Scaling Phenomena in Gravity and Yang-Mills Theories, or BH Formation and its Unitarization

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General Considerations

More than fourty years have elapsed since the general theorems on singularities in GR and BH formation have been studied in detail

Apart from BH-Thermodynamics, we have learned throught the work of Bekenstein and Hawking that once QM is brought to bare, we find rather puzzling problems

Hawking went futher, and argued that thermal production of radiations in BH emission implied the fundamental loss of coherence in Quantum Mechanics. We lose information and also unitarity

Most people (including Hawking) believe that the problem is basically solved within the AdS/CFT correspondence. The question is how?, what when wrong in Hawking's original argument?

Cosmic censorship?, no-hair theorems?, numerical relativity?...

Proposed Analyses

Here we cannot be exhaustive, just a few highlights

Entropy counts: D5-D1-P systems

Fuzzballs

Mathur's program

ACV (Amati, Ciafaloni, Veneziano, et al)

Giddings et al

Giddings et al

According to Giddings we should take the problem very seriously

The solution to the puzzle will contain deep messages about Quantum Gravity

There seems to be a clash between locality, Quantum Mechanics, and Diff invariant General Relativity

The issue is: how do we cut the Gordian knot?

Is string non-locality enough? Do we need to advocate other types of non-localities, yet unknown at the level of the horizon? Do BH in the classical sense really form?

Mathur program and its variations

Perhaps trivializing a little, the idea is to construct the micro-states of a BH with configurations without horizons. After coarse graining you may get a picture that simulates for all practical purposes the existence of a horizon.

You can construct fuzzballs (Skenderis Taylor)

Or the explicit constructions of Bena et al.

Or the holographic search of horizon of for instance Hubeny and Rangamany using Eddington-Finkelstein-like coordinates instead of Feffermann-Graham

Holographic renormalization group...

ACV string, unitary S-Matrix

They propose a unitarity S-matrix based on string theory computations, and in the eikonal approximation, leading to an effective action also proposed by Lipatov and his group. There are three regions to discuss:

Eikonal → **Black Hole** ← **Stringy**

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' \, \Delta p$$

Their effective action contains non-local terms due to resummation of diagrams and also to string computations of graviton emission. The S-matrix by construction is unitary, and they can see glimpses of BH formation, with properties that are analogous to expectations from more classical arguments. More work remains...

A Basic Puzzle

The truly difficult thing to understand is to make compatible the picture of the outside and the incoming observers, if big BHs really form. At the horizon of such an object, the curvature is not big, and the in-falling observer should not find anything unusual there according to standard (faulty?) reasoning. What is the holographic description of the in-falling observer?

Since string theory presumably solves the singularity at the center, what is the mechanism that emits the information carried by the in-falling person?

A Missing Link

This is perhaps old fashioned, but one thing that is missing is the analogue of the Oppenheimer-Snyder solutions within String Theory

In the collapse of a pressureless fluid we can follow everthing analytically, see the formation of the horizon, and the fate of the collapsing matter in detail

Formulated differently, and at least in the context of AdS/CFT we should try to relate holographically, situations on the boundary requiring unitarization from the field theory point of view, whose gravity duals exhibit, at least semiclassically the formation of a BH

General philosophy

One of the most important lessons we have learned from the Maldacena conjecture is that the QCD string, is the fundamental string in some higher dimensional geometry. Accumulated evidence points to the relevance of BH's in the holographic description

There are two approximations to YM theories where the string picture appears naturally. One is the 't Hooft large-N limit. For strong 't Hooft coupling, we have a geometric description, at least with enough supersymmetry

The Regge limit, historically at the origin of string theory, with finite g and N, but with very large Log s. In this kinematic regime, resummation of diagrams in terms of (g2 Log s) leads to a stringy behavior of the form $s^{\alpha(t)}$
Is there a holographic interpretation of this behavior?

Basic aim

Scaling phenomena

Gravity ⟷ Yang-Mills

Holography

We believe that the Maldacena conjecture should be extended beyond supersymmetry

So far most scenarios explored have been static, i.e. thermal HP...

We want to find dynamical phenomena that could be related and also observable:

Weakly coupled gauge theories vs strong curvature gravity

...continued

We compare universal properties of BH formation
in the scaling region

with

The scaling properties of the Regge region in YM,
a weakly coupled, but non-perturbative regime.
The hard pomeron world

We will also consider trapped surface formation in
the collision of gravitational shock waves

Perhaps too ambitious...

Scaling in gravitational collapse

Studying rigorously gravitational collapse for a m=0 field coupled to gravity

to answer questions of black hole formation from regular initial conditions,

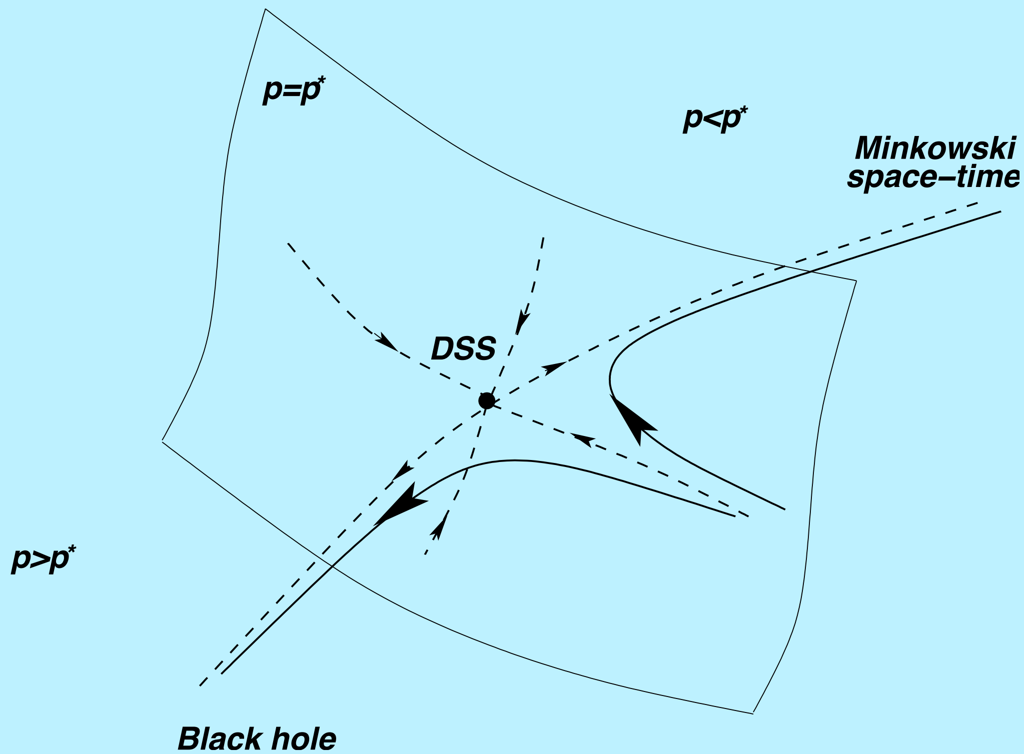
and also the possible appearance of naked singularities, Christodoulou asked:

Is it possible to create black holes with

arbitrarily small mass?

