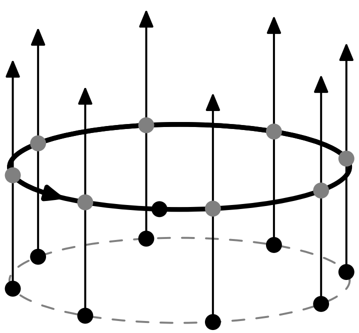
Integrability in the AdS/CFT Correspondence



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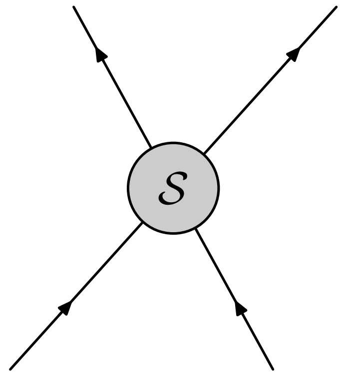

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References: . . . , `hep-th/0511082`, . . .

Introduction

Review methods of integrability for planar spectrum in AdS/CFT.

Outline:

- AdS/CFT, two main examples & spectrum.
- planar limit, spin chains & integrability.
- AdS/CFT as a particle model.
- S-Matrix & asymptotic Bethe equations.
- BES equation & twist-two dimensions.
- Wilson loops & dual superconformal symmetry.
- finite coupling, wrapping.

Why?!

- Understand all secrets of (planar) scattering amplitudes $\to$ LHC.
- Analytical non-perturbative method $\to$ quantum gravity.
- Exciting connections to condensed matter theory $\to$ superconductivity.

AdS/CFT Correspondence

Introduction to AdS/CFT

AdS/CFT duality: Conjectured exact duality of [Maldacena hep-th/9711200] [Gubser Klebanov Polyakov] [Witten hep-th/9802150]

- string theory on $AdS_{D+1} \times M^{9-D}$ background and
- a particular D-dimensional conformal field theory.

Two qualitatively different models. Why should they be equivalent?

Symmetry groups match: $\widetilde{\mathrm{SO}}(D,2)$.

Holography: Boundary of AdS_{D+1} is conformal $\mathbb{R}^{D-1,1}$ or $\mathbb{R} \times S^{D-1}$.

Prospects:

- More general AdS/CFT may explain aspects of QCD strings.
- Study aspects of quantum gravity with QFT means.

Main examples of highly supersymmetric AdS/CFT pairs:

⋆ strings on $AdS_5 \times S^5$, ◦ strings on $AdS_4 \times \mathbb{CP}^3$, [Aharony, Bergman Jafferis, Maldacena]

⋆ 4D $\mathcal{N} = 4$ Yang–Mills theory, ◦ 3D $\mathcal{N} = 6$ Chern–Simons theory.

Here: Use integrability to compare spectra of both pairs of models.

Strings on $AdS_5 \times S^5$ and $AdS_4 \times \mathbb{CP}^3$

$AdS_5 \times S^5$ is a coset space

$$AdS_5 \times S^5 \times 32 \text{ fermi} = \frac{\widetilde{\mathrm{PSU}}(2,2|4)}{\mathrm{Sp}(1,1) \times \mathrm{Sp}(2)}\,.$$

$AdS_4 \times \mathbb{CP}^3$ (with partial kappa gauge fixing) is a coset space[Arutyunov Frolov][Stefański 0806.4948]

$$AdS_4 \times \mathbb{CP}^3 \times 24 \text{ fermi} = \frac{\widetilde{\mathrm{OSp}}(6|4,\mathbb{R})}{\mathrm{U}(3) \times \mathrm{SL}(2,\mathbb{C})}\,.$$

Worldsheet non-linear sigma model (Green-Schwarz superstring) [Metsaev Tseytlin]

$$S_\sigma \simeq \frac{\sqrt{\lambda}}{2\pi} \int_{M^{1,1}} \left(\tfrac{1}{2}\,\mathrm{STr}\, P \wedge *P - \tfrac{1}{2}\,\mathrm{STr}\, Q_1 \wedge Q_2\right).$$

Above coset spaces are $\mathbb{Z}^4$-graded symmetric spaces:
H, Q_1, P, Q_2 are $0, 1, 2, 3$-graded components of Maurer–Cartan form.

Supersymmetric CFT's

4D $\mathrm{SU}(N)$ $\mathcal{N}=4$ superconformal Yang–Mills theory:

⋆ $\mathrm{SU}(4)$ flavour symmetry, $\mathrm{SU}(2,2)$ conformal symmetry $\to \widetilde{\mathrm{PSU}}(2,2|4)$,

⋆ coupling constant g_{YM}, theta angle θ, rank N,

⋆ gauge field, 6 adjoint scalars, $4+\bar{4}$ adjoint fermions,

$$\mathcal{L} \sim \frac{1}{g_{\mathrm{YM}}^2}\left(\mathcal{F}^2 + (\mathcal{D}\Phi)^2 + \Psi\mathcal{D}\Psi + \Phi\Psi^2 + \Phi^4\right).$$

3D $\mathrm{SU}(N)\times\mathrm{SU}(M)$ $\mathcal{N}=6$ superconformal Chern–Simons theory: [Aharony Bergman Jafferis]

∘ $\mathrm{SU}(4)$ flavour symmetry, $\mathrm{Sp}(4,\mathbb{R})$ conformal symmetry $\to \widetilde{\mathrm{OSp}}(6|4,\mathbb{R})$,

∘ CS level k_{CS}, rank N, M,

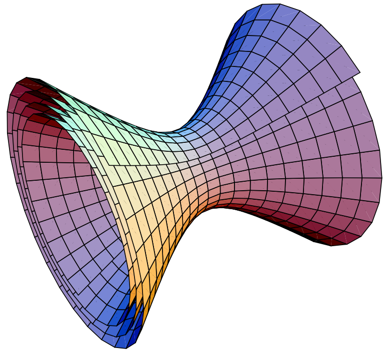
∘ gauge field (non-dynamical), $4\ N\times\bar{M}$ and $\bar{4}\ M\times\bar{N}$ scalars/fermions,

$$\mathcal{L} \sim k_{\mathrm{CS}}\left(\mathcal{A}\mathcal{F} + \mathcal{A}^3 + (\mathcal{D}\Phi)^2 + \Psi\mathcal{D}\Psi + \Phi^2\Psi^2 + \Phi^6\right).$$

Spectrum

Spectrum of AdS/CFT

String Theory:



States: Solutions X of classical equations of motion
plus quantum corrections.

Energy: Charge $E_X(\lambda)$ for translation along AdS-time.

Conformal Field Theory:

States: Local operators. Gauge-inv. combinations (glueballs), e.g.

$$\mathcal{O} = \operatorname{Tr} \Phi_1 \Phi_2 (\mathcal{D}_1 \mathcal{D}_2 \Phi_2)(\mathcal{D}_1 \Psi_2) + \ldots .$$

Energy: Scaling dimensions, e.g. two-point function in conformal theory

$$\langle \mathcal{O}(x)\, \mathcal{O}(y) \rangle = C|x-y|^{-2D_{\mathcal{O}}(\lambda)}.$$

AdS/CFT: String energies and CFT dimensions match, $E(\lambda) = D(\lambda)$?!

Strong/Weak Duality

Problem: Models have coupling constant: λ. Strong/weak duality.

- Perturbative regime of strings at $\lambda \to \infty$.

$$E(\lambda) = \sqrt{\lambda}\, E_0 + E_1 + E_2/\sqrt{\lambda} + \ldots .$$

E_ℓ: Contribution at ℓ (world-sheet) loops. Limit: 1 or 2 loops.

- Perturbative conformal field theory at $\lambda \to 0$.

$$D(\lambda) = D_0 + \lambda D_1 + \lambda^2 D_2 + \ldots .$$

D_ℓ: Contribution at ℓ (gauge) loops. Limit: 3 or 4 (or 5?!) loops.

Tests impossible unless quantities are known at finite λ.

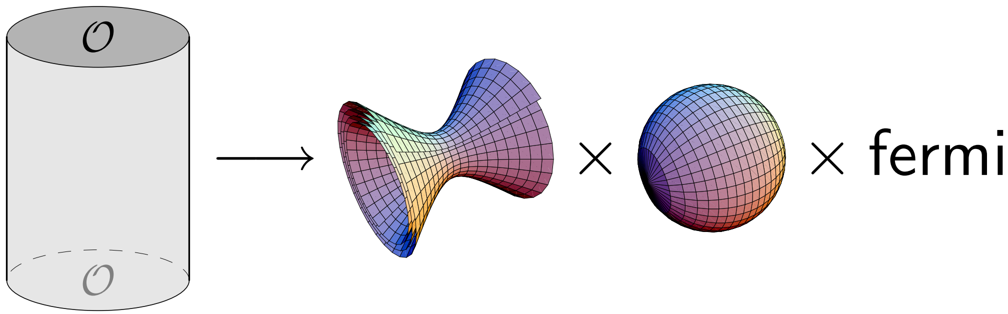
Cannot compare, not even approximately.

