# Large Twist Limit for Any Operator in $\mathcal{N} = 4$ SYM

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# Introduction and Motivation

- holography: explicit dictionary, many tests but no proof,
- ideal exemple: $\mathcal{N} = 4$ SYM in the planar limit, but still too complicated, many results remain conjectural,
- further simplification: fishnet theory. Origin of integrability is better understood, holography has been derived. [Gürdoğan and Kazakov (2015)] [Gromov, Kazakov, Korchemsky, Negro, and Sizov (2018)] [Gromov and Sever (2019)]

How to progressively go back to $\mathcal{N} = 4$ SYM?

# Outline

1. A Few Facts About the Fishnet Theory

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2. Short Operators in $\mathcal{N} = 4$ SYM
3. Generic Operators and Mixing

# A Few Facts About the Fishnet Theory

# From $\mathcal{N} = 4$ SYM to The Fishnet Theory

Start from $\gamma$-deformed $\mathcal{N} = 4$ SYM:

$$\mathcal{L} = -N_c \mathrm{Tr}\left[\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2\alpha}(\partial^\mu A_\mu)^2 + D^\mu \phi_i^\dagger D_\mu \phi^i + \psi^\dagger_{\dot{\alpha} A} \not{D}^{\dot{\alpha}\alpha} \psi_\alpha^A\right] + \mathcal{L}_{int} \,,$$

where

$$D_\mu = \partial_\mu + \mathrm{i}\, g [A_\mu, \cdot] \,,$$

$$F_{\mu\nu} = -\frac{\mathrm{i}}{g}[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu + \mathrm{i}\, g [A_\mu, A_\nu] \,,$$

and

$$\begin{aligned}
\mathcal{L}_{int} = N_c g^2 \,\mathrm{Tr}&\left[2\, e^{-\mathrm{i}\,\epsilon^{ijk}\gamma_k} \phi_i^\dagger \phi_j^\dagger \phi^i \phi^j - \frac{1}{2}\left\{\phi_i^\dagger, \phi^i\right\}\left\{\phi_j^\dagger, \phi^j\right\}\right] \\
+ \sqrt{2} N_c g\, \mathrm{Tr}&\left[e^{\frac{\mathrm{i}}{2}\gamma_j^-} \psi_4^\dagger \phi^j \psi_j^\dagger - e^{-\frac{\mathrm{i}}{2}\gamma_j^-} \psi_j^\dagger \phi^j \psi_4^\dagger + \mathrm{i}\, \epsilon_{ijk} e^{\frac{\mathrm{i}}{2}\epsilon_{imk}\gamma_m^+} \psi^i \phi^j \psi^k \right. \\
&\left. - e^{\frac{\mathrm{i}}{2}\gamma_j^-} \psi^4 \phi_j^\dagger \psi^j + e^{-\frac{\mathrm{i}}{2}\gamma_j^-} \psi^j \phi_j^\dagger \psi^4 + \mathrm{i}\, \epsilon^{ijk} e^{\frac{\mathrm{i}}{2}\epsilon_{imk}\gamma_m^+} \psi_i^\dagger \phi_j^\dagger \psi_k^\dagger \right] .
\end{aligned}$$

Then, set $\gamma_1 = \gamma_2 = 0$ and take the double-scaling limit

$$e^{-\,\mathrm{i}\,\gamma_3} \to \infty\,, \quad g \to 0\,, \quad \xi^2 = \frac{g^2 e^{-\,\mathrm{i}\,\gamma_3}}{8\pi^2} \quad \text{fixed}\,.$$

Denoting $\phi_1 = X\,, \phi_2 = Z$, the fishnet Lagrangian is

$$\mathcal{L}_{\text{fishnet}} = -N_c\,\mathrm{Tr}\left(\partial^\mu X^\dagger \partial_\mu X + \partial^\mu Z^\dagger \partial_\mu Z - (4\pi)^2 \xi^2 X^\dagger Z^\dagger X Z\right)\;.$$

[Gürdoğan and Kazakov (2015)]

Single, chiral interaction vertex:

We will work in the planar limit $N_c \to +\infty$.

