# XIV. International Workshop

# LIE THEORY AND ITS APPLICATIONS IN PHYSICS 20 - 26 June 2021, Sofia, Bulgaria

# **ABSTRACTS**

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# Cristian Anghel Lie algebroids and weight systems

We intend to put the Rozansky-Witten weight systems obtained from Lie algebroids by Voglaire & Xu, into the general machine provided by Konsevich in the context of foliations and formal geometry. If time permits, we will provide also some applications from holomorphic foliations. This is a joint work with Dorin Cheptea.

## Lilia Anguelova Primordial black holes from rapid turns in two-field models

We study rapid-turn trajectories in a class of two-field cosmological models, whose scalar manifold is the Poincaré disk. Background solutions in multi-field inflation, with field-space trajectories exhibiting sharp turns, can seed primordial black hole (PBH) formation. We investigate a class of exact solutions with hidden symmetry and show that they exhibit the kind of transient rapid-turn period, needed to induce PBH generation. Furthermore, we relax the symmetry condition and find, in a certain regime, modified solutions with improved behavior of the Hubble  $\eta$ -parameter, which preserve the desired shape of the turning rate function. Interestingly, the modified solutions describe a brief ultra-slow roll phase, followed by long-term slow roll inflation. It is notable that slow roll occurs near the center (not near the boundary) of the Poincaré disk, unlike in the standard  $\alpha$ -attractor case.

#### Sinya Aoki

## Bulk reconstruction from a scalar CFT at the boundary by the smearing with the flow equation

We first introduce our method proposed some time ago to construct a d+1 dimensional bulk field from a primary scalar field in an arbitrary d dimensional conformal field theory using the smearing with the free flow equation. We show that an induced metric defined from the flowed field generally corresponds to the quantum information metric, called the Bures or Helstrom metric, if the flowed field is normalized appropriately. We verify that the induced metric computed explicitly with this smearing always becomes the Euclidean Anti-de-Sitter (AdS) metric when the theory is conformal, even though the bulk isometry is slightly broken. As a new attempt, we modify the flow equation so that the conformal symmetry at the boundary turns into the bulk isometry exactly under the smearing. We apply this new flow equation to derive the bulk-to-boundary propagator, which agrees with the known result. We also derive the metric modified in the presence of the scalar primary state at the boundary, which is shown to be described by the pure gravity plus the bulk massive scalar field in the AdS. We finally discuss further application of the smearing with this new flow equation.

#### Koichi Arashi

# Coherent state representations of the holomorphic automorphism group of a bounded homogeneous domain

Coherent state representations are generalizations of the highest weight representations of semisimple Lie groups to a wider class of Lie groups. Generic coherent state rep-

resentations of connected unimodular Lie groups were studied and classified by W. Lisiecki. In this talk, we classify irreducible coherent state representations of a specific non-unimodular Lie group, which is the holomorphic automorphism group of the tube domain over the dual of the Vinberg cone. The restrictions of the representations to a maximal connected split solvable subgroup play important roles in our classification.

#### Hulya Arguz

# Donaldson–Thomas invariants of quivers with potentials from the flow tree formula

A categorical notion of stability for objects in a triangulated category was introduced by Bridgeland. Donaldson–Thomas (DT) invariants are then defined as virtual counts of semistable objects. We will focus attention on a natural class of triangulated categories defined via the representation theory of quivers with potentials, and explain how to compute DT invariants in this case from a smaller subset of "attractor invariants" which are known in many cases. For this we investigate wall-crossing in the space of stability conditions, and prove a flow tree formula conjectured by Alexandrov-Pioline in this setup. This is joint work with Pierrick Bousseau.

### **Paolo Aschieri**

## Levi-Civita connection and noncommutative gravity

We study the differential geometry of noncommutative manifolds when their Hopf algebras of infinitesimal diffeomorphisms are triangular. This is a generalization of the Drinfeld twist deformation approach to noncommutative differential geometry and gravity. We study the Cartan calculus of Lie derivatives, contraction operators and arbitrary connections, and obtain the Cartan structure equations and the Bianchi identities for curvature and torsion. Existence and uniqueness of the Levi-Civita connection for arbitrary braided symmetric (pseudo)Riemannian metrics is proven. We thus obtain Einstein equations on a quite general class of noncommutative (pseudo)Riemannian manifolds.

# Tatyana Barron

### Entanglement of mixed states in Kahler quantization

We consider the semiclassical limit of entanglement of formation for a sequence of certain mixed states associated with a submanifold of product of two compact integral Kahler manifolds. We make a specific calculation for the 2-dimensional sphere.

## Vladimir Bavula Simplicity criteria for rings of differential operators

Simplicity criteria are found for rings of differential operators (in arbitrary characteristic) which gives an answer to an old question of finding such criterion.

## Almut Beige Local quantum field theories and the Casimir effect

By avoiding certain assumptions, we recently succeeded in quantising the electromagnetic field in position space [1]. Starting from the assumption that the basic building blocks of the electromagnetic field in one dimension are localised bosonic wave packets with a clear direction of propagation, we identified the relevant Schroedinger equation and constructed Lorentz covariant electric and magnetic field observables. Our description contains the standard description of the quantised electromagnetic field which is shown to apply to a subspace of states. In a recent paper, we already showed that our local description of the quantised electromagnetic field can be used to design locally-acting mirror Hamiltonians which reproduce the well-known classical dynamics of light scattering experiments with two-sided semi-transparent mirrors [2]. Here we have a closer look at the Casimir effect and show that a local description of the electromagnetic field provides new insight into the origin of this effect.

[1] D. Hodgson, J. Southall, R. Purdy and A. Beige, Quantising the electromagnetic field in position space, arXiv:2104.04499 (2021). [2] J. Southall, D. Hodgson, R. Purdy and A. Beige, Locally-acting mirror Hamiltonians, arXiv:1908.07597 (2021).

#### Asmus Bisbo

# **PBW-type bases for simple** osp(1|2n)-modules

The Lie superalgebra osp(1|2n) describes the interactions of parabosonic particles, relevant in several areas of theoretical physics. We construct polynomial PBW-type bases for an important class of simple osp(1|2n)-modules know as parabosonic Fock spaces. We present two complementary expressions for the basis elements. Firstly, they are written as polynomials in the odd generators of osp(1|2n) acting on the lowest weight vector. Secondly, they are written as monomials in the generators of the u(n) subalgebra of osp(1|2n) acting on the u(n)-highest weight vectors. Matrix elements corresponding to this basis will also be discussed. This is joint work with Hendrik De Bie and Joris Van der Jeugt.

#### **Nikolay Bobev**

# The unreasonable effectiveness of higher-derivative supergravity in $AdS_4$ holography

I will describe the four-derivative corrections to four-dimensional N=2 minimal gauged supergravity and show that they are controlled by two constants. Interestingly, the solutions of the equations of motion in the two-derivative theory are not modified by the higher-derivative corrections. I will use this to arrive at a general formula for the regularized on-shell action for any asymptotically locally  $AdS_4$  solution of the theory and show how the higher-derivative corrections affect black hole thermodynamic quantities in a universal way. I will employ these results in the context of holography to derive new explicit results for the subleading corrections in the large N expansion of supersymmetric partition functions on various compact manifolds for a large class of three-dimensional SCFTs arising from M2- and M5-branes. I will also briefly discuss possible extensions and generalizations of these results.

# Eugenia Boffo Graded Poisson algebra and effective string actions

A graded Poisson structure of degree 2 together with a homological vector field are equivalent to a Courant algebroid on the generalized tangent bundle. We will show how to construct the effective action for the gravity multiplet of sting theory deforming the 2-graded Poisson algebra and using elements of differential geometry of the generalized tangent bundle.