The answer is yes: Type I,II collapse

Critical behavior in phase space



Choptuik's (93) showed the existence of a co-dimension one critical surface.

For generic one parameter families of initial data, parameterized by p, there is a critical value p* where it crosses the critical surface.

There are two possible large time evolutions, or fixed points:

A BH forms with arbitrarily small mass

Or the system bounces and it is radiated away to infinity leaving behind M4

The critical solution has an unstable mode, or relevant direction.

The eigenvalue of the relevant direction leads to the BH critical exponent.

Basic results

For the spherical collapse of the massless scalar field,
the metric takes the form:

$$ds^2 = -\alpha^2(t,r)dt^2 + a^2(t,r)dr^2 + r^2 d\Omega^2_{\mathrm{d}-2}$$

By looking at one-parameter families of initial conditions, Choptuik found the existence of a critical solution. There are two basic properties:

The critical solution is independent of the initial conditions. On the supercritical side, the size of the small BH satisfies a universal scaling law. The critical solution exhibits DSS:

$$r_{BH} \sim (p - p^*)^\gamma \quad ; \quad Z_*(e^{n\Delta}\, t, e^{n\Delta}\, r) = Z_*(t,r)$$

Choptuik vs Liapounov

The numerical values obtained depend only on the type of matter considered, they are "pure" numbers. They do not depend on initial conditions. The critical solution is characterized by having a single unstable direction. Hence computing the Choptuik exponent is related to computing the Liapounov exponent of the small perturbations around the critical solution

$$Z_p(\tau,\zeta) \approx Z_*(\tau,\zeta) + \sum_{k=1}^{\infty} C_k(p) e^{\lambda_k \tau} \delta_k Z(\tau,\zeta)$$

D	Δ	γ
4	$3.37 \pm 2\%$	$0.372 \pm 1\%$
5	$3.19 \pm 2\%$	$0.408 \pm 2\%$
6	$3.01 \pm 2\%$	$0.422 \pm 2\%$
7	$2.83 \pm 2\%$	$0.429 \pm 2\%$
8	$2.70 \pm 2\%$	$0.436 \pm 2\%$
9	$2.61 \pm 2\%$	$0.442 \pm 2\%$
10	$2.55 \pm 3\%$	$0.447 \pm 3\%$
11	$2.51 \pm 3\%$	$0.44 \pm 3\%$

$$\gamma = -\frac{1}{\lambda_1}$$

Perfect fluid collapse

In the relevant scaling limit in YM, there is no echo parameter.

We want a similar symmetry in gravity. This is achieved by studying the collapse of perfect fluids.

The critical solution will have CSS rather than DSS. A region of the space time before the singularity forms has homothety, i.e. a conformal Killing vector of weight 2.

We choose comoving coordinates to describe the spherical collapse of the fluid. The equations are simpler.

Cahill-Taub, Bicknell-Henriksen, Coleman-Evans, Hara-Koike-Adachi, Harada-Maeda. We follow and complete these authors in any d

$$ds^2 = -\alpha(t,r)^2 dt^2 + a(t,r)^2 dr^2 + R(t,r)^2 d\Omega_{d-2}^2$$

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p\, g_{\mu\nu}$$

$$p = k\,\rho, \qquad 0 \le k \le 1$$

Equations of motion

$$
\begin{aligned}
2m_{,r} &= \frac{16\pi}{d-2}\,\rho R_{,r} R^{d-2}, \\
2m_{,t} &= -\frac{16\pi}{d-2}\,p R_{,t} R^{d-2}, \\
2G_N m &= R^{d-3}\left(1+\frac{R_{,t}^2}{\alpha^2}-\frac{R_{,r}^2}{a^2}\right), \\
\frac{\alpha_{,r}}{\alpha} &= -\frac{p_{,r}}{\rho+p}, \\
\frac{a_{,t}}{a} &= -\frac{\rho_{,t}}{\rho+p}-(d-2)\frac{R_{,t}}{R},
\end{aligned}
$$

It is easy to understand physically each of these equations

CSS conditions

$$\tau = -\log(-t), \qquad z = -\frac{r}{t} \qquad \begin{aligned} \eta(\tau,z) &= 8\pi r^2 \rho(t,r), \\ S(\tau,z) &= \frac{R(t,r)}{r}, \\ m(t,r) &= r^{d-3} M(t,r), \end{aligned} \qquad y = (d-2)(d-3)\frac{M}{\eta S^{d-1}}$$

$$\alpha = c_\alpha(\tau)\left(\frac{z^2}{\eta}\right)^{\frac{k}{k+1}}, \qquad a = \eta^{-\frac{1}{k+1}} S^{2-d} \qquad V_z = -\frac{a\,z}{\alpha}$$

$$\begin{aligned} \frac{M'}{M} + (d-3) &= \frac{d-3}{y}\left(1 + \frac{S'}{S}\right) \\ \frac{\dot{M}}{M} + \frac{M'}{M} &= -\frac{(d-3)k}{y}\left(\frac{S'}{S} + \frac{S'}{S}\right) \\ a^2 S^{-2}\left(\frac{2M}{S^{d-3}} - 1\right) &= V_z^2\left(\frac{\dot{S}}{S} - \frac{S'}{S}\right)^2 - \left(1 + \frac{S'}{S}\right)^2 \end{aligned}$$

