Black hole microstate counting

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Varna, Sept 12, 2008

Outline

Overview of approaches to black hole microstate counting

Brief overview of some recent developments

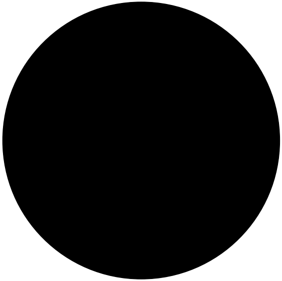
BPS bound states and wall crossing for dummies

The OSV conjecture

Wall crossing for non-dummies

Overview of some approaches

Bekenstein-Hawking entropy

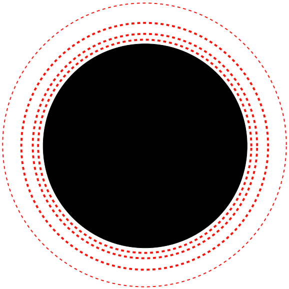


$$S_{BH} = \frac{A}{4G}$$

- Schwarzschild: $S = 4\pi GM^2$
- Extremal Reissner-Nordström: $S = \pi GM^2$
- N=2 BPS: $S = \min \pi GM^2$

$\rightsquigarrow$ stat. mech. explanation?

1. Quantizing field fluctuations in black hole background



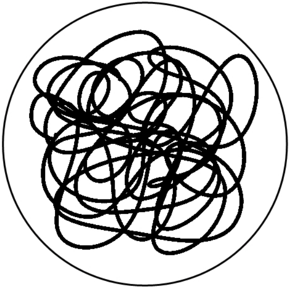
['t Hooft 85]

- the good: $S = \mathrm{const.} \times A$
- the bad: $\mathrm{const.} = \infty \quad (= \Lambda_{UV} \sim 1/G)$
- the puzzling: depends on cutoff, number of species and other details

(related: interpretation as entanglement entropy [Srednicki 93, Bombelli *et al* 86])

2. Highly excited fundamental string states

[Susskind 93]



- (Free) string microscopic entropy: $S_{micro} \sim \ell_s M$
- But $S_{\rm BH} \sim GM^2$??
- Redshift factor at distance ℓ_s from horizon ("stretched horizon") $= \ell_s/GM$

$$\Rightarrow \qquad M_{loc} \sim GM^2/\ell_s \qquad S_{micro} \sim \ell_s M_{loc} \sim GM^2 \quad \checkmark$$

3. BPS black holes

Saturating BPS bound for given charge Q:

$$M = \frac{|Z(Q)|}{\sqrt{G}}$$

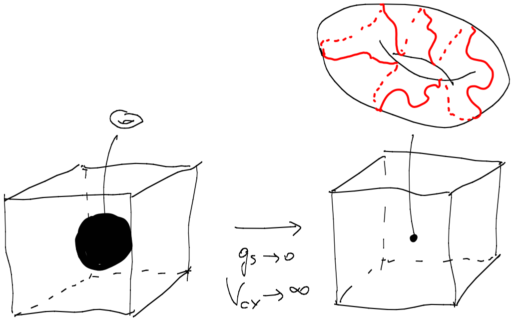
Here $Z(Q) =$ central charge; depends on vacuum moduli.

Remains true with $M \to M_{loc}$ for any local static observer, even with varying moduli and metric! Unlike non-BPS case, mass *not* renormalized between near and far!

$\Rightarrow$ entropy $=$ log $\#$ states of charge Q and energy $E = \frac{|Z(Q)|}{\sqrt{G}}|_{hor}$.

$Q \neq 0 \quad \Rightarrow \quad$ Need to count (BPS) D-brane states.

3. BPS black holes (continued)



Use:

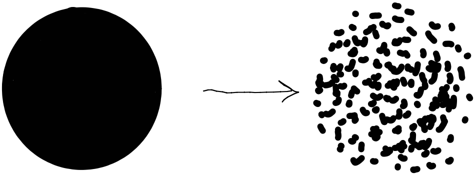
1. generically: degeneracy at finite coupling $\sim$ (good) index at finite coupling
2. index invariant under change of couplings

and compute semiclassically in weakly coupled regime.

$\rightsquigarrow$ $S_{micro} = S_{BH}$ [Stroninger-Vafa 96, Maldacena-Strominger-Witten 97]

[However all successful cases dual to D4-D0/M5-P/AdS_3 systems!]

