Wilson loops in 5D SYM and Holography

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Based on [2111.15493] with Valentina Giangreco M. Puletti

New Mathematical Methods in Solvable Models and Gauge/String Dualities
Varna

August 19th, 2022

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The story will be very similar to the one for the 1/2 BPS (circular) WL in $\mathcal{N} = 4$ SYM

Erickson, Semenoff, Zarembo (2000)

$$\langle \mathcal{W} \rangle = \frac{2N}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) \approx N\lambda^{-3/4}\sqrt{\frac{2}{\pi}}\mathrm{e}^{\sqrt{\lambda}}\,.$$

However...

Our theory is not conformal. The gravity dual is more complicated. The dilaton is non-trivial. Only one other non-conformal example has been explored in this detail in the literature.

Chen-Lin, Medina-Rincon, Zarembo (2017)

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Besides, extracting finite answers for the WL vev on the string side is still an open problem [Giombi, Tseytlin (2020)]. A successful strategy that has emerged in the past few years is to compute ratio of WLs.

But there is only one loop operator that is under control in 5D SYM at strong coupling.

Field theory

5D maximal SYM

Consider the Euclidean maximal SYM in five dimensions:

$$\mathcal{L} = -\frac{1}{2g_{\mathrm{YM}}^2}\mathrm{Tr}\left(|F|^2 - |D\Phi_m|^2 + \bar{\Psi}\not{D}\Psi - \frac{1}{2}[\Phi_m, \Phi_n]^2 + \bar{\Psi}\Gamma^m[\Phi_m, \Psi]\right).$$

We are using 10D language to write down the 5D fermions (Ψ has 16 components but should be decomposed into a pair of 5D spinors). The indices are $m = 0, 1, \ldots, 4$ and R-symmetry is $\mathrm{SO}(1, 4)$.

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In fact the theory is non-renormalizable. Nevertheless, 5D MSYM is expected to be UV completed in the 6D (2,0) SCFT.

Douglas (2010), Lambert, Papageorgakis, Schmidt-Sommerfeld (2010)

5D MAXIMAL SYM ON S^5

When we place euclidean SYM on S^5, we can preserve SUSY by adding terms to the Lagrangian

Blau (2000)

$$\delta\mathcal{L} = -\frac{1}{\mathcal{R}^2}\mathrm{Tr}\left(3\Phi_m\Phi^m+\Phi_a\Phi^a\right)+\frac{1}{2\mathcal{R}}\mathrm{Tr}\left(\bar{\Psi}\Gamma_{012}\Psi-8\Phi_0\left[\Phi_1,\Phi_2\right]\right),$$

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The global symmetry group is $\mathrm{SU}(4|1,1)$ (the R-symmetry is $\mathrm{SU}(1,1) \times \mathrm{U}(1)$.)

At large N a special role will be played by a 't Hooft like coupling constant

$$\xi = \frac{g_{\mathrm{YM}}^2 N}{2\pi\mathcal{R}}.$$

Localization

After introducing auxiliary fields to make one of the supercharge manifest off-shell this theory can be localized in much as the same way as 4D $\mathcal{N} = 2$ by Pestun.

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This maps the problem of computing a class of observables to a Matrix Model computation. At large N the partition function of the matrix model can be written

$$Z = \frac{1}{N!} \int \prod_{i=1}^{N} \mathrm{d}\mu_i \mathrm{e}^{-S_{\text{eff}}}\,, \quad S_{\text{eff}} = \frac{2\pi^2 N}{\xi} \sum_{i=1}^{N} \mu_i^2 - \sum_{j \neq i}^{N} \log|\sinh(\pi(\mu_i - \mu_j))|$$

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In the saddle point expansion (for large N) we can find the distribution of eigenvalues

Mariño (2004)

$$\rho(\mu) = \frac{2}{\xi} \arctan \frac{\sqrt{\mathrm{e}^{\xi} - \cosh^2 \pi\mu}}{\cosh \pi\mu} .$$

LOCALIZATION

Localization allows us to compute the Free energy (= log Partition function). At strong coupling we find

$$\frac{F}{N^2} = \frac{-\xi}{6} + \frac{\pi^2}{3\xi} - \frac{2\zeta(3)}{\xi^2} + \mathcal{O}(\mathrm{e}^{-\xi}) + \mathcal{O}(N^{-1})\,.$$

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In this talk we will focus on the expectation value of a supersymmetric Wilson loop that is compatible with localization. The large N result is valid for all values of the coupling ξ and takes the form

$$\langle \mathcal{W} \rangle = \frac{N}{\xi}(\mathrm{e}^{\xi} - 1) + \mathcal{O}(N^{-1})\,.$$

Our goal is to reproduce and verify this result (its strong coupling expansion) using holography.