Regularity conditions

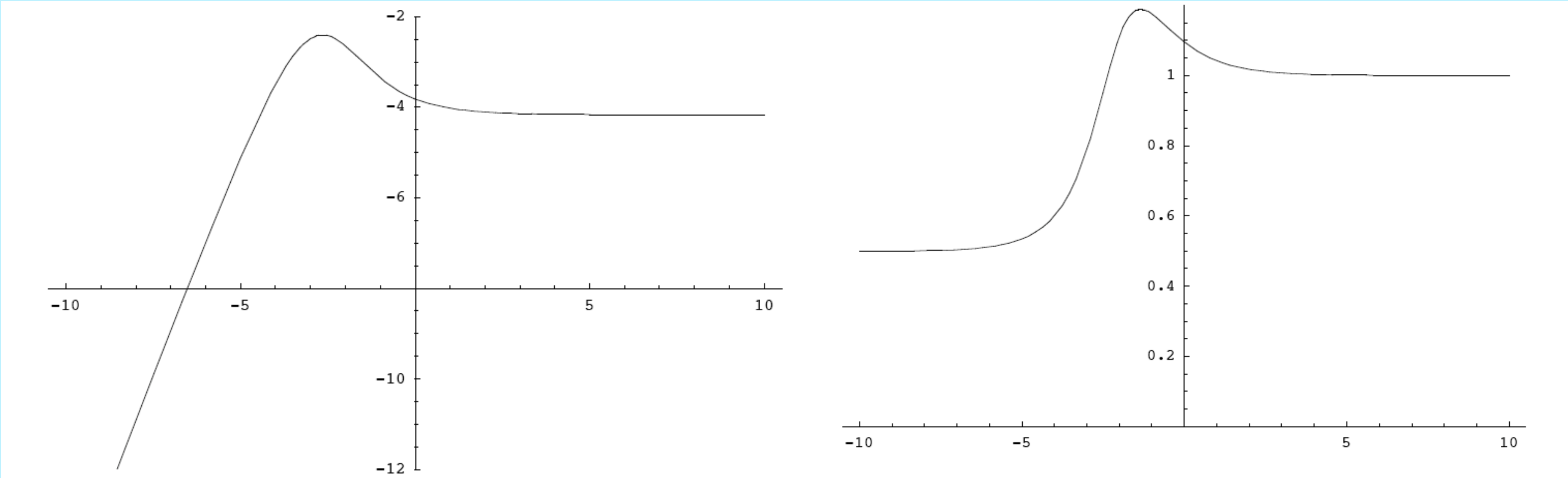
$$\frac{d\log M}{d\log z} = \frac{(d-3)k}{k+1}\left(\frac{1}{y}-1\right)$$
$$\frac{d\log S}{d\log z} = \frac{1}{k+1}(y-1)$$
$$\frac{d\log \eta}{d\log z} = \frac{1}{V_z^2-k}\left[\frac{(1+k)^2}{d-2}\eta^{\frac{k-1}{k+1}}S^{4-2d}-(d-2)(y-1)V_z^2-2k\right]$$

$$y(0^+) = \frac{d-3}{d-1}$$

$$M(z) \simeq \frac{(2D)^{\frac{k}{k+1}}}{(d-2)}\left[\frac{k+1}{(d-1)k+d-3}\right]z^{\frac{2k}{k+1}}$$
$$S(z) \simeq \left[\frac{(2D)^{\frac{1}{k+1}}}{k+1}\left(k+\frac{d-3}{d-1}\right)\right]^{\frac{1}{1-d}} z^{-\frac{2}{(d-1)(k+1)}}$$

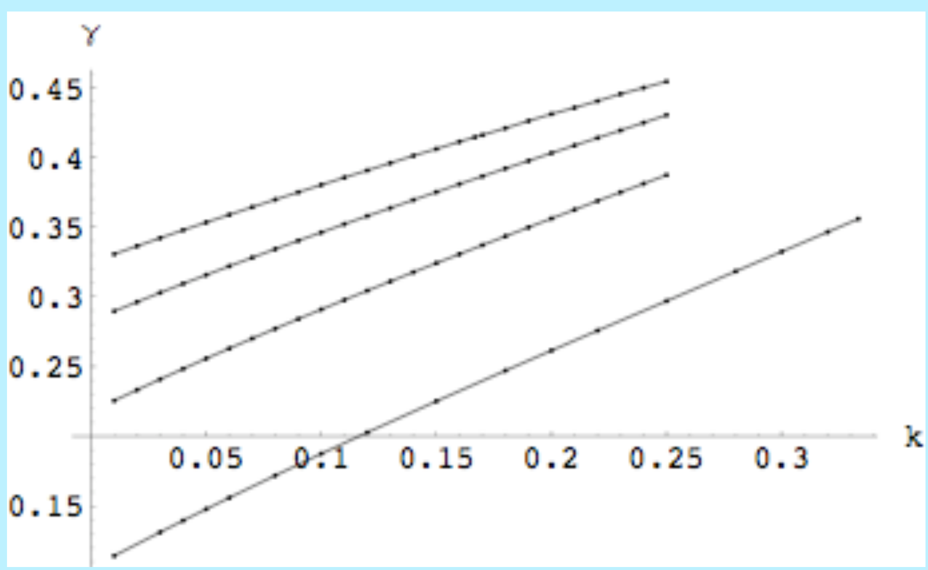
Analyticity at the origin and sonic surface determines a discrete set of D's only functions of k,d. For each such D we can uniquely determine the Choptuik exponent by analyzing the perturbations. Here we have gone beyond what is in the literature, where a crucial equation is missing

Sample numerics



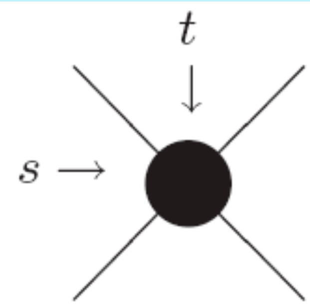
Plot of the critical solution for k=.05, d=5, D=80.62

In the analysis of the perturbations, and the computation of exponents, it is important also to take into account analyticity at the sonic point



BFKL the Regge limit of YM

BFKL is an equation which describes the high-energy limit of weakly coupled YM



$$s >> |t|\,, Q^2$$
$$\alpha_s(Q^2) << 1$$

Large $\ln s$ compensate small α_s: $\alpha_s \ln s \sim 1$ [Balitsky–Fadin–Kuraev–Lipatov]

$$\mathcal{A}(s,t) \sim s^{\alpha(t)} \quad ; \quad s >> 1 \qquad \sigma^{\text{total}} \sim s^{\alpha(0)-1}$$

$$\alpha(0) = 1 + (4 \log 2)\, \alpha_s + \mathrm{O}[\alpha_s^2]$$

Unitarity violation, Froissart-Martin bound. We will be looking at weak coupling. Reggeization processes have also been studied at strong coupling by Brower, Polchinski, Strassler and Tan