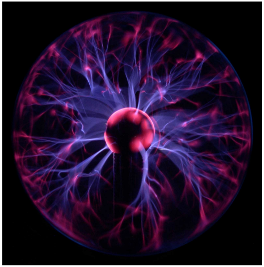
4. (BPS) Fuzzballs



[Mathur, Benna-Warner, Balasubramanian-Gimon-Levi, de Boer *et al*, Strominger *et al*, Skenderis-Taylor *et al*, X *et al*]

- There exist very intricate 4d multiparticle BPS bound states.
- "Scaling" solutions: asymptotically indistinguishable from black hole.
- Huge internal degeneracies. Partial quantitative successes, however so far not shown to be huge enough to reproduce full 4d S_{BH}.
- Adding spherical D2-branes seems to $\pm$ reproduce S_{BH}.

5. AdS_5-CFT_4

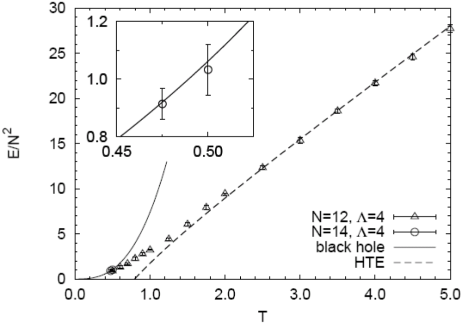


Comparison $\mathcal{N} = 4$ $SU(N)$ YM field theory entropy S_0 computed at zero 't Hooft coupling g^2N and black hole entropy at large 't Hooft coupling S_∞:

$$S_\infty = \frac{3}{4}\, S_0$$

Renormalization effect quite well understood by now.

6. Computer simulations



[Anagnostopoulos-Hanada-Nishimura-Takeuchi 07]

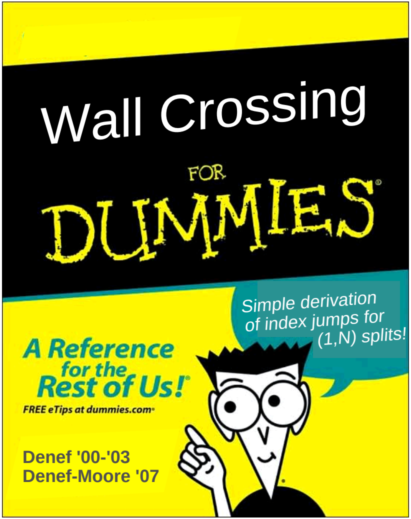
Monte Carlo simulations of $U(N)$ susy QM with 16 supercharges, dual to (nonextremal) black hole with D0 charge N in IIA.

Perhaps most convincing confirmation ever that string theory gets quantum gravity right!

Some recent developments in (non-fuzzball) microstate counting:

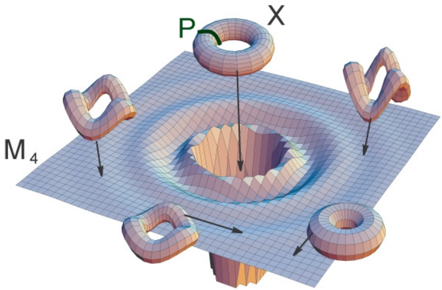
- [de Wit-Mohaupt-Lopes Cardoso-Kappeli 99-00]: Wald R^2 corrections to $S_{\rm BH}$ match subleading corrections microscopic M5-P entropy.
- [Ooguri-Strominger-Vafa 04] All order refinement — the OSV conjecture: $Z_{BH} \approx |Z_{top}|^2$.
- [Dabholkar 04,Sen 05] Entropy of "small" black holes
- [Sen 05] Non-BPS extremal BH attractors in arbitrary higher derivative gravity; Entropy function formalism.
- [Gaiotto-Strominger-Yin 06,de Boer-Cheng-Dijkgraaf-Manschot-Verlinde 06] Relation AdS_3-CFT_2 elliptic genus to OSV.
- [Gaiotto-Strominger-Yin 06] Exact M5 elliptic genera
- [Gaiotto Shih Strominger Yin Sen Dabholkar Murthy Narayan Banerjee Nampuri David Jatkar Srivastava Mukherjee Mukhi Nigam ...] ((re-)re-)re-counting N=4 dyons
- [Denef-Moore 07] Partial proof version of OSV; wall crossing formulae
- [Sen 07-08,Cheng-Verlinde 07-08]: General N=4 wall crossing + alg
- [Kontsevitch-Soibelman,Gaiotto-Moore-Neitzke]: General N=2 wall crossing formula