# Jean-Emile Bourgine Algebraic engineering and integrable hierarchies

The algebraic engineering consists in constructing observables of supersymmetric gauge theories within the representation theory of a quantum group. It is based on the branes system realization in string theory, this system being mapped to a network of modules on which act intertwining operators. The algebraic construction brings new perspectives on many important properties of gauge theories (e.g. AGT- correspondence, dualities, integrability,...). In this talk, I will briefly review the recent advances on this topic, and then use the underlying algebra to revisit the relation between topological strings and the KP integrable hierarchy. This talk will be based on the preprint arXiv:2101.09925.

# Tomasz Brzezinski Beyond rings and algebras

An associative algebra is a vector space with a bilinear, associative multiplication. In this talk we describe a similar algebraic structure consisting of an affine space with a bi-affine, associative multiplication. We explain how such systems encompass algebras (or, more generally and fundamentally, rings) and braces, introduced recently to describe and fully classify solutions of the (set-theoretic) quantum Yang-Baxter equation.

# Ĉestmír Burdík Trace formula for the RTT algebra of sp(4) type

This talk continues our recent results on the Bethe vectors for the RTTalgebra of sp(4) type. We show how it is possible to rewrite the Bethe vectors in a different which is similar to the traceformula for the Bethe vectors of the RTT-algebra of the gl(3) type.

## Marijana Butorac

## Principal subspaces of standard modules of untwisted affine Lie algebras

By using the theory of vertex operator algebras G. Georgiev constructed quasi-particle bases of principal subspaces of standard modules of rectangular highest weights in the case of affine Lie algebra of type  $A_l^{(1)}$ . In this talk we describe quasi-particle bases of principal subspaces of standard modules of rectangular highest weights for the remaining untwisted affine Lie algebras. From quasi-particle bases, we obtain characters of principal subspaces. This talk is based on a joint work with Slaven Kozic.

#### Guillem Cazassus Donaldson theory as a Hamiltonian field theory

We will define an extended field theory in dimensions 1, 2, 3. In dimensions 1 and 2 it is formally similar with the Moore-Tachikawa field theory, but involves different moduli spaces introduced by Huebschmann and Jeffrey.

Our construction is expected to extend to dimension four and to correspond to Donaldson's invariants (ASD instanton counting). We will outline how one could possibly define such an extension, by using a new construction of equivariant Lagrangian Floer homology defined in a joint work with Paul Kirk, Artem Kotelskiy, Mike Miller-Eismeier, and Wai-Kit Yeung.

#### Hadewijch De Clercq

#### Serre presentation for quantum symmetric pair coideals of Kac-Moody type

Quantum symmetric pairs consist of a quantum group  $U_q(g)$ , together with a one-sided coideal subalgebra. The structure theory of these quantum symmetric pair coideals has been developed by Letzter for semisimple Lie algebras g and was later extended to symmetrizable Kac-Moody algebras by Kolb. A crucial tool in the development of their representation theory is their presentation by generators and relations. Up to recently, the most intricate of these defining relations, which are of inhomogeneous quantum Serre type, were only known explicitly for lower rank cases. In this talk I will present novel closed expressions for these quantum Serre relations, valid without restrictions on the Kac-Moody algebra g. These complete the set of defining relations. I will explain how they can be derived from Letzter's projection technique and a binary distributive expansion. Special attention will be drawn to the generalized q-Onsager algebras, a class of split quantum symmetric pair coideals which arise naturally in certain integrable models.

# Branko Dragovich Nonlocal gravity and its cosmology

General relativity (GR), i.e. Einstein theory of gravity, is recognized as one of the best physical theories – with nice theoretical properties and significant phenomenological confirmations. Nevertheless, GR is not a complete theory of gravity and there are many attempts to modify it. One of the actual approaches towards more complete theory is nonlocal modified gravity. Nonlocal gravity model, which I consider here without matter, is given by the action  $S = \frac{1}{16\pi G} \int \sqrt{-g} (R - 2\Lambda + P(R)\mathcal{F}(\Box)Q(R)) d^4x$ , where R is scalar curvature and  $\Lambda$  - cosmological constant. P(R) and Q(R) are some differentiable functions of R.  $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$  is an analytic function of the corresponding d'Alambertian  $\Box$ . I plan to present a brief review of general properties, and then consider cosmological solutions for two concrete functions P(R) and Q(R). Derivation of equations of motion is presented in [1].

The first case is  $P(Q) = R(Q) = \sqrt{R - 2\Lambda}$ . One of the exact cosmological solu-

tions is  $a(t) = At^{\frac{2}{3}} e^{\frac{\Lambda}{14}t^2}$ , which mimics the dark matter and the dark energy [2]. In the second case  $P(Q) = R(Q) = R - 4\Lambda$ . It contains exact cosmological solution  $a(t) = A\sqrt{t}e^{\frac{\Lambda}{4}t^2}$  that mimics properties similar to the radiation and the dark energy [3].

[1] I. Dimitrijevic, B. Dragovich, Z. Rakic and J. Stankovic, "Variations of infinite derivative modified gravity", Springer Proc. Math. Stat. **263** (2018) 91–111.

[2] I. Dimitrijevic, B. Dragovich, A.S. Koshelev, Z. Rakic and J. Stankovic, "Cosmological solutions of a nonlocal square root gravity," Phys. Lett. B **797** (2019) 134848.
[3] I. Dimitrijevic, B. Dragovich, A.S. Koshelev, Z. Rakic and J. Stankovic, "Some cosmological solutions of a new nonlocal gravity model", Symmetry **2020**, 12, 917 (2020).

# Ferruccio Feruglio

#### Automorphic forms and fermion masses

Symmetry principles have long been applied to the flavour puzzle. In a bottom-up approach, the variety of possible symmetry groups and symmetry breaking sectors is huge, the predictability is reduced and the number of allowed models diverges. A relatively well-motivated and more constrained framework is provided by supersymmetric theories where a discrete subgroup  $\Gamma$  of a non-compact Lie group G plays the role of flavour symmetry and the symmetry breaking sector spans a coset space G/K, K being a compact subgroup of G. For a general choice of G, K,  $\Gamma$  and a general superpotential, for both rigid and local N = 1 supersymmetric theories. We also describe a concrete model of lepton masses, specializing the construction to the case G = Sp(2g,R), K = U(g) and  $\Gamma$  = Sp(2g, Z).

#### **Veselin Filev**

#### **Backreacted D0/D4 background**

We construct a type II A supergravity background corresponding to a backreacted D0/D4-brane system. The background is holographically dual to the Berkooz–Douglas matrix model, which was recently simulated on a lattice.

#### **Tamar Friedmann**

#### Euler's reflection formula, infinite product formulas, and the correspondence principle of quantum mechanics

We show how to obtain Euler's reflection formula and a systematic set of infinite product formulas from a variational computation of the spectrum of the Hydrogen atom. We explain how Bohr's correspondence principle allows us to carry out our derivations, which involve various combinations of gamma functions, without the need for the limit definition of the gamma function.

## Chih-Hao Fu BCJ duality as a physical realisation of the quantum algebra

The Bern-Carrasco-Johansson (BCJ) duality suggests a symmetry structure for the Yang-Mills and gravity scattering amplitudes which plays an important role in practical amplitude calculations as well as inspiring new formulations. I will briefly review its applications in amplitudes and discuss the recent explanation from quantum groups.

## Alexander Ganchev About bidirectionality in learning, control, inference, physics, etc.

After a short overview of bidirectional transformations, i.e., categorical optics, and their applications in supervised learning, game theory, or Bayesian inference, we speculate on their use in control theory, open dynamical systems and port-Hamiltonian systems, and quantum physics.

#### **Richard Garavuso**

## Curvature constraints in heterotic Landau-Ginzburg models

In this paper, we study a class of heterotic Landau-Ginzburg models. We show that the action can be written as a sum of BRST-exact and non-exact terms. The nonexact terms involve the pullback of the complexified Kähler form to the worldsheet and terms arising from the superpotential, which is a Grassmann-odd holomorphic function of the superfields. We then demonstrate that the action is invariant on-shell under supersymmetry transformations up to a total derivative. Finally, we extend the analysis to the case in which the superpotential is not holomorphic. In this case, we find that supersymmetry imposes a constraint which relates the nonholomorphic parameters of the superpotential to the Hermitian curvature. Various special cases of this constraint have previously been used to establish properties of Mathai-Quillen form analogues which arise in the corresponding heterotic Landau-Ginzburg models. There, it was claimed that supersymmetry imposes those constraints. Our goal in this paper is to support that claim. The analysis for the nonholomorphic case also reveals a constraint imposed by supersymmetry that we did not anticipate from studies of Mathai-Quillen form analogues.