Reggeized particles

The notion of reggeization is crucial in the soft pomerons, and it is fundamenal in the BFKL and BK equations

A particle of spin j and mass m reggeizes if when exchanged in the t-channel, the amplitude behaves as

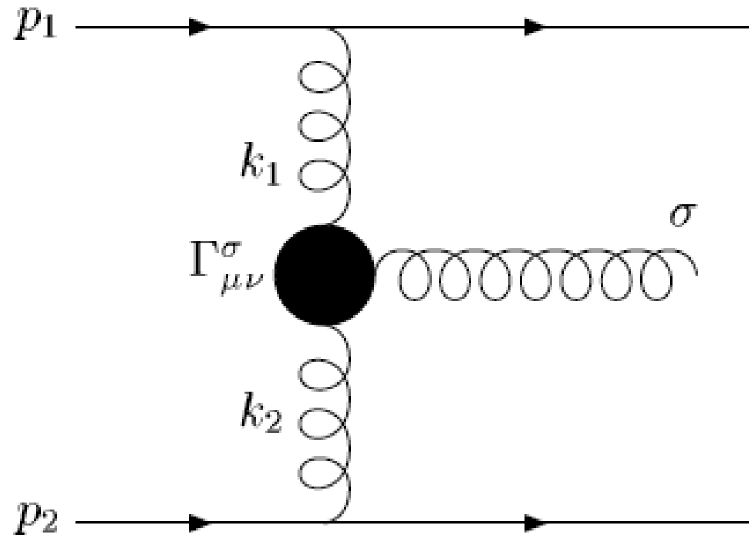
$$s^{\alpha(t)}$$

where in the exponent we have the corresponding Regge trajectory.

A substantial amount of work shows that the gluon reggeizes in the Regge limit

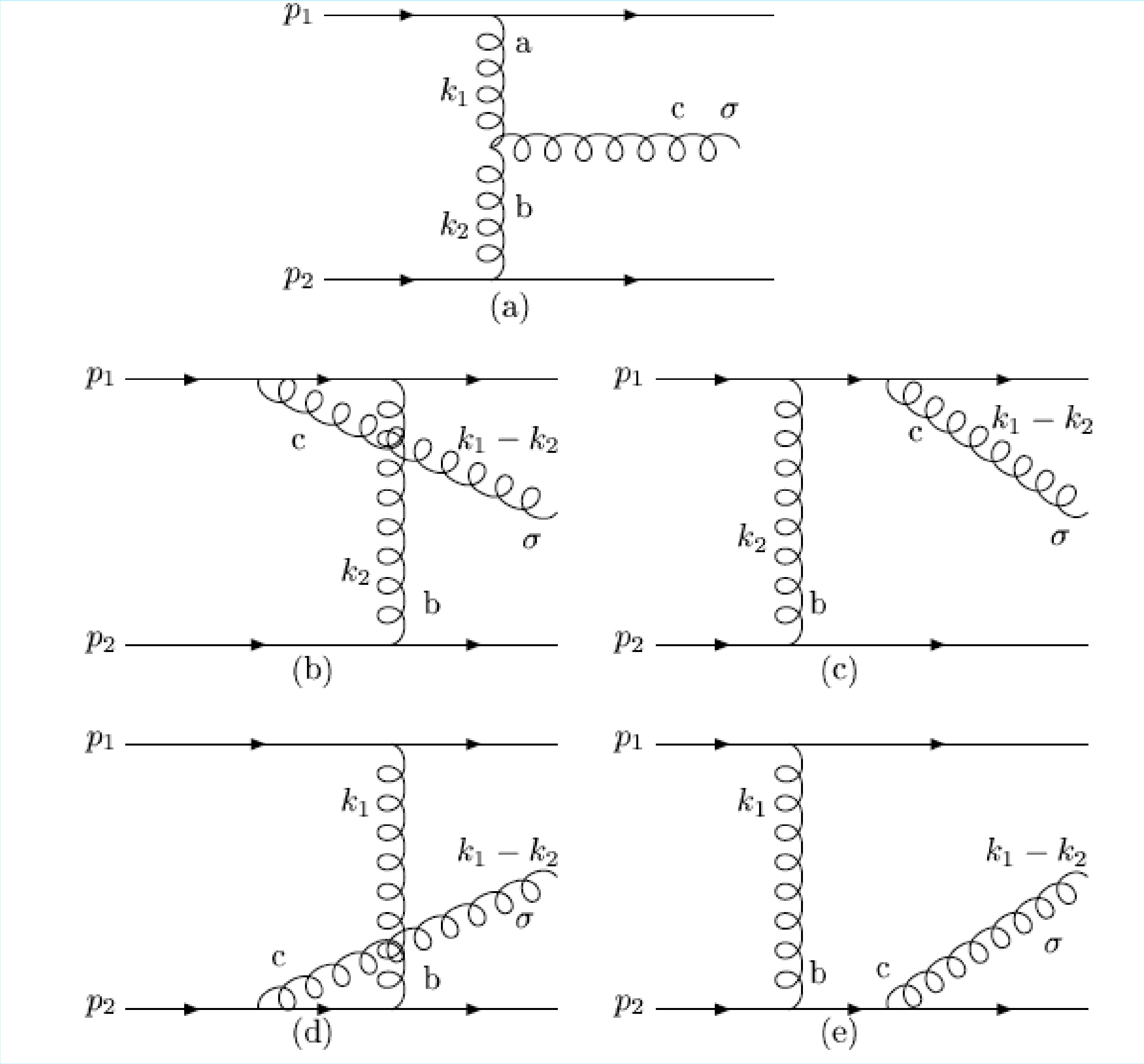
Use of the cutting rules is crucial. In the next few pages we assume that we are computing the absorptive part, and later one can discuss the real part as well