BPS bound states and wall crossing for dummies



Realizations of BPS states in string theory

Setting



- IIA on Calabi-Yau X $\rightsquigarrow$ 4d $\mathcal{N} = 2$ supergravity $+(h^{1,1} + 1)$ gauge fields
- D6-D4-D2-D0 BPS bound st. (D-branes + gauge flux) $\rightsquigarrow$ BPS black holes with magn. and el. charges (p^0, p^A, q_A, q_0)

BPS states and stability

- BPS bound for mass of particle with charge $\Gamma = (p^0, p, q, q_0)$ in vacuum with complexified Kähler moduli $t \equiv B + iJ$:

$$M \geq M_{BPS} = |Z|\, M_p$$

where

$$Z = \left(\frac{(\mathrm{Im}\, t)^3}{6}\right)^{-1/2} \left(p^0 \frac{t^3}{6} - p \cdot \frac{t^2}{2} + q \cdot t - q_0\right) + \text{inst. corr.}$$

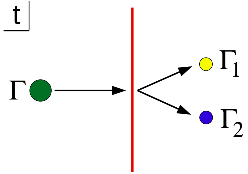
- For generic t: $|Z(\Gamma_1 + \Gamma_2, t)| < |Z(\Gamma_1, t)| + |Z(\Gamma_2, t)|$
$\Rightarrow$ BPS states absolutely stable.

Decay at marginal stability

- Stability argument fails when t such that

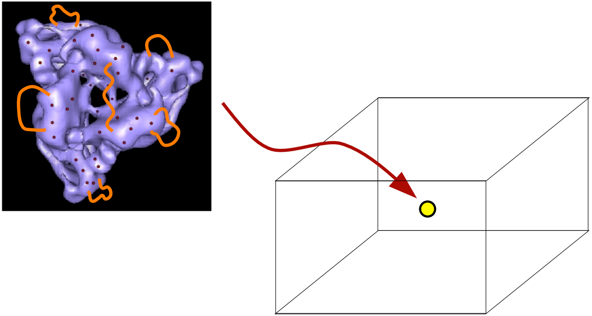
$$\arg Z(\Gamma_1, t) = \arg Z(\Gamma_2, t)$$

  since then $|Z(1+2)| = |Z(1)| + |Z(2)|$: marginal stability.
- $\Rightarrow$ BPS states can disappear from spectrum when crossing walls of marginal stability.



BPS particle splits in two BPS particles conserving different susies. Even index of BPS states jumps!

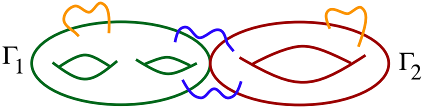
BPS states at $g_s \to 0$ and $V_{CY} \to \infty$

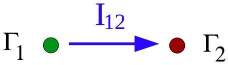


- Localized at single point in noncompact space.
- Wrapped on holomorphic cycles.
- Bound states with lower dim branes:
  - gauge flux: holomorphic vector bundles
  - brane "gas": ideal sheaves

BPS states at $g_s \to 0$ near marginal stability

- Decay $\Gamma \to \Gamma_1 + \Gamma_2$ at marginal stability often invisible in IIA large volume geometrical D-brane picture.
- Stringy microscopic description [Kachru-McGreevy]:





Light $1 \to 2$ open string modes ϕ_i, $i = 1, \ldots, I_{12}$ have D-term potential:

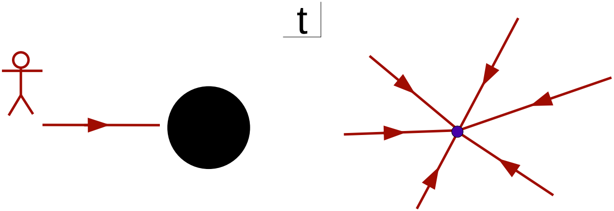
$$V_D \sim \sum_i (|\phi_i|^2 - \xi)^2 \quad \Rightarrow \quad \mathcal{M}_{\rm susy} = \mathbb{CP}^{I_{12}-1}.$$

- FI term ξ changes sign when crossing MS wall $\Rightarrow$ susy config. exists on one side, not on other: $\exists$ “tachyon glue” iff $\xi > 0$.

BPS states in 4d supergravity ($g_s|\Gamma| \gg 1$)

Simplest possibility: spherically symmetric BPS black hole of charge $\Gamma \equiv (p^0, p, q, q_0)$:

$$ds^2 = -e^{2U(r)}dt^2 + e^{-2U(r)}d\vec{x}^2$$



Solutions $\Leftrightarrow$ attractors [Ferrara-Kallosh-Strominger]:

Radial inward flow of moduli $t(r)$ is gradient flow of $\log|Z(\Gamma, t)|$.

