambda)$ are taken 1 for a polarization along **a** and -1 for a polarization in the orthogonal direction in the same plane; similarly for $B(\mathbf{b}, \lambda)$ and **b**

- **IMPORTANT (Bell's locality assumption):**
The result $A(\mathbf{a},\lambda)$ does not depend on the setting b nor $B(\mathbf{b},\lambda)$ on a

- averaging with respect to the analyzers' parameters λ' one should replace $A(\mathbf{a},\lambda)$ and $B(\mathbf{b},\lambda)$ by their mean values which satisfy

$$-1 \leq \bar{A}(\mathbf{a},\lambda) \leq 1 \ , \qquad -1 \leq \bar{B}(\mathbf{b},\lambda) \leq 1$$

- introduce further a normalized probability measure

$$d\,\mu(\lambda) \geq 0, \qquad \int d\,\mu(\lambda) = 1 \ .$$

- the statistical correlation function (expectation value) is given by the mean value of their product, satisfying

$$-1 \le E(\mathbf{a},\mathbf{b}) = \langle\, A(\mathbf{a})\, B(\mathbf{b})\,\rangle_{HV} := \int d\mu(\lambda)\, \bar{A}(\mathbf{a},\lambda)\, \bar{B}(\mathbf{b},\lambda) \le 1\ .$$

Bell-CHSH INEQUALITY:

Let $\mathbf{a}$, $\mathbf{a}'$ and $\mathbf{b}$, $\mathbf{b}'$ be two pairs of different choices of directions for Alice and Bob analyzers, respectively. Then in any local HV theory the following inequality holds:

$$|\, S(\mathbf{a},\mathbf{b},\mathbf{a}',\mathbf{b}')\,| \le 2 \quad \text{for}$$

$$S(\mathbf{a},\mathbf{b},\mathbf{a}',\mathbf{b}') := E(\mathbf{a},\mathbf{b}) - E(\mathbf{a},\mathbf{b}') + E(\mathbf{a}',\mathbf{b}) + E(\mathbf{a}',\mathbf{b}')\ .$$

Proof of Bell-CHSH inequality:

$$
\begin{aligned}
&|E(\mathbf{a},\mathbf{b}) - E(\mathbf{a},\mathbf{b}')| = \\
&= |\int d\mu(\lambda)\bar{A}(\mathbf{a},\lambda)\bar{B}(\mathbf{b},\lambda)\,(1 \pm \bar{A}(\mathbf{a}',\lambda)\bar{B}(\mathbf{b}',\lambda)) - \\
&- \bar{A}(\mathbf{a},\lambda)\bar{B}(\mathbf{b}',\lambda)\,(1 \pm \bar{A}(\mathbf{a}',\lambda)\bar{B}(\mathbf{b},\lambda))\,| \;\leq \\
&\leq \int d\mu(\lambda)\,(1 \pm \bar{A}(\mathbf{a}',\lambda)\bar{B}(\mathbf{b}',\lambda)) + \\
&+ \int d\mu(\lambda)\,(1 \pm \bar{A}(\mathbf{a}',\lambda)\bar{B}(\mathbf{b},\lambda)) = \\
&= 2 \pm (E(\mathbf{a}',\mathbf{b}') + E(\mathbf{a}',\mathbf{b})) \qquad \Rightarrow
\end{aligned}
$$

$$|E(\mathbf{a},\mathbf{b}) - E(\mathbf{a},\mathbf{b}')| + |E(\mathbf{a}',\mathbf{b}') + E(\mathbf{a}',\mathbf{b})| \leq 2\ .$$

Теоретичната възможност за нарушаване на неравенствата на Бел в квантовата механика

The experiment with entangled photons - QM setting

Polarization ("qubit") Hilbert space $\mathcal{H} \simeq \mathbb{C}^2$

Two-photon polarization space $\mathcal{H} \otimes \mathcal{H}$ ($\equiv \mathcal{H}_{Alice} \otimes \mathcal{H}_{Bob}$)

Entangled states - non-factorizable vectors in $\mathcal{H} \otimes \mathcal{H}$

Polarization basis $|+\rangle$, $|-\rangle$ of $\mathcal{H}$
such that $+$ corresponds to polarization along a certain axis, and $-$, along a perpendicular one.