$$k_1^+ \gg k_2^+,\; k_2^- \gg k_1^-$$

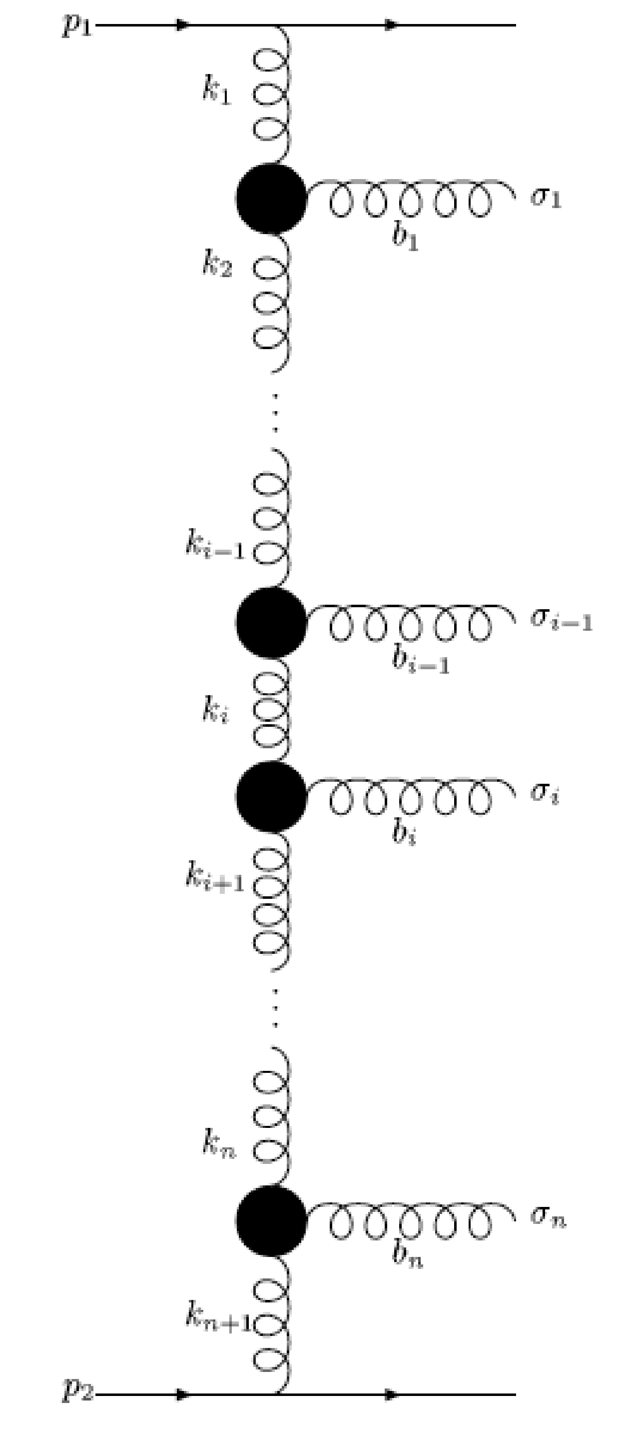


$$\Gamma^{\sigma}_{+-}(k_1,k_2) \;=\; 2gf^{abc}\left(k_1^+ + \frac{2\mathbf{k_1}^2}{k_2^-}, k_2^- + \frac{2\mathbf{k_2}^2}{k_1^+}, -(\mathbf{k_1}+\mathbf{k_2})\right)$$

Graphs contributing to the effective vertex



Effective ladder graphs

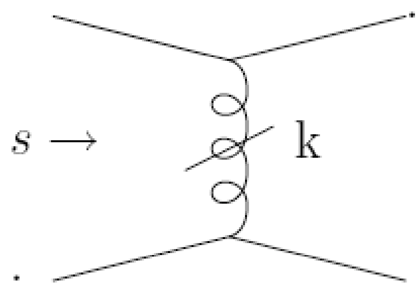


Generalizes to all order in the leading log approximation as is the case in simpler examples keeping the original vertices

Using large amounts of algebra, one can prove reggeization of the gluon. Each vertex is replaced by the effective vertex.

It can be shown that effective planarity is recovered, but now the effective N is related to the original N, and log s

Reggeized gluon

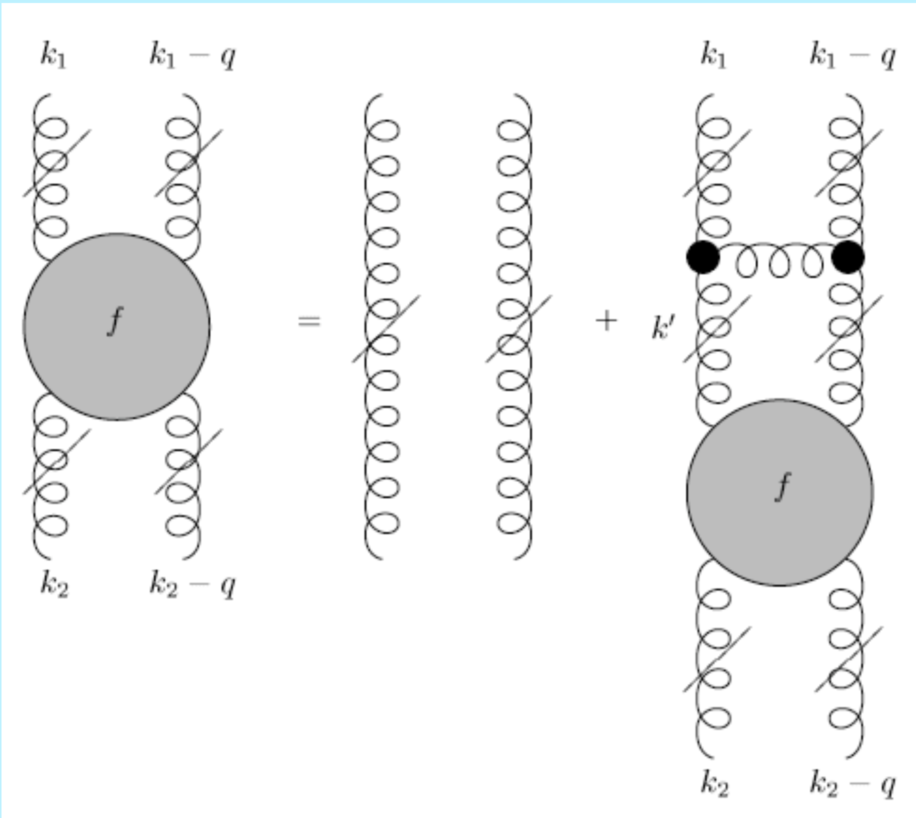


$$\frac{1}{\mathbf{k}^2}\left(\frac{s}{\mathbf{k}^2}\right)^{\varepsilon_G(\mathbf{k}^2)}$$

$$\epsilon_G(\mathbf{q}^2) \;=\; -\frac{\alpha_s C_A}{4\pi^2}\int d^2\mathbf{k}\,\frac{\mathbf{q}^2}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2}$$