Existence of spherically symmetric BPS black holes

Three possibilities [Moore]:

1. Gradient flow ends in minimum $t = t_*(\Gamma)$ with $Z(\Gamma, t_*) \neq 0$.
   $\Rightarrow$ Regular black hole with horizon area $A = 4\pi |Z(\Gamma, t_*)|^2$.

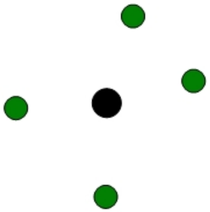
2. Flow ends in boundary point $t = t_0$ with $Z(\Gamma, t_0) = 0$.
   $\Rightarrow$ Zero area black hole, but still BPS solution (e.g. pure D6, D2-D0; note: regular after uplifting to 5d).

3. Flow ends in interior point $t = t_0$ with $Z(\Gamma, t_0) = 0$.
   $\Rightarrow$ No BPS black hole solution.

BPS black hole molecules

More general BPS solutions exist: multi-centered bound states:

$$ds^2 = -e^{2U(\vec{x})}\left(dt - \omega_i dx^i\right)^2 + e^{-2U(\vec{x})} d\vec{x}^2.$$



- Centers have nonparallel charges.
- Bound in the sense that positions are constrained by gravitational, scalar and electromagnetic forces.
- Stationary but with intrinsic spin from e.m. field

Explicit multicentered BPS solutions

- N-centered solutions characterized by harmonic function $H(\vec{x})$ from 3d space into charge space:

$$H(\vec{x}) = \sum_{i=1}^{N} \frac{\Gamma_i}{|\vec{x} - \vec{x}_i|} + H_\infty$$

with H_∞ determined by $t_{|\vec{x}|=\infty}$ and total charge Γ.

- Positions constrained by

$$\sum_{j=1}^{N} \frac{\langle \Gamma_i, \Gamma_j \rangle}{|\vec{x}_i - \vec{x}_j|} = 2\,\mathrm{Im}\left(e^{-i\alpha} Z(\Gamma_i)\right)_{|\vec{x}|=\infty}$$

where $\langle \Gamma_1, \Gamma_2 \rangle = \Gamma_1^{\mathrm{m}} \cdot \Gamma_2^{\mathrm{e}} - \Gamma_1^{\mathrm{e}} \cdot \Gamma_2^{\mathrm{m}}$ and $\alpha = \arg Z(\Gamma)$.

- All fields can be extracted completely explicitly from the entropy function $S(\Gamma)$ on charge space, e.g.

$$e^{2U(\vec{x})} = \frac{\pi}{S(H(\vec{x}))}$$

Decay at marginal stability

2-centered case:

- Equilibrium distance from position constraint:

$$|\vec{x}_1 - \vec{x}_2| = \frac{\langle \Gamma_1, \Gamma_2 \rangle}{2} \left. \frac{|Z_1 + Z_2|}{\mathrm{Im}(Z_1 \overline{Z_2})} \right|_{|\vec{x}|=\infty}$$

- When MS wall is crossed: RHS $\to \infty$ and then becomes negative: decay

Example: pure $D4 = D6 - \overline{D6}$ molecule

- Pure D4 with D4-charge P has

$$Z \sim -P \cdot \frac{t^2}{2} - \frac{P^3}{24}.$$

$Z(t) = 0$ at $t \sim i\,P$ $\Rightarrow$ No single centered solution.

- Instead: realized as bound state of single D6 with $U(1)$ flux $F = P/2$ and anti-(single D6 with flux $F = -P/2$):

Stable for $\mathrm{Im}\, t > \mathcal{O}(P)$.

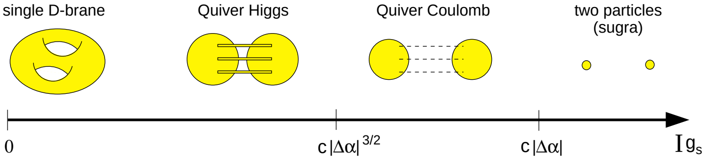
- M-theory uplift: smooth "bubbling" geometry.
[Benna-Warner,Cheng]

Transition between $g_s|\Gamma| \gg 1$ and $g_s|\Gamma| \ll 1$ pictures

- Mass squared lightest bosonic modes of open strings between Γ_1 and Γ_2:

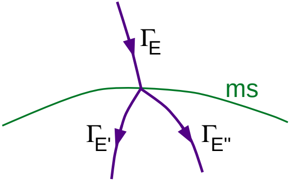
$$
\begin{aligned}
M^2/M_s^2 \quad &\sim \quad \frac{|\vec{x}_1 - \vec{x}_2|^2}{\ell_s^2} + \Delta\alpha \\
&= \quad c(t)\, g_s^2 + \Delta\alpha
\end{aligned}
$$

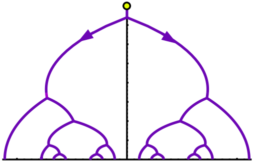
- On stable side of MS wall $\Delta\alpha < 0$, so if g_s gets sufficiently small, open strings become tachyonic and branes condense into single centered D-brane. [FD qqhh]



The flow tree - BPS state correspondence

- Establishing existence of multicentered BPS configurations not easy: position constraints, $S(H(\vec{x})) \in \mathbb{R}^+ \ \forall \vec{x}$, ... However:
- Theorem/conjecture: Branches of multicentered configuration moduli spaces in 1-1 correspondence with attractor flow trees:



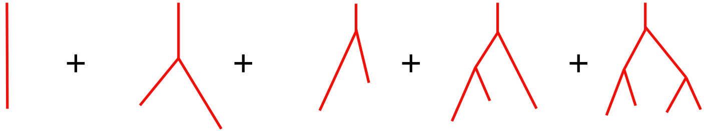


- Much simpler to check & enumerate!

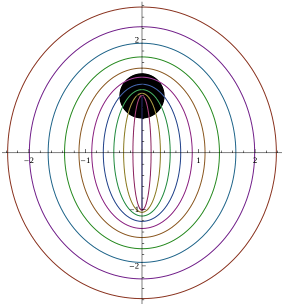
Flow tree decomposition of BPS Hilbert space

- Flow trees can also be given microscopic interpretations (decay sequences / tachyon gluing).
- $\Rightarrow$ Hilbert space of BPS states of charge Γ in background t has canonical decomposition in attractor flow tree sectors:

$$\mathcal{H}(\Gamma, t) =$$



Uplift to M-theory on $\mathrm{AdS}_3 \times S^2$



Flow trees which start at infinite CY volume can be realized as multicentered bound states in asymptotic $\mathrm{AdS}_3 \times S^2$:

[de Boer-(Denef-)El Showk-Messamah-Van den Bleeken 08]

$\Rightarrow$ relevant for $(0,4)$ CFT_2s.

Wall crossing formulae

The BPS index

Hilbert space of BPS states in 4d $\mathcal{N}=2$ theories:

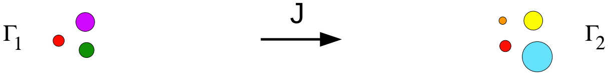
$$\mathcal{H}(\Gamma, t) = (\tfrac{1}{2}, 0, 0) \otimes \mathcal{H}'(\Gamma, t)$$

Index:

$$\Omega(\Gamma, t) = \mathrm{Tr}_{\mathcal{H}'(\Gamma, t)} (-1)^{2J_3'}$$

$\rightsquigarrow$ How does Ω change when t crosses MS wall?

Wall crossing formula for primitive splits



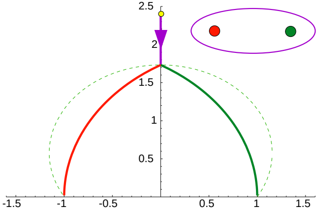
- Near marginal stability wall $\Gamma \to \Gamma_1 + \Gamma_2$ (with Γ_1 and Γ_2 primitive), the decaying part of $\mathcal{H}'(\Gamma, t)$ has following factorized form:

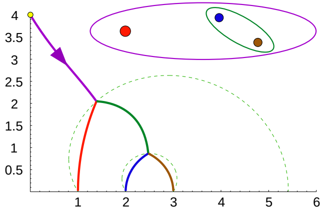
$$\Delta\mathcal{H}'(\Gamma, t) = (J) \otimes \mathcal{H}'(\Gamma_1, t) \otimes \mathcal{H}'(\Gamma_2, t)$$

  with $J = \frac{1}{2}(\langle\Gamma_1, \Gamma_2\rangle - 1)$.
- Spin J factor:
  - macroscopically from intrinsic angular momentum monopole-electron system ($-1/2$ from relative c.o.m. spin)
  - microscopically from quantizing open string tachyon moduli space $\mathcal{M}_{\rm susy} = \mathbb{CP}^{2J}$.
- Implies index jump

$$\Delta\Omega = (-)^{2J}(2J+1)\,\Omega(\Gamma_1, t_{\rm ms})\,\Omega(\Gamma_2, t_{\rm ms}).$$