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- Integrable: related to a non-compact $SO(1,5)$ spin chain,
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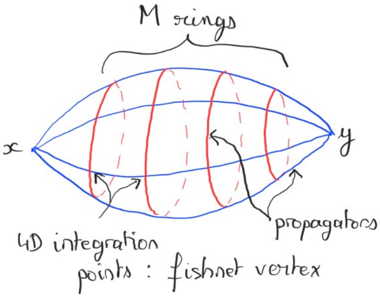
- Holographic dual derived from first principles: chain of point particles with local interactions. [Gromov and Sever (2019)]

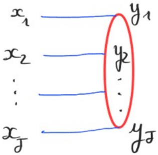
# Graph-Building Operators

Conformal dimension of $Tr(Z^J(x))$: the 2-point function has an iterative structure

$$\left\langle Tr(Z^J(x))\, Tr(Z^{\dagger J}(y))\right\rangle \leftrightarrow \sum_{M=0}^{+\infty} \xi^{2JM}$$



The graph-building operator $\widehat{H}$ is an integral operator with kernel



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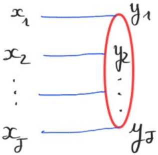


Its action on an arbitrary function $\Phi$ is

$$\left[\widehat{H}\Phi\right](x_1,\ldots,x_J) = \int \frac{\Phi(y_1,\ldots,y_J)}{\prod_{k=1}^{J}(x_k - y_k)^2 y_{k,k+1}^2} \mathrm{d}^4 y_1 \ldots \mathrm{d}^4 y_J$$

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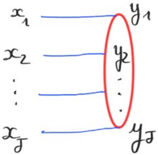


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The 2-point function is essentially reduced to the computation of

$$\sum_{M=0}^{+\infty} \xi^{2MJ} \widehat{H}^M = \frac{1}{1 - \xi^{2J}\widehat{H}} .$$

$\Longrightarrow$ one needs to diagonalise $\widehat{H}$

# Conformal Symmetry and Integrability

$\widehat{H}$ commutes with the generators of $\mathfrak{so}(5,1)$ in scalar principal series representations at each site.

Actually, it is part of the conserved charges of an integrable spin chain with conformal symmetry.

Remark: Principal series representations of the conformal group will be denoted $(\Delta, \ell, \bar{\ell})$ with $\Delta \in \mathbb{C}$ and $(\ell, \bar{\ell}) \in \mathbb{N}^2$.

For arbitrary representation $\rho$ at a given site of the chain, the Lax matrix with 4-dimensional auxiliary space is

$$L^{(\rho;4)}(u) = u\,\mathrm{Id} - \frac{1}{2} q^{(\rho)}_{MN} \otimes \Sigma^{MN}\,,$$

where $\Sigma^{MN}$ are $4 \times 4$ matrices. It satisifies the Yang-Baxter relation

$$R^{(4;4)}(u-v) L^{(\rho;4)}_1(u) L^{(\rho;4)}_2(v) = L^{(\rho;4)}_2(v) L^{(\rho;4)}_1(u) R^{(4;4)}(u-v)\,.$$

with

$$R^{(4;4)}(u) = L^{(4;4)}\left(u + \frac{1}{4}\right) = u\,\mathrm{Id} + \mathbb{P}\,,$$

One can then use the fusion procedure to construct the Lax matrix with 6-dimensional auxiliary space. The result is

$$L^{(\rho;6)}(u) = \Bigg[ u^2 \eta_{MN} - u q^{(\rho)}_{MN} + \frac{1}{2}\Bigg( q^{(\rho),}{}_M{}^P q^{(\rho)}_{PN} - 2 q^{(\rho)}{}_{MN} - \frac{C_\rho + 2}{4}\eta_{MN} - \frac{1}{8}\epsilon_{MN}{}^{ABCD} q^{(\rho)}_{AB} q^{(\rho)}_{CD}\Bigg)\Bigg] \otimes e^{MN} \,,$$

where $\eta_{MN} = \mathrm{Diag}(1,1,1,1,1,-1)$ and $\{e_M{}^N\}$ is a basis of $6 \times 6$ matrices. The quadratic Casimir operator is

$$q^{(\rho),MN} q^{(\rho)}_{NM} = C_\rho \, \mathrm{Id} \ .$$

For the graph-building operator: chain of length $J$ with $\rho_k = (\Delta_k, \ell_k, \bar{\ell}_k) = (1, 0, 0)$ for each site.

