The Superconformal Gaugings in Three Dimensions

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. based on `arXiv:0804.2201, 0806.2584 and 0807.2841`

. with O. Hohm, D. Roest, M. de Roo, H. Samtleben and E. Sezgin

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The M2-brane

Sezgin, Townsend + E.B. (1987)

covariant formulation : $(X^\mu, \theta^\alpha) : 8 + 8$

lightcone gauge : 8 scalars X^I, $I = 1, \cdots 8$

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- multiple D2-branes : $U(N)$ YM theory $\rightarrow$ CFT_3 ?

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M2-branes versus D2-branes

- multiple D2-branes : $U(N)$ YM theory $\rightarrow$ CFT_3 ?
- what is CFT_3 in AdS_4/CFT_3 ?

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Multiple M2-branes

conformal susy gauge theory in three dimensions ?

YM deformation $g \sim m^{1/2}$ versus

CS deformation $g \sim m^{0}$ Schwarz (2004)

$$\mathscr{L} \sim (DX)^2 + \underbrace{g\ A \wedge \partial A + g^2\ A \wedge A \wedge A}_{\text{Chern-Simons term}} \quad \rightarrow \quad {}^*F(A) \sim DX$$

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The BLG Model

Bagger, Lambert; Gustavsson

$$f^{ab}{}_c \quad \Rightarrow \quad f^{abc}{}_d \quad \text{and} \quad h_{ab} \qquad a = 1, \ldots, N$$

reproduces Basu-Harvey equation

- linear constraint
  - $f^{abcd} = f^{[abcd]}$
- quadratic constraint
  - $f^{abe}{}_g f^{cdg}{}_f - f^{cde}{}_g f^{abg}{}_f - f^{abc}{}_g f^{dge}{}_f + f^{abd}{}_g f^{cge}{}_f = 0$

the tensor f^{abcd} is an embedding tensor !

Nicolai, Samtleben

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The Embedding Tensor

$$D_\mu \;=\; \partial_\mu - g\,\Theta_{\alpha\beta}A_\mu{}^\alpha t^\beta \;, \qquad\qquad \alpha,\beta = 1,\ldots,\dim G$$

$$\Theta_{\alpha\beta} = \Theta_{\beta\alpha}\ : \qquad\qquad \mathscr{L}_{\text{CS}} \;\sim\; \Theta_{\alpha\beta}\,A^\alpha dA^\beta$$

- The generators $\Theta_{\alpha\beta}t^\beta$ generate a closed subalgebra $G_0 \subset G$
- Θ is invariant under G_0

$\Rightarrow$ $\qquad \Theta_{\alpha\epsilon}\Theta_{\delta(\beta}f^{\delta\epsilon}{}_{\gamma)} \;=\; 0$: quadratic constraint

- SUSY $\qquad \Rightarrow \qquad$ linear constraint

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Example: $\mathcal{N} = 8$ SUSY

scalars X^{aI} : $\quad G \times H_R = SO(N) \times SO(8)$

$\Theta_{\alpha\beta} \quad \rightarrow \quad \Theta_{ab,cd} = \Theta_{[ab,cd]}$: 

SO(n) adjoint $\subset$ SO(N) : $\quad \delta_{c[a}\,\delta_{b]d} \qquad a = 1, \cdots, n$

SO(4) adjoint: $\quad \epsilon_{abcd} \quad \rightarrow \quad \Theta_{ab,cd} = g\,\epsilon_{abcd}$

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$G_2 \subset SO(7)$?

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- $\Theta_{abcd} = C_{abcd}$

linear constraint : ✓ closed G_2 algebra : × invariant tensor : ✓

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- $\Theta_{abcd} = C_{abcd}$

  linear constraint : $\checkmark$ closed G_2 algebra : $\times$ invariant tensor : $\checkmark$

- $\Theta_{ab,cd} \;=\; \delta_{c[a}\delta_{b]d} + \frac{1}{4}C_{abcd}$

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- what about $\mathcal{N} \leq 8$ SUSY ?