Indices from wall crossing: examples





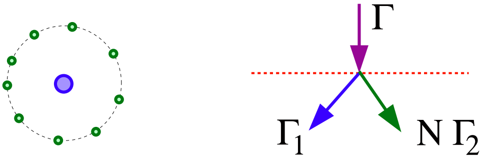
1. Pure D4 on $P > 0$ with pulled back flux S splits in D6 with flux $P/2 + S$ and anti-(D6 with flux $-P/2 + S$), so

$$|\Omega| = |\langle \Gamma_1, \Gamma_2 \rangle| = \frac{P^3}{6} + \frac{c_2 P}{12} =: I_P = \chi(\mathcal{M}_P). \quad \checkmark$$

2. D6-D2 "brane gas" bound state with D2 $= U \cap V$, $V > U$, first splits off D6 with flux U, remainder next splits in D6 with flux V and anti-(D6 with flux $U + V$), so

$$|\Omega| = |\langle \Gamma_1, \Gamma_2 + \Gamma_3 \rangle \langle \Gamma_2, \Gamma_3 \rangle| = |I_V - I_{V-U}| I_U = \chi(\mathcal{M}_{U \cap V}) \quad \checkmark$$

Nonprimitive splits 1: halos



Halo = bound state of one Γ_1 particle ("core") with N Γ_2 particles.

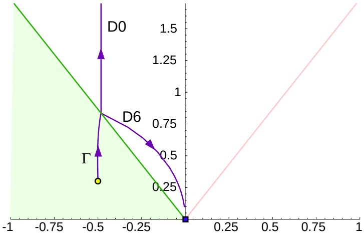
Wall crossing formula for index from generating function:

$$\sum_N \Delta\Omega(\Gamma_1 + N\Gamma_2, t)\, u^N = \Omega(\Gamma_1)\left(1 - (-1)^{\langle \Gamma_1, \Gamma_2 \rangle} u\right)^{|\langle \Gamma_1, \Gamma_2 \rangle| \Omega(\Gamma_2, t)}$$

Most general nonprimitive splits: $\Gamma \to (M\Gamma_1, N\Gamma_2) \rightsquigarrow$??

Example: D6-D0 bound states in large B-field

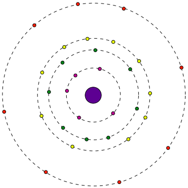
For $-B \sim -\mathrm{Re}\, t$ sufficiently large: D6 - N D0 BPS bound states exist as halos:

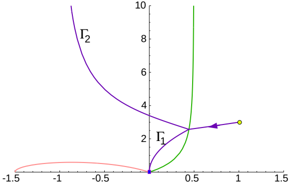


$$\mathcal{Z}_{D6-D0}(u;t) := \sum_n \Omega(D6+nD0,t)\, u^n = \prod_k (1-(-u)^k)^{-k\chi(X)} \quad \checkmark$$

(product is over halos of D0-particles of D0-charge k.)

D2-D0 halos and D6-DT-GV correspondence





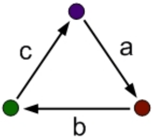
Similarly: D2D0 halos around D6-$\cdots$ cores. MS lines asymptoting to vertical lines $B \sim q_{D0}/q_{D2}$, $J \to \infty$.

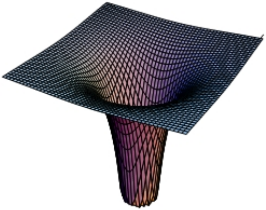
$\Rightarrow$ only in suitable $B \to \infty$ limit identification:

$$\mathcal{Z}_{D6D2D0} = \mathcal{Z}_{DT} = \mathcal{Z}_{GV}$$

Genus $r = 0$ part of $\mathcal{Z}_{GV}$ counts halo states, genus $r > 0$ core states. Asymmetry under inversion D0-charge of $r = 0$ part due to different signs halo D0 charge stable for different signs B-field.

