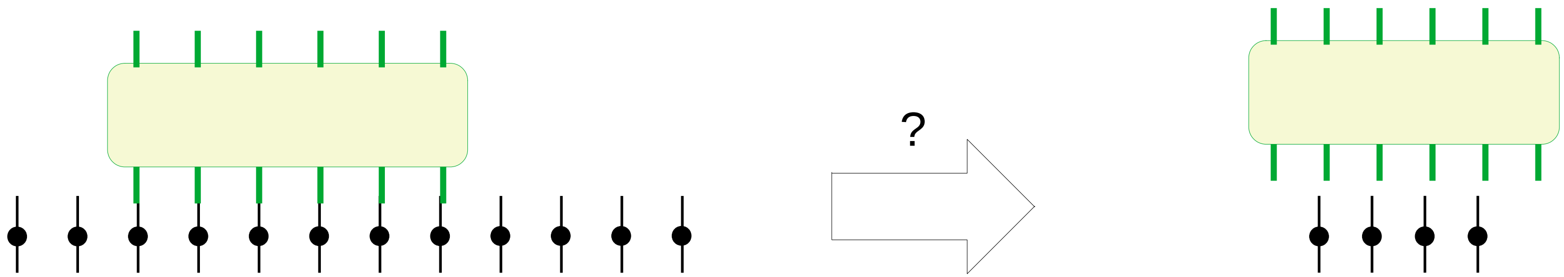
New Mathematical Methods in Solvable Models and Gauge/String Dualities
Varna, 2022



Wrapping corrections for long range spin chains

Tamas Gombor

Based on: 2206.08679

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Motivation (long range spin chains in AdS/CFT, wrapping corrections)

Preliminaries (Definition of asymptotic long range spin chains)

Construction of the transfer matrix for long range spin chains

Finite size long range Hamiltonians

Requirements from AdS/CFT duality

N=4 SYM at the planar limit

Two-point function in the planar limit at weak coupling

$$\langle \mathcal{O}_I(x)\mathcal{O}_{I'}(y)\rangle = \frac{\delta_{I,I'}}{|x-y|^{2\Delta}} = \frac{\delta_{I,I'}}{|x-y|^{2\Delta_0}}(1-2\gamma\log|x-y|+\dots) \qquad \Delta(\lambda)=\Delta_0+\gamma(\lambda)$$

$$\mathcal{O}_I = \mathcal{F}_I^{i_1\dots i_J}\,\mathrm{Tr}\,[\phi_{i_1}\dots\phi_{i_J}] \longrightarrow \Delta_0 = J$$

Single-trace operators

$\mathrm{span}\,\{\phi_i\} = V$ one site

$\mathcal{F}_I \in V^{\otimes J}$ spin chain with length J

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle = \frac{C_{123}}{x_{12}^{\Delta_{12}}x_{13}^{\Delta_{13}}x_{23}^{\Delta_{23}}}$$

$$\langle \mathcal{D}(x)\mathcal{D}(y)\mathcal{O}(z)\rangle = (\dots)C_{\mathcal{O}}$$

Kinetical factor

Structure constant

Y. Jiang, S. Komatsu E. Vescovi ‘19

$x_3 = 0$

Defect theory

x_3

$$\langle \mathcal{O}(x)\rangle = \frac{C_{\mathcal{O}}}{x_3^{\Delta}}$$

Tree-level:

M. de Leeuw, C. Kristjansen, G. Linardopoulos ‘18

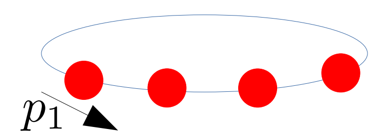
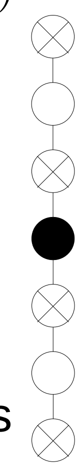
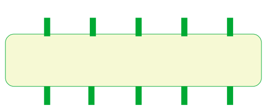
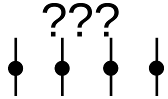
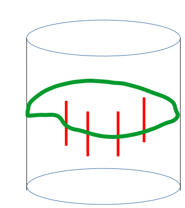
Asymptotic: TG, Z. Bajnok ‘20

S. Komatsu, Y. Wang" ‘20

$$C_{\mathcal{O}} = \langle B|\mathcal{F}_{\mathcal{O}}\rangle$$

Open question: $\longrightarrow$ Wrapping corrections for $C_{\mathcal{O}}$??