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$1 \leq \mathcal{N} \leq 8$ SUSY

$\mathcal{N}$	G	H_{R}	# scalars
8	$\mathrm{SO}(N)$	$\mathrm{SO}(8)$	$8N$
6	$\mathrm{U}(N)$	$\mathrm{SU}(4)$	$8N$
5	$\mathrm{Sp}(N)$	$\mathrm{Sp}(2)$	$8N$
4	$\mathrm{Sp}(N) \times \mathrm{Sp}(\mathrm{N}')$	$\mathrm{Sp}(1) \times \mathrm{Sp}(1)$	$4N + 4N'$
3	$\mathrm{Sp}(\mathrm{N})$	$\mathrm{Sp}(1)$	$4N$
2	$\mathrm{U}(N)$	$\mathrm{U}(1)$	$2N$
1	$\mathrm{SO}(N)$	1	N

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Linear Constraints

- $\mathscr{N} = 8$: $\Theta_{ab,cd} = \Theta_{[ab,cd]}$: 

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Linear Constraints

- $\mathcal{N} = 8$: $\quad \Theta_{ab,cd} = \Theta_{[ab,cd]}$: $\quad \begin{array}{c}\Box\\\Box\\\Box\\\Box\end{array}$

- $\mathcal{N} = 6$: $\quad \Theta_{a}{}^{b}{}_{,c}{}^{d} = \Theta_{[a}{}^{b}{}_{,c]}{}^{d}$: $\quad 1 \oplus \Box\,\overline{\Box} \oplus \begin{array}{c}\Box\\\Box\end{array}\,\overline{\begin{array}{c}\Box\\\Box\end{array}}$

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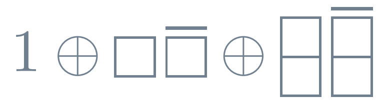
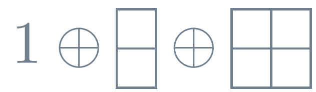
- $\mathcal{N} = 6$: $\Theta_{a}{}^{b}{}_{,c}{}^{d} = \Theta_{[a}{}^{b}{}_{,c]}{}^{d}$: 

- $\mathcal{N} = 5$: $\Theta_{(ab,cd)} = 0$: 

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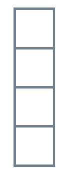

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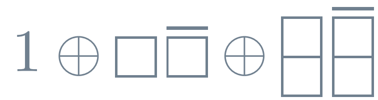

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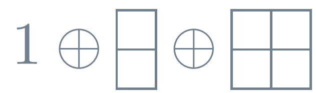

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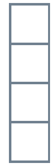
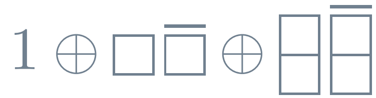
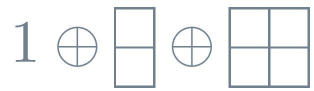
- $\mathcal{N} = 8:$ $\Theta_{ab,cd} = \Theta_{[ab,cd]}:$ 
- $\mathcal{N} = 6:$ $\Theta_{a}{}^{b}{}_{,c}{}^{d} = \Theta_{[a}{}^{b}{}_{,c]}{}^{d}:$ 
- $\mathcal{N} = 5:$ $\Theta_{(ab,cd)} = 0:$ 
- $\mathcal{N} = 4:$ $\Theta_{ab,cd}$, $\Theta_{a'b',c'd'}$ and $\Theta_{ab,c'd'}$
- $\mathcal{N} \leq 3:$ no constraint

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Building Blocks

- SO(n) adjoint: $\delta_{c[a}\,\delta_{b]d}$

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- SU(n) adjoint: $(\delta_c{}^b\,\delta_a{}^d - \frac{1}{n}\delta_a{}^b\,\delta_c{}^d)$

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- special tensors:

$$SO(4):\quad \epsilon_{abcd} \qquad SO(7):\quad \Gamma^{mn}_{ab}\Gamma^{mn}_{cd}$$
$$G_2:\quad C_{abcd}$$

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- special tensors:

  $SO(4)$: ϵ_{abcd} $SO(7)$: $\Gamma^{mn}_{ab}\Gamma^{mn}_{cd}$

  G_2 : C_{abcd}
- $a \to (i, \bar{i})$: bi-fundamental representation

  . allow singlets like $\delta_{ab}\delta_{cd}$ etc.

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The ABJM Model

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$$\Theta_a{}^b{}_{,c}{}^d = g\,\delta_{[a}{}^{[d}\,\delta_{c]}{}^{b]} \;=\; \frac{g}{2}\,\underbrace{\left(\delta_c{}^b\,\delta_a{}^d - \frac{1}{n}\,\delta_a{}^b\,\delta_c{}^d\right)}_{\mathsf{SU}(n)} - \frac{(n-1)}{n}\,\frac{g}{2}\,\underbrace{\delta_a{}^b\,\delta_c{}^d}_{\mathsf{U}(1)}$$