#### **Esther García**

#### Ad-nilpotent elements in semiprime algebras with involution

In this talk we classify nilpotent inner derivations in semiprime associative algebras R with involution \* and in the Lie algebra of skew-symmetric elements Skew(R, \*). The classification depends on the equivalece class their indices of nilpotency modulo 4, and in certain cases produces a GPI in R that forces the structure of R itself.

## Miguel Angel Gómez Lozano

#### sl<sub>2</sub>-triple and von Neumann regular elements in associative rings

Let R be an associative ring and let  $a \in R$  be a nilpotent element. We show that a is part of a  $\mathfrak{sl}_2$ -triple, i.e., there exists a, h, f where h = [a, f], [h, a] = -2a and

[h, f] = 2f, if and only if for every  $k \in \mathbb{N}$ ,  $a^k$  is a von Neumann regular element.

## Falk Hassler Poisson-Lie T-duality, integrability and quantum corrections

Poisson-Lie groups emerge naturally in the classical limit of quantum groups. Besides their important role in mathematics, they are also central to the phenomena of T-duality in physics. Originally, T-duality arises in the context of string theory but over that last decade it has also become an essential tool to study integrable two-dimensional  $\sigma$ models. While this approach works very well in the classical regime, we only started to understand its implications for quantum corrections last year. After giving an introduction to Poisson-Lie T-duality and integrable  $\sigma$ -models, I will discuss these recent developments and their implications.

#### Yang-Hui He

#### Machine-learning mathematical structures

We review a variety of recent experiments on extracting structure from machine-learning mathematical data that have been compiled over the years. Focusing on supervised machine-learning on labeled data from different fields ranging from geometry to representation theory, from string theory to number theory, we present a comparative study of the accuracies on different problems. The paradigm should be useful for conjecture formulation, finding more efficient methods of computation in mathematics and theoretical physics, as well as probing into certain hierarchy of "randomness" in mathematics.

#### Malte Henkel

# Non-equilibrium dynamics of the quantum spherical model: exact solution

The collective and purely relaxational dynamics of quantum many-body systems after a quench at temperature T = 0, from a disordered state to various phases is studied through the exact solution of the quantum Langevin equation of the spherical and the O(n)-model in the limit  $n \to \infty$ . The stationary state of the quantum dynamics is shown to be a non-equilibrium state. The quantum spherical and the quantum O(n)model for  $n \to \infty$  are in the same dynamical universality class. The long-time behaviour of single-time and two-time correlation and response functions is analysed and the universal exponents which characterise quantum coarsening and quantum ageing are derived. The importance of the non-Markovian long-time memory of the quantum noise is elucidated by comparing it with an effective Markovian noise having the same scaling behaviour and with the case of non-equilibrium classical dynamics.

## Jiří Hrivnák Quantum particle on lattices in Weyl alcoves

Description of a free quantum particle that propagates on rescaled and shifted Weyl group invariant lattices inside closures of Weyl alcoves is discussed. The amplitudes

of the particle's propagation to adjoined positions are induced by finitely supported hopping functions. Specifically situated on the borders of the Weyl alcoves, Dirichlet and Neumann walls trap the particle by reflecting its hopping amplitudes. The discrete Hamiltonians of the quantum systems are obtained as summations of particularly constructed hopping operators. Using the generalized Fourier-Weyl transforms, the eigenenergies together with the vectors of momentum bases which represent the solutions of the time-independent Schrödinger equations are exactly specified.

# Kwalombota Ilwale Twisted connections on projective modules

A twisted derivation is a linear map on an algebra satisfying a twisted kind of Leibniz rule. Similarly on a module a connection satisfying a kind of twisted Leibniz rule can be constructed. Given an associative algebra and a set of twisted derivations on an algebra, a structure consisting of this pair is introduced. A corresponding left module over such a structure is constructed. In this work we define twisted connections on the constructed module and show that such connections exists on projective module.

# Tsukasa Ishibashi Wilson lines and their Laurent positivity

On the moduli space of G-local systems with some boundary data called the framings and pinnings on a marked surface, we introduce a new class of G-valued functions associated with the arcs between boundary intervals which we call the "Wilson line" functions. These functions are regular G-valued functions, and their matrix coefficients in a finite-dimensional representation of G give rise to Laurent polynomials in each cluster Poisson chart. We show that, for a suitable choice of matrix coefficients, these Laurent polynomials have positive integral coefficients in the cluster Poisson chart associated with any decorated triangulation. This talk is based on a joint work with Hironori Oya.

### **Palle Jorgensen**

#### Unitary representations of Lie groups with reflection symmetry

In its original form, the Osterwalder-Schrader idea served to link Euclidean field theory to relativistic quantum field theory. It links the abelian property of Gaussian processes/fields in the Euclidean setting to the non-commutativity of quantum fields. Since its inception, reflection positivity has been generalized and extended, it has become a powerful tool in non-commutative harmonic analysis, in the theory of unitary representations of Lie groups. Co-authors in this subject include G. Olafsson, and K.-H. Neeb.

## Tekin Karadag Lie structure on Hopf algebra cohomology

We calculate the Gerstenhaber bracket (Graded Lie bracket) on Hopf algebra and Hochschild cohomologies of a Taft algebra which is a nonquasi-triangular Hopf algebra. We show that the bracket is indeed zero on Hopf algebra cohomology of a Taft algebra, as in all known quasi-triangular Hopf algebras. This example is the first known bracket computation for a nonquasi-triangular algebra. We also give a different technique, homotopy lifting, to calculate the bracket on Hopf algebra cohomologies.

## Mariana Kirchbach

### Color confinement on the compactified Minkowski spacetime

At the compactified Minkowski spacetime, whose geometry is  $S^1 \times S^3$ , and which can host only charge neutral  $2^n$  poles, we consider a conformally coupled massless dipole, both as free and interacting. The interaction, a harmonic function to the  $S^3$  Laplace-Beltrami operator, is introduced by a suited conformal deformation of the round metric, which we let depend on two fundamental QCD parametres, the running coupling and the number of colors. In effect, motion within the harmonic potential becomes equivalent to free motion on the deformed metric. Identifying the charge as the colorelectric charge degree of freedom in QCD, and the dipole by a quark-antiquark, i.e. by a meson, we show that the conformal wave operator on the deformed metric provides a pretty good description of a broad range of strong interaction phenomena, such like hadron spectra, electromagnetic form factors, and phase transitions, all reasons for which we view the wave operator under discussion as a realistic quantum mechanical limit to QCD. Once admitted as a new scale next to Lambda, the compactification radius, which remains stable between 0 and about  $3000 MeV^2/c^2$ , and equals 0.56 fm, allows for a re-parametrization of the squared transferred momentum in such a way that the divergence in the strong coupling is removed. Instead, it starts freezing out, thus opening the conformal window. In this fashion, QCD could be regularized in the infrared and treated perturbatively. At the root of the above scheme, we find the dichotomy between Jordan algebras and null-ray cones, which, upon being promoted to a first principle, predicts a flat light cone for QED with propagation of single charges, and a compactified one, for propagation of confined color electric charges. Joint work with T. Popov, J.A. Vallejo.

#### **Roland Kirschner**

#### **Representations of orthogonal and symplectic Yangians**

Exteded Yangian algebras of orthogonal and symplectic types are defined by the Yang-Baxter RLL relation involving the fundamental R-matrix with so(n) or sp(2m) symmetry. We study representations of highest weight characterized by weight function ratios. We consider the algebra relations for the linear and the quadratic evaluations and the resulting conditions imposed on the representation weights. We present expressions of L-operators constructed on underlying Clifford and Heisenberg algebras and characterize their representations.