In particular, the transfer matrix with 6-dimensional auxiliary space gives

$$\widehat{\mathbb{T}}^{(6)}(0) = \mathrm{Tr}\Big(L_N^{(\rho_N;6)}(0) L_{N-1}^{(\rho_{N-1};6)}(0) \cdots L_1^{(\rho_1;6)}(0)\Big)$$

$$= \frac{1}{4^J} \prod_{k=1}^{J} x_{k,k+1}^2 \prod_{k=1}^{J} \Box_k = \widehat{H}^{-1}\,.$$

# Physical Eigenvectors

Eigenvectors of $\widehat{H}$ with eigenvalue $E = \xi^{-2J}$ represent primary operators of the fishnet theory (and their descendents). This is given by the representation of the conformal group $(\Delta(\xi^2), \ell, \bar{\ell})$ under which the eigenvector tranforms.

Example: $J = 2$, eigenvectors can be written explicitly, physical states correspond to symmetric traceless tensors of arbitrary rank $\ell \geqslant 0$, their dimensions are

$$\Delta_{\ell,\pm} = 2 + \sqrt{(\ell+1)^2 + 1 \pm 2\sqrt{(\ell+1)^2 + 4\xi^4}}\,.$$

[Grabner, Gromov, Kazakov, and Korchemsky (2017)]

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- The previous results are exact, they are not perturbative. In particular, for $\ell = 0$,

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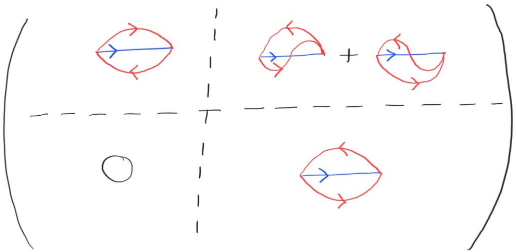
- We find only two operators for each $\ell$; this means that many operators are protected in the fishnet theory.

- The fishnet theory is a logarithmic CFT: the dilatation operator is not diagonalisable.

  Example: Mixing between $\mathrm{Tr}(ZXX^\dagger)$ and $\mathrm{Tr}(ZX^\dagger X)$.



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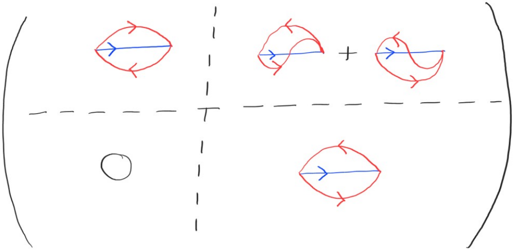


- Neither fermions nor gauge boson in the fishnet theory.

  [Gürdoğan and Kazakov (2015)]

How can one incorporate back these protected or logarithmic operators?

# Short Operators in $\mathcal{N} = 4$ SYM

# New Double-Scaling Limits

We consider $\gamma$-twisted $\mathcal{N} = 4$ SYM with $\gamma_1 = \gamma_2 = 0$, and focus on the following short operators:

$$\mathrm{Tr}\big(ZXX^\dagger\big)\,, \quad \mathrm{Tr}(Z\psi_A)\,, \quad \mathrm{Tr}(ZF)\,.$$

Proposal: consider each 2-point function separately. At order $n$ in perturbation theory the contribution is a finite sum of the form

$$g^n \sum_{k=-k_n^*}^{k_n^*} c_{n,k}\, \mathrm{e}^{\mathrm{i}\, k\gamma_3}\ .$$

Then, choose a double-scaling limit such that $g \to 0$ while $g^n\, \mathrm{e}^{-ik_n^*\gamma_3}$ remains fixed.

$\Longrightarrow$ huge simplification and simple iterative structure

# $\mathrm{Tr}(ZXX^\dagger)$ and $\mathrm{Tr}(ZX^\dagger X)$

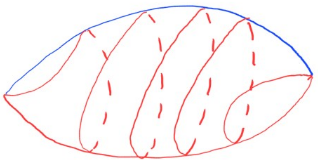
Double-scaling limit:

$$e^{-\mathrm{i}\gamma_3} \to \infty\,, \quad g \to 0\,, \quad \tilde{\xi}_X^4 = \frac{g^4 e^{-\mathrm{i}\gamma_3}}{64\pi^4} \quad \text{fixed}\,.$$

For comparison, recall that the fishnet limit was $\xi^4 \propto g^4\, \mathrm{e}^{-2\mathrm{i}\gamma_3}$.
Relevant interactions:

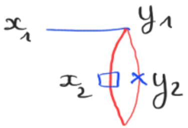
$$N_c g^2\, \mathrm{Tr}(X^\dagger X^\dagger X X) \quad \text{and} \quad 2N_c g^2 e^{-\mathrm{i}\gamma_3}\, \mathrm{Tr}(X^\dagger Z^\dagger X Z)\,.$$