Some details, and RG

Like a Bethe-Salpeter equation at leading log



$$f(\sqrt{s}, \mathbf{k_1}, \mathbf{k_2}) = \delta^2(\mathbf{k_1} - \mathbf{k_2}) - \frac{\alpha C_A}{2\pi^2} \int d^2\mathbf{k}' \int^{\sqrt{s}} dk^+ f(k^+, \mathbf{k}', \mathbf{k_2})$$

$$\times \left(\frac{k_1^+}{k^{+\prime}} \right)^{2\varepsilon_G(\mathbf{k}'^2)} \frac{\Gamma^{\sigma}_{+-}(k_1, k')\Gamma^{\sigma}_{+-}(k_1, k')}{\mathbf{k}'^4}$$

This is similar to integrating out the fast, longitudinal degrees of freedom and working with the effective transverse hamiltonian.

This hamiltonian exhibits scale (SL(2,C)) invariance

Eigenfunctions and eigenvalues

Eigenfunctions:, $\phi_{n,\nu}(\mathbf{k}) = (k^2)^{-1/2+i\nu} e^{in\theta}$

Eigenvalues: $\frac{\alpha_s C_A}{\pi} \chi_n(\nu)$

$$\chi(\nu) = 2\Psi(1) - \Psi\left(\frac{(n+1)}{2} + i\nu\right) - \Psi\left(\frac{(n+1)}{2} - i\nu\right)$$

General solution $(\mathbf{k} = (k, \theta))$

$$\tilde{f}(\omega, \mathbf{k_1}, \mathbf{k_2}) = \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{d\nu}{2\pi^2 \mathbf{k_1} \mathbf{k_2}} \left(\frac{\mathbf{k_1}^2}{\mathbf{k_2}^2}\right)^{i\nu} \frac{e^{in(\theta_1 - \theta_2)}}{\omega - \overline{\alpha_s} \chi_n(\nu)}$$

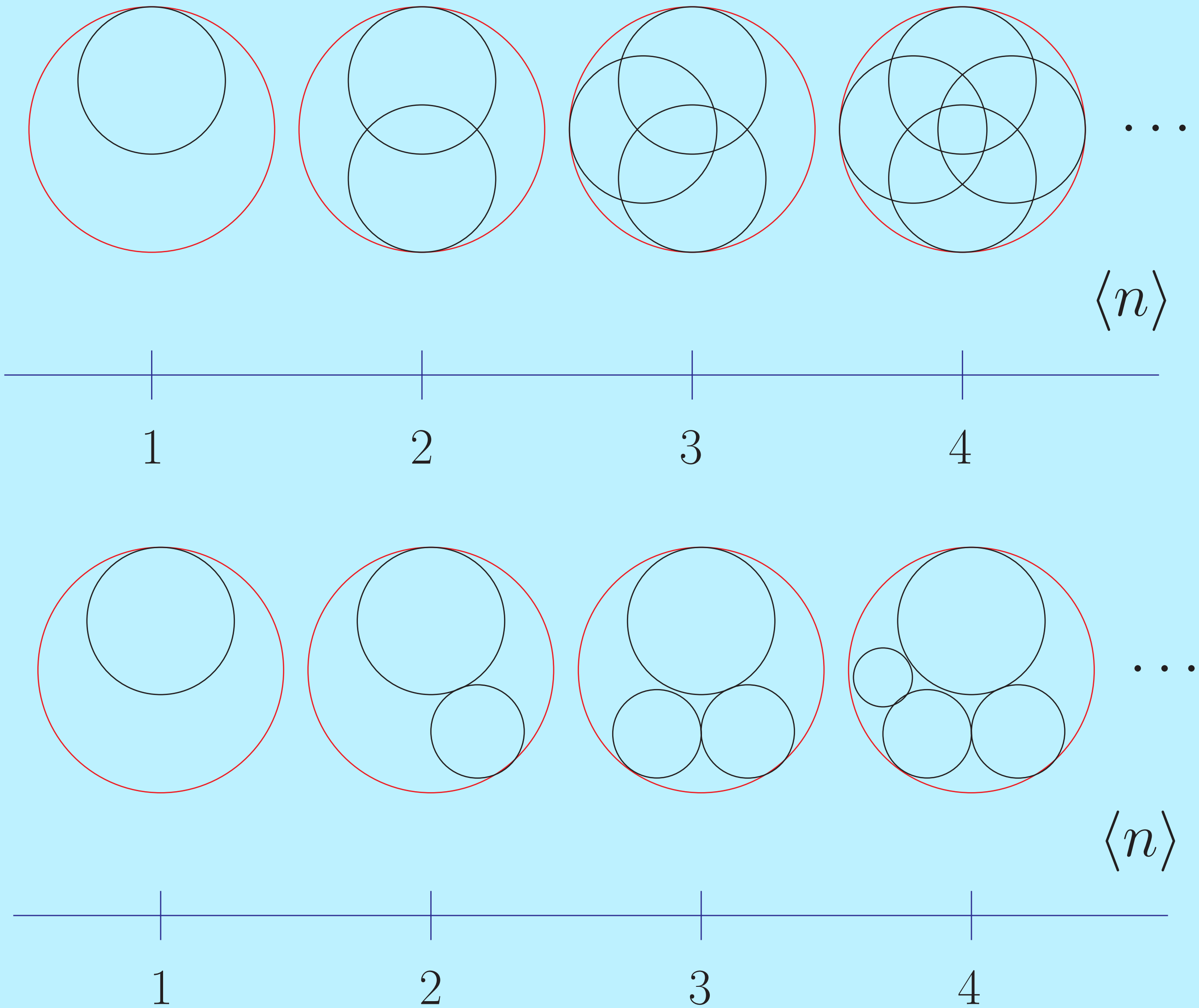
where

$$\overline{\alpha_s} \equiv \frac{\alpha_s C_A}{\pi}$$

Therefore BFKL is valid up to a “saturation scale’’ after which nonlinear effects from overlapping wave function of gluons and partons cannot be neglected.