# Toshiyuki Kobayashi Limit algebras and tempered representations

I plan to discuss the new connection between the following four (apparently unrelated) topics:

- 1. (analysis) Tempered unitary representations on homogeneous spaces
- 2. (combinatorics) Convex polyhedral cones
- 3. (topology) Limit algebras
- 4. (symplectic geometry) Quantization of coadjoint orbits

based on a series of joint papers with Y. Benoist "Tempered homogeneous spaces I-IV".

## Slaven Kožić

## Parafermionic bases of standard modules for affine Lie algebras

The parafermionic currents present a remarkable class of nonlocal vertex operators with variables in fractional powers which go back to A.B. Zamolodchikov and V.A. Fateev. In this talk, we present a construction of combinatorial bases of parafermionic spaces associated with the standard modules of the rectangular highest weights for the non-simply-laced untwisted affine Lie algebras. The construction relies on the quasiparticle bases of the corresponding principal subspaces found by M. Butorac and the author. As an application, we obtain a proof of the character formulae of A. Kuniba, T. Nakanishi and J. Suzuki. This is a joint work with Marijana Butorac and Mirko Primc.

#### **Folkert Kuipers**

#### Space-time stochasticity and second order geometry

In this talk, I will discuss Schwartz-Meyer second order geometry and its relevance to theories of quantum gravity that incorporate a notion of space-time stochasticity. I will illustrate the framework by extending the Nelsonian stochastic quantization scheme to pseudo-Riemannian geometry. The result is a non-perturbative relativistic quantum theory of scalar test particles on Lorentzian space-times. I will conclude with a discussion of several physical predictions of stochastic quantization regarding a theory of quantum gravity.

## Alexis Langlois-Rémillard Finite dimensional representations of the 3D dihedral Dunkl-Dirac symmetry algebra

Given a reflection group and a function invariant under its action, the Dunkl operators form a commutative family of differential operators generalising partial derivatives. The Dunkl-Dirac operator is then obtained by using Dunkl operators instead of partial derivative. With its dual symbol, it realises the superalgebra osp(1|2). We study the symmetry algebra of this osp(1|2) realisation for the reflection groups linked to reducible root systems of rank 3. We construct the finite-dimensional irreducible representations of the symmetry algebra and exhibit one family thereof as the polynomial solutions of the Dunkl-Dirac equation. This is joint work with Hendrik De Bie, Roy Oste and Joris Van der Jeugt.

# Adela Latorre Construction of hypersymplectic Lie algebras

Let (g, J) be a Lie algebra endowed with a complex structure. A symplectic form F on g is said to be pseudo-Kaehler if F(J, J) = F(,) and it is called complex symplectic when F(J, J) = -F(,). Both types of structures have been broadly studied in the literature, and it has been shown that their existence is unrelated. In this talk, we are interested in those pairs (g, J) that simultaneously admit a pseudo-Kaehler form and a complex symplectic one. We will present a method to determine when these forms come from a hypersymplectic structure on g (in the sense of Hitchin). Moreover, the method will allow us to recover the hypersymplectic structure on g from the initial data given by J, the pseudo-Kaeler form, and the complex symplectic form. As an application, we provide a 4-step nilpotent Lie algebra with hypersymplectic structures. To our knowledge, this is the first example with such nilpotency step. Joint work with G. Bazzoni and A. Gil-Garcia.

#### **Robin Lautenbacher**

#### Representations of involutory subalgebras of affine Kac-Moody algebras

We consider the subalgebras of split real, non-twisted affine Kac-Moody Lie algebras that are fixed by the Chevalley involution. These infinite-dimensional Lie algebras are not of Kac-Moody type and admit finite-dimensional unfaithful representations. We exhibit a formulation of these algebras in terms of N-graded Lie algebras that allows the construction of a large class of representations using the techniques of induced representations. We study how these representations relate to previously established spinor representations as they arise in the theory of supergravity. Joint work with: Axel Kleinschmidt, Ralf Köhl, Hermann Nicolai (arXiv:2102.00870 [math.RT])

## Karyn R.J. Le Hur Spin-1/2 models in curved space, light response and quantum transport in topological states

Topological states of matter are characterized by a gap in the bulk of the system referring to an insulator or a superconductor and topological edge modes as well which find various applications in transport and spintronics. The bulk-edge correspondence is associated to a topological number. The table of topological states include the quantum Hall effect and the quantum anomalous Hall effect, topological insulators and topological superconductors in various dimensions and lattice geometries. Here, we discuss classes of states which can be understood from mapping onto a spin-1/2 particle in the reciprocal space of wave-vectors. We develop a geometrical approach on the associated Poincare-Bloch sphere, developing smooth fields, which shows that the topology can be encoded from the poles only. We show applications for the light-matter coupling when coupling to circular polarizations and develop a relation with quantum transport and the quantum Hall conductivity. The formalism allows to include interaction effects. We show our recent developments on a stochastic approach to englobe these interaction effects and discuss applications for the Mott transition of the Haldane and Kane-Mele models. Then, we develop a model of coupled spheres and show the possibility of fractional topological numbers as a result of interactions between spheres and entanglement allowing a superposition of two geometries, one encircling a topological charge and one revealing a Bell or EPR pair. Then, we show applications of the fractional topological numbers C=1/2 in bilayer honeycomb models describing topological semi-metals characterized by a quantized  $\pi$  Berry phase at one Dirac point.

## Andre A. Lima Ramond states of the D1-D5 CFT away from the free orbifold point

The free orbifold point of the D1-D5 CFT must be deformed with a scalar marginal operator driving it to the region in moduli space where the holographic supergravity description of fuzzball microstates becomes available. We discuss the effects of the deformation operator on the twisted Ramond ground states of the CFT by computing four-point functions. One can thus extract the OPEs of the deformation operator with these Ramond fields to find the conformal dimensions of intermediate non-BPS states and the relevant structure constants. We also compute the anomalous dimensions at second order in perturbation theory, and find that individual single-cycle Ramond fields are renormalized, while the full N-cycle ground states of the  $S_N$  orbifold remain protected at leading order in the large-N expansion.

Joint work with G.M. Sotkov and M. Stanishkov.

# Florian Loebbert Yangian symmetry for the masses

Since the rise of the AdS/CFT duality, integrability has proven to be an important tool to advance our understanding of massless QFT. In this talk we demonstrate that integrability is also present in massive QFT in D > 2 spacetime dimensions. We show that large classes of massive Feynman integrals are highly constrained by an infinite dimensional Yangian symmetry. When translated to momentum space, this leads to a novel massive generalization of conformal symmetry. Finally, we argue that these features of Feynman integrals can be understood as the integrability of planar scattering amplitudes in a massive version of the so-called fishnet theory, which is obtained as a double-scaling limit of N=4 super Yang-Mills theory on the Coulomb branch.

### **David Lowe**

## Holography in asymptotically flat spacetimes

I will describe an approach to representing quantum fields in asymptotically flat spacetimes via a holographic construction with a global conformal symmetry on the celestial sphere.

## S.N. Hazel Mak

# Explorations of 10D & 11D superfield supergravity via Lie algebraic methods

For the first time in the physics literature, the Lorentz representations of all compo-

nent fields in 10D, N = 2A, N = 2B, and 11D, N = 1 unconstrained scalar superfields are found. While obstacles that have never been surmounted appear in the traditional Grassmann coordinate expansions in 10D & 11D, our new findings are enabled by expressing the dependence of superfield component expansions on the branching rules of irreducible representations in ordinary Lie algebras, which is remarkably efficient. Interpretations of component fields by Young tableaux methods will be presented. Alongside, we introduce a new graphical method, "tying rules," that provides an alternative to Littlewood's 1950 mathematical results which involve a specific Schur function series. An analogue of Breitenlohner's approach is implemented to scan for superfields that contain graviton(s) and gravitino(s), which are the candidates for the superconformal prepotential superfields of 10D and 11D off-shell supergravity theories. We found for 10D, N = 1 supergravity, a superfield with four symmetrized Lorentz indices would be a supergravity prepotential candidate. For 11D, N = 1, surprisingly the scalar superfield alone can serve as a semi-prepotential superfield, with the superfield with one spinor index being the prepotential superfield.

