D-branes on T-folds with few T's

Massimo BIANCHI

Università di Roma "Tor Vergata" - INFN
CERN Theory Division
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Foreword

- Most mechanisms of moduli stabilization require (R-R) fluxes that are poorly understood in string theory
- Yet, in asymmetric orbifolds or alike, chiral twists freeze untwisted moduli, non-geometric shifts prevent massless twisted moduli [Dabholkar, Hull; Dine, Silverstein; Kumar, Vafa; ...M.B., Morales, Pradisi]
- The combined effect of twists and shifts has not been systematically explored in the context of unoriented strings
- Expect duality between (D-branes in) non-geometric vacua and (D-branes in) flux compactifications, at least for extended susy ($\mathcal{N} \geq 3$)

Plan

Part I: Unoriented twists and shifts [Anastasopoulos, MB, Morales, Pradisi (w.i.p.)]

- Unoriented T-folds with few T's
- Two simple examples based on $T^6_{SO(12)}/Z_2^L \times Z_2^R \times Z'^L_2 \times Z'^R_2$:
  - "h_{11}" $=$ "h_{21}" $= 1$ (self-mirror)
  - "h_{11}" $= 0$, "h_{21}" $= 6$ and its "mirror" (discrete torsion)
- Partial moduli stabilization, rank reduction, non-chiral ...

Part II: Bound-states of D-branes in L-R asymmetric vacua [MB]

- Residual susy and R-R couplings
- $\mathcal{N} = 6 = 2_L + 4_R$ and other extended susy cases
- Open string excitations

Outlook and Announcements

Part I: Unoriented twists and shifts

Unoriented T-folds with few T's

CDMP model [Camara, Dudas, Maillard, Pradisi; Vafa, Witten]: standard geometric freely acting orbifold $T^6/Z_2 \times Z_2$, Type I / Heterotic dual pairs All twisted moduli are massive. Only untwisted moduli T_I, U_I, Combine with gaugino condensate(s) in open string sector and/or 3-form fluxes to partially stabilize dilaton and other moduli.

DJK 'minimal' model [Dolivet, Julia, Kounnas]: (non-magic) hyper-free model, fermionic construction

$$G = \psi^\mu \partial X_\mu + \chi^i y^i w^i$$

Fermionic basis sets [Antoniadis, Ellis, Hagelin, Nanopoulos; ... Faraggi, Kounnas; ...] :
$F, S, \bar{S}, \bar{b}_1, b_1 = \{\psi^\mu, \chi^{1,2}; y^{3,4,5,6}, y^1 w^1 | \bar{y}^5 \bar{w}^5\}$,
$b_2 = \{\psi^\mu, \chi^{3,4}; y^{1,2}, w^{5,6} y^3 w^3 | \bar{y}^6 \bar{w}^6\}$,
$b_3 = \{\psi^\mu, \chi^{5,6}; w^{1,2,3,4} y^6 w^6 | \bar{y}^6 \bar{w}^6\}$
Only dilaton vector-multiplet survives. $\mathcal{N}_L = 0$, L-R asymmetric $(-)^{F_L}\sigma$ freely acting orbifold of $T^6_{SO(12)}$, $y^i w^i \approx$ shifts.

Combining DJK and CDMP

Replace $\bar{b}_3$ with b_2, get a L-R symmetric asymmetric orbifold.
Geometric (freely acting) projections associated to $b_1\bar{b}_1$ and $b_2\bar{b}_2$.
Non geometric (freely acting) projections associated to $b_1, b_2, \ldots b_1\bar{b}_2$.
All untwisted moduli except dilaton hypermultiplet are projected out ($\mathcal{N}_L = \mathcal{N}_R = 1, \mathcal{N}_{tot} = 2$).
Massless multiplets only from $b_1 b_2 \bar{b}_1 \bar{b}_2$ twisted sector: one hyper and one vector, "h_{11}" $=$ "h_{21}" $= 1$ ($\mathcal{HOLY}\,\mathcal{GRAIL}$ [Szendroi, Gross])
Unoriented projection produces $1_u + 2_t - n$ chiral-plets and n vector-plets ($n = 0, 1$)
Open string spectrum: non chiral, rank reduction from B-field.
Systematic study under way. For MSSM embeddings [Kiritsis, Schellekens]
Combine with (non) anomalous $U(1)$'s, fluxes, (non)geometric D-brane instantons (gaugino condensation, ADS-like superpotentials, ...) [Blumenhagen's, Dudas's, Cvetic's talks]