Side remark: scaling solutions





- $\exists$ multicentered "scaling" solutions *without* MS lines, asymptotically indistinguishable from single black hole
- E.g. multicenter configuration corresponding to three node closed loop quiver. Microscopic index:

$$\Omega = \int_0^\infty ds\, e^{-s}\, L^1_{a-1}(s)\, L^1_{b-1}(s)\, L^1_{c-1}(s) \; + \cdots$$

  - Polynomial in (a, b, c) in regime without scaling solutions, ✓
  - Exponential in regime with scaling solutions:

$$\Omega \sim (abc)^{-1/3}\, 2^{a+b+c} \qquad \rightsquigarrow \text{why?}$$

The OSV conjecture

The OSV conjecture

[Warning: $2 \equiv \pi \equiv i \equiv 1$.]

Defining

$$\mathcal{Z}_{osv}(\phi) \equiv \sum_q \Omega(p,q)\, e^{\phi \cdot q}$$

[Ooguri-Strominger-Vafa] conjectured:

$$\mathcal{Z}_{osv}(\phi) \sim \mathcal{Z}_{top}(g_{top}, t)\, \overline{\mathcal{Z}_{top}(g_{top}, t)}$$

with identifications:

$$g_{\rm top} = \frac{1}{\phi^0 + i\, p^0}, \qquad t^A = \frac{\phi^A + i\, p^A}{\phi^0 + i\, p^0}.$$

Inverting:

$$\Omega(p,q) \sim \int d\phi\, e^{-\phi \cdot q}\, |\mathcal{Z}_{top}|^2(p,\phi).$$

RHS in leading saddle point approx. $= e^{S_{\rm BH}(p,q)}$.

Deriving OSV for $p^0 = 0$: rough outline

1. Identify $\mathcal{Z}_{osv} = \lim_{\beta\to 0} \mathcal{Z}_{D4}(\beta, C_1, C_3)$.
2. Use $SL(2,\mathbb{Z})$ TST-duality to rewrite $\mathcal{Z}_{D4}$ at $t = i\infty$ as Fareytail/Rademacher series built on polar part $\mathcal{Z}^{-}_{D4}$:

$$\mathcal{Z}_{osv} = \sum_{A\in SL(2,\mathbb{Z})} f(A,\phi^0)\, \mathcal{Z}^{-}_{D4}(A\cdot(\phi^0,\phi))$$

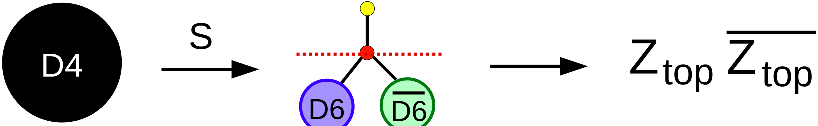
3. Polar BPS states split: no attractor point, $\Omega_*(\Gamma^-) = 0$, so

$$\Omega(\Gamma^-, t) = \sum_{\Gamma^-\to\Gamma_1+\Gamma_2} (-)^{\langle\Gamma_1,\Gamma_2\rangle - 1} |\langle\Gamma_1,\Gamma_2\rangle|\, \Omega(\Gamma_1, t_{\rm ms})\, \Omega(\Gamma_2, t_{\rm ms})$$

4. At large P, $SL(2,\mathbb{Z})$ element $A = S : \phi^0 \mapsto 1/\phi^0$, and splits into Γ_1 (Γ_2) = single (anti-)D6 with dilute D2-D0 gas dominate nonpolar part of $\mathcal{Z}_{D4}$ in Fareytail sum, provided $\phi^0(\sim 1/g_{\rm top})$ not too large (and P large).
5. Dilute gasses fully factorize $\Rightarrow$ in suitable regime:

$$\mathcal{Z}_{osv} \sim \mathcal{Z}_{D6}\, \mathcal{Z}_{\overline{D6}} \sim \mathcal{Z}_{DT}\, \overline{\mathcal{Z}_{DT}} = |\mathcal{Z}_{\rm top}|^2.$$

Pictorial summary



Final result

Index of nonpolar charge $\Gamma = (0, P, Q, q_0)$:

$$\Omega(\Gamma; t_\infty = i\infty) = \int d\phi\, \mu(P,\phi)\, e^{-2\pi q_\Lambda \phi^\Lambda}\, e^{\mathcal{F}^\epsilon(P,\phi) + \delta\mathcal{F}},$$

where, with substitutions $g \equiv \frac{2\pi}{\phi^0}$, $t^A \equiv \frac{1}{\phi^0}(\phi^A + i\frac{P^A}{2})$:

$$\begin{aligned}
\mu(P,\phi) &= \frac{4\pi}{g^2}\, e^{-K^\epsilon(g,t,\bar{t})} \\
\mathcal{F}^\epsilon(P,\phi) &= F^\epsilon_{\rm top}(g,t) + \overline{F^\epsilon_{\rm top}(g,t)} \\
\delta\mathcal{F} &= \mathcal{O}(e^{-\epsilon g P^3})
\end{aligned}$$

with $F^\epsilon_{\rm top}$ topological string free energy cut off by taking only DT invariants $N_{DT}(\beta, n)$ with $\beta \cdot P < \epsilon P^3$, $|n| < \epsilon P^3$, and K^ϵ "Kähler potential" derived from this.