## Alessio Marrani

#### Theory of invariants in Maxwell-Einstein gravity

I will discuss the interplay of, on one side, the theory of representations and invariants of Lie groups and algebras and, on the other side, the simplest black hole solutions to Maxwell-Einstein gravity coupled to non-linear sigma models, in four space-time dimensions. Recent advances on theories obtained by Kaluza-Klein reduction from five dimensions will be highlighted.

#### John Mashford

#### An eightfold way for QCD through 4-D conformal geometry

We show that the Lie algebra u(2,2) of the structure group U(2,2) of the underlying natural principal bundle associated with any locally conformally flat space-time can be decomposed into an 8 dimensional algebra associated with the electroweak force and another 8 dimensional algebra describing quantum chromodynamics (QCD). The predictions of the QCD sector for tree level quark-quark scattering are shown to coincide precisely with the predictions of the standard QCD theory based on su(3).

#### Toru Masuda

# A new S-matrix formula and extension of the state space in open string field

In this contribution, I will describe a new S-matrix formula in Witten's open string field theory. This formula is a gauge invariant combination of a classical solution  $\Psi$ , a reference tachyon vacuum solution  $\Psi_T$ , on-shell vertex operators  $\{O_j\}$ , and a formal object A which satisfies  $Q_{\Psi}A = 1$  with  $Q_{\Psi}$  the BRST operator around the classical solution  $\Psi$ . By considering an extension of the state space, one can interpret this formula from the viewpoint of Feynman rules with what we call the unconventional propagator. I also plan to comment on the Murata-Schnabl solution, which is a hypothetical classical solution for multiple D-branes that was proposed in 2011 but has yet to be realized, and of which a topological interpretation was claimed by some authors. I argue that the Murata-Schnabl solution is realized in this extended state space, though its topological interpretation is still not clear.

# Volodymyr Mazorchuk

# Bigrassmannian permutations and Verma modules

In this talk I will try to describe an unexpected connection between bigrassmannian permutations and the cokernel of inclusions between Verma modules (over the special linear Lie algebra). An application (and the original motivation) is a complete description of the first extension space from a simple highest weight module to a Verma modules. This is a report on a joint work with Hankyung Ko and Rafael Mrden.

#### Haruka Mori

# Doubled aspects of algebroids and gauge symmetry in double field theory

The metric algebroid proposed by Vaisman (the Vaisman algebroid) governs the gauge symmetry algebra generated by the C-bracket in double field theory (DFT). Moreover, there is a family of algebroids described by the bracket, the pre- and the ante-Courant algebroids etc. We collectively call these the DFT algebroids. We show that the DFT algebroids are obtained by an analogue of the Drinfel'd double of Lie algebroids. We examine geometric implementations of these algebroids in the para-Hermitian manifold, which is a realization of the doubled space-time in DFT.

#### Patrick Moylan

### Velocity reciprocity in flat and curved space-time

Despite efforts by Levy Leblond and others, e.g. V. Berzi, V. Gorini, "Reciprocity Principle and the Lorentz Transformations" J. Math. Phys. 10, 1518-1524 (1969), there still remains even today a widely held misconception that velocity reciprocity follows solely from the relativity of motion principle, where by velocity reciprocity is meant that the velocity of a reference frame S' relative to a given stationary frame S is equal and opposite to the velocity of S relative to S'. With the aim to uphold Levy Leblond and others as to why this misconception is wrong, we consider examples in both curved and flat spacetimes illustrating it.

## Rubén José Muñoz Alcázar A filtration associated to an abelian inner ideal and the speciality of the subquotient of a Lie algebra

Given an abelian inner ideal B of a Lie algebra L such that  $ad_{[B,Ker_L(B)]}^{n-1}([B,Ker_L(B)]) \subseteq B$  for some  $n \in \mathbb{N}$ , we can construct a chain of subsets of L

 $\cdots \subseteq B = \mathcal{F}_{-n} \subseteq \mathcal{F}_{-n+1} \subseteq \cdots \subseteq \mathcal{F}_0 \subseteq \cdots \subseteq \mathcal{F}_{n-1} \subseteq \mathcal{F}_n = L \subseteq \cdots$ 

This chain is a bounded filtration of L, so we have a new Lie algebra

 $\hat{L} = \mathcal{F}_{-n} \oplus \mathcal{F}_{-n+1} / \mathcal{F}_{-n} \oplus \cdots \oplus \mathcal{F}_n / \mathcal{F}_{n-1}$ 

such that its associated Jordan pair,  $V = (\mathcal{F}_{-n}, \mathcal{F}_n/\mathcal{F}_{n-1})$ , is the subquotient  $(B, L/Ker_L(B))$ . Thanks to this filtration, we can prove that the subquotient  $(B, L/Ker_L(B))$  is a special strongly prime Jordan pair when the Lie algebra L is strongly prime and  $Ker_L(B)$  is not a subalgebra of L.

#### Yuta Nasuda Several exactly solvable quantum mechanical systems and the SWKB quantization condition

Supersymmetric quantum mechanics (SUSY QM) has been extensively investigated for many years. In the context of SUSY QM, a WKB-like quantization condition called supersymmetric WKB (SWKB) condition is known. It was supposed to be exact for all known exactly solvable quantum mechanical systems with the shape invariance. Recently, it was claimed that the SWKB condition was not exact for the extended radial oscillator, whose eigenfunctions consisted of the exceptional orthogonal polynomial, even the system possesses the shape invariance. In this talk, we examine the SWKB condition for several "novel" classes of exactly solvable systems, including systems with the multi-indexed Laguerre and Jacobi polynomials or the Krein–Adler Hermite, Laguerre and Jacobi polynomials as the main parts of the eigenfunctions. For all of them, the condition equation is not exactly but approximately satisfied. We also discuss higher order corrections for the SWKB condition for those cases.

#### Fabrizio Nieri

#### Intersecting defects and supergroup gauge theory

We consider 5d supersymmetric gauge theories with unitary groups in the  $\Omega$ -background and study codim-2/4 BPS defects supported on orthogonal planes intersecting at the origin along a circle. We explore the relations between instanton and generalized vortex calculus, pointing out a duality between intersecting defects subject to the  $\Omega$ background and a deformation of supergroup gauge theories, the exact supergroup point being achieved in the self-dual or unrefined limit. When the parent 5d theory is Abelian, we identify the dual to the intersecting defect as the supergroup version of refined Chern-Simons theory via open/closed duality in refined topological string. We also discuss the integrability or BPS/CFT side of the correspondence, finding an interesting large rank duality with super-instanton counting.

#### **Herve Partouche**

# Wave function of the universe: Diffeomeorphism invariance and field redefinitions

We reconsider the wave function of the universe defined as a path integral by Hartle and Hawking, in the case of a homogenous and isotropic universe with positive cosmological constant. We pay special attention to the gauge fixing of time reparametrizations by taking into account the moduli space of lapse functions. However, field redefinitions of the scale factor yield inequivalent diffeomorphism-invariant path-integral measures. For each choice, we lift a long standing ambiguity in the canonical derivation of the Wheeler-DeWitt equation. Remarkably, it turns out that the Hilbert spaces corresponding to all prescriptions yield identical observable predictions at least at the semi-classical level.

## Mateo Paulisic Gauging the higher-spin-like symmetries by the Moyal product

Well established approaches to gauging U(1) transformations or spacetime translations lead to theories of interacting bosons of spin 1 or spin 2. We describe a novel approach to gauging their higher derivative generalizations (i.e. higher-spin-like symmetries), leading to a Yang-Mills like theory defined over a symplectic manifold dubbed "master space". The theory incorporates the starting symmetries by using the Moyal product, has a weakly non-local action functional and it is perturbatively stable. The non-commutative structure allows for a more general, covariant formulation, similar to matrix models. Coupling to matter in various representations is displayed. In the spin-2 sector we find a geometric description reminiscent of teleparallelism, with the induced linear connection opposite to Weitzenböck's.