Directions - by the corresponding azimuthal angles:

$$\mathbf{a} = (\cos\theta_1, \sin\theta_1) \ , \quad \mathbf{b} = (\cos\theta_2, \sin\theta_2)$$

A state of polarization $\varepsilon = \pm$ rotated to angle θ is given then by

$$|\,\theta, \varepsilon\rangle := \cos\theta\,|\varepsilon\rangle + \varepsilon\,\sin\theta\,|-\varepsilon\rangle \ , \quad \varepsilon = \pm\,.$$

The analyzer $A(\theta)$ is defined as an operator in $\mathcal{H}$
à la Bell, i.e. through $(A(\theta)-\varepsilon)\,|\theta,\varepsilon\rangle = 0$

In the basis $\{|+\rangle\,,\,|-\rangle\}$ its matrix is

$$A(\theta) = \cos 2\theta\,\sigma_3 + \sin 2\theta\,\sigma_1\,, \quad \sigma_3\,|\varepsilon\rangle = \varepsilon\,|\varepsilon\rangle\,, \quad \sigma_1\,|\varepsilon\rangle = |-\varepsilon\rangle\,.$$

The operators corresponding to the analyzers of the first and the second photon - $A_i(\theta_i)\,,\ i=1,2$:

$$A(\mathbf{a}) \to A_1(\theta_1) = A(\theta_1)\otimes 1\,, \qquad B(\mathbf{b}) \to A_2(\theta_2) = 1\otimes A(\theta_2)\,,$$

A maximally entangled ("Bell") state in the 4-dimensional complex Hilbert space $\mathcal{H}\otimes\mathcal{H}$ is given by

$$\Psi = \frac{1}{\sqrt{2}}\left(\,|+\rangle\otimes|+\rangle + |-\rangle\otimes|-\rangle\,\right) \equiv \frac{1}{\sqrt{2}}\sum_{\varepsilon=\pm}|\varepsilon\rangle\otimes|\varepsilon\rangle\,.$$

The 2-point correlation function of the product $A_1 A_2$ in the state Ψ is then given by

$$E(\theta_1, \theta_2) := \langle \Psi | A_1(\theta_1) A_2(\theta_2) | \Psi \rangle \ .$$

Result: $E(\theta_1, \theta_2) = \cos 2\theta_{12} \ , \quad \theta_{12} := \theta_1 - \theta_2$

Calculation - several ways (elementary but instructive)

(1) directly (matrix Kronecker product, in block form)

$$A(\theta_1) \otimes A(\theta_2) = \begin{pmatrix} \cos 2\theta_1 \, A(\theta_2) & \sin 2\theta_1 \, A(\theta_2) \\ \sin 2\theta_1 \, A(\theta_2) & -\cos 2\theta_1 \, A(\theta_2) \end{pmatrix} \ ,$$

$$\Psi^t = \frac{1}{\sqrt{2}} (1 \ 0 \ 0 \ 1)$$

So $E(\theta_1, \theta_2)$ equals $\frac{1}{2}$ the sum of the corner elements,

$$\frac{1}{2} .2 \left(\cos 2\theta_1 \ \cos 2\theta_2 + \sin 2\theta_1 \ \sin 2\theta_2\right) = \cos 2\theta_{12} \ .$$

(2) Using the spectral decomposition

$$A(\theta) = \sum_{\varepsilon=\pm} \varepsilon\, |\theta, \varepsilon\rangle\langle\theta, \varepsilon|$$

for $A_1(\theta_1)A_2(\theta_2)$. **This gives**

$$E(\theta_1,\theta_2) = \langle\Psi|A_1(\theta_1)A_2(\theta_2)|\Psi\rangle = \sum_{\varepsilon_1,\varepsilon_2} \varepsilon_1\varepsilon_2\, |\langle\theta_1,\varepsilon_1|\otimes\langle\theta_2,\varepsilon_2|\Psi\rangle|^2 =$$

$$= P_{++}(\theta_{12}) + P_{--}(\theta_{12}) - P_{+-}(\theta_{12}) - P_{-+}(\theta_{12})$$