AdS/CFT duality

	Gauge theory side	String theory side
	Long range spin chain with length J	1+1 dimensional sigma model with length J
$J \to \infty$	Dilatation operator: $\mathcal{H}(\lambda) = H^{(0)} + \lambda H^{(1)} + \lambda^2 H^{(2)} + \dots$ Integrability: $[\mathcal{H}(\lambda), \mathcal{Q}_3(\lambda)] = 0$ $\mathcal{Q}_3(\lambda) = Q_3^{(0)} + \lambda Q_3^{(1)} + \lambda^2 Q_3^{(2)} + \dots$ $H^{(0)}$:2-site int. $Q_3^{(0)}$: 3-site int. $H^{(1)}$:3-site int. $Q_3^{(1)}$: 4-site int. $H^{(2)}$:4-site int. $Q_3^{(2)}$: 5-site int. Asymptotic Bethe Ansatz N=4 SYM: inf. local dimension, “dynamical” Toy models: gl(N) long range spin chains	Anomalous dimension = string energies Dispersion relation: $e(p)$ Scattering matrix: $\mathbb{S}(p_1,p_2) \sim S(p_1,p_2)^{\otimes 2}$ $\mathrm{su}(2\|2)_c$ scattering matrix $E = \sum e(p_j)$   Asymptotic Bethe Ansatz Toy models: simpler 1+1d field theories
Finite J	 ???  Wrapping corrections Toy models: ???	Lüscher corrections (finite size corrections) TBA, Y-, T-, Q-systems Toy models: 1+1d field theories in finite volume 

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Integrability at higher loops

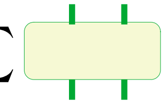
N. Beisert, C. Kristjansen, M. Staudacher '03

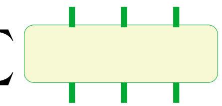
one-loop: nearest neighbor interaction (2-site)

two-loop: next-to-nearest neighbor interaction (3-site)

...

k-loop: (k+1)-site interaction

$H^{(0)} = \sum_j$ 
j

$H^{(1)} = \sum_j$ 
j

$\mathcal{H}(\lambda) = H^{(0)} + \lambda H^{(1)} + \lambda^2 H^{(2)} + \dots$ $H^{(\ell)}$ is defined only for $J \geq \ell + 2$ (asymptotic region)

Integrability: $[\mathcal{H}(\lambda), \mathcal{Q}_3(\lambda)] = 0$ $\mathcal{Q}_3(\lambda) = Q_3^{(0)} + \lambda Q_3^{(1)} + \lambda^2 Q_3^{(2)} + \dots$

0th-order: $[H^{(0)}, Q_3^{(0)}] = 0$

1th-order: $[H^{(1)}, Q_3^{(0)}] + [H^{(0)}, Q_3^{(1)}] = 0$

2th-order: $[H^{(2)}, Q_3^{(0)}] + [H^{(1)}, Q_3^{(1)}] + [H^{(0)}, Q_3^{(2)}] = 0$