#### **Anne Pilkington**

# Reflection representations of Coxeter groups over non-commutative rings

We study a class of reflection representations of Coxeter groups on modules over special non-commutative rings arising as quotients of path algebras. We give an alternative description of the rings as subrings of matrix rings over free products of rings determined by the Coxeter matrix, which implies that the ring has certain domain-like properties.

## Elena Poletaeva Representations of principal W-algebra for the superalgebra Q(n)and the super Yangian YQ(1)

We classify irreducible representations of finite W-algebra for the queer Lie superalgebra Q(n) associated with the regular even nilpotent coadjoint orbits. We also obtain a classification of irreducible finite-dimensional representations of the super Yangian YQ(1). Joint work with V. Serganova.

# Todor Popov H-atom and AdS5 symmetry

The hydrogen atom is a simple quantum mechanical system based on a scalar massless field living in a flat Minkowski space. However the full beauty of the conformal symmetry of hydrogen atom is revealed only when we embed the scalar field in 6 dimensions. The conformal symmetry stems from a simple Jordan algebra generated by the Pauli matrices. The bound states of the hydrogen live on a de Sitter space with a constant positive curvature,  $dS_4$ , whereas the scattering states live on a hyperbolic space AdS4. The Hydrogen atom, simplest QED system is based on a conformal triality: three conformally equivalent spaces, the Minkowski space, the deSitter space  $dS_4$  and

the hyperbolic space  $AdS_4$ ; each one is a boundary in  $AdS_5$ . This observation opens the way to consider mesons as bounded states in a system of a quark and anti-quark living on a compactified Minkowski space, a closed space where only neutral charge pairs are allowed, such as the color neutral mesons in QCD. In such a way mesons are modeled as excited states in a simple Hydrogen-like system with their electric charges identified with colors in QCD.

#### **Anton Pribytok**

# $AdS_n$ integrable deformations, algebras and free fermion consistency

We study regular solution space of YBE, where the sector is spanned by spin chains up to 8 vertices with nearest-neighbour interaction. In this framework one finds four solutions, two of which are associated to 6- and 8-vertex models with R-matrices of difference form dependence, whereas overall approach accounts for arbitrary spectral dependence to investigate AdS integrable models and generalisations. The other two appear to be novel one-parameter deformed models that allow embeddings and admit  $AdS_{2,3}$  S-matrices as special cases, we also address their relation to deformed supercoset and Sigma models. In addition, we look into generalised integrable models with 2-, 3- and 4-dimensional local space (including 15-vertex). In the latter case, we look for  $AdS_5$  deformations (generalised Hubbard model) by different methods, which take into account generalisation of  $U_q(psu(2,2|4))$ , we provide discussion on relation to Korepanov generic construction of scattering matrix for Zamolodchikov algebra and potential connection with bi-layer construction of Mitev-Staudacher-Tsuboi and Shiroishi-Wadati. For the aforementioned AdS models we also demonstrate freefermion condition emergence and we prove that it is satisfied for novel  $AdS_{2,3}$  models, further direction includes FF derivation for generalised  $AdS_5$  sector from above. In addition, extended deformations of sl(2) will be shown (extension of Kulish type model), however their BA analysis, cohomological inverse scattering and relation to non-unitary Fishnet-type models are of upcoming investigation. Several other results on  $AdS_{2.3}$  deformed algebras and limiting models expected to appear by June. Based on arXiv: 2003.04332, 2010.11231, 2011.08217, 2105.xxxxx (AdS integrability and deformed algebras).

#### Sanjaye Ramgoolam

#### Four dimensional conformal field theory using representation theory and two dimensional topological field theory

I will explain how operators and correlators of higher dimensional conformal field theories, with a focus on theories at or near four dimensions, can be constructed using two dimensional topological field theory and representation theory. The talk will be based on: arXiv:1403.6646, arXiv:1806.01085, arXiv:2003.08173.

# Lucrezia Ravera On the hidden symmetries of D=11 supergravity

I will present some recent developments regarding supersymmetric Free Differential

Algebras describing the vacuum structure of higher-dimensional supergravity theories. I will focus on the case of D=11 supergravity and, in particular, on the emergence of a hidden superalgebra underlying the theory, explaining the group-theoretical role played by the nilpotent fermionic generator naturally appearing for consistency of the model. I will also discuss the relation between this hidden superalgebra and other superalgebras with 32 supercharges.

# Susanne Reffert

### The large charge expansion

Working in sectors of large charge leads to important simplifications when studying otherwise strongly coupled systems. It is particularly easy to implement in CFTs. I will explain the approach for the simple case of the O(2) model and then go on to show features that appear only in theories with non-Abelian symmetry groups.

#### **Igor Salom**

## Bethe states and Knizhnik-Zamolodchikov equations of the trigonometric Gaudin model with triangular boundary

We will discuss the non-periodic trigonometric sl(2) Gaudin model with triangular boundary, with an emphasis on specific freedom found in the local realization of the generators, as well as in the creation operators used in the algebraic Bethe ansatz. First, we will find Bethe vectors of the Gaudin model and the off-shell action of the generating function of the trigonometric Gaudin Hamiltonians with general boundary terms. Next, by careful choice of the arbitrary functions appearing in our more general formulation, we obtain the solutions to the Knizhnik-Zamolodchikov equations.

#### José María Sánchez

#### On the structure of graded Lie algebras of order 3

Lie algebras of order 3 constitute one of the generalizations of Lie theory which has been proven to have important applications in physics, for instance. This talk will focus on the structure of graded Lie algebras of order 3 with arbitrary dimension and over an arbitrary base field. Under certain conditions, we show that any of such algebras L with a symmetric G-support is the direct sum of its gr-simple ideals  $I_k$  satisfying  $[I_j, I_k^{\bar{0}}] = \{I_j^{\bar{i}}, I_k^{\bar{i}}, L^{\bar{i}}\} = 0$ , with  $\bar{i} \in \{\bar{1}, \bar{2}\}$ , if  $j \neq k$ . This is joint work with E. Barreiro, A.J. Calderón and R.M. Navarro.

## Shin Sasaki

## Integration of DFT algebroids and pre-rackoid in doubled geometry

The integration problem of algebroids, known the coquecigrue problem, in double field theory (DFT) is studied. We focus on the Vaisman (or metric, pre-DFT) and the Courant algebroids that govern the gauge symmetry of DFT. We introduce a notion of the pre-rackoid as a global group-like object for an infinitesimal algebroid structure. We propose a geometric realization of the pre-rackoid structures defined by cotangent paths along doubled foliations in a para-Hermitian manifold. We show that the pre-

rackoid reduces to a rackoid that is the integration of the Courant algebroid when the strong constraint of DFT is imposed. We also comment on the relation between the strong constraint and the Yang-Baxter equation in the rackoid.

#### Naoki Sasakura

# Emergence of Lie group symmetries in oscillatory integrations in matrix and tensor models

In this talk, I will explain coherence phenomena which may be called emergence of Lie-group symmetries as peak structure of oscillatory integrations. I have encountered the phenomena in my past studies of tensor and matrix models, but the phenomena can easily and intuitively be understood to occur in many cases that a target quantity is expressed by an oscillatory integration and has tunable external parameters which can make the integration symmetric under Lie-groups. I will give the intuitive argument which assures the generality of the phenomena, and give some concrete examples encountered in the matrix and tensor models.