Gravity dual

Holographic dual in flat space

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$$\mathrm{d}s_{10}^2 = H^{-1/2}\mathrm{d}s_{||}^2 + H^{1/2}\mathrm{d}s_{\perp}^2 \ .$$

where $\mathrm{d}s_{||}^2$, $\mathrm{d}s_{\perp}^2$ are the metrics on flat 5D spacetimes and H is harmonic on $\mathrm{d}s_{\perp}^2$.

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For the case at hand there is a quick way to obtain this solution by uplifting the near-horizon metric around flat D4s to 11D where one obtaines $\mathrm{AdS}_7 \times S^4$. Then we can change coordinates and reduce back to 10D carefully making sure supersymmetry is not broken.

Spherical D4 solution

Bobev, Bomans, FFG (2018)

$$\mathrm{d}s_{10}^2 = \ell_s^2 (N\pi \mathrm{e}^{\Phi})^{2/3} \Big[\frac{4(\mathrm{d}\sigma^2 + \mathrm{d}\Omega_5^2)}{\sinh^2\sigma} + \mathrm{d}\theta^2 + \cos^2\theta \mathrm{d}\Omega_2^2 + \frac{\sin^2\theta}{1 - \frac{1}{4}\tanh^2\sigma \sin^2\theta} \mathrm{d}\phi^2 \Big] \,,$$

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The form fields B_2, C_1, and C_3 are also nontrivial but their form is not important.

This background exhibits $\mathrm{SU}(4|2)$ symmetry just like the QFT.

Evaluating the renormalized supergravity action we obtain a leading order match with the QFT answer for the free energy obtained by localization.

Bobev, Bomans, FFG, Minahan, Nedelin (2019)

Holographic Wilson loop

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To leading order the string partition function is just the area of a string sitting at its saddle point. Next to leading order is given by the one-loop partition function* of the string.

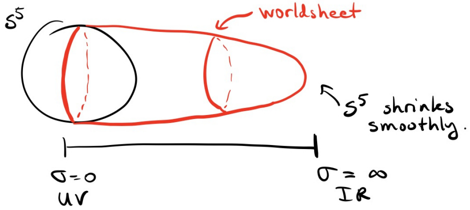
Recall the answer from localization

$$\log\langle\mathcal{W}\rangle = \xi + \log\frac{N}{\xi} + \mathcal{O}(\mathrm{e}^{-\xi})\,.$$

And compare with the string expansion

$$\log Z_{\text{string}} = -\text{Area} - W + \mathcal{O}(\mathrm{e}^{-\xi})\,.$$

THE CLASSICAL SADDLE POINT



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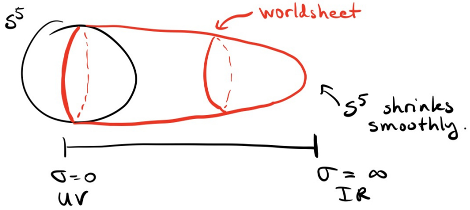

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The worldsheet is a two-dimensional surface with metric

$$\mathrm{d}s_2^2 = \mathrm{e}^{2\rho}(\mathrm{d}\sigma^2 + \mathrm{d}\tau^2)\,, \quad \mathrm{e}^{2\rho} = \frac{4\xi \ell_s^2}{\tanh\sigma\ \sinh^2\sigma}\,.$$

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There is a standard procedure to regularize the divergent area of string worldsheets such as ours. We must identify the correct boundary variables by performing a Legendre transform. This introduces a new term in the action which can be thought of as a counterterm action.

Drukker, Gross, Ooguri (1999)

Adding the counterterm to the classical action gives the regularized area of the string:

$$S_{\text{classical}} + S_{\text{ct}} = \text{Area} = -\xi$$

One loop expansion

Lets now consider small fluctuations of the string about its saddle point.

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We have 8 (physical) bosonic modes ζ^a, and 8 fermionic modes θ^a

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$$\mathcal{K}_a = (\mathcal{K}_x, \mathcal{K}_x, \mathcal{K}_x, \mathcal{K}_x, \mathcal{K}_y, \mathcal{K}_y, \mathcal{K}_z, \mathcal{K}_z) \,.$$

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$$\mathcal{K}_a = \mathrm{e}^{-2\rho}\tilde{\mathcal{K}}_a\,, \qquad \mathcal{D} = \mathrm{e}^{-3\rho/2}\tilde{\mathcal{D}}\mathrm{e}^{\rho/2}\,,$$
$$\tilde{\mathcal{K}}_a = -\partial_\sigma^2 - \partial_\tau^2 + E_a\,, \quad \tilde{\mathcal{D}} = i\partial\!\!\!/ + \tau_3 a + v\,.$$

Here E_a, a, and v are potentials that depend on σ.