BMN type comparison suffers from order-of-limits.

Planar Limit

- Simplifications & surprises [’t Hooft Nucl. Phys. B72, 461] [Lipatov hep-th/9311037] [Faddeev Korchemsky] [Anastasiou, Bern Dixon, Kosower] [Alday Maldacena] . . .
- AdS/CFT integrability [Lipatov ICTP 1997] [Mandal Suryanarayana Wadia] [Minahan Zarembo] [NB Kristjansen Staudacher] [Bena Polchinski Roiban] [NB, Staudacher hep-th/0307042] . . .

String Theory:



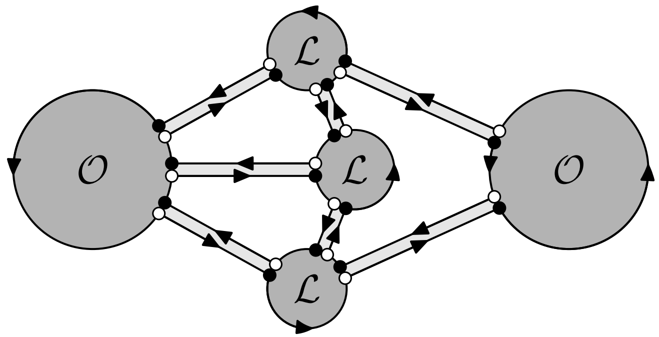
- No string coupling $g_\mathrm{s} = 0$.
- Strictly cylindrical worldsheet.
- No string splitting or joining.
- Worldsheet coupling λ free.

Conformal Field Theory:

⋆ $\mathcal{N} = 4$ SYM: $N = \infty, g_\mathrm{YM} = 0$,
finite ’t Hooft coupling $\lambda = g_\mathrm{YM}^2 N$.

∘ $\mathcal{N} = 6$ SCS: $N, M, k_\mathrm{CS} = \infty$,
finite ’t Hooft coupling $\lambda = \sqrt{NM}/k_\mathrm{CS}$.

- Only planar Feynman diagrams.



Spin Chains

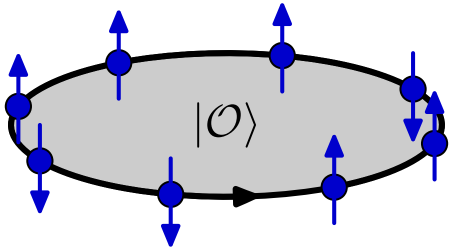
Planar limit: only single-trace operators relevant.

Translate single-trace operators to spin chain states, e.g. [Minahan Zarembo] [Minahan Zarembo]

$\mathfrak{su}(2)$ sector of $\mathcal{N}=4$ SYM:

$$\mathcal{O} = \operatorname{Tr} \phi_1\phi_1\phi_2\phi_1\phi_1\phi_1\phi_2\phi_2$$

identify $\phi_1 \to |\uparrow\rangle$, $\phi_2 \to |\downarrow\rangle$

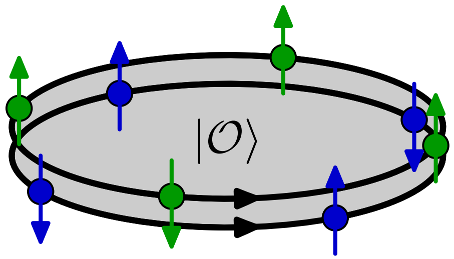


$$|\mathcal{O}\rangle = |\uparrow\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow\downarrow\rangle$$

$\mathfrak{su}(2)\times\mathfrak{su}(2)$ sector of $\mathcal{N}=6$ SCS:

$$\mathcal{O} = \operatorname{Tr} \Phi_1\bar\Phi^3\Phi_2\bar\Phi^3\Phi_1\bar\Phi^3\Phi_2\bar\Phi^4$$

identify $\Phi_{1,2} \to |\updownarrow\rangle$, $\bar\Phi^{3,4} \to |\bar\updownarrow\rangle$

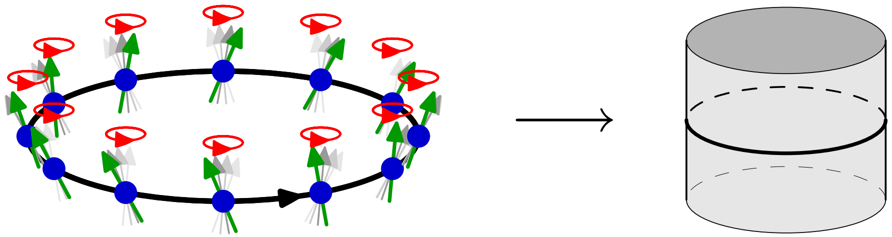


$$|\mathcal{O}\rangle = |\uparrow\bar\uparrow\downarrow\bar\uparrow\uparrow\bar\uparrow\downarrow\bar\downarrow\rangle$$

Energy spectrum: Eigenvalues of spin chain Hamiltonian.

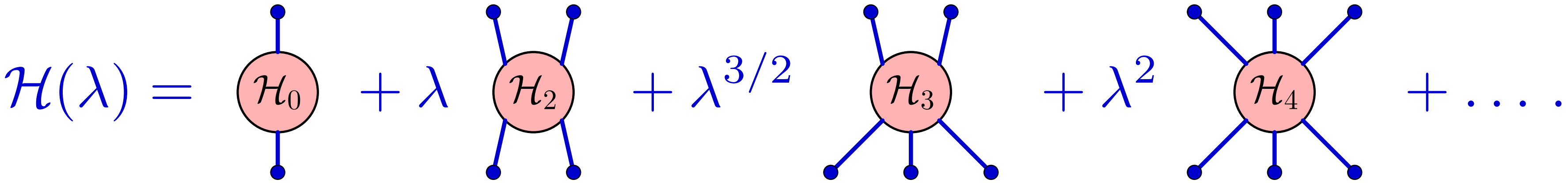
Qualitative similarity with strings:

- Many coherent spins can trace out snapshot of string embedding.

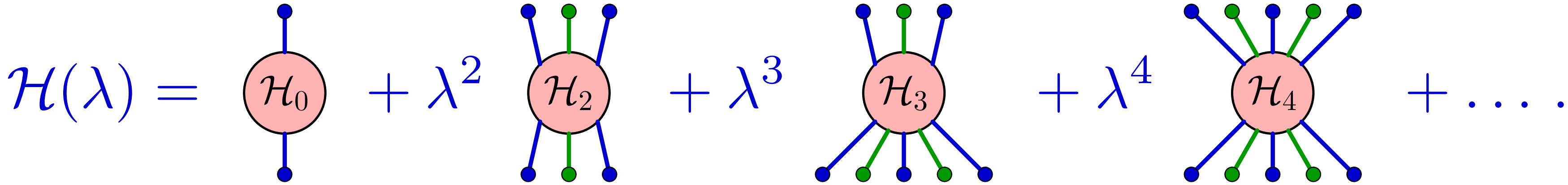


Perturbative Spin Chain Hamiltonian

Perturbative Hamiltonian (planar Feynman diagrams) in $\mathcal{N}=4$ SYM:

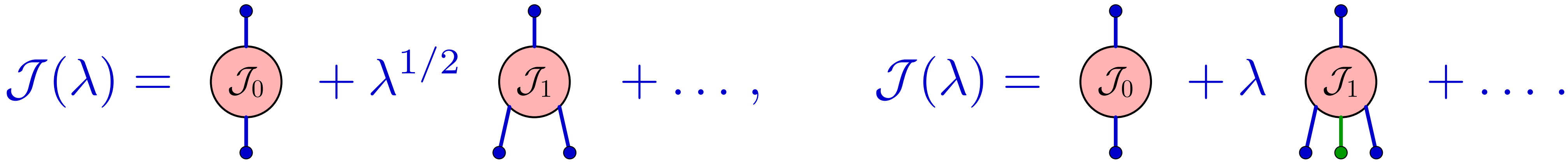


Perturbative Hamiltonian in $\mathcal{N}=6$ SCS:



Properties: • perturbatively short-ranged, • homogeneous, • dynamic.