Must take $\epsilon < \mathcal{O}(P^{-1})$ for factorization and DT id., so error

$$\delta\mathcal{F} \sim e^{-gP^2} \sim e^{-({\rm Im}\, t)^2/g}$$

Range of validity

Unless freaky cancelations of contributions to indices occur, we find restriction

$$g > \mathcal{O}(1)$$

i.e. strong topological string coupling!

Equivalently (as $g|_{\mathrm{saddle}} \sim \sqrt{\widehat{q}_0/P^3}$):

$$\widehat{q}_0 > \mathcal{O}(P^3).$$

Technical reason: only when $g > g_{\mathrm{crit}} \sim \mathcal{O}(1)$ are non-factorizable terms in fareytail series sufficiently suppressed (entropy overwhelms Boltzmann suppression $\rightsquigarrow$ phase transition).

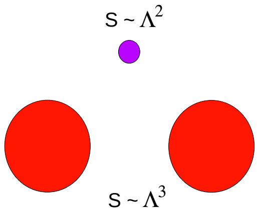
Not artifact of derivation, but related to "entropy enigma".

The Entropy Enigma

For $\Gamma = \Lambda(0, P, Q, Q_0)$ in large Λ limit, and in background with $\mathrm{Im}\, t \gg O(\Lambda)$, there always exists two centered D6-anti-D6 type black hole configuration such that

$$S_{\rm BH,2} := S_{\rm BH,1}(\Gamma_1) + S_{\rm BH,1}(\Gamma_2) \sim \Lambda^3$$

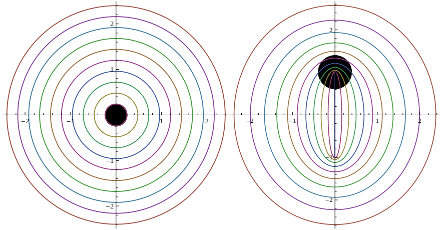
while leading order OSV prediction is $\log \Omega \sim S_{\rm BH,1}(\Gamma) \sim \Lambda^2$.



[Denef-Moore 07]

The Entropy Enigma demystified

M-theory uplift [de Boer-(Denef-)El Showk-Messamah-Van den Bleeken 08]:



EE = Susy version of [Banks-Douglas-Horowitz-Martinec] thermodynamic instability of Schwarzschild-AdS to localization on sphere.

Does this mean OSV is wrong?

Does entropy enigma imply that large Λ / weak g ($\sim 1/\Lambda$) OSV conjecture is wrong?

No, physically one only really expects the conjecture to be valid at attractor point, i.e. finite t_∞:

$$\Omega(\Gamma; t_*(\Gamma)) \overset{?}{\sim} \int d\phi\, e^{-\phi\cdot q} |\mathcal{Z}_{top}|^2(p,\phi).$$

[No troubling Λ^3 solutions there.]

But: interpretation lost of counting large volume D-brane ground states; direct microscopic (D-brane) counting beyond reach at this point.

Wall crossing for non-dummies








Kontsevich-Soibelman formula

- Consider two charges Γ_1 and Γ_2 near a wall of marginal stability. Let, for positive m, n:

$$\Omega_{\pm}(m,n) := \Omega(m\,\Gamma_1 + n\,\Gamma_2\,,\ t_{\pm})$$

  where $t_{\pm}$ are moduli immediately on left/right of wall.

- Let $k := \langle \Gamma_1, \Gamma_2 \rangle$ and define the maps (symplectomorphisms)

$$T_{m,n} : (x,y) \mapsto \left(x(1-(-1)^{mn}x^m y^n)^n\,,\ y(1-(-1)^{mn}x^m y^n)^{-m} \right).$$

- Then the KS wall crossing formula states

$$\prod_{m/n\downarrow} T_{m,n}^{k\Omega_+(m,n)} = \prod_{m/n\uparrow} T_{m,n}^{k\Omega_-(m,n)}.$$

- Example: $k = 1$: $T_{10}\,T_{01} = T_{01}\,T_{11}\,T_{10}$.
- KS reproduces and extends (1,n) w.c. formula to (m,n)!