$T^6/(Z_{2L} \times Z'_{2L} \times Z_{2R} \times Z'_{2R})$ model with "h_{11}" = "h_{21}" = 1

Generators of the orbifold group specified by the fermionic sets

$$
\begin{array}{rcl}
g_1 & = & \{\chi_{3456}\,;\; y_{13456}\,;\; w_1 \;|\; \;;\; \tilde{y}_5\,;\; \tilde{w}_5\,\} \\
g_2 & = & \{\chi_{1256}\,;\; y_{123}\,;\; w_{356} \;|\; \;;\; \tilde{y}_6\,;\; \tilde{w}_6\,\} \\
\tilde{g}_1 & = & \{\;;\; y_5\,;\; w_{15} \;|\tilde{\chi}_{3456}\,;\; \tilde{y}_{13456}\,;\; \tilde{w}_1\,\} \\
\tilde{g}_2 & = & \{\;;\; y_6\,;\; w_6 \;|\tilde{\chi}_{1256}\,;\; \tilde{y}_{123}\,;\; \tilde{w}_{356}\,\}
\end{array}
$$

Introducing

$$
\begin{array}{rcl}
g_3 & = & g_1 g_2 = \{\chi_{1234}\,;\; y_{2456}\,;\; w_{1356} \;|\; \;;\; \tilde{y}_{56}\,;\; \tilde{w}_{56}\,\} \\
\tilde{g}_3 & = & \tilde{g}_1 \tilde{g}_2 = \{\;;\; y_{56}\,;\; w_{56} \;|\tilde{\chi}_{1234}\,;\; \tilde{y}_{2456}\,;\; \tilde{w}_{1356}\,\}
\end{array}
$$

the generic orbifold group element can be written as $g_m^a \tilde{g}_n^b$ with $m, n = 1, 2, 3,\ a, b = 0, 1$.

Torus partition function

$$\mathcal{T} = \tfrac{1}{16} \sum_{a,b,c,d=0}^{3} \rho_{ac} \, \bar{\rho}_{bd} \, \mathbf{\Lambda}_{ab,cd}$$

with ρ_{ab} chiral orbifold amplitudes and $\mathbf{\Lambda}_{ab,cd}$ invariant (sub)lattices of $SO(12)$
Only massless states come from the $g_3\tilde{g}_3$-twisted sector:

$$\mathcal{T} \;\; = \;\; |V - S - C|^2 + |2O - S - C|^2 + \ldots$$

$\mathcal{N} = 2$ supergravity coupled to 1+1 hypers and 1 vector
"h_{11}" = "h_{21}" $= 1$, "χ" $= 0$ (self-mirror) !!

Unoriented projection and open strings

Klein bottle projection

$$\mathcal{K} = \frac{1}{32}\sum_{a,b,c=0}^{3} \mathrm{Tr}_{g_a\tilde{g}_a}\Omega g_b\tilde{g}_c (q\bar{q})^{H_{cl}} = \frac{1}{8}\sum_{a,d=0}^{3} \mathrm{Tr}_{g_a\tilde{g}_a}\Omega\hat{g}_d (q\bar{q})^{H_{cl}}$$
$$= \frac{1}{8}\sum_{a,d=0}^{3} \rho_{a,d}\hat{\Lambda}_{a,d}(2\tau_2)$$

only 64 'effective' characters $\hat{g}_a$: $SO(12) \to SO(4)^2 \times SO(2)^2$
Transverse channel $\tilde{\mathcal{K}} \approx \chi_1$ (identity), as a consequence $\tilde{\mathcal{M}} = \hat{\chi}_1$ and the Möbius-strip projection is simply

$$\mathcal{M} = +\hat{\chi}_1 - \hat{\chi}_4 - \hat{\chi}_5 - \hat{\chi}_8$$

Symplectic Chan-Paton group, tadpole condition $Sp(1)^8$ with uninteresting non-chiral matter

Intermezzo: B-field and no vector structure

Most 'rational' models allow/require discrete non zero values for the 'frozen' moduli