Explicit operators

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$$E_x = \frac{7 + 8\cosh 2\sigma}{\sinh^2 2\sigma}\,, \quad E_y = \frac{1 + 2\cosh 2\sigma}{\sinh^2 2\sigma}\,, \quad E_z = \frac{3}{\sinh^2 2\sigma}\,,$$
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Since the model is Gaussian the partition function is given by determinants

$$\Gamma_{\mathbb{K}} = \frac{1}{2}\log\frac{(\det\mathcal{K}_x)^4(\det\mathcal{K}_y)^2(\det\mathcal{K}_z)^2}{(\det\mathcal{D})^8}$$

Weyl rescaling

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The combined object W is Weyl invariant: $W[\rho] = W[0]$.

WEYL RESCALING

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This is the 'center' of the worldsheet, the σ coordinate breaks down there

$$\mathrm{d}s_2^2 \to 16\xi\ell_s^2(\mathrm{d}r^2 + r^2\mathrm{d}\tau^2)\,, \qquad r = \mathrm{e}^{-\sigma}\,.$$

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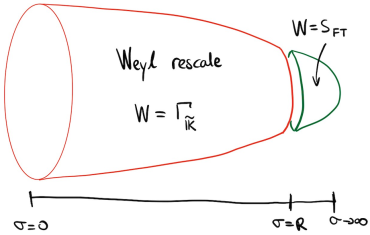
$$\mathrm{d}s_2^2 \to 16\xi\ell_s^2(\mathrm{d}r^2 + r^2\mathrm{d}\tau^2)\,, \qquad r = \mathrm{e}^{-\sigma}\,.$$

Blindly Weyl rescaling the metric away changes the topology of the worldsheet from a disc to a cylinder. This issue was first pointed out in a different but related context.

Cagnazzo, Medina-Rincon, Zarembo (2017)

WEYL RESCALING

Before Weyl rescaling, we cut the worldsheet in two pieces



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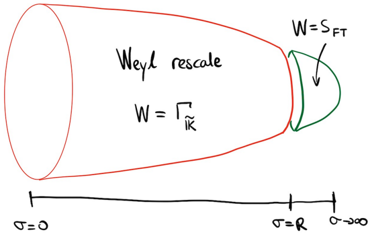

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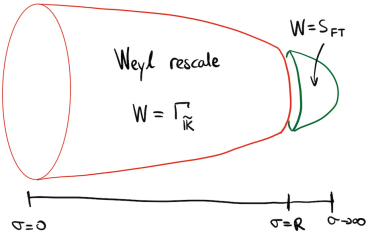

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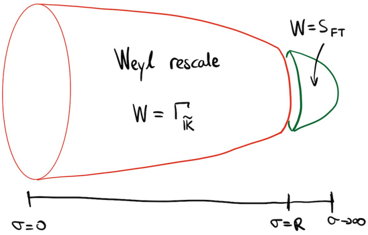

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Combining the two answers, we have

$$W = \Gamma_{\tilde{\mathbb{K}}}(R) - \log \frac{N\pi}{\xi^{3/2}} \,.$$

One-loop result

The one-loop partition function can be computed using various methods, we use the phase shift method first used in this context in

Chen-Lin, Medina-Rincon, Zarembo (2017)

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But it prevents a comparison with the QFT.

We need to construct a finite ratio of string disc partition functions for which the universal divergences cancel.

Ratio

The only other type IIA string partition function we know the answer for corresponds to Wilson loops in ABJM (dual to a string in $\mathrm{AdS}_4 \times \mathbf{C}P^3$)

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$$A = \frac{2\pi}{\ell_s^2} \int_R^\infty \mathrm{e}^{2\rho} \mathrm{d}\sigma \sim T\mathrm{e}^{-2R} \,.$$

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Sidenote: This replacement introduces the factor $\sim \sqrt{T}$ as discussed by Giombi and Tseytlin.

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The two type IIA partition functions are

$$\log Z_{\text{string}}^{\text{SYM}} = \xi + \log \frac{4N_{\text{SYM}}}{\xi\sqrt{\pi}} - \log(\Lambda\sqrt{A})\,,$$

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This matches perfectly the QFT answers!

OUTLOOK

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- Latitude WL in 5D – Can we compute their vev on matrix model or string side?
- SYM in other dimensions – QFT answer exist for 3D, can we reproduce the answer from string side?

  WIP. w. Pieter Bomans and Valentina Giangreco M. Puletti
- $\mathcal{N} = 2^*$ for arbitrary mass, The QFT prediction is a highly nontrivial function of the mass

  Chen-Lin, Gordon, Zarembo (2014)

  With current technology we should be able to reproduce it/verify it in string theory

  WIP. w. Valentina Giangreco M. Puletti and Konstantin Zarembo
- We should understand the cancellation of divergences on the string side without computing ratios.