Perturbative expansion of non-manifest symmetries $\mathcal{J}=\mathcal{Q},\mathcal{S},\mathcal{P},\mathcal{K}$



Problems of Hamiltonian Approach

- Contributions at $\mathcal{O}(\lambda^0)$: trivial; counting spins.
- Contributions at $\mathcal{O}(\lambda^1)/\mathcal{O}(\lambda^2)$: NN/NNN Hamiltonian.
- Contributions at higher orders hard to compute in field theory.
- Scaling dimension $D_{\mathcal{O}}$ as eigenvalue of the Hamiltonian $\mathcal{H}(\lambda)$.
  Diagonalisation of NN/NNN Hamiltonian hard: combinatorics/calculus.
- Complete chains: infinitely many spins $\mathcal{W} \in \{\mathcal{D}^n\Phi, \mathcal{D}^n\Psi, \mathcal{D}^n\mathcal{F}\}$.
  Irreducible modules $\mathbb{V}_{\mathrm{F}}$ of $\mathfrak{psu}(2,2|4)$ or $\mathbb{V}_{\mathrm{F}}$ and $\mathbb{V}_{\bar{\mathrm{F}}}$ of $\mathfrak{osp}(6|4,\mathbb{R})$.
- Hamiltonian defined modulo similarity transformations. Ambiguous!
- Part of symmetry algebra (eigenvalues label representations):
  Representation theory of $\mathfrak{psu}(2,2|4)/\mathfrak{osp}(6|4,\mathbb{R})$ useless.
- Perturbative string theory requires all orders/finite coupling...

How to obtain spectrum?

Integrability

Classical String Integrability

Integrability: Existence of family of flat connections [Mandal Suryanarayana Wadia] [Bena Polchinski Roiban]

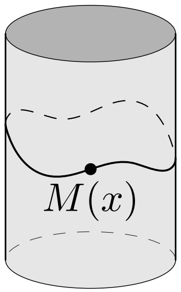
$$A(x) = H + \tfrac{1}{2}(z^{-2} + z^2)P + \tfrac{1}{2}(z^{-2} - z^2)*P + z^{-1}Q_1 + z\,Q_2.$$

Flatness equivalent to Maurer–Cartan equations and equations of motion:

$$0 = dA(x) - A(x) \wedge A(x).$$

Monodromy of flat connection around worldsheet cylinder

$$M(x) = \mathrm{P}\exp\oint_\gamma A(x).$$



Eigenvalues of $M(z)$ (as a function of z)

- are independent of path γ, base point $\gamma(0)$ and WS diffeomorphisms,
- define spectral curve (action variables), [Kazakov, Marshakov Minahan, Zarembo] [NB, Kazakov Sakai, Zarembo]
- fully classify classical solutions in terms of conserved charges.

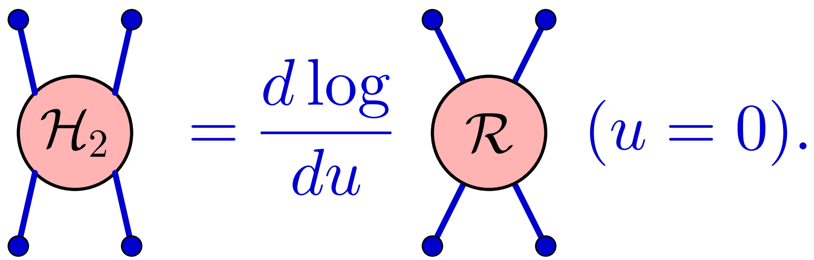
NN/NNN Spin Chain Integrability

$\mathcal{N}=4$ SYM: Complete one-loop Hamiltonian $\mathcal{H}_2$. [Minahan Zarembo] [NB hep-th/0307015] [NB hep-th/0407277]

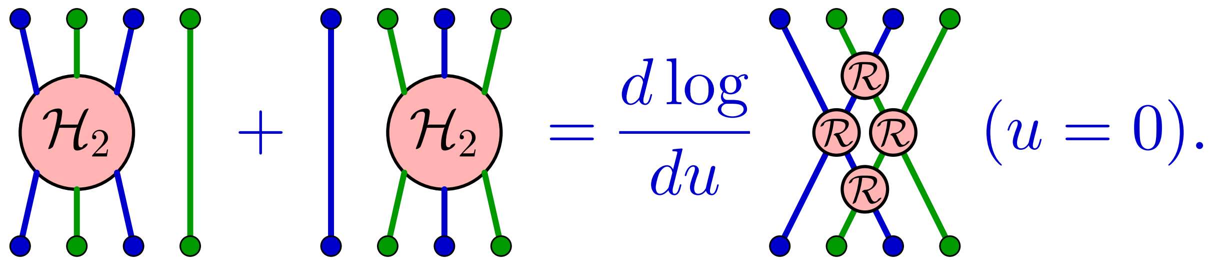
Planar one-loop Hamiltonian completely integrable! [Lipatov ICTP 1997] [Minahan Zarembo] [NB, Staudacher hep-th/0307042]

(To some extent in 4D gauge theory.) [Lipatov ICTP 1997] [Braun Derkachov Manashov] [Belitsky hep-ph/9902361] [Belitsky, Braun Gorsky, Korchemsky]

NN-Hamiltonian from $\mathfrak{psu}(2,2|4)$ R-matrix ($\mathcal{R}_{12}\mathcal{R}_{13}\mathcal{R}_{23}=\mathcal{R}_{23}\mathcal{R}_{13}\mathcal{R}_{12}$).



$\mathcal{N}=6$ SCS: NNN-Hamiltonian from $\mathfrak{osp}(6|4,\mathbb{R})$ R-matrices [Minahan Zarembo] [Bak Rey]



Quantum Integrability in $AdS_5 \times S^5 / \mathcal{N} = 4$ SYM

Higher-loop (non)compact Hamiltonian. [NB Kristjansen Staudacher] [NB hep-th/0310252] [Zwiebel hep-th/0511109] [NB McLoughlin Roiban]
Various string quantum corrections. [Frolov Tseytlin] [Callan, Lee, McLoughlin Schwarz, Swanson, Wu] . . .

Notion of quantum integrability unclear (R-matrix?). **Indications:**

- Degenerate pairs with opposite parity. [NB Kristjansen Staudacher] [NB hep-th/0310252]
- Local commuting charges $[\mathcal{H}, \mathcal{Q}_r] = [\mathcal{Q}_r, \mathcal{Q}_s] = 0$. [NB, Dippel Staudacher] [Agarwal Ferretti]
- Yangian symmetry. [Dolan Nappi Witten] [Agarwal Rageev] [Zwiebel hep-th/0610283] [NB Erkal]
- Four-loop gluon scattering and integrability. [Bern, Czakon, Dixon Kosower, Smirnov] [NB, Eden Staudacher]
- Four-loop wrapping effects from integrability. [Fiamberti Santambrogio Sieg, Zanon] [Bajnok Janik]
- Bethe equations work for quantum strings. [Arutyunov Frolov Staudacher] [Schäfer-Nameki Zamaklar, Zarembo] [NB, Tseytlin hep-th/0509084]
- Pure spinor BRST consistency of flat connection. [Berkovits hep-th/0411170]

Useful working hypothesis. Assumption! Path towards proof unclear; but:

- Recursion relation for algebra in $\mathfrak{su}(1,1|2)$ sector. [Zwiebel 0806.1786]
- Integrability-preserving recursion relation in $\mathfrak{su}(2)$ sector. [Bargheer NB, Loebbert]

Asymptotic Bethe Ansatz and Beyond

Procedure to obtain the spectrum of an integrable model.

- Relax periodicity: Consider model on infinite line instead of circle.
- Find particles and determine their properties.
- Derive 2-particle S-matrix. Integrability: Factorised scattering!
- Diagonalise S-matrix.
- Impose Bethe equations as periodicity conditions for eigenstates.
- Read off eigenvalues as sums over particles.
- Add finite-size corrections (S-matrix on a circle, not line). TBA.

Particle Model

$AdS_5 \times S^5$ Strings: Light Cone Gauge

Obtain spectrum of $\mathcal{H}(\lambda)$ from perturbative string theory at large λ.

Perform light cone gauge using time from AdS_5 and great circle from S^5.

- Vacuum: Point-particle moving along time and great circle. [Berenstein Maldacena Nastase]
- Excitations: 4 coordinates on AdS_5 and 4 coordinates on S^5.
- Fermions: $1/2$ are gauged away, $1/2$ are momenta, 8 remain.

QM particle model of 8 bosonic and 8 fermionic flavours on the circle.

Residual symmetry

Unbroken symmetries of the vacuum state:

- $\widetilde{\mathrm{SO}}(4,2) \simeq \widetilde{\mathrm{SU}}(2,2)$ reduces to $\mathrm{SO}(4) \times \widetilde{\mathrm{SO}}(2) \simeq \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathbb{R}$.
- $\mathrm{SO}(6) \simeq \mathrm{SU}(4)$ reduces to $\mathrm{SO}(4) \times \mathrm{SO}(2) \simeq \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)$.
- $\widetilde{\mathrm{PSU}}(2,2|4)$ reduces to $\mathrm{U}(1) \ltimes (\mathrm{PSU}(2|2) \times \mathrm{PSU}(2|2)) \ltimes \mathbb{R}$.