where

$$P_{\varepsilon_1\varepsilon_2}(\theta_{12}) := |\langle\theta_1,\varepsilon_1| \otimes \langle\theta_2,\varepsilon_2|\Psi\rangle|^2 \;= \frac{1}{4}(1+\varepsilon_1\varepsilon_2\cos 2\theta_{12})\ ,$$

$$\text{i.e., } P_{++}(\theta_{12}) = P_{--}(\theta_{12}) = \frac{1}{2}\cos^2\theta_{12}\ ,$$

$$P_{+-}(\theta_{12}) = P_{-+}(\theta_{12}) = \frac{1}{2}\sin^2\theta_{12}\ .$$

(3) Reduction to a subsystem
Start with

$$E(\theta_1, \theta_2) = \langle \Psi | A_1(\theta_1) A_2(\theta_2) | \Psi \rangle \equiv \mathrm{tr} A_1(\theta_1) A_2(\theta_2) | \Psi \rangle \langle \Psi |$$

and use (with w_{ij} the 2×2 Weyl matrices)

$$|\Psi\rangle\langle\Psi| = \frac{1}{2} \sum_{i,j=\pm} w_{ij} \otimes w_{ij} \quad \text{and} \quad \mathrm{tr} A \otimes B = \mathrm{tr} A \, \mathrm{tr} B$$

obtaining $E(\theta_1, \theta_2)$ as trace of a 2×2 matrix product,

$$E(\theta_1, \theta_2) = \frac{1}{2} \mathrm{tr} A(\theta_1) A(\theta_2) = \mathrm{tr} A(\theta_1) A(\theta_2) \rho = \cos 2\, \theta_{12}$$

where $\rho = \frac{1}{2} \mathbb{1} = \frac{1}{2} \sum_\varepsilon |\varepsilon\rangle\langle\varepsilon|$ is the 2×2
***reduced density matrix* corresponding to the *maximally entangled* pure state $|\Psi\rangle\langle\Psi|$ of the composite system.**

It follows that the quantum mechanical counterpart of $S(\mathbf{a},\mathbf{b},\mathbf{a}',\mathbf{b}')$ is

$$S(\theta_1,\theta_2,\theta_3,\theta_4) = E(\theta_1,\theta_2) - E(\theta_1,\theta_4) + E(\theta_3,\theta_2) + E(\theta_3,\theta_4) =$$

$$= \cos 2\,\theta_{12} - \cos 2\,\theta_{14} + \cos 2\,\theta_{23} + \cos 2\,\theta_{34} \;.$$

Setting the consecutive differences equal to each other, $\theta_{ii+1} = \theta\,,\;\; i = 1,2,3$ **we find the following global extremal points of the function** $S(0,\theta,2\theta,3\theta)$**:**

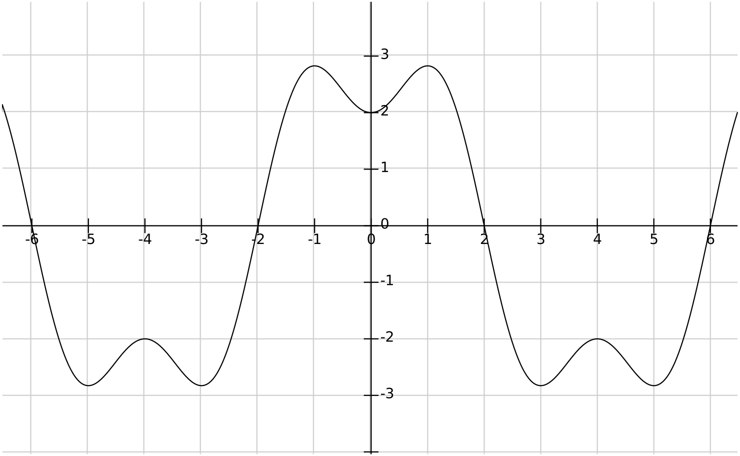
$$\theta = \frac{\pi}{8}\,(2k+1)\;, \qquad k \in \mathbb{Z}\;.$$