$Q_3^{(0)}$: 3-site interaction

$Q_3^{(1)}$: 4-site interaction

$Q_3^{(2)}$: 5-site interaction

N. Beisert, T. Klose '06

Starting from an integrable 2-site spin chain $\longrightarrow$ We obtain $H^{(k)}, Q_3^{(k)}$ order by order which contains unfixed parameters

$$\frac{d}{d\lambda}\mathcal{Q}_r(\lambda) = [\mathcal{X}(\lambda), \mathcal{Q}_r(\lambda)]$$

$$[\mathcal{Q}_r(0), \mathcal{Q}_s(0)] = 0$$

$$\longrightarrow \quad [\mathcal{Q}_r(\lambda), \mathcal{Q}_s(\lambda)] = 0$$

$$\mathcal{H}(\lambda) = \mathcal{Q}_2(\lambda)$$

T. Bargheer, N. Beisert, and F. Loebbert '08

$$\mathcal{X}(\lambda) = \sum_r \alpha_r(\lambda)\mathcal{B}[\mathcal{Q}_r(\lambda)] + \sum_{r,s} \beta_{r,s}(\lambda)[\mathcal{Q}_r(\lambda)|\mathcal{Q}_s(\lambda)]$$

$$\tilde{\mathcal{Q}}_r = e^{\mathcal{U}}\mathcal{Q}_r e^{-\mathcal{U}} \qquad \mathcal{U} = \sum_k \epsilon_k(\lambda)\mathcal{L}^k$$

$$\tilde{\mathcal{Q}}_r = \sum_s \gamma_{r,s}(\lambda)\mathcal{Q}_s$$

Physical paramters: $\alpha_r(\lambda), \beta_{r,s}(\lambda)$

Unphysical paramters: $\gamma_{r,s}(\lambda), \epsilon_k(\lambda)$

Asymptotic Bethe Ansatz

Example: long range deformation of the XXX spin chain.

$$e^{ip} = \frac{u+i/2}{u-i/2} \qquad Q_r = \sum_{k=1}^{N} q_r(u_k)$$

$$\left(\frac{u_j+i/2}{u_j-i/2}\right)^J = -\prod_{k=1}^{K} \frac{u_j-u_k+i}{u_j-u_k-i} \qquad q_r(u) = \frac{1}{r-1}\left(\frac{i}{(u+i/2)^r} - \frac{i}{(u-i/2)^r}\right)$$

N. Beisert, T. Klose ‘06

$$e^{ip} = \frac{x(u+i/2)}{x(u-i/2)} \qquad \mathcal{Q}_r = \gamma_{r,0}(\lambda)J + \sum_{s}^{\infty} \gamma_{r,s}(\lambda)\bar{\mathcal{Q}}_s \qquad \bar{\mathcal{Q}}_r = \sum_{k=1}^{N} \bar{q}_r(u_k)$$

$$\bar{q}_r(u) = \frac{1}{r-1}\left(\frac{i}{x(u+i/2)^r} - \frac{i}{x(u-i/2)^r}\right)$$

$$\left(\frac{x(u_j+i/2)}{x(u_j-i/2)}\right)^J = -\prod_{k=1}^{K} \exp(2i\Theta(u_j,u_k))\frac{u_j-u_k+i}{u_j-u_k-i}$$

$$u(x) = x + \sum_{\ell=0}^{\infty} \frac{\alpha_\ell(\lambda)}{x^{\ell+1}} \qquad \Theta(u,v) = \sum_{r=2}^{\infty}\sum_{s=r+1}^{\infty} \beta_{r,s}(\lambda)(\bar{q}_r(u)\bar{q}_s(v) - \bar{q}_s(u)\bar{q}_r(v))$$