Typical diagram:



Graph-building operator $\widehat{H}_X$ has inverse

$$\widehat{H}_X^{-1} = \frac{1}{16} x_{12}^2 \Box_2 x_{12}^2 \Box_1 \,.$$



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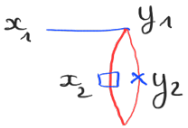


Integrability: length-2 spin chain with a scalar of dimension 1 and a scalar of dimension 2. Once again

$$\widehat{\mathbb{T}}^{(6)}(0) = \widehat{H}_X^{-1} \,.$$

Spectrum: $(\Delta_{\ell,\pm}, \ell, \ell)$ for $\ell \geqslant 0$ and

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are the dimensions of two linear combinations of $\mathrm{Tr}\big(ZXX^\dagger\big)$ and $\mathrm{Tr}\big(ZX^\dagger X\big)$. Operator mixing is resolved.

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Structure constants can also be computed in this limit.

# $\mathrm{Tr}(ZF)$

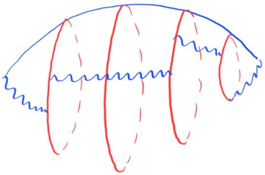
Double-scaling limit:

$$e^{-\mathrm{i}\gamma_3} \to \infty\,, \quad g \to 0\,, \quad \tilde{\xi}_F^4 = \frac{g^4 e^{-\mathrm{i}\gamma_3}}{64\pi^4} \quad \text{fixed}\,.$$

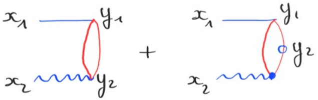
Same limit as for the previous case.
Relevant interactions:

$$-\mathrm{i}\, N_c g\, \mathrm{Tr}\big(\partial_\mu X^\dagger [A^\mu, X] + \partial_\mu X [A^\mu, X^\dagger]\big)\,,$$
$$2N_c g^2\, \mathrm{Tr}\big(X^\dagger A_\mu X A^\mu\big)\,, \quad \text{and} \quad 2N_c g^2 e^{-\mathrm{i}\gamma_3}\, \mathrm{Tr}\big(X^\dagger Z^\dagger X Z\big)\,.$$

Typical diagram:



Graph-building operator $\widehat{H}_A$ depends on the gauge-fixing parameter $\alpha$.



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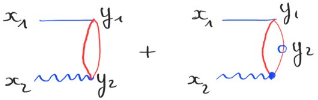

However, there exists a gauge-independent operator $\widehat{H}_F$ acting on antisymmetric tensors $\Psi_F^{\mu\nu}$ and such that: if $\Psi_F^{\mu\nu} = \partial_2^\mu \Psi_A^\nu - \partial_2^\nu \Psi_A^\mu$, then

$$\left[\widehat{H}_F \Psi_F\right]^{\mu\nu} = \partial_2^\mu \left[\widehat{H}_A \Psi_A\right]^\nu - \partial_2^\nu \left[\widehat{H}_A \Psi_A\right]^\mu .$$

This shows explicitly that the 2-point function $\left\langle \mathrm{Tr}(ZF)(x)\, \mathrm{Tr}\left(Z^\dagger F\right)(y) \right\rangle$ is gauge-independent in the double-scaling limit.

One can invert $\widehat{H}_F$ when acting on antisymmetric tensors of the form $\Psi_F^{\mu\nu} = \partial_2^\mu \Psi_A^\nu - \partial_2^\nu \Psi_A^\mu$, the inverse is

$$\left[\widehat{H}_F^{-1}\Psi_F\right]^{\mu\nu} = \frac{1}{16}\left(\partial_2^\mu x_{12}^4 \Box_1 \partial_2^\rho \Psi_{F,\rho}{}^{\nu} - (\mu \leftrightarrow \nu)\right) \, .$$

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Integrability: length-2 spin chain with a scalar of dimension 1 and a rank-2 antisymmetric tensor of dimension 2. When restricted to tensors coming from a vector dimension 1,

$$\widehat{\mathbb{T}}^{(6)}(0) = \widehat{H}_F^{-1} .$$

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The dimension of $\mathrm{Tr}(ZF)$ is $\Delta'_{0,-}$.