B-field in Type I, odd under Ω, $B = 1/2$ allowed, rank reduction in toroidal compactifications with flat bdles (susy) $N = 32 \times 2^{-r_B/2}$ [MB, Sagnotti; MB, Pradisi, Sagnotti], for $SO(12)$ $r_B = 4$

Heterotic dual description: CHL strings [Chaudhuri, Hockney, Lykken]

Compactifications without vector structure $B = \tilde{w}_2$, non commuting Wilson lines [Sen, Sethi; Berkooz et al; MB; Witten; Dijkgraaf et al]

For non-susy (non-tachyonic) toroidal models with magnetic fields and 't Hooft fluxes, rank reduction not necessary

Model C [Bachas] $G = U(5) \times U(3) \times U(4) \times U(4)'$ with $(5^*, 3)$ and 3 copies of 10. Non susy. Magnetic field without vector structure, requires non zero B-field for consistency [Bachas, MB, Blumenhagen, Lüst, Weigand]

$T^6/(Z_{2L} \times Z'_{2L} \times Z_{2R} \times Z'_{2R})$ model with "h_{21}" $= 6$, "h_{21}" $= 0$ and its mirror

Generators of the orbifold group specified by the fermionic sets

$$
\begin{array}{rcl}
g'_1 & = & \{\chi_{3456}\ ;\ y_{1235}\ ;\ w_{1246}\ |\ \ ;\ \tilde{y}_{35}\ ;\ \tilde{w}_{35}\ \} \\
g'_2 & = & \{\chi_{1256}\ ;\ y_{2346}\ ;\ w_{1345}\ |\ \ ;\ \tilde{y}_{15}\ ;\ \tilde{w}_{15}\ \} \\
\tilde{g}'_1 & = & \{\ \ ;\ y_{35}\ ;\ w_{35}\ |\tilde{\chi}_{3456}\ ;\ \tilde{y}_{1235}\ ;\ \tilde{w}_{1246}\ \} \\
\tilde{g}'_2 & = & \{\ \ ;\ y_{15}\ ;\ w_{15}\ |\tilde{\chi}_{1256}\ ;\ \tilde{y}_{2346}\ ;\ \tilde{w}_{1345}\ \}
\end{array}
$$

All 'geometric' twisted sectors $g_a\tilde{g}_a$ contribute massless states, depending on discrete torsion $\epsilon = \pm 1$

- $\epsilon = +1$: $6 = 3 \times 2$ vector multiplets
- $\epsilon = -1$: $6 = 3 \times 2$ hyper multiplets

Including open and unoriented strings

Klein bottle projection, similar to previous case, yet
$\hat{g}'_a : SO(12) \to SO(4)^4$.
In the transverse channel $\tilde{K} \approx \chi_1 + \mathrm{massive}$.
(So far) only viable possibility: $\tilde{M} \approx \hat{\chi}_1$.
Uninteresting $\mathcal{O}(N_{Avogadro}/10^{23})$ non-chiral open string spectra.
Identification of D-branes under T-duality twists ... loss of chirality

Part II: Bound-states of D-branes in L-R asymmetric vacua

Type II superstring vacua with extended susy

[Ferrara, Kounnas; Dabholkar, Harvey]

$$\mathcal{N} = 8 \quad \leftrightarrow \quad \mathcal{N}_L = 4 \, , \; \mathcal{N}_R = 4$$
$$\mathcal{N} = 6 \quad \leftrightarrow \quad \mathcal{N}_L = 2 \, , \; \mathcal{N}_R = 4$$
$$\mathcal{N} = 5 \quad \leftrightarrow \quad \mathcal{N}_L = 1 \, , \; \mathcal{N}_R = 4$$
$$\mathcal{N} = 4 \quad \leftrightarrow \quad \mathcal{N}_L = 2 \, , \; \mathcal{N}_R = 2 \quad \textbf{or} \quad \mathcal{N}_L = 0 \, , \; \mathcal{N}_R = 4$$
$$\mathcal{N} = 3 \quad \leftrightarrow \quad \mathcal{N}_L = 1 \, , \; \mathcal{N}_R = 2$$
$$\mathcal{N} = 2 \quad \leftrightarrow \quad \mathcal{N}_L = 1 \, , \; \mathcal{N}_R = 1 \quad \textbf{or} \quad \mathcal{N}_L = 0 \, , \; \mathcal{N}_R = 2$$