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Medium range spin chains

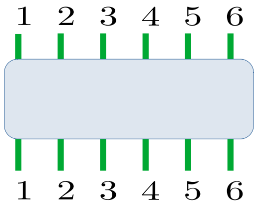
B. Pozsgay '21

TG, B. Pozsgay '21

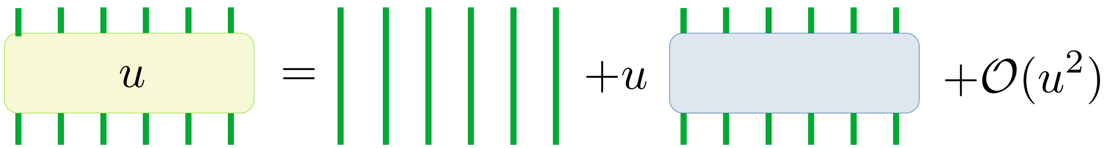
Interaction range $\ell + 2$

Length J $\quad J \geq \ell + 2$

$$H_J^{(\ell)} = \sum_{j=1}^{J} h_{j,j+1,\dots,j+\ell+1} = \sum_j h_j^{(\ell)}$$

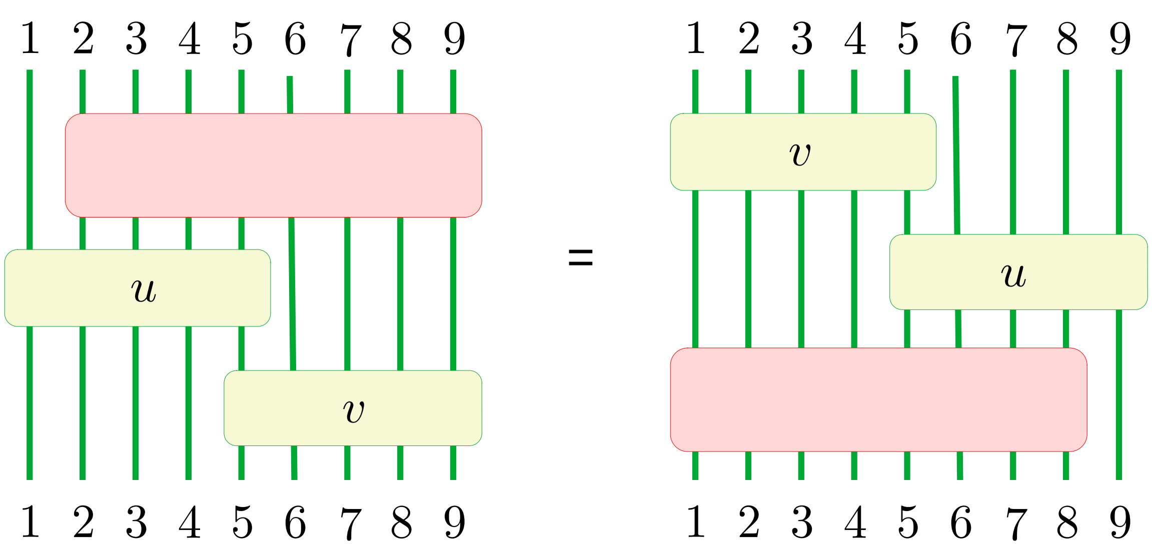


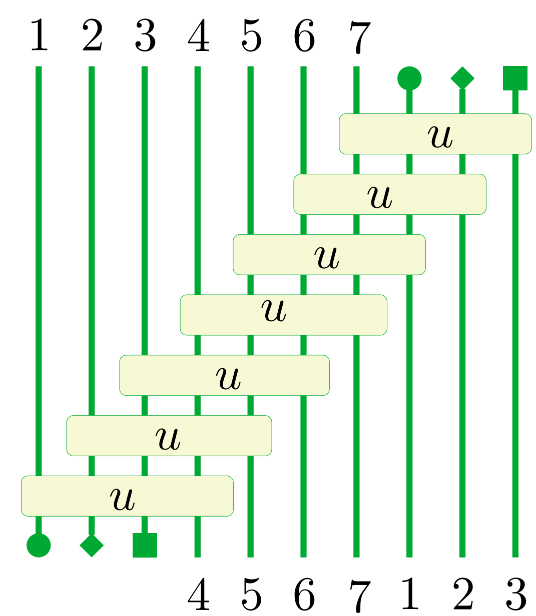
$$\check{L}_1^{(\ell)}(u) = \check{L}_{1,2,\dots,\ell+2}^{(\ell)}(u) = 1 + u h_1^{(\ell)} + \mathcal{O}(u^2),$$