#### Gizem Sengor

# Principal and complementary series representations at the late-time boundary of de Sitter

The group SO(d+1,1) is both the conformal symmetry group of d-dimensional Euclidean space and the isometry group of d+1 dimensional de Sitter spacetime. This observation suggests that the de Sitter spacetime, which is one of the maximally symmetric solutions to vacuum Einstein equations, may have a holographic nature. However further properties of this observation are not yet fully understood. On the other hand, the de Sitter spacetime has relevance in primordial and late-time Cosmology. In this talk we will demonstrate how free massive scalar fields in the set up that usually appears in early universe inflationary studies, correspond to the principal series and complementary series representations of the group SO(d+1,1). The representation theory of the group SO(d+1,1) is well established. We will establish the connection between the cosmological fields and group theory representations by introducing late-time operators and conclude with remarks on their two-point functions.

#### Artur Sergyeyev

#### New (3+1)-dimensional integrable system with an algebraic nonisospectral Lax pair

In this talk we present a system which is, to the best of our knowledge, the first known example of an integrable (3+1)-dimensional dispersionless system with nonisospectral Lax pair involving algebraic, rather than rational, dependence on the spectral parameter, which shows that the class of integrable (3+1)-dimensional dispersionless systems with nonisospectral Lax pairs is significantly more diverse than it appeared before. The Lax pair in question is of a novel type, discovered in our earlier work, and is related to contact geometry; we will review the construction in question to make the talk more self-contained. For further details please see A. Sergyeyev, Integrable (3+1)-dimensional system with an algebraic Lax pair, Appl. Math. Lett. 92 (2019), 196–200,

arXiv:1812.02263 and A. Sergyeyev, New integrable (3+1)-dimensional systems and contact geometry, Lett. Math. Phys. 108 (2018), 359–376, arXiv:1401.2122

## **Ood Shabtai**

## Pairs of spectral projections of spin operators

We study the semiclassical behavior of an arbitrary bivariate polynomial, evaluated on certain spectral projections of spin operators, and contrast it with the behavior of the polynomial when evaluated on random pairs of projections.

## Malin Sjodahl The chirality-flow formalism for standard model calculations

Scattering amplitudes are often split up into their color (su(N)) and kinematic (two copies of complexified su(2)) components. Since the su(N) gauge part can be dealt with using flows of color, it should similarly be possible to describe the  $su(2) \oplus su(2)$  kinematics of an amplitude in terms of flows of chirality. In two recent papers (hep-ph:2003.05877 and hep-ph:2011.10075) we showed that this is indeed the case, introducing the chirality-flow formalism for standard model calculations. Using the chirality-flow method — which simplifies the spinor-helicity formalism — Feynman diagrams can be directly written down in terms of Lorentz-invariant spinor inner products, allowing the simplest and most direct path from Feynman diagram to complex number. In this talk, I will introduce this method and show some examples.

#### Andrei Smilga

#### **Spin(7)** instantons in eight dimensions

We explicitly construct topologically nontrivial 8-dimensional gauge field configurations that belong to the algebra spin(7) and are associated with the homotopy group  $\pi 7[\text{Spin}(7)] = \mathbb{Z}$ . Similar constructions for other algebras in different dimensions are briefly discussed.

#### Ilia Smilga

#### Representations with vectors fixed by a Levi subgroup

Let G be a semisimple complex Lie group, and L a Levi subgroup. The main question that will interest us is the following: classify the finite-dimensional irreducible representations (rho, V) of G in which the subspace  $V^L$  of L-invariant vectors is nontrivial. This question is motivated by some geometric problem (about affine manifolds), in which only Levi subgroups of a special form occur: namely, those that arise as the centralizer of the maximal split torus of some real form of G. We will present a complete answer to this question for these particular Levi subgroups. The set of highest weights of representations with this property turns out to be the intersection of the root lattice with some convex polyhedral cone (except for L corresponding to the real group SO(p,q) where there is also a parity condition). The proof involves some generalized versions of Young tableaux, found by Littelmann and slightly improved by myself; similar methods can probably be applied to the other Levi subgroups as well. We will also evoke the following further question (coming from the same geometric problem): among these representations, which ones have the property that the longest word  $w_0$  of the restricted Weyl group (corresponding to the real form defining L) acts nontrivially on this space  $V^L$ ? So far, we have found a complete answer only in the case where the real form is split (i.e. L is just the full maximal torus, and  $V^L$  is the zero-weight space), that I presented at the previous Lie Theory conference. In the general case, I have only experimental results.

#### Denitsa Staicova Testing late time cosmic acceleration with uncorrelated baryon acoustic oscillations dataset

Baryon Acoustic Oscillations (BAO) involve measuring the spatial distribution of galaxies to determine the growth rate of cosmic structure. We derive constraints on cosmological parameters from 17 uncorrelated BAO measurements that were collected from 333 published data points in the effective redshift range  $0.106 \le z \le 2.36$ . We test the correlation of the subset using random covariance matrix. The  $\Lambda$ CDM model fit yields the cosmological parameters:  $\Omega m = 0.261 \pm 0.028$  and  $\Omega \Lambda = 0.733 \pm 0.021$ . Combining the BAO data with the Cosmic Chronometers data, the Pantheon Type Ia supernova and the Hubble Diagram of Gamma Ray Bursts and Quasars, the Hubble constant yields  $69.85 \pm 1.27 km/sec/Mpc$  and the sound horizon distance gives:  $146.1 \pm 2.15 Mpc$ . Beyond the  $\Lambda$ CDM model we test  $\Omega$ KCDM and wCDM. The spatial curvature is  $\Omega k = -0.076 \pm 0.012$  and the dark energy equation of states:  $w = -0.989 \pm 0.049$ . We perform AIC test to compare the 3 models and see that  $\Lambda$ CDM scores best.

# Stoimen Stoimenov

#### **Regime of meta-conformal invariance in spherical model**

We consider an application of meta-conformal invariance to ferromagnetic spherical model, with a biased dynamics and spatially long-ranged initial correlations. It is turned out that the two-point response function of the model, obtained by direct calculations, in long time-limit coincide with a two-point function covariant under ageing subalgebra of an hybrid algebra, called meta-conformal Schrodinger algebra, which we define and explore.

## Axel Tiger Norkvist Minimal embeddings in noncommutative geometry

In noncommutative geometry, one may use real calculi to encode information about certain noncommutative manifolds by pairing a \*-algebra A with a set of derivations and a module over A as analogues of vector fields. We discuss the concept of homomorphisms of real calculi, which can be used to give a general definition of embeddings in noncommutative geometry. Several classical results in differential geometry can then be shown to have analogues in the noncommutative context and we define the notion of mean curvature and minimal embeddings. Using this framework, we show that the noncommutative torus can be minimally embedded into the noncommutative 3-sphere

for a class of perturbed metrics.

#### Ivan Todorov Quantum algebra of the standard model

We explore the  $\mathbb{Z}_2$  graded product  $C\ell_{10} = C\ell_4 \hat{\otimes} C\ell_6$  as a finite quantum algebra of the Standard Model of particle physics. The gamma matrices generating  $C\ell_{10}$  are expressed in terms of left multiplication by the imaginary octonion units and the Pauli matrices. The subgroup of Spin(10) that fixes an imaginary unit (and thus allows to write  $\mathbb{O}=\mathbb{C}\oplus\mathbb{C}^3$  expressing the quark-lepton splitting) is the Pati-Salam group  $G_{PS} = Spin(4) \times Spin(6)/\mathbb{Z}_2 \subset Spin(10)$ . If we identify the preserved imaginary unit with the  $C\ell_6$  pseudoscalar  $\omega_6 = \gamma_1 \dots \gamma_6$ ,  $\omega_6^2 = -1$ , then  $\mathcal{P}_{ex} = \frac{1}{2}(1 + i\omega_6)$ will play the role of the projector on the extended particle subspace including the right-handed (sterile) neutrino. We express the generators of  $C\ell_4$  and  $C\ell_6$  in terms of fermionic oscillators  $a_{\alpha}, a_{\alpha}^*, \alpha = 1, 2$  and  $b_j, b_j^*, j = 1, 2, 3$  describing flavour and colour, respectively. The internal space observable algebra (an analog of the algebra of real functions on space-time) is then defined as the Jordan subalgebra of hermitian elements of the complexified Clifford algebra  $\mathbb{C} \otimes C\ell_{10}$  that commute with the weak hypercharge  $\frac{1}{2}Y = \frac{1}{3}\sum_{j=1}^{3} b_j^* b_j - \frac{1}{2}\sum_{\alpha=1}^{2} a_{\alpha}^* a_{\alpha}$ . We only distinguish particles from antiparticles if they have different eigenvalues of Y. Thus the sterile neutrino and antineutrino (with Y = 0) are allowed to mix into Majorana neutrinos. Restricting  $C\ell_{10}$  to the particle subspace which consists of leptons with Y < 0 and quarks with  $Y \neq 0, tr(\mathcal{P}Y) = 0$  allows a natural definition of the Higgs field  $\Phi$ , the scalar of Quillen's superconnection, as an element of  $C\ell_4^1$ , the odd part of the first factor in  $C\ell_{10}$ . As an application we express the ratio  $\frac{m_H}{m_W}$  of the Higgs and the W-boson masses in terms of the cosine of the theoretical Weinberg angle.