Generically, L-R asymmetric and thus non-geometric but 'exact' vacuum configurations based on (rational) CFT's

When massless R-R states (eg graviphotons) survive there MUST be bound-states of D-branes they couple to

Some fraction of extended susy is preserved, BPS condition

Use boundary states to determine open string excitations

$\mathcal{N} = 6 = 2_L + 4_R$ case

Spontaneous breaking $\mathcal{N} = 8 \to \mathcal{N} = 6$ via chiral Z_2 twist of the L-movers ('T-duality' on four internal directions, T^4_t)

$$X^i_{_L} \to -X^i_{_L} \quad , \quad \Psi^i_{_L} \to -\Psi^i_{_L} \quad , \quad i = 6, 7, 8, 9$$

accompanied by an order two shift along untwisted T^2_s
Unbroken susy's satisfy

$$\mathcal{Q}_{_L} = \Gamma_{6789} \mathcal{Q}_{_L}$$

no conditions on $\mathcal{Q}_{_R}$.
After dualizing all masseless 2-forms into axions, the $30 = 2_{_{NS-NS}} + 12_{_{NS-NS}} + 16_{_{R-R}}$ scalar moduli parameterize the space $\mathcal{M}^{D=4}_{\mathcal{N}=6} = SO^*(12)/U(6)$. The $16 = 8_{_{NS-NS}} + 8_{_{R-R}}$ vectors together with their magnetic duals transform according to the **32** dimensional chiral spinor representation of $SO^*(12)$.

$\mathcal{N} = 6$ BPS branes, susy

Surviving $16_{R-R} = 2_{(1|5)} + 4_{(1|3)} + 6_{(3|3)} + 4_{(5|3)}$ R-R charges carried by D-brane bound-states invariant under twist and shift

$$q_1^a + \frac{1}{4!}\varepsilon_{ijkl} q_5^{aijkl} \quad , \quad q_1^i + \frac{1}{3!}\varepsilon^i{}_{jkl} q_3^{jkl} \quad ,$$

$$q_3^{aij} + \frac{1}{2!}\varepsilon^{ij}{}_{kl} q_3^{akl} \quad , \quad q_5^{abijk} + \varepsilon^{ijk}{}_l q_3^{abl}$$

Eg 1/3 BPS state: D5 wrapped along twisted $T_t^4 \times S_s^1$ and D1 along same S_s^1,

$$\mathcal{Q}_R = \Gamma_{04}\Gamma_{6789}\mathcal{Q}_L = \Gamma_{04}\mathcal{Q}_L$$

Different analysis for BPS states carrying NS-NS charges eg two massive gravitini and superpartners form a complex 1/2 BPS multiplet

D-branes in T-folds [Brunner, Rajaraman, Rozali; Gutperle; MB, Morales, Pradisi; Gaberdiel, Schafer-Nameki; Lawrence, Schulz, Wecht; Kawai, Sugawara]

Other $\mathcal{N} = 6$ cases

- Z_n chiral projection acting on 4 super-coordinates as

  $$(Z^1, Z^2)_L \to (\omega Z^1, \omega^{-1} Z^2)_L \quad , \quad (\Psi^1, \Psi^2)_L \to (\omega \Psi^1, \omega^{-1} \Psi^2)_L$$

  with $\omega^n = 1$. In order to avoid massless twisted states, combine with an order n shift along the 'untwisted' directions $(Z^3_L; Z^i_R)$
- maximal torus of $SU(3)^3$ with chiral Z_3 projection and no shift. $\mathcal{N} = 5$ supergravity in untwisted sector. Twisted sector produces the extra massless gravitino multiplet to complete the spectrum of $\mathcal{N} = 6$ supergravity [Dabholkar, Harvery]