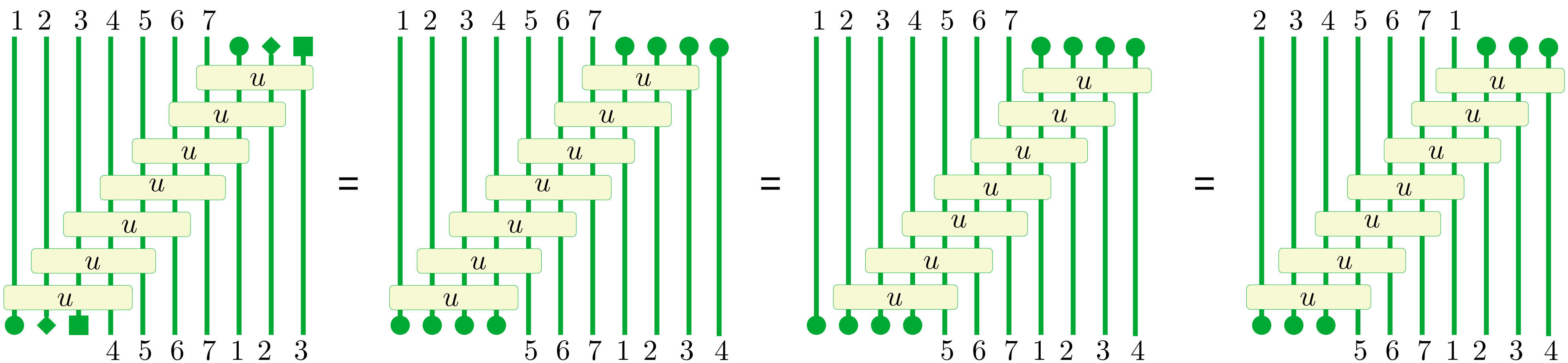
$$\check{R}_1^{(\ell)}(u,v) = \check{R}_{1,2,\dots,2\ell+2}^{(\ell)}(u,v), \qquad \check{R}_2^{(\ell)}(u,v)\check{L}_1^{(\ell)}(u)\check{L}_{\ell+2}^{(\ell)}(v) = \check{L}_1^{(\ell)}(v)\check{L}_{\ell+2}^{(\ell)}(u)\check{R}_1^{(\ell)}(u,v).$$

$$T_J^{(\ell)}(u) = \widehat{\mathrm{Tr}}_J\left[\check{L}_J^{(\ell)}(u)\dots\check{L}_1^{(\ell)}(u)\right] \qquad T_J^{(\ell)}(0) = \mathbf{1} \qquad \left.\frac{d}{du}T_J^{(\ell)}(u)\right|_{u=0} = H_J^{(\ell)}$$