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$$\Theta_{(i,\bar{i})}{}^{(k,\bar{k})}{}_{,(j,\bar{j})}{}^{(l,\bar{l})} \;=\; g\,\delta_{\bar{i}}{}^{\bar{k}}\delta_{\bar{j}}{}^{\bar{l}}\underbrace{\left(\delta_i{}^l\delta_j{}^k - \frac{1}{m}\delta_i{}^k\delta_j{}^l\right)}_{\mathsf{SU}(m)} - g\delta_j{}^l\delta_i{}^k\underbrace{\left(\delta_{\bar{j}}{}^{\bar{k}}\delta_{\bar{i}}{}^{\bar{l}} - \frac{1}{n}\delta_{\bar{i}}{}^{\bar{k}}\delta_{\bar{j}}{}^{\bar{l}}\right)}_{\mathsf{SU}(n)}$$

$$-\frac{(m-n)}{mn}g\,\underbrace{\delta_i{}^k\delta_j{}^l\delta_{\bar{i}}{}^{\bar{k}}\delta_{\bar{j}}{}^{\bar{l}}}_{\mathsf{U}(1)}$$

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Symplectic Gaugings

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$$\Theta_{ab,cd} \;=\; g\, \underbrace{\Omega_{ab}\Omega_{cd}}_{\mathsf{U}(1)} - g\, \underbrace{\left(\Omega_{ca}\Omega_{bd} \;+\; \Omega_{cb}\Omega_{ad}\right)}_{\mathsf{Sp}(n)}$$

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Superconformal Gaugings

$\mathcal{N}$	gauge group
8	$\mathrm{SU}(2) \times \mathrm{SU}(2)$
$4,6$	$\mathrm{SU}(m) \times \mathrm{SU}(n) \times \mathrm{U}(1)$
$4,6$	$\mathrm{SO}(2) \times \mathrm{Sp}(n)$
$4,5$	$\mathrm{SO}(m) \times \mathrm{Sp}(n)$
$4,5$	$\mathrm{SO}(7) \times \mathrm{SL}(2)$
$4,5$	$G_2 \times \mathrm{SL}(2)$
$4,5$	$\mathrm{SO}(4) \times \mathrm{Sp}(2)$

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Superconformal Gaugings

Gaiotto, Witten

$\mathcal{N}$	gauge group	Lie superalgebra
8	$\mathrm{SU}(2)\times\mathrm{SU}(2)$	$\mathrm{U}(2\vert 2)$
4, 6	$\mathrm{SU}(m)\times\mathrm{SU}(n)\times\mathrm{U}(1)$	$\mathrm{U}(m\vert n)$
4, 6	$\mathrm{SO}(2)\times\mathrm{Sp}(n)$	$\mathrm{OSp}(2\vert n)$
4, 5	$\mathrm{SO}(m)\times\mathrm{Sp}(n)$	$\mathrm{OSp}(m\vert n)$
4, 5	$\mathrm{SO}(7)\times\mathrm{SL}(2)$	$F(4)$
4, 5	$G_2\times\mathrm{SL}(2)$	$G(3)$
4, 5	$\mathrm{SO}(4)\times\mathrm{Sp}(2)$	$OSp(4\vert 2;\alpha)$

Generalizations

- $\mathcal{N} = 1$: interactions without gauging

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- $\mathcal{N} > 8$?
  - CS deformations : $\frac{\mathrm{SO}(8,N)}{\mathrm{SO}(8)\times\mathrm{SO}(N)} \cdots \frac{E_{7(-5)}}{\mathrm{SO}(12)\times\mathrm{Sp}(1)} \cdots \frac{E_{8(8)}}{SO(16)}$

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  - massive deformations : $\mathcal{N} \leq 16$ ✓

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- $\mathcal{N} = 1$: interactions without gauging
- massive CS deformations : m and g
- $\mathcal{N} > 8$?
  - CS deformations : $\frac{\mathrm{SO}(8,N)}{\mathrm{SO}(8)\times\mathrm{SO}(N)} \cdots \frac{E_{7(-5)}}{\mathrm{SO}(12)\times\mathrm{Sp}(1)} \cdots \frac{E_{8(8)}}{SO(16)}$
  - massive deformations : $\mathcal{N} \leq 16$ ✓

$$\mathcal{N} = 4 : \quad \{Q^i_\alpha, Q^j_\beta\} = 2(\gamma^\mu C)_{\alpha\beta} P_\mu \delta^{ij} + \underbrace{2mC_{\alpha\beta}\,\epsilon^{ijkl}\, M_{kl}}_{\text{non-central term}}$$

Hohm + E.B., in preparation

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Outlook

- new superconformal gauge theories in $D = 3$ dimensions

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- M5-branes (N^3 behaviour) ?

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