# Generic Operators and Mixing

# Fishnet Contributions

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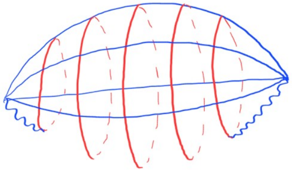
Let us consider the 2-pt function $\big\langle \mathrm{Tr}\big(Z^J F\big)(x)\, \mathrm{Tr}\big((Z^\dagger)^J F\big)(y)\big\rangle$. When $e^{-\mathrm{i}\gamma_3} \to +\infty$, the dominant contributions are



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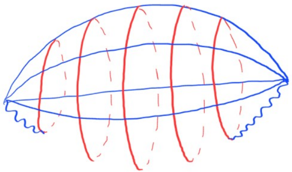

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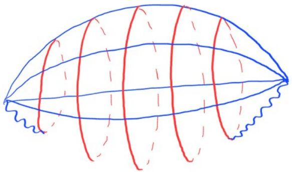

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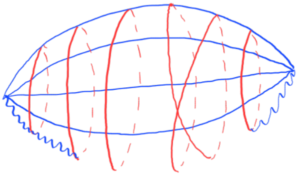
But $\mathrm{Tr}\big(Z^J F\big)$ is absent from the fishnet theory, so more graphs need to be taken into account.

# Mixing

For the same twist $e^{-\mathrm{i}\,JM\gamma_3}$, we now include diagrams of order between $g^{2JM+2}$ and $g^{2JM+2M}$. These subleading contributions generically look like



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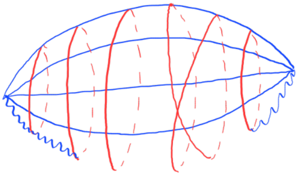

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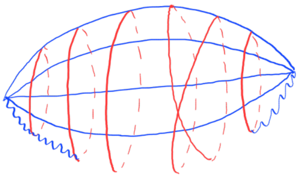
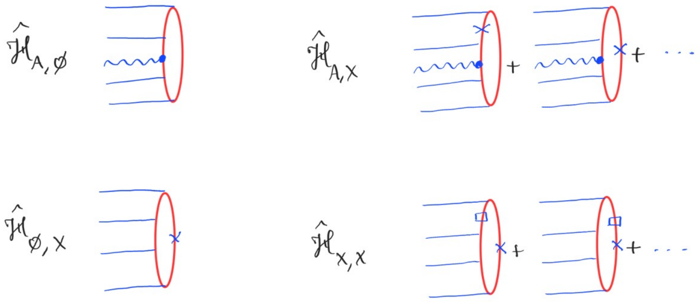


In the bulk, there can be either $Z^J$ or $Z^J A$ or $Z^J XX^\dagger$.

Remark: There exist more diagrams with the same scaling but we claim that they do not contribute to the anomalous dimensions in the limit we are going to focus on:

$$e^{-\mathrm{i}\,\gamma_3} \to \infty\,, \quad g \to 0\,, \quad \xi^{2(J+1)} = \left(\frac{g^2}{8\pi^2}\right)^{J+1} e^{-\mathrm{i}\,J\gamma_3} \quad \text{fixed}\,.$$

There is still an iterative structure: the graph-building operator is now a matrix $\widehat{\mathcal{H}}$ with one row (and one column) for each intermediate state.



$\widehat{\mathcal{H}}$ is defined such that 2-point functions are essentially matrix elements of $\frac{1}{1-\widehat{\mathcal{H}}}$

Example:

$$\left\langle \mathrm{Tr}(A^\mu(x_0)Z(x_1)\dots Z(x_J))\,\mathrm{Tr}\big(Z^\dagger(z_J)\dots Z^\dagger(z_1)\big)\right\rangle$$

$$= -\frac{\mathrm{i}}{2}\int \frac{\langle x_0, x_1, \dots, x_J|\left(\frac{1}{1-\widehat{\mathcal{H}}}\right)^{\mu}_{A\emptyset}|y_1, \dots, y_J\rangle}{(4\pi^2)^J\prod_{i=1}^J (y_i - z_i)^2}\,\frac{\prod_{i=1}^J \mathrm{d}^4 y_i}{\pi^{2J}}\,.$$

The problem is still to diagonalise $\widehat{\mathcal{H}}$, and physical states correspond to those with eigenvalue equal to 1.