Other extended susy cases with L≠R

- $\mathcal{N} = 5 = 1_L + 4_R$, unique massless spectrum, non-geometric, uncorrected LEEA (as for $\mathcal{N} = 6, 8$)
- $\mathcal{N} = 4 = 2_L + 2_R$ uncorrected LEEA, (non)geometric, $SL(2) \times SO(6, N_v)$ symmetry
- $\mathcal{N} = 3 = 1_L + 2_R$ uncorrected LEEA, non-geometric / fuxes, $U(3, N_v)$ symmetry
- $\mathcal{N} = 2 = 1_L + 1_R$, (non) geometric, quantum corrections absent in special cases ($\chi = 0$, eg FHSV, octonionic magic)
- $\mathcal{N} = 4 = 0_L + 1_R$, $\mathcal{N} = 4 = 0_L + 2_R$, $\mathcal{N} = 1 = 0_L + 1_R$ NO massless R-R graviphotons, NO BPS D-branes

Focus on $\mathcal{N} = 5, 3$

$\mathcal{N} = 5 = 1_L + 4_R$ case

Simple(st) realization [Ferrara, Kounnas] $Z_2^L \times Z_2^L$ which acts by T-duality along T^4_{6789} and T^4_{4589} combined with order two shifts
1/5 BPS bound states of D-branes carrying
$8_{R-R} = 6_{(1533)} + 2_{(3333)}$ R-R charges (invariant orbits)

$$q^I_{(1335)} = q^I_1 + \frac{1}{4!}\varepsilon_{i_I j_I k_I l_I} q_5^{I i_I j_I k_I l_I} + \frac{1}{3!}\varepsilon^I{}_{J,K'L'} q_3^{JK'L'} + \frac{1}{3!}\varepsilon^I{}_{J,K"L"} q_3^{JK"L"}$$

where i_I, j_I, k_I, l_I run over the four directions orthogonal to T^2_I while K', L' and $K", L"$ run over the two sets of two directions orthogonal to T^2_I and

$$q^{I_1 I_2 I_3}_{(3333)} = q_3^{I_1 I_2 I_3} + \frac{1}{2!}\varepsilon^{I_2 I_3}{}_{J_2 J_3} q_3^{I_1 J_2 J_3} + \frac{1}{2!}\varepsilon^{I_3 I_1}{}_{J_3 J_1} q_3^{J1 I_2 J_3} + \frac{1}{2!}\varepsilon^{I_1 I_2}{}_{J_1 J_2} q_3^{J_1 J_2 I_3}$$

$\mathcal{N} = 3 = 1_L + 2_R$ case

The simplest $\mathcal{N} = 3$ model with 3 matter vector-plets. Two steps

- 'geometric' Z_2 freely acting orbifold (locally equivalent to $K3 \times T^2$). The Z_2 action combines a twist breaking $\mathcal{N} = 8 = 4_L + 4_R$ to $\mathcal{N} = 4 = 2_L + 2_R$ and a shift preventing massless twisted states
- non geometric chiral (say Left-) projection combined with a shift along the orthogonal directions $a = 4, 5$

Surviving NS-NS charges: p^a_R and their magnetic duals $\hat{P}^a_R$.
Surviving R-R charges: T-duality invariant combinations

$$q^a_1 + \frac{1}{3!}\varepsilon^a_{bij} q^{bij}_3 \quad , \quad q^{aij}_3 + \frac{1}{3!}\varepsilon^a_{bkl} q^{bijkl}_5$$

One can consider 1/3 BPS states.
Massive gravitino in $\mathcal{N} = 4 \to \mathcal{N} = 3$ long multiplet.
No 1/2 BPS states

Boundary states for L-R asymmetric branes

[Abouelsaood, Callan, Lovelace, Nappi, Yost; Billò, Di Vecchia, Frau, Lerda, Pesando].

Bosonic coordinates

$$|B_a\rangle^{(X)} = \sqrt{\det(\mathcal{G}_a + \mathcal{F}_a)}\, \exp(-\sum_{n>o} a^i_{-n} R_{ij}(F_a) \tilde{a}^j_{-n})|0_a\rangle$$

where $R_a = (1 - F_a)/(1 + F_a)$ and $|0_a\rangle \leftrightarrow p_{_L} = -R_a p_{_R}$

NS-NS sector (no fermionic zero-modes)

$$|B_a, \pm\rangle^{(\psi)}_{NS-NS} = \exp(\pm i \sum_{n\geq 1/2} \psi^i_{-n} R_{ij}(F_a) \tilde{\psi}^j_{-n})|\pm\rangle$$