Excitations in $(2|2) \times (2|2)$ representations of $(\mathrm{PSU}(2|2) \times \mathrm{PSU}(2|2)) \ltimes \mathbb{R}$.

$\mathcal{N}=4$ SYM: Coordinate Space Bethe Ansatz

Gauge spectrum of $\mathcal{H}$? Consider spin chain states with few "excitations".

Ferromagnetic vacuum: protected state with scalar $\mathcal{Z} = \Phi_5 + i\Phi_6$

$$|0\rangle = |\ldots \mathcal{Z}\mathcal{Z}\mathcal{Z}\ldots\rangle, \qquad \delta\mathcal{H}\,|0\rangle = 0.$$

Residual symmetry of $(\mathfrak{psu}(2|2) \times \mathfrak{psu}(2|2)) \ltimes \mathbb{R}$ stabilises $\mathcal{Z}$).

One-excitation states with excitation $\mathcal{A}$ at position a, momentum p

$$|\mathcal{A}, p\rangle = \sum_a e^{ipa} |\ldots \mathcal{Z} \ldots \overset{\overset{a}{\downarrow}}{\mathcal{A}} \ldots \mathcal{Z} \ldots\rangle, \qquad \delta\mathcal{H}\,|\mathcal{A}, p\rangle = \delta E_{\mathcal{A}}(p) |\mathcal{A}, p\rangle.$$

$(4+4|\,4+4)$ flavours of excitations $\mathcal{A} \in \{\phi_i, \mathcal{D}_\mu \mathcal{Z} |\, \psi_a, \dot{\psi}_{\dot{a}}\}$. [Berenstein Maldacena Nastase]

Other spin orientations in module $\mathbb{V}_{\mathrm{F}}$ are multiple coincident excitations.

Coordinate space Bethe ansatz leads to a particle model
with $(2|2) \times (2|2)$ flavours transforming under $(\mathfrak{psu}(2|2) \times \mathfrak{psu}(2|2)) \ltimes \mathbb{R}$.

Residual Extended $\mathfrak{psu}(2|2)$ Algebra

Residual algebra $\mathfrak{su}(2|2)$ preserves particle number. Generators: [NB hep-th/0511082]

- $\mathcal{R}^a{}_b$ flavour $\mathfrak{su}(2)$ rotation generator,
- $\mathcal{L}^\alpha{}_\beta$ spacetime $\mathfrak{su}(2)$ rotation generator,
- $\mathcal{Q}^\alpha{}_b$ supersymmetry generator,
- $\mathcal{S}^a{}_\beta$ superboost generator,
- $\mathcal{H}$ central charge.

Algebra: $\mathcal{R}^a{}_b$, $\mathcal{L}^\alpha{}_\beta$ transform indices. Anticommutator of supercharges

$$\{\mathcal{Q}^\alpha{}_a, \mathcal{S}^b{}_\beta\} = \delta^b_a \mathcal{L}^\alpha{}_\beta + \delta^\alpha_\beta \mathcal{R}^b{}_a + \delta^b_a \delta^\alpha_\beta \mathcal{H},$$
$$\{\mathcal{Q}^\alpha{}_a, \mathcal{Q}^\beta{}_b\} = \varepsilon^{\alpha\beta}\varepsilon_{ab}\mathcal{P},$$
$$\{\mathcal{S}^\alpha{}_a, \mathcal{S}^b{}_\beta\} = \varepsilon^{ab}\varepsilon_{\alpha\beta}\mathcal{K}.$$

- Additional central generators $\mathcal{P}, \mathcal{K}$ are gauge transformations.
- Exceptional threefold central extension $\mathfrak{psu}(2|2) \ltimes \mathbb{R}^3$.

Fundamental Representation of $\mathfrak{psu}(2|2) \ltimes \mathbb{R}^3$

Excitations should transform in $(\mathbf{2}|\mathbf{2})$ representation of extended $\mathfrak{psu}(2|2)$.

Ansatz for $(\mathbf{2}|\mathbf{2})$ representation with canonical action of $\mathcal{R}^a{}_b, \mathcal{L}^\alpha{}_\beta$:

$$
\begin{aligned}
\mathcal{Q}^\alpha{}_a|\phi^b\rangle &= a\,\delta^b_a|\psi^\alpha\rangle, & \mathcal{H}|\mathcal{X}\rangle &= E|\mathcal{X}\rangle,\\
\mathcal{Q}^\alpha{}_a|\psi^\beta\rangle &= b\,\varepsilon^{\alpha\beta}\varepsilon_{ab}|\phi^b\mathcal{Z}^+\rangle, & \mathcal{P}|\mathcal{X}\rangle &= P|\mathcal{X}\mathcal{Z}^+\rangle,\\
\mathcal{S}^a{}_\alpha|\phi^b\rangle &= c\,\varepsilon^{ab}\varepsilon_{\alpha\beta}|\psi^\beta\mathcal{Z}^-\rangle, & \mathcal{K}|\mathcal{X}\rangle &= K|\mathcal{X}\mathcal{Z}^-\rangle.\\
\mathcal{S}^a{}_\alpha|\psi^\beta\rangle &= d\,\delta^\beta_\alpha|\phi^a\rangle,
\end{aligned}
$$

Closure requires $ad - bc = 1$, $E = \frac{1}{2}(ad + bc)$, $P = ab$, $K = cd$.

Shortening/atypicality condition for central charges: $E^2 - PK = \frac{1}{4}$.

Gauge Transformations and Extended $\mathfrak{su}(2|2)$

Generator $\mathcal{P}$ generates gauge transformation (similarly for $\mathcal{K}$)

$$\mathcal{P}|\mathcal{X}\rangle \sim |[\mathcal{Z}^+,\mathcal{X}]\rangle = |\mathcal{Z}^+\mathcal{X}\rangle - |\mathcal{X}\mathcal{Z}^+\rangle = (1-e^{ip})|\mathcal{Z}^+\mathcal{X}\rangle.$$

$\mathfrak{su}(2|2)$ symmetry recovered for physical states (annihilated by $\mathcal{P},\mathcal{K}$).
P, K are algebraically fixed functions of momentum p

$$P = g\alpha(1-e^{ip}), \quad K = g\alpha^{-1}(1-e^{-ip}), \qquad g = \frac{\sqrt{\lambda}}{4\pi}\,.$$

Cyclicity condition $P = K = 0$ for physical states with zero momentum.
Dispersion relation $E(p)$ algebraically determined from atypicality

$$E = \sqrt{\tfrac{1}{4} + PK} = \tfrac{1}{2}\sqrt{1 + 16g^2\sin^2(\tfrac{1}{2}p)}\,.$$

$AdS_4 \times \mathbb{CP}^3$ Strings: Light Cone Gauge

Does this work for $AdS_4 \times \mathbb{CP}^3$ strings as well?

Perform light cone gauge using time from AdS_4 and geodesic from $\mathbb{CP}^3$.

- Vacuum: Point-particle moving along time and geodesic.
- Excitations: 3 coordinates on AdS_4 and $1+4$ coordinates on $\mathbb{CP}^3$.
- Fermions: $1/3$ are gauged away, $1/2$ are momenta, $4+4$ remain.

Residual symmetry

Unbroken symmetries of the vacuum state:

- $\widetilde{\mathrm{SO}}(3,2) \simeq \widetilde{\mathrm{Sp}}(4,\mathbb{R})$ reduces to $\mathrm{SO}(3) \times \widetilde{\mathrm{SO}}(2) \simeq \mathrm{SU}(2) \times \mathbb{R}$.
- $\mathrm{SO}(6) \simeq \mathrm{SU}(4)$ reduces to $\mathrm{SU}(2) \times \mathrm{U}(1)^2$.
- $\widetilde{\mathrm{OSp}}(6|4,\mathbb{R})$ reduces to $\mathrm{U}(1) \ltimes (\mathrm{PSU}(2|2) \times \mathrm{U}(1)) \ltimes \mathbb{R}$.

Excitations in $(2|2)^+ + (2|2)^- + (4|4)^0$ representations.

$\mathcal{N}=6$ SCS: Coordinate Space Bethe Ansatz

What about $\mathcal{N}=6$ SCS? Pick an alternating vacuum using Φ_1 and $\bar{\Phi}^4$

$$|0\rangle = |\ldots \Phi_1 \bar{\Phi}^4 \Phi_1 \bar{\Phi}^4 \Phi_1 \bar{\Phi}^4 \ldots\rangle.$$

Residual symmetry of $(\mathfrak{psu}(2|2) \times \mathfrak{u}(1)) \ltimes \mathbb{R}$ stabilises Φ_1 and $\bar{\Phi}^4$.
Single excitations $(\mathbf{2}|\mathbf{2})^+$: $\Phi_1 \to \Phi_{2,3}, \psi_\alpha$ and $(\mathbf{2}|\mathbf{2})^-$: $\bar{\Phi}^4 \to \bar{\Phi}^{2,3}, \bar{\psi}_\alpha$.
Multiplet $(\mathbf{4}|\mathbf{4})^{\mathbf{0}}$ is double excitation. Not bound! Problem?!