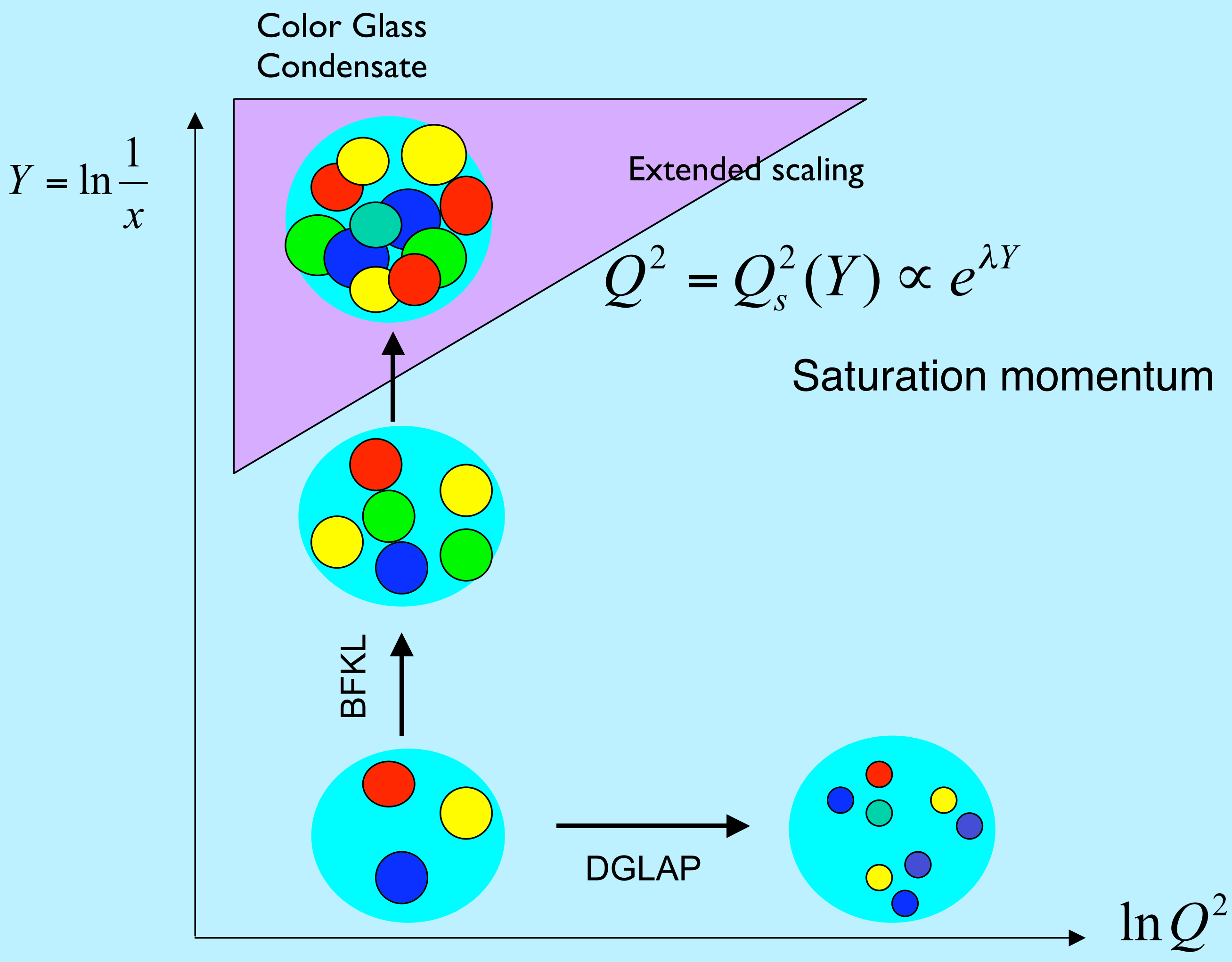
The BFKL can be modified to introduce nonlinear effects to restore unitarity. These lead to the saturation phenomena, easier to explain in terms of pictures. This is the BK behavior of the gluon distribution function