For $\theta = \frac{\pi}{8} = 22.5^o$ **and** $\theta = \frac{3\pi}{8} = 67.5^o$ **this gives**

$$S(0,\theta,2\theta,3\theta) = \begin{cases} 3\,\cos\frac{\pi}{4} - \cos\,\frac{3\pi}{4} = 4\,\frac{1}{\sqrt{2}} = 2\sqrt{2} \\ 3\,\cos\frac{3\pi}{4} - \cos\,\frac{9\pi}{4} = -4\,\frac{1}{\sqrt{2}} = -2\sqrt{2} \end{cases} \;.$$

At these angles QM predicts violation of the Bell-CHSH inequality by more than 40%.

Violation of the Bell inequalitiy for the Bell state Ψ



Plot of the function $3\cos\frac{\pi}{4}x - \cos\frac{3\pi}{4}x$

"Tsirelson's bound" (B.S. Tsirelson 1980): Quantum Mechanics predicts a maximum value for $|S|$ of $2\sqrt{2}$

Експерименти със сплетени фотони.

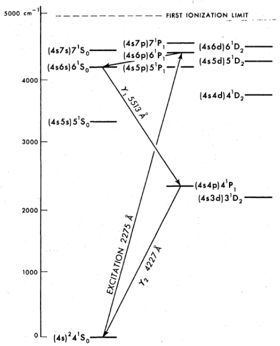
Запушване на "пробойните"(loopholes).

Приложения

J. Clauser - S. Freedman - L. Berkeley Nat. Lab, 1972

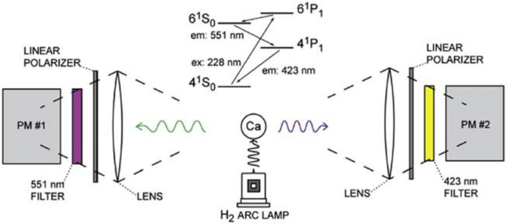
C.A. Kocher, E.D. Commins, Polarization correlation of photons emitted in an atomic cascade, Phys. Rev. Lett. **18** (1967) 575-577

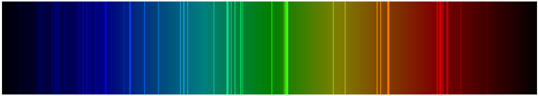
Energy level diagram for calcium (Kocher-Commins)



John Clauser - Stuart Freedman Bell-CHSH experiment: Kocher-Commins Ca cascade setting

Bengt Nordén, Entangled photons from single atoms and molecules, Chemical Physics **507** (2018) 28–33





- two lasers instead of an arc lamp to directly excite the $6\,^1S_0$ state by means of two-photon absorption – much more effective
- better polarizers – leading to excellent statistics and the largest violation of Bell inequalities at the time
- first addressing (with collaborators) the locality loophole in 1982 experiments with acousto-optical devices to rotate the orientation of the two polarizers (in < 20 ns)

LOOPHOLES

(1) spatial correlation loophole

(2) locality (communication) loophole

(3) detection efficiency loophole

(1) the spatial correlation loophole

The original Bell inequality experiments employing two photon atomic cascades suffered from spatial correlation: the two entangled cascade photons were produced in the presence of a third particle, the atom, that carried off some momentum and smeared out the spatial correlation between them.

Problem efficiently eliminated by employing experiments based on two-body processes

LOOPHOLES closing

(2) the locality (communication) loophole

first proposed (in 1976) by Alain Aspect and

performed by him, with collaborators, in 1982

experiments with acousto-optical devices

to rotate the orientation of the polarizers

in less than the 20 ns

(the photon flight time for ~ 6 m distance)

...