Above particle **multiplet construction** applies here as well:
Gauge transformation generators $\mathcal{P}$, $\mathcal{K}$ act like

$$\mathcal{P}\mathcal{X} \sim \Phi_1 \bar{\Phi}^4 \mathcal{X} - \mathcal{X} \bar{\Phi}^4 \Phi_1, \qquad \mathcal{P}\bar{\mathcal{X}} \sim \bar{\Phi}^4 \Phi_1 \bar{\mathcal{X}} - \bar{\mathcal{X}} \Phi_1 \bar{\Phi}^4.$$

Dispersion $E_{\mathbf{2}|\mathbf{2}} = \frac{1}{2}\sqrt{1+16h^2\sin^2 p}$, $E_{\mathbf{4}|\mathbf{4}} = \frac{1}{2}\sqrt{4+16h^2\sin^2 p}$.
h is function of coupling: $h \sim \sqrt{\lambda}$ (strings) or $h \sim \lambda$ (CFT). Physical?!

S-Matrix

Scattering Matrix

Consider multi-particle states on non-compact worldsheet.

If particles are well-separated: plane wave partial eigenstates, e.g.

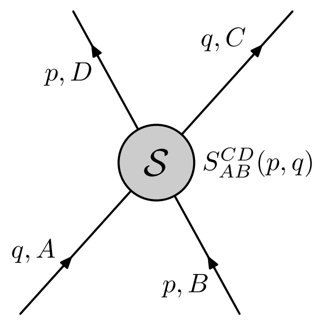
$$|\mathcal{X}_1 < \mathcal{X}_2 < \mathcal{X}_3\rangle = \sum_{k_1 \ll k_2 \ll k_3} e^{ip_1k_1 + ip_2k_2 + ip_3k_3} |\ldots \overset{\overset{k_1}{\downarrow}}{\mathcal{X}_1} \ldots \overset{\overset{k_2}{\downarrow}}{\mathcal{X}_2} \ldots \overset{\overset{k_3}{\downarrow}}{\mathcal{X}_3} \ldots\rangle$$

Construct two-particle eigenstates by fitting asymptotic regions [Staudacher hep-th/0412188]

$$|\mathcal{X}_1^A \mathcal{X}_2^B\rangle \simeq |\mathcal{X}_1^A < \mathcal{X}_2^B\rangle + \mathsf{UV} + S_{CD}^{AB}(p,q)|\mathcal{X}_2^D < \mathcal{X}_1^C\rangle.$$

S-matrix $S_{AB}^{CD}(p,q)$ encodes phase shift
for permuting particles with momenta p, q.

Energy $\mathcal{H}|\mathcal{X}_1^A \mathcal{X}_2^B\rangle = (E(p) + E(q))|\mathcal{X}_1^A \mathcal{X}_2^B\rangle$.



Integrable Scattering Matrix

Scattering matrix in a generic field theory

$$|\mathcal{X}_1^A \mathcal{X}_2^B \mathcal{X}_3^C\rangle \simeq \int dp' S_{DEF}^{ABC}(p,p')|\mathcal{X}_{1'}^D < \mathcal{X}_{2'}^E < \mathcal{X}_{3'}^F\rangle + \text{UV}.$$

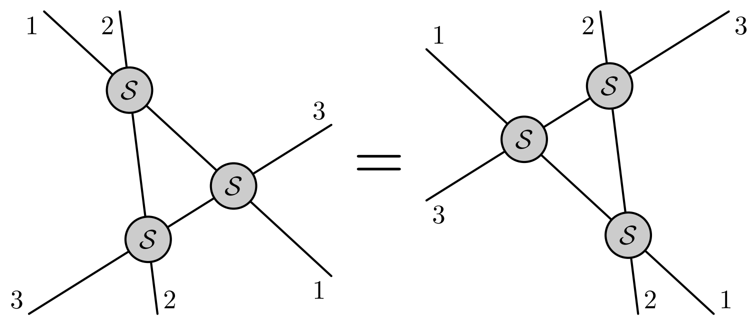
Integrability: set of particle momenta $\{p_k\}$ conserved (integrals of motion)

$$|\mathcal{X}_1^A \mathcal{X}_2^B \mathcal{X}_3^C\rangle \simeq \sum_{\pi \in S_3} (S_\pi)_{DEF}^{ABC}(p)|\mathcal{X}_{\pi(1)}^D < \mathcal{X}_{\pi(2)}^E < \mathcal{X}_{\pi(3)}^F\rangle + \text{UV}.$$

Permutation group S_K generated by pairwise permutations $\mathcal{S}_{k,l}$.

Need only two-particle S-matrix!

Consistency: Yang–Baxter equation $\mathcal{S}_{12}\mathcal{S}_{13}\mathcal{S}_{23} = \mathcal{S}_{23}\mathcal{S}_{13}\mathcal{S}_{12}$.



S-Matrix Construction

Use $\mathfrak{su}(2|2)$ invariance to construct two-particle S-matrix. [NB hep-th/0511082]

From manifest $\mathfrak{su}(2)\times\mathfrak{su}(2)$ symmetries

$$\mathcal{S}_{12}|\phi_1^a\phi_2^b\rangle = A_{12}|\phi_2^{\{a}\phi_1^{b\}}\rangle + B_{12}|\phi_2^{[a}\phi_1^{b]}\rangle + \tfrac{1}{2}C_{12}\varepsilon^{ab}\varepsilon_{\alpha\beta}|\psi_2^\alpha\psi_1^\beta\mathcal{Z}^-\rangle,$$

$$\mathcal{S}_{12}|\psi_1^\alpha\psi_2^\beta\rangle = D_{12}|\psi_2^{\{\alpha}\psi_1^{\beta\}}\rangle + E_{12}|\psi_2^{[\alpha}\psi_1^{\beta]}\rangle + \tfrac{1}{2}F_{12}\varepsilon^{\alpha\beta}\varepsilon_{ab}|\phi_2^a\phi_1^b\mathcal{Z}^+\rangle,$$

$$\mathcal{S}_{12}|\phi_1^a\psi_2^\beta\rangle = G_{12}|\psi_2^\beta\phi_1^a\rangle + H_{12}|\phi_2^a\psi_1^\beta\rangle,$$

$$\mathcal{S}_{12}|\psi_1^\alpha\phi_2^b\rangle = K_{12}|\psi_2^\alpha\phi_1^b\rangle + L_{12}|\phi_2^b\psi_1^\alpha\rangle.$$

with ten coefficient functions $A(p_1,p_2),\ldots,L(p_1,p_2)$.

- Supersymmetry fixes all functions up to one overall factor: $\sigma(p_1,p_2)$.
- Yang–Baxter equation automatically satisfied.
- Crossing relation constrains factor. [Janik hep-th/0603038] [NB hep-th/0511082]
- Proposal for strong/weak interpolating phase factor. [NB Hernández López] [NB, Eden Staudacher]

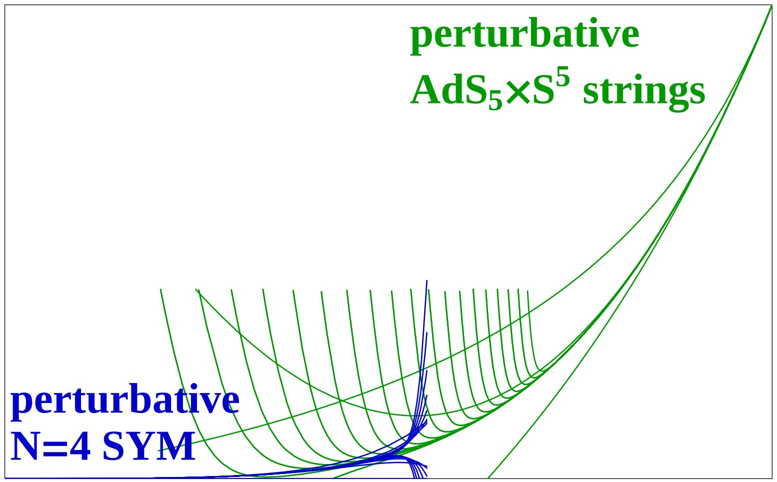
Phase Factor

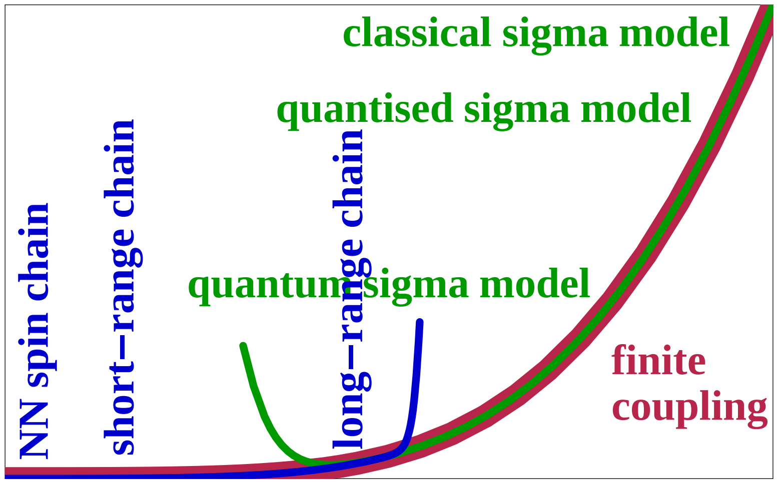
Educated guess for crossing-symmetric phase [NB Hernández López][NB, Eden Staudacher]