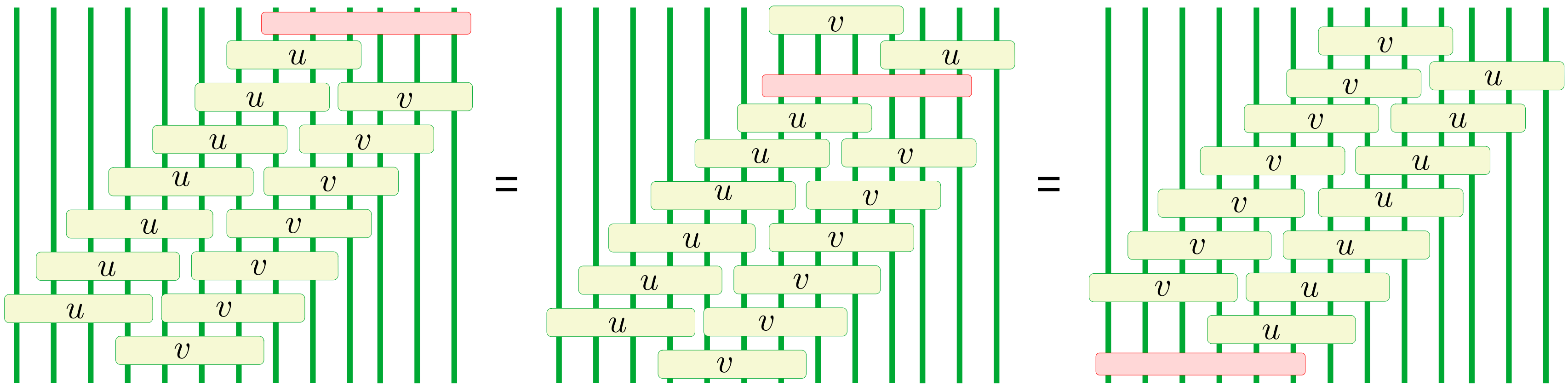


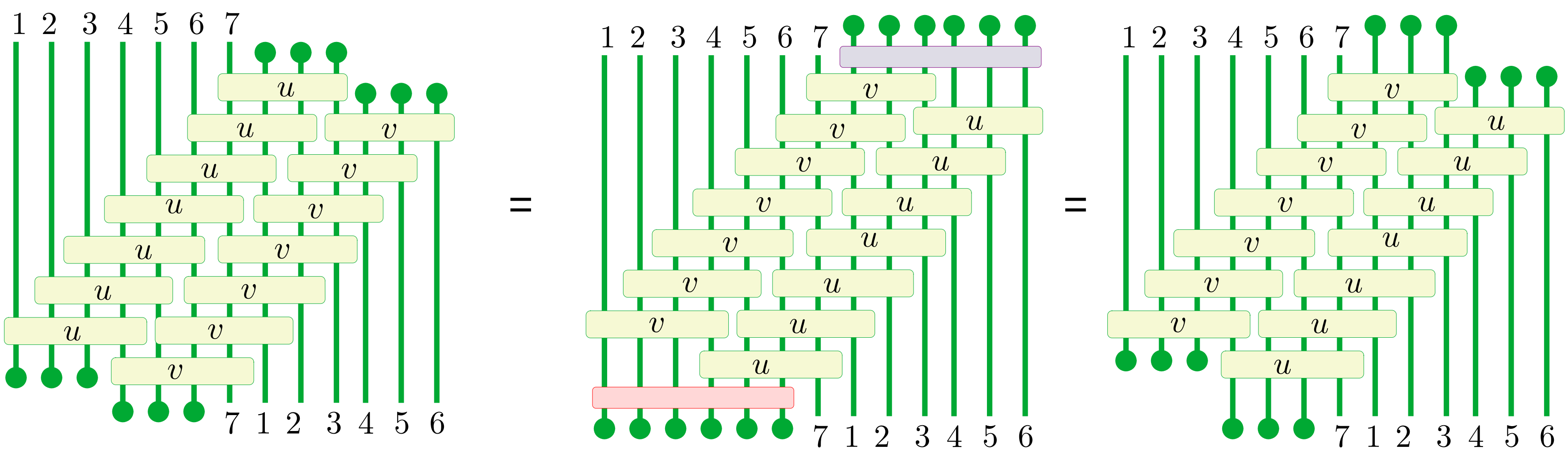
Properties of transfer matrices



$$T_J^{(\ell)}(u) = U T_J^{(\ell)}(u) U^{-1}$$

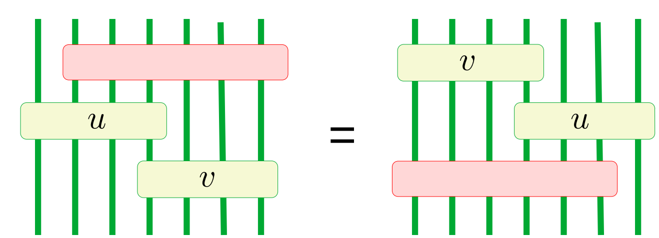
Properties of transfer matrices





$$T_J^{(\ell)}(u)U^{\ell+1}T_J^{(\ell)}(v)U^{-\ell-1} = T_J^{(\ell)}(v)U^{\ell+1}T_J^{(\ell)}(u)U^{-\ell-1}$$