A pictorial description



Dilute to dense transition, with a fractal dim equal to the BFKL exponent

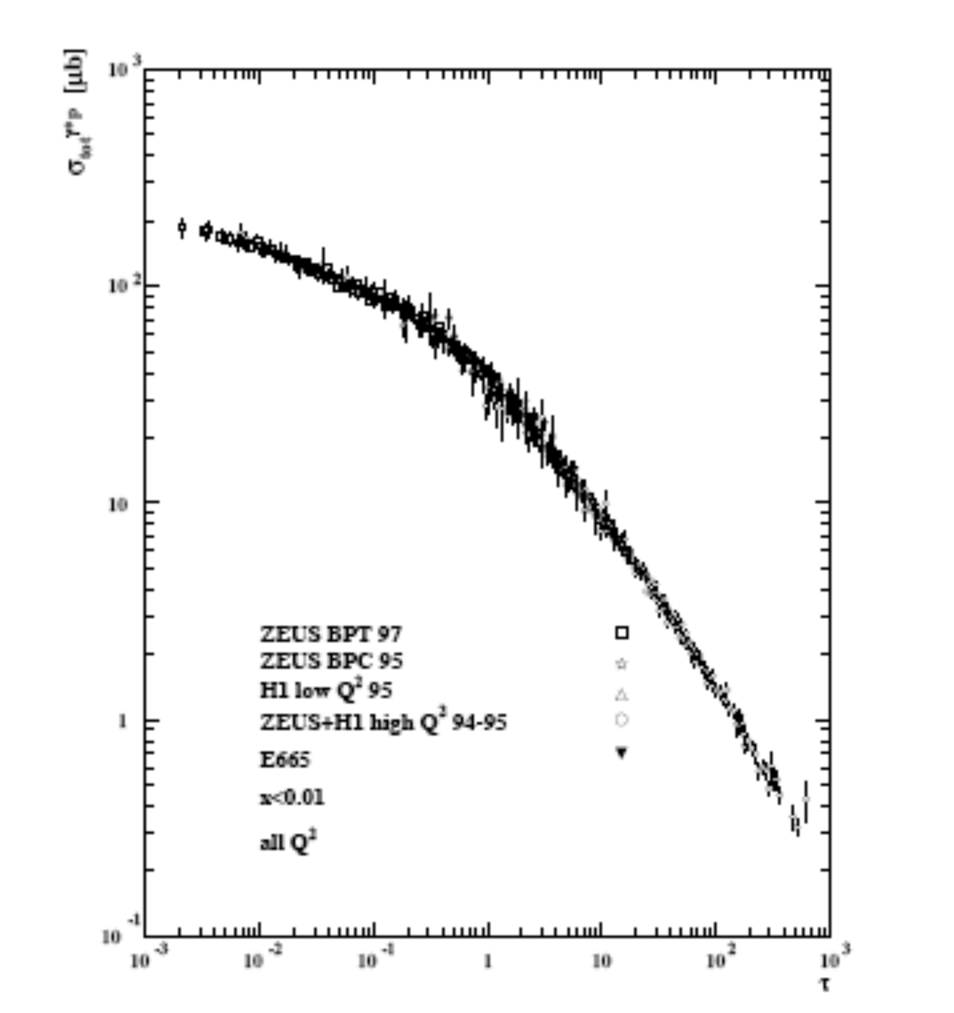
Saturation in QCD



Geometric scaling

$$\sigma^{\rm tot}_{|_{\gamma^*\,\rm P}}(Y,Q)=\sigma^{\rm tot}_{|_{\gamma^*\,\rm P}}(\tau)\ \ ;\ \ \tau=\frac{Q^2}{Q_s^2(Y)}=Q^2x^{\lambda_s}$$

HERA data for $\sigma_{\gamma^* p}$ with $x < 0.01$ versus τ



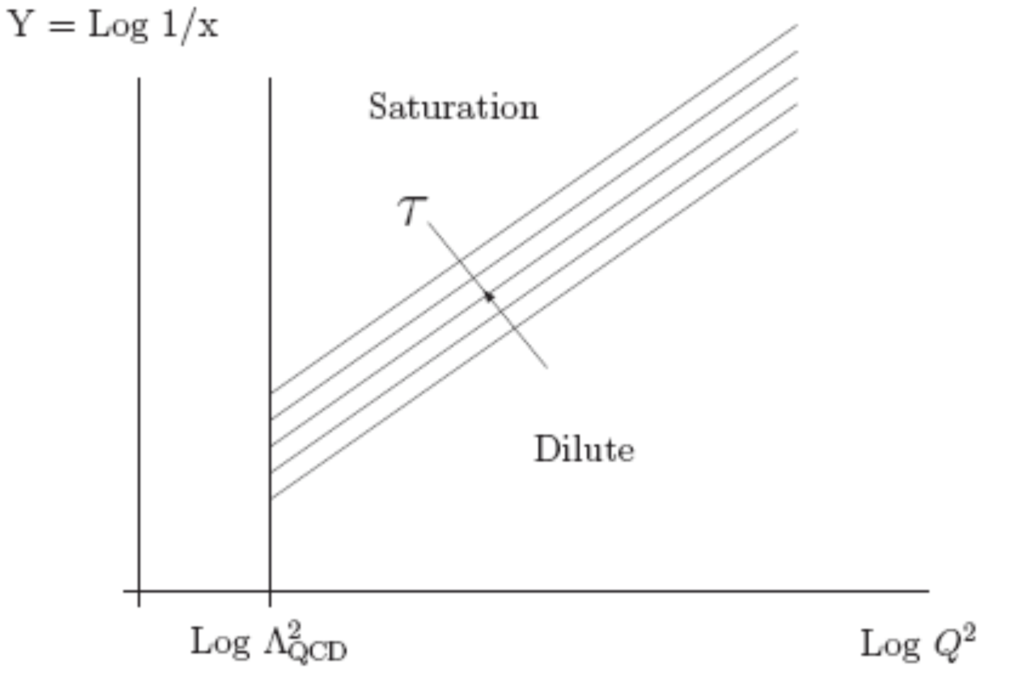


Fig. 5: Continuous self–similarity on the $(Y, \log Q^2)$ plane.

Some more speculation

BFKL to BK is like going from single particle Hilbert space to the Fock space

Klein paradox in RQM

Horowitz-Pochinski transition for small BHs

The unitarization in the Regge region may help, through holography to understand what happens at least with small BHs

$$\gamma_{\rm cr} = .6275 \qquad \lambda_{\rm cr} = \chi'(\gamma_{\rm cr})/2 = 2.44...$$

Rough dictionary

4D YM	5D Gravity
Weakly coupled	Strongly coupled
Large s	Large velocities
Geometric Scaling	CSS perfect fluid
$(\ \alpha_s N_c Y\ ,\ \log\ Q^2\ )$	(t, r)
Scaling symmetry of BFKL/BK	Conformal perfect fluid k=1/4

Some comments

Effective planarity at leading order in the Regge or multi-regge region

$$N_{eff} \sim \frac{\alpha N}{4\pi} \log s$$

Robustness based on the fact that we compute critical exponents, replaces BPS and holomorphy Not as robust but reasonably strong

The corresponding exponents of geometric scaling and CSS are reasonably close.

$$\lambda_{\rm BFKL} \approx 2.44 \approx \lambda_{\rm Ch}^{d=5,k=1/4}$$

The LHC will begin exploring systematically the Regge region, the exponents will be measured accurately

The fate of the BH singularity. SHP transitions one-to-multi string transition

$$\lambda_{\rm SBH} = 2.58$$

More work needed...

Speculation galore...

Our computation relied on spherical symmetry

There are better ways to look at holographic representations of Regge collisions, like the collision of plane gravitational waves (Abraham and Evans)

In the BK equation we have two fixed points, one at low density is described by the BFKL equation with an unstable direction

At high densities we have a stable fixed point that restores unitarity and energy conservation. This is the saturation region.

In the gravitational context we have also two fixed points:

The Choptuik critical solution with a single unstable direction

and the AdS BH that becomes static after all available energy has been absorbed.

There is a cross over between the two, where scaling should show up.
The analogy is irresistible

....Continued

Type IIB collapse

Small vs big AdS black holes

Horowitz-Hubeny Solution

....

A promising perspective

The previous work is a bit too hard. Perhaps it pays to look at a simpler scenario.

Recently Gubser et al. have analyzed a holographic dual of entropy generation in heavy ion collisions in terms of shock wave collisions. Although the analysis is still in preliminary stages, we can use their approach to study critical phenomena in the formation of trapped surfaces. This is a far simpler analysis, and has lead to some interesting early results to be published soon.

In the Regge region, both in the boundary and the bulk, we can describe soft emission in terms of relatively simple, but non-local Hamiltonians with very similar properties.

There are several types of duality that can be called upon: open-close string duality, gravity=|gauge|^2, AdS/CFT, ... Perhaps a combination of them will give us a glimpse of the correct unitarity description in the formation of BHs

Enough!

Thank you for your patience