$$\sigma(p_1,p_2) = \exp\left(i \sum_{r=2}^{\infty} \sum_{s=r+1}^{\infty} c_{r,s}(\lambda)\,\bigl(q_r(p_1)\,q_s(p_2) - q_r(p_2)\,q_s(p_1)\bigr)\right),$$

$$c_{r,s}(\lambda) = 2\sin[\tfrac{1}{2}\pi(s-r)]\,(r-1)(s-1)\int_0^\infty \frac{dt}{t}\,\frac{\mathrm{J}_{r-1}(t)\,\mathrm{J}_{s-1}(t)}{e^{2\pi t/\sqrt{\lambda}}-1}\,.$$

Strong/weak expansion agrees with expectations. E.g. plot of $c_{23}(\lambda)$



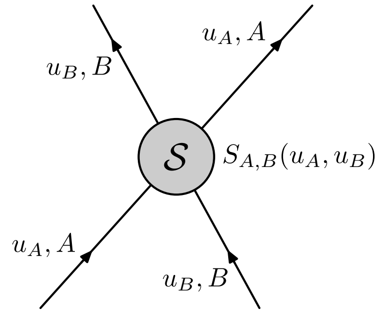


Asymptotic Bethe Equations

[NB hep-th/0511082]

Diagonalisation of S-matrix (nested Bethe ansatz):
Introduce several flavours $A = \mathrm{I}, \ldots, R$ (rank) of Bethe excitations $u_{A,k}$ with $k = 1, \ldots, K_A$

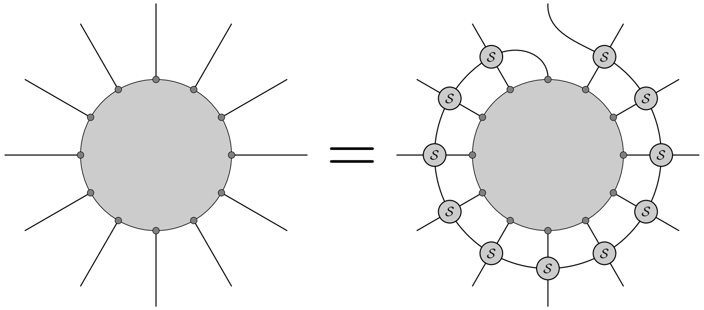
- main excitations carry momentum & energy.
- auxiliary roots $u_{A,k}$ carry spin waves,



[Sutherland Rocky Mountain J. Math. 8, 413]

Equation for asymptotically L-periodic wave function

$$1 = e^{-ip_A(u_{A,j})L} \prod_{B=\mathrm{I}}^{R} \prod_{\substack{j=1 \\ (B,j)\neq(A,k)}}^{K_B} S_{A,B}(u_{A,k}, u_{B,j}).$$



Charge eigenvalues:

$$\exp iP = \prod_{A=\mathrm{I}}^{R} \prod_{j=1}^{K_A} \exp ip_A(u_{A,k}) = 1, \qquad E = \sum_{A=\mathrm{I}}^{R} \sum_{j=1}^{K_A} E_A(u_{A,k}).$$

Bethe Equations for $AdS_5 \times S^5$ and $\mathcal{N} = 4$ SYM

Complete asymptotic Bethe equations

[NB, Staudacher hep-th/0504190] [NB hep-th/0511082]

coupling constant

$$g^2 = \frac{\lambda}{16\pi^2}$$

relations between u and $x^\pm$

$$u = x^+ + \frac{1}{x^+} - \frac{i}{2g} = x^- + \frac{1}{x^-} + \frac{i}{2g}$$

magnon momentum and energy

$$e^{ip} = \frac{x^+}{x^-}, \qquad e = -igx^+ + igx^- - \tfrac{1}{2}$$

total energy

$$E = L + \tfrac{1}{2}N + \tfrac{1}{2}\dot{N} + 2\sum_{j=1}^{K}\left(\frac{ig}{x_j^+} - \frac{ig}{x_j^-}\right)$$

local charges

$$q_r(x^\pm) = \frac{1}{r-1}\left(\frac{i}{(x^+)^{r-1}} - \frac{i}{(x^-)^{r-1}}\right), \qquad Q_r = \sum_{j=1}^{K} q_r(x_j^\pm)$$

scattering factor σ with coefficients $c_{r,s}$

$$\sigma_{12} = \exp\left(i\sum_{r=2}^{\infty}\sum_{s=r+1}^{\infty} c_{r,s}(g)\,\big(q_r(x_1^\pm)\,q_s(x_2^\pm) - q_r(x_2^\pm)\,q_s(x_1^\pm)\big)\right)$$

Bethe equations

$$1 = \prod_{j=1}^{K} \frac{x_j^+}{x_j^-}$$

$$1 = \prod_{\substack{j=1\\ j\neq k}}^{\dot{M}} \frac{\dot{w}_k - \dot{w}_j - ig^{-1}}{\dot{w}_k - \dot{w}_j + ig^{-1}} \prod_{j=1}^{\dot{N}} \frac{\dot{w}_k - \dot{y}_j - 1/\dot{y}_j + \frac{i}{2}g^{-1}}{\dot{w}_k - \dot{y}_j - 1/\dot{y}_j - \frac{i}{2}g^{-1}}$$

$$1 = \prod_{j=1}^{\dot{M}} \frac{\dot{y}_k + 1/\dot{y}_k - \dot{w}_j + \frac{i}{2}g^{-1}}{\dot{y}_k + 1/\dot{y}_k - \dot{w}_j - \frac{i}{2}g^{-1}} \prod_{j=1}^{K} \frac{\dot{y}_k - x_j^+}{\dot{y}_k - x_j^-}$$

$$1 = \left(\frac{x_k^-}{x_k^+}\right)^L \prod_{\substack{j=1\\ j\neq k}}^{K} \left(\frac{u_k - u_j + ig^{-1}}{u_k - u_j - ig^{-1}}\,\sigma^2(u_k, u_j)\right) \prod_{j=1}^{\dot{N}} \frac{x_k^- - \dot{y}_j}{x_k^+ - \dot{y}_j} \prod_{j=1}^{N} \frac{x_k^- - y_j}{x_k^+ - y_j}$$

$$1 = \prod_{j=1}^{M} \frac{y_k + 1/y_k - w_j + \frac{i}{2}g^{-1}}{y_k + 1/y_k - w_j - \frac{i}{2}g^{-1}} \prod_{j=1}^{K} \frac{y_k - x_j^+}{y_k - x_j^-}$$

$$1 = \prod_{\substack{j=1\\ j\neq k}}^{M} \frac{w_k - w_j - ig^{-1}}{w_k - w_j + ig^{-1}} \prod_{j=1}^{N} \frac{w_k - y_j - 1/y_j + \frac{i}{2}g^{-1}}{w_k - y_j - 1/y_j - \frac{i}{2}g^{-1}}$$

magic coefficients

$$c_{r,s}(g) = 2\sin[\tfrac{1}{2}\pi(s-r)]\,(r-1)(s-1)\int_0^\infty \frac{dt}{t}\,\frac{\mathrm{J}_{r-1}(t)\,\mathrm{J}_{s-1}(t)}{e^{t/2g} - 1}$$

Asymptotic nature: Valid only up to terms $\mathcal{O}(e^{-*L})$ or $\mathcal{O}(\lambda^L)$. [NB, Dippel Staudacher]

Same as wrapping order where Hamiltonian can wrap the chain fully.