Each matrix element scales differently:

$$\widehat{\mathcal{H}}_{\text{res}} = \tilde{\xi}^{2(J+1)} \begin{pmatrix} g^{-2}\widehat{\mathcal{H}}_{\emptyset\emptyset} & g^{-1}\widehat{\mathcal{H}}_{\emptyset A} & g^{-1}\widehat{\mathcal{H}}_{\emptyset X} \\ g^{-1}\widehat{\mathcal{H}}_{A\emptyset} & \widehat{\mathcal{H}}_{AA} & \widehat{\mathcal{H}}_{AX} \\ g^{-1}\widehat{\mathcal{H}}_{X\emptyset} & \widehat{\mathcal{H}}_{XA} & \widehat{\mathcal{H}}_{XX} \end{pmatrix} + O(g)\,.$$

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In the double-scaling limit we have chosen, some eigenvalues will diverge, some will go to zero. We focus on those which remain finite:

$$\widehat{\mathcal{H}}\Psi = E\Psi\,, \quad \text{with} \quad E = E_0 + O(g)\,, \quad E_0 \neq 0\,. \tag{1}$$

At leading order, only the above $3 \times 3$ submatrix is relevant. Writing

$$\Psi = \begin{pmatrix} \Psi_{\emptyset,0}(x_1, \ldots, x_J) \\ \Psi^{\mu}_{A,0}(x_0, x_1, \ldots, x_J) \\ \Psi_{X,0}(x_0, x_1, \ldots, x_J) \end{pmatrix} + O(g)\,,$$

we get $\Psi_{\emptyset,0} = 0$ and

$$\tilde{\xi}^{2(J+1)} \widehat{\mathfrak{H}} \begin{pmatrix} \Psi_{F,0} \\ \Psi_{X,0} \end{pmatrix} = E_0 \begin{pmatrix} \Psi_{F,0} \\ \Psi_{X,0} \end{pmatrix}$$

for $\Psi^{\mu\nu}_{F,0} = \partial^{\mu}_0 \Psi^{\nu}_{A,0} - \partial^{\nu}_0 \Psi^{\mu}_{A,0}$, and some $2 \times 2$ matrix $\widehat{\mathfrak{H}}$ depending on all 9 matrix elements of $\widehat{\mathcal{H}}_{\text{res}}$.

$\widehat{\mathfrak{H}}$ is a complicated matrix of integral operators but it is local (contrary to $\widehat{\mathcal{H}}_{\mathrm{res}}$) and can be inverted:

$$\widehat{\mathfrak{H}}^{-1} = \begin{pmatrix} \theta \cdot \partial_0 \, x_{J0}^2 x_{10}^2 \, \partial_0 \cdot \partial^{(\theta)} & 2\,\theta \cdot \partial_0 \left( \frac{\theta \cdot x_{J0}}{x_{J0}^2} - \frac{\theta \cdot x_{10}}{x_{10}^2} \right) x_{J0}^2 x_{10}^2 \\ 2 \left( \frac{x_{10} \cdot \partial^{(\theta)}}{x_{10}^2} - \frac{x_{J0} \cdot \partial^{(\theta)}}{x_{J0}^2} \right) x_{J0}^2 x_{10}^2 \, \partial_0 \cdot \partial^{(\theta)} & \partial_{0,\mu} \, x_{J0}^2 x_{10}^2 \, \partial_0^\mu + 8\, x_{10} \cdot x_{J0} \end{pmatrix}$$
$$\times \frac{\prod_{i=1}^{J-1} x_{i,i+1}^2 \prod_{i=1}^{J} \Box_i}{(-4)^{J+1}} \,,$$

where $\theta^\mu$ is a polarisation vector such that $\{\theta^\mu, \theta^\nu\} = 0$. It encodes the tensor structure: $\Psi^{\mu\nu} \mapsto \Psi = \theta^\mu \theta^\nu \Psi_{\mu\nu}$.

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There is conformal symmetry, but we have not yet been able to prove integrability.

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- In most cases, this involves mixing with other operators having the same R-symmetry charges, including fishnet operators.
- Regarding holography, the fishchain picture appears to be generic.
- The graph-building operator $\widehat{\mathcal{H}}$ can also be used to study corrections in $g$. For instance, corrections to the fishnet limit.
- For uncharged operators, one needs to modify the scheme since one cannot start from $\gamma$-deformed $\mathcal{N}=4$. But it should be possible to directly twist the correlators. [Cavaglià, Grabner, Gromov, and Sever (2020)]

Thank you for your attention!