Dominik Rauch,..., A. Zeilinger, Cosmic Bell test using random measurement settings from high-redshift quasars-PRL 121 (2018) 080403

(3) the detection efficiency loophole

(closed by Anton Zeilinger and others):

no detector has 100 percent efficiency,

some photons are invariably lost,

and if Nature is cruel, they can conspire

to fake a violation of Bell inequality

Only in the years 2015-2017 several groups managed to simultaneously close both the locality and detection loopholes

Entanglement between many particles

Daniel Greenberger - Horne - Zeilinger (GHZ) states 1989 (+ Shimony 1990)

$$\Psi_{GHZ} = \frac{1}{\sqrt{2}}(|+++\rangle + |---\rangle)$$

allows to perform a Bell test without a statistical analysis (D. Mermin 1990); Zeilinger et al: in 1999, realization of Ψ_{GHZ}; in 2000, Bell inequality violation

Entanglement as a resource

Entangled particles – from just an instrument to main object of interest

Overcoming the "no cloning" theorem

((W.K. Wootters, W.H. Zurek, Nature **299** (1982) 802; D. Dieks, Phys. Lett. **A** (1982) 271):

No operator $\hat{U}$ exists s.t.

$$\hat{U}\,\psi \otimes \phi = \psi \otimes \psi$$

for a given ϕ and any ψ.

Problem: on average, every second photon is lost in a 10 km long optical fibre,
so how to amplify quantum signals ??

'Quantum teleportation'
Despite the no-cloning theorem, the possibility to 'teleport' an arbitrary quantum state from one position to another remains, so long as the original copy is destroyed (C.H. Bennett, G. Brassard et al., 1993). First experiments were performed in 1997 by the groups of Anton Zeilinger and Francesco de Martini.

Essential idea: sharing a Bell pair

'Entanglement swapping'
Similar idea, of practical importance for quantum telecommunication (first experiment, in 1998, by Jian-Wei Pan, Anton Zeilinger and others).

Quantum networks

(Jian-Wei Pan et al 2017, Chinese quantum communication satellite Micius)
Survival of two-photon entanglement observed and a violation of Bell inequality by 2.37 ± 0.09 using quantum communication satellite; later, with Zeilinger group – distributing entanglement through satellite communication China - Austria

Quantum Key Distribution

- In 2006 Zeilinger et al. used the Eckert (1991) QKD protocol to establish a secure key btw La Palma and Tenerife (144 km apart) using polarization entangled photon pairs. The obtained CHSH $S = 2.508 \pm 0.037$ demonstrating violation of the "local realistic"limit by more than 13 standard deviations. ...
- In 2022 three groups used loophole-free Bell tests to experimentally realize device independent QKD protocols (closing loopholes this time in order to fight eavesdropping) .

Additional references

Sergio Doplicher, The measurement process in Local Quantum Physics and the EPR paradox, Commun. Math. Phys. 357 (2017) 407–420; arXiv: 0908.0480

Lee A. Rozema et al., Violation of Heisenberg's measurement-disturbance relationship by weak measurements, arXiv: 1208.0034

Tim Maudlin (Professor of Philosophy at NY Univ. Author of Quantum Non-Locality and Relativity. Founder and director of the John Bell Institute for the Foundations of Physics), What the Nobel prize gets wrong about quantum mechanics (06.10.2022)

Shlomo Sternberg, Group theory and physics, Cambridge University Press (1994), Appendix F. A history of 19th century spectroscopy

Shlomo Sternberg, Group Theory and Physics, Cambridge 1994; Appendix F.
A history of 19th century spectroscopy

1802, William H. Wollaston (1766-1828)
and
1814, Joseph von Fraunhofer (1787-1826)
discover dark (absorption) lines
in the solar spectrum

After 1821 Fraunhofer developed the diffraction grating method

1868, Anders Jonas Angstrom
precision spectral measurements

1885, Johann Jakob Balmer (1825-1898)
Balmer H-series (m= 3,4,5,6, then m=7)

$$\lambda_m = \frac{m^2}{m^2-4}\, b \ , \qquad b = 3645.6\ \overset{o}{A} \quad (\text{later } 3646.13\ \overset{o}{A})$$

Johannes Robert Rydberg (1854-1919)
expressed results in terms of wave number,
e.g. for Balmer series

$$\frac{1}{\lambda_m} = R\left(\frac{1}{2^2} - \frac{1}{m^2}\right) \ , \ \ R = \frac{4}{b} = 1.097.10^7 m^{-1}$$

1913, Bohr model of the hydrogen atom

+ Planck black-body radiation law (1900)
– main prerequisites to QM

БЛАГОДАРЯ ЗА ВНИМАНИЕТО!