Bethe Equations for $AdS_4 \times \mathbb{CP}^3$ and $\mathcal{N} = 6$ SCS

Complete asymptotic Bethe equations. [Gromov, Vieira 0807.0777]

coupling constant

$$h \overset{\lambda\to 0}{\sim} \lambda \qquad h \overset{\lambda\to\infty}{\sim} \sqrt{\lambda}$$

relations between u and $x^\pm$

$$u = x^+ + \frac{1}{x^+} - \frac{i}{2h} = x^- + \frac{1}{x^-} + \frac{i}{2h}$$

magnon momentum and energy

$$e^{ip} = \frac{x^+}{x^-}, \qquad e = -ihx^+ + ihx^- - \tfrac{1}{2}$$

total energy

$$E = 2L + \tfrac{1}{2}N + 2\sum_{j=1}^{K}\left(\frac{ih}{x_j^+} - \frac{ih}{x_j^-}\right) + 2\sum_{j=1}^{\dot K}\left(\frac{ih}{\dot x_j^+} - \frac{ih}{\dot x_j^-}\right)$$

local charges

$$q_r(x^\pm) = \frac{1}{r-1}\left(\frac{i}{(x^+)^{r-1}} - \frac{i}{(x^-)^{r-1}}\right), \qquad Q_r = \sum_{j=1}^{K} q_r(x_j^\pm)$$

scattering factor σ with coefficients $c_{r,s}$

$$\sigma_{12} = \exp\left(i\sum_{r=2}^{\infty}\sum_{s=r+1}^{\infty} c_{r,s}(h)\,(q_r(x_1^\pm)\,q_s(x_2^\pm) - q_r(x_2^\pm)\,q_s(x_1^\pm))\right)$$

Bethe equations

$$1 = \prod_{j=1}^{K}\frac{x_j^+}{x_j^-}\prod_{j=1}^{\dot K}\frac{\dot x_j^+}{\dot x_j^-}$$

$$1 = \left(\frac{x_k^-}{x_k^+}\right)^L \prod_{\substack{j=1\\ j\neq k}}^{K}\left(\frac{u_k - u_j + ig^{-1}}{u_k - u_j - ig^{-1}}\,\sigma(u_k,u_j)\right)\prod_{j=1}^{\dot K}\sigma(u_k,\dot u_j)\prod_{j=1}^{N}\frac{x_k^- - y_j}{x_k^+ - y_j}$$

$$1 = \left(\frac{\dot x_k^-}{\dot x_k^+}\right)^L \prod_{\substack{j=1\\ j\neq k}}^{\dot K}\left(\frac{\dot u_k - \dot u_j + ig^{-1}}{\dot u_k - \dot u_j - ig^{-1}}\,\sigma(\dot u_k,\dot u_j)\right)\prod_{j=1}^{K}\sigma(\dot u_k,u_j)\prod_{j=1}^{N}\frac{\dot x_k^- - y_j}{\dot x_k^+ - y_j}$$

$$1 = \prod_{j=1}^{M}\frac{y_k + 1/y_k - w_j + \frac{i}{2}g^{-1}}{y_k + 1/y_k - w_j - \frac{i}{2}g^{-1}}\prod_{j=1}^{K}\frac{y_k - x_j^+}{y_k - x_j^-}\prod_{j=1}^{\dot K}\frac{y_k - \dot x_j^+}{y_k - \dot x_j^-}$$

$$1 = \prod_{\substack{j=1\\ j\neq k}}^{M}\frac{w_k - w_j - ih^{-1}}{w_k - w_j + ih^{-1}}\prod_{j=1}^{N}\frac{w_k - y_j - 1/y_j + \frac{i}{2}h^{-1}}{w_k - y_j - 1/y_j - \frac{i}{2}h^{-1}}$$

magic coefficients

$$c_{r,s}(h) = 2\sin[\tfrac{1}{2}\pi(s-r)]\,(r-1)(s-1)\int_0^\infty \frac{dt}{t}\,\frac{\mathrm{J}_{r-1}(t)\,\mathrm{J}_{s-1}(t)}{e^{t/2h}-1}$$

Based on LO Bethe equations and classical spectral curve. [Minahan Zarembo] [Gromov, Vieira 0807.0437]

Finite-Twist Operators

Finite-Twist Operators

Consider deep inelastic scattering (in any 4D gauge theory)

- Operators of lowest twist $T_{\mathcal{O}} = D_{\mathcal{O}} - S_{\mathcal{O}}$ are dominant: $\mathcal{O} = \operatorname{Tr} \mathcal{D}^S \Phi^T$.
- Scaling violations: Anomalous dimensions $\delta D_{\mathcal{O}}$ of twist-two operators.

$D_{\mathcal{O}}$ can be expressed through generalised harmonic sums cf. [Vermaseren hep-ph/9806280]

$$S_{\pm n}(k) = \sum_{j=1}^{k} \frac{(\pm 1)^j}{j^n}, \qquad S_{\pm n,m,\ldots}(k) = \sum_{j=1}^{k} \frac{(\pm 1)^j}{j^n} S_{m,\ldots}(j).$$

Calculation of twist-2 dimension [Gross Wilczek] [Georgi Politzer] [Sterman NPB281,310] [Kotikov Lipatov] [Kotikov, Lipatov Onishchenko, Velizhanin]

$$\delta D_S \sim \lambda S_1(S) + \lambda^2 \big(S_3(S) + \ldots \big) + \ldots .$$

High-spin limit: cusp dimension $D_S \sim D_{\text{cusp}} \log S$ [Moch Vermaseren Vogt] [Kotikov, Lipatov Onishchenko, Velizhanin]

$$\pi^2 D_{\text{cusp}} = \tfrac{1}{2}\lambda - \tfrac{1}{96}\lambda^2 + \tfrac{11}{23040}\lambda^3 + \ldots .$$

BES Equation

Compute cusp dimension using Bethe equations. Integral eq.: [Eden Staudacher]

$$\psi(x) = K(x,0) - (K * \psi)(x), \qquad A * B = \int_0^\infty A(y)\,\frac{dy\,y}{e^{2\pi y/\sqrt{\lambda}} - 1}\,B(y).$$

Cusp dimension: $\pi^2 D_{\text{cusp}} = \lambda\psi(0)$. Kernel $K = K_0 + K_1 + K_\text{d}$ [NB, Eden Staudacher]

$$K_{0,1}(x,y) = \pm\frac{x\,\mathrm{J}_{1,0}(x)\,\mathrm{J}_{0,1}(y) - y\,\mathrm{J}_{0,1}(x)\,\mathrm{J}_{1,0}(y)}{x^2 - y^2}, \qquad K_\text{d} = 2K_1 * K_0.$$

Prediction for cusp dimension from integrability [NB, Eden Staudacher][Basso Korchemsky Kotański]

$$\pi^2 D_{\text{cusp}}(\lambda) = \frac{\lambda}{2} - \frac{\lambda^2}{96} + \frac{11\lambda^3}{23040} - \left(\frac{73}{2580480} + \frac{\zeta(3)^2}{1024\pi^6}\right)\lambda^4 \pm \ldots,$$

$$\pi E_{\text{cusp}}(\lambda) = \sqrt{\lambda} - 3\log 2 - \frac{\beta(2)}{\sqrt{\lambda}} + \ldots,$$

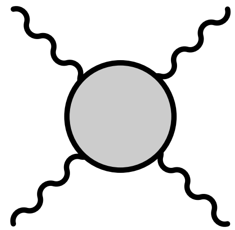
agreement! [Roiban Tirziu Tseytlin]

Gluon Scattering Amplitudes

Use gluon scattering to verify conjecture at weak coupling.

Four-gluon scattering amplitude obeys "iteration" relation [Anastasiou, Bern, Dixon, Kosower][Bern, Dixon, Smirnov]

$$A(p,\lambda) \simeq A^{(0)}(p)\exp\Big(2D_{\text{cusp}}(\lambda)M^{(1)}(p)\Big)\,.$$



Only required data: • tree level, • one loop, • cusp dimension.

Gluon scattering amplitudes constrained by unitarity.

Higher-loop supersymmetric amplitude constructible by suitable ansatz.