R-R sector

$$|B_a, \pm\rangle^{(\psi)}_{R-R} = \frac{1}{\sqrt{\det(\mathcal{G}_a + \mathcal{F}_a)}} \exp(i\pm\sum_{n>0} \psi^i_{-n} R_{ij}(F_a) \tilde{\psi}^j_{-n}) \mathcal{U}^{\pm}_{A\tilde{B}}(F_a)|A, \tilde{B}\rangle$$

$$\mathcal{U}^{\pm}_{A\tilde{B}}(F_a) = \left[\mathrm{AExp}(-F^a_{ij}\Gamma^{ij}/2) C\Gamma_{11} \frac{1 \pm i\Gamma_{11}}{1 \pm i}\right]_{A\tilde{B}} \quad .$$

Partition functions and tree channel

Magnetized / Intersecting D-branes in L-R symmetric orbifolds

[Angelantonj, Sagnotti; Blumenhagen, Cvetic, Langacker, Shiu; Blumenhagen, Körs, Lüst, Stieberger]

Generalize to $Z^L_{N_L} \times Z^R_{N_R}$ action, invariant boundary states would then be of the form

$$|B,F\rangle_g = \frac{1}{\sqrt{N_L N_R}}\left(1 + g_L + g_R + \dots + g_L^{N_L-1} g_R^{N_R-1}\right)|B,F\rangle = $$

$$= \frac{1}{\sqrt{N_L N_R}} \sum_{l,r} |B, F_{(l,r)}\rangle$$

where the 'induced' magnetic field $F_{(l,r)}$ is determined by the condition

$$R(F_{(l,r)}) = R(g_L^l) R(F) R^t(g_R^r)$$

$$\mathcal{A}_{g,h} = \Lambda(g,h)\mathcal{I}(g,h) \sum_\alpha c_\alpha^{GSO} \frac{\vartheta_\alpha(0)}{\eta^3} \prod_I \frac{\vartheta_\alpha(\epsilon_I(g,h)\tau)}{\vartheta_1(\epsilon(g,h)\tau)}$$

Example: $\mathcal{N} = 5$ model on T^6/Z_3^L torus of $SU(3)^3$

Prior to twists and shifts, 27 boundary states

$$\mathcal{A}_{\vec{r},\vec{s}} = N_{\vec{r},\vec{s}}^{\vec{t}} \mathcal{X}_{\vec{t}}$$

where $\mathcal{X}_{\vec{t}} = (V_8 - S_8)\chi_{t_1}\chi_{t_2}\chi_{t_3}$ correspond to branes with magnetic quantum number $(n, m) = (1, 0), (-1, 1), (0, -1)$
After twist and shift: branes rotated and displaced wrt one another

$$\mathcal{A}_{Z_3^{L\neq R}} = \frac{1}{6} \sum_{a,b\in Z_3} \Lambda_{(a,b)} \mathcal{I}_{(a,b)} \sum_{\alpha} c_\alpha^{GSO} \frac{\vartheta_\alpha(0)}{\eta^3} \prod_I \frac{\vartheta_\alpha(a\tau + b)}{\vartheta_1(a\tau + b)}$$

Both ‘untwisted’ and ‘twisted’ strings are present [MB, Morales, Pradisi; Blumehagen, Gorlich, Kors, Lüst; ...], yet non chiral spectra

Outlook

Twists and shifts can conveniently combine with other mechanisms, *e.g.* open and closed string fluxes, non-anomalous $U(1)$'s, instanton effects, ... of moduli stabilization.

Explicit computations are feasible. Model with only 1+1 twisted moduli ... some tension with chirality ($\chi = 0$)

D-branes in L-R asymmetric vacua, very promising ... yet relation with (non) geometric fluxes [Lawrence, Schulz, Wecht], [Hull, ...], [Berman, ...], [Villadoro, Zwirner], ... to be understood and interacting CFT's (WZW, Gepner or alike) to be worked out [Kawai, Sugawara]

Announcements

- *Strings '09*

  Rome, 22-26 June 2009

  Angelicum - Pontificia Università S. Tommaso
  http://people.roma2.infn.it/ strings2009/

- *New Perspectives in String Theory*

  GGI, Arcetri (Florence), 6 April - 19 June 2009
  http://ggi-
  www.fi.infn.it/index.php?p=events.inc&id=25