The talk is based on the paper I. Todorov, Superselection of the weak hypercharge and the algebra of the Standard Model, *JHEP*04(2021)164.

#### Olena Vaneeva

#### Generalization of the algebraic method for solving group classification problems

Enhancing and essentially generalizing previous results on a class of (1+1)-dimensional nonlinear wave and elliptic equations  $u_{tt} = f(x, u)u_{xx} + g(x, u)$  ( $(f_u, g_{uu}) \neq (0, 0)$ ), we apply several new techniques to classify admissible point transformations within this class. After extending the algebraic method of group classification to non-normalized classes of differential equations, we solve the complete group classification problem for the class under study up to both usual and general point equivalences. The solution includes the complete preliminary group classification of the class and the construction of singular Lie-symmetry extensions, which are not related to subalgebras of the equivalence algebra. The talk is based on the paper: O.O. Vaneeva, A. Bihlo and R.O. Popovych, Generalization of the algebraic method of group classification with application to nonlinear wave and elliptic equations, Commun. Nonlinear. Sci. Numer. Simulat. 91 (2020), 105419, 28 pp.

### Guillermo Vera de Salas Ad-nilpotent elements in prime associative superalgebras

Let R be an associative prime superalgebra with superinvolution \*. We can consider the Lie superalgebras  $R^- = (R, [,])$  and the skew-symmetric elements K = Skew(R, \*) with the new product  $[x, y] = xy - (-1)^{|x||y|}yx$ . We say that a is ad-nilpotent of  $R^-$  or K if there exists  $n \in \mathbb{N}$  such that  $ad_a^n = 0$  where  $ad_a(x) = [a, x]$ . In this work we have described homogeneous ad-nilpotent elements a of  $R^-$  or K depending on its index of ad-nilpotence. The descriptions depend on the parity of the element: if a belongs to  $R^-$  or K and it is even then the description follows the description in the algebra settings [1]. If a is odd we have studied  $a^2$  appearing many other different possibilities. Finally, we present examples on matrices to reach each one of the cases of the descriptions. This work has been accepted to be published ([2]).

[1] J. Brox, E. García, M. Gómez Lozano, R. Muñoz Alcázar, and G. Vera de Salas. A description of ad-nilpotent elements in semiprime rings with involution. *Bull. Malays. Math. Sci. Soc.*, 2021.

[2] E. García, M. Gómez Lozano and G. Vera de Salas. Nilpotent superderivations in prime superalgebras. *Linear and Multilinear Algebra*, 2021.

# Patrizia Vitale Jacobi sigma models

I will discuss a two-dimensional sigma model associated with a Jacobi manifold. The model is a generalisation of a Poisson sigma model providing a topological open string theory. It is a constrained system, with first class constraints, which generate gauge invariance of the model under diffeomorphisms. I will discuss the reduced phase space and show that it is finite-dimensional, with dimension related to that of the target Jacobi manifold.

## **Yihong Wang**

## Examples of applications of quantum algebra in scattering amplitudes

I will present several recently discovered applications of quantum algebra in the study of scattering amplitudes, including the formulation of Z-amplitudes as solutions of KZ equations, momentum kernel as Shapovalov forms. Also I present a few generally interesting problems for the amplitudes community such as the seemingly puzzle of alpha' =0 and alpha' =infinity limit of string amplitudes, and the algebraic connection of BCJ numerators.

#### S.R. Eric Yang

#### Topologically ordered interacting disordered graphene zigzag nanoribbon

While a disorder-free zigzag graphene nanoribbon (ZGNR) is a symmetry protected topological insulator, an interacting disordered ZGNR is a topologically ordered insulator. It displays e/2 fractional edge charges, spin-charge separation, and doubly degenerate ground states [1, 2]. The charge fractionalization is protected against quantum charge fluctuations by a soft gap. Its topological entanglement entropy is small but universal, independent of interaction and disorder strengths [3]. The obtained result of the topological entanglement entropy shows that the disorder-free phase is critical and becomes unstable in the presence of infinitely small disorder. There is also an important difference in the entanglement spectrum between symmetry protected and topologically ordered ZGNRs: degeneracy of zero eigenvalue of the reduced density matrix is split in the topologically ordered phase. There are several experimental implications. Due to the presence of e/2 fractional charges the tunneling DOS is predicted to be linear at the critical disorder strength. Quantum shot noise may directly measure the tunneling fractional charge of a zigzag graphene nanoribbon. Resonant tunneling measurement through a quantum dot structure made of a rectangular zigzag graphene nanoribbon may also be explored. It would be interesting to investigate other zigzag nanoribbon systems that exhibit antiferromagnetism, e.g., silicene and boron nitride nanoribbons. [1] Y. H. Jeong, S.-R. Eric Yang, and M. C. Cha, J. Phys.: Condens. Matter 31, 265601 (2019). [2] S.-R. Eric Yang, M. C. Cha, H. J. Lee, and Y. H. Kim, Phys. Rev. Res. 2, 033109 (2020). [3] Y. H. Kim, H. J. Lee, and S.-R. Eric Yang, Phys. Rev. B 103,115151(2021).

#### Shuichi Yokoyama

#### Charge conservation, entropy current, and gravitation

I will speak about the summary of my recent works on the proposal of a precise definition of charges with the form of a volume integral on a general curved space-time and its consequences. Employing this definition we conclude that i) the matter energy momentum tensor distributes non-trivially at singularity of a black hole and any block hole cannot be regarded as a vacuum solution black, ii) a mass for a compact star with internal structure contains not only the gravitational mass known as the Misner-Sharp mass, which may be written by a surface integral, but also gravitational binding energy. Subsequently I will explain the proposal of a new class of vector fields to construct a conserved charge in a general field theory whose energy momentum tensor is covariantly conserved. I claim that there always exists such a vector field in a given field theory even without global symmetry and that the conserved current constructed from the vector field can be identified with the entropy current of the system. As pieces of evidence I will explain that the conserved charge defined therefrom satisfies the first law of thermodynamics for an isotropic system, and that the proposed formulation of entropy reproduces a couple of classic results in a more unified viewpoint.

#### George Zoupanos

# N = 1 trinification from dimensional reduction of N = 1, 10D $E_8$ over $SU(3)/U(1) \times U(1) \times Z_3$

We present an extension of the Standard Model that results from the dimensional reduction of the N = 1, 10D  $E_8$  gauge theory over a  $M_4 \times B_0/Z_3$  space, where  $B_0$  is the nearly-Kaehler manifold  $SU(3)/U(1) \times U(1)$  and  $Z_3$  is a freely acting discrete group on  $B_0$ . Using the Wilson flux breaking mechanism we are left in four dimensions with a softly broken N = 1,  $SU(3)^3$  Grand Unified Theory. Below the unification scale we have a two Higgs doublet model in a split-like supersymmetric version of the Standard Model, which yields third generation quark and light Higgs masses within the experimental limits and predicts the LSP 1500GeV.