4-loop result in agreement with Bethe equations. [Bern, Czakon, Dixon, Kosower, Smirnov][NB, Eden, Staudacher]

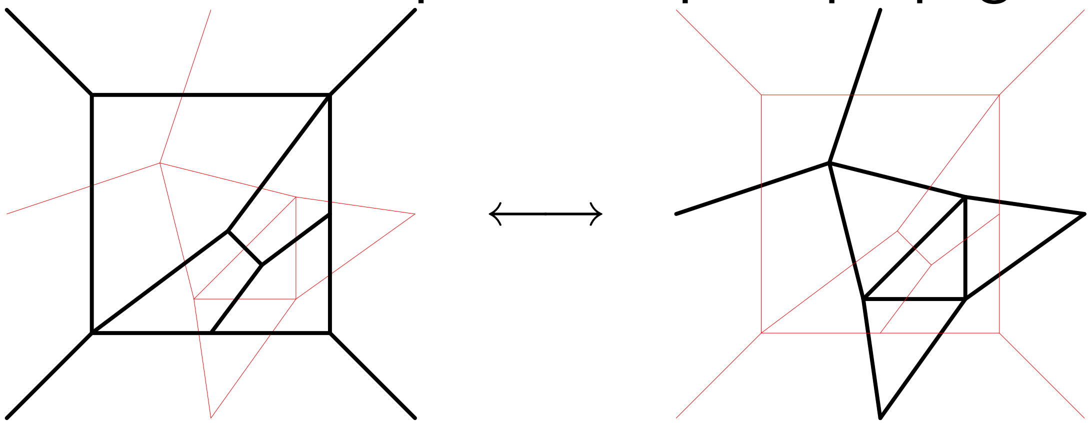
Short detour on scattering amplitudes

- Dual superconformal symmetry.
- Scattering amplitudes and light-like Wilson loops.

Detour: Dual Superconformal Symmetry

Planar amplitudes and integrals simpler than expected:

- No bubbles, no triangles, typically scalar boxes.
- Dual amplitudes and integrals are conformal. [Drummond Korchemsky Sokatchev][Drummond, Henn Korchemsky Sokatchev]...
- Similarity of momentum and position space propagators in $D = 4$.

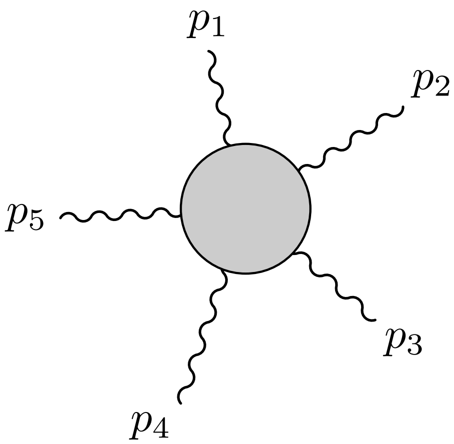


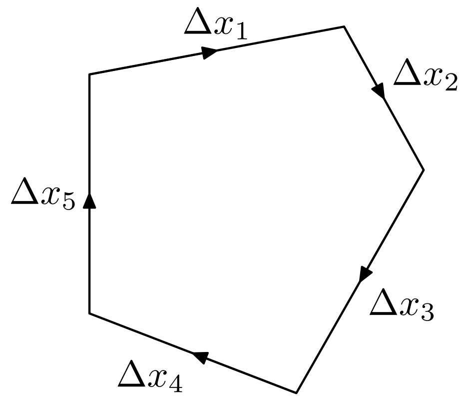
Underlying symmetry:

- T-dual string model equivalent to original model. T-self-duality! [Alday Maldacena]
- Fermions require also fermionic T-duality (bosonic!). [Berkovits Maldacena]
- Dual (super)conformal symmetry $=$ symmetry of dual model.
- Dual superconformal symmetry from string integrability. [NB, Ricci Tseytlin, Wolf][Berkovits Maldacena]

Detour: Light-Like Wilson Loops

How to relate gluon scattering to a Wilson loop? [Alday, Maldacena] [Drummond, Korchemsky, Sokatchev]





- light-like momenta $p_k^2 = 0$
- momentum conservation $\sum_k p_k = 0$
- polarisations

- light-like separations $\Delta x_k^2 = 0$
- closure $\sum_k \Delta x_k = 0$
- ? (only MHV prefactor)

Set $p_k = \Delta x_k$ and match Wilson loop expectation value with amplitude.

- Follows from T-duality in string theory. [Alday, Maldacena]
- Generic one-loop agreement. [Drummond, Korchemsky, Sokatchev]
- Four legs at two loops: Agreement (adjust renormalisation). [Drummond, Henn, Korchemsky, Sokatchev]

Finite Spins

Back: Consider operators with finite S, T.

Twist-2 scaling dimensions [Kotikov, Lipatov Onishchenko, Velizhanin] [Eden Staudacher]

$$D_S = T + \frac{\lambda}{2\pi^2} S_1 - \frac{\lambda^2}{16\pi^4} (S_3 + S_{-3} - 2S_{-2,1} + 2S_1 S_2 + 2S_1 S_{-2}) + \frac{\lambda^3}{64\pi^6} (S_5 + \ldots) + \ldots .$$

Asymptotic Bethe ansatz only valid to three loops.

Further related results:

- Easily diagonalisable for two-parameter family. [NB, Bianchi Morales Samtleben]
  Three loop results from Bethe equations. [Beccaria 0704.3570] [Kotikov, Lipatov Rej, Staudacher Velizhanin] . . .
- Generalised BES equation with soft twist-dependence. [Freyhult, Rej Staudacher]

Finite Coupling

Twist-2, spin-2 state is part of Konishi multiplet.

Four-loop Feynman diagrams [Fiamberti Santambrogio Sieg, Zanon] [NB McLoughlin Roiban] [Eden 0712.3513] [Fiamberti Santambrogio Sieg, Zanon] [Keeler Mann]

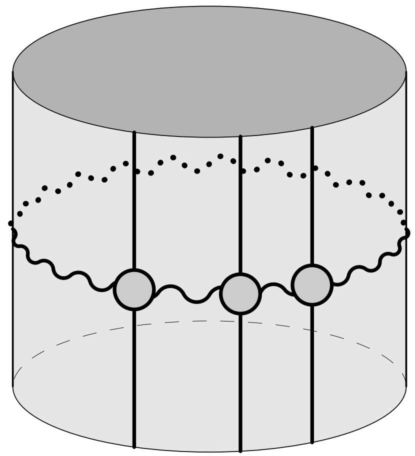
$$D = 2 + \frac{3\lambda}{4\pi^2} - \frac{3\lambda^2}{16\pi^4} + \frac{21\lambda^3}{256\pi^6} - \frac{\big(156 - 36\zeta(3) + 90\zeta(5)\big)\lambda^8}{4096\pi^8} + \ldots .$$

Prediction from asymptotic Bethe ansatz correct to 3 loops.

Include Lüscher terms at four loops [Bajnok Janik]

Sum over bound states propagating around circle.

Precise agreement at four loops.



Agreement also at strong coupling. [Gromov Schäfer-Nameki Vieira] [Heller, Janik Łokowski] [Zarembo 0802.3681] [Penedones Vieira] [Hatsuda Suzuki]

Cusp for $AdS_4 \times \mathbb{CP}^3/\mathcal{N} = 6$ CSC?

What about finite-twist operators for $AdS_4 \times \mathbb{CP}^3/\mathcal{N} = 6$ CSC?

- Bethe equations almost the same.
  BES equation applies with proper definition of g. [Gromov Vieira]

$$2\pi E_{\text{cusp}}(\lambda) = \sqrt{g} - 3\log 2 + \ldots .$$

- Semi-classical string theory result [McLoughlin Roiban] [Alday Arutyunov Bykov] [Krishnan 0807.4561]

$$\pi E_{\text{cusp}}(\lambda) = \sqrt{g} - 5\log 2 + \ldots .$$

- Discrepancy caused by different regularisation of sum over modes.
- Apparent resolution: finite renormalisation of g between schemes. [work in progress]

Problems in $AdS_4 \times \mathbb{CP}^3/\mathcal{N} = 6$ CSC?

Coupling constant:

- Superficially physical because $\lambda = N/k_{\mathrm{CS}}$ is rational.
- Apparently unphysical at strong coupling (regularisation-dependent).
- Note: function $g(\lambda)$ also unphysical in $AdS_5 \times S^5/\mathcal{N} = 4$ SYM!
- What is this function (in which scheme)? Does it interpolate nicely?

Lack of data/results and integrability assumption

- Algebraic constructions at weak coupling? Multiplets? Wrapping?
- Dressing phase the same as for $AdS_5 \times S^5/\mathcal{N} = 4$ SYM?
- Absence of 8/16 magnons at weak coupling?! Problem?

Conclusions

Conclusions

⋆ **Planar AdS/CFT Correspondence**

- String theory & coordinate space Bethe ansatz for gauge theory: Exciting particle model.
- Residual symmetry: involves $\mathfrak{psu}(2|2)$ with central extensions.
- Unique S-matrix constructed. YBE automatically satisfied.
- Proposal for interpolating phase factor at strong and weak coupling.
- Cusp anomalous dimension computed at finite coupling.
- Full agreement with AdS/CFT!

⋆ **Open Questions**

- Promote integrability in $AdS_4 \times \mathbb{CP}^3/\mathcal{N} = 6$ CSC to solid ground.
- Find exact finite-size equations.
- Mathematical structure of integrable system.