$$[T_J^{(\ell)}(u), T_J^{(\ell)}(v)] = 0$$



Generalization to long range I

Let us start with the nearest neighbor model

$$\check{R}^{(0,0)}_{1,2}, \check{L}^{(0)}_{1,2} \longrightarrow \boxed{\check{R}^{(0,0)}_{2,3}\check{L}^{(0)}_{1,2}(u)\check{L}^{(0)}_{2,3}(v) = \check{L}^{(0)}_{1,2}(v)\check{L}^{(0)}_{2,3}(u)\check{R}^{(0,0)}_{1,2}}$$

First order ⟶ 3-site interaction ⟶ $\check{\mathcal{L}}^{(1)}_{1,2,3}(u) = \check{L}^{(0)}_{1,2}(u) + \lambda \check{L}^{(1)}_{1,2,3}(u)$

Truncated RLL: $\check{\mathcal{R}}^{(1)}_{2,3,4,5}\check{\mathcal{L}}^{(1)}_{1,2,3}(u)\check{\mathcal{L}}^{(1)}_{3,4,5}(v) = \check{\mathcal{L}}^{(1)}_{1,2,3}(v)\check{\mathcal{L}}^{(1)}_{3,4,5}(u)\check{\mathcal{R}}^{(1)}_{1,2,3,4} + \mathcal{O}(\lambda^2)$

Ansatz for R: $\check{\mathcal{R}}^{(1)}_{1,2,3,4} = \check{R}^{(1,0)}_{1,2,3,4} + \lambda \check{R}^{(1,1)}_{1,2,3,4}$

$$\lambda^0: \quad \boxed{\check{R}^{(1,0)}_{2,3,4,5}\check{L}^{(0)}_{1,2}(u)\check{L}^{(0)}_{3,4}(v) = \check{L}^{(0)}_{1,2}(v)\check{L}^{(0)}_{3,4}(u)\check{R}^{(1,0)}_{1,2,3,4}}$$

$$\lambda^1: \quad \check{R}^{(1,1)}_{2,3,4,5}\check{L}^{(0)}_{1,2}(u)\check{L}^{(0)}_{3,4}(v) + \check{R}^{(1,0)}_{2,3,4,5}\check{L}^{(1)}_{1,2,3}(u)\check{L}^{(0)}_{3,4}(v) + \check{R}^{(1,0)}_{2,3,4,5}\check{L}^{(0)}_{1,2}(u)\check{L}^{(1)}_{3,4,5}(v) =$$
$$\check{L}^{(0)}_{1,2}(v)\check{L}^{(0)}_{3,4}(u)\check{R}^{(1,1)}_{1,2,3,4} + \check{L}^{(0)}_{1,2}(v)\check{L}^{(1)}_{3,4,5}(u)\check{R}^{(1,0)}_{1,2,3,4} + \check{L}^{(1)}_{1,2,3}(v)\check{L}^{(0)}_{3,4}(u)\check{R}^{(1,0)}_{1,2,3,4}$$

In this order we have a new equation for $\check{L}^{(0)}_{1,2}$ but it turns out it follows from the previous one.

$$\check{R}^{(1,0)}_{1,2,3,4} := \check{L}^{(0)}_{2,3}(u)\check{R}^{(0,0)}_{1,2}\check{L}^{(0)}_{2,3}(v)^{-1}$$

$$\left(\check{L}^{(0)}_{3,4}(u)\check{R}^{(0,0)}_{2,3}\check{L}^{(0)}_{3,4}(v)^{-1}\right)\check{L}^{(0)}_{1,2}(u)\check{L}^{(0)}_{3,4}(v) = \check{L}^{(0)}_{3,4}(u)\check{R}^{(0,0)}_{2,3}\check{L}^{(0)}_{1,2}(u) = \check{L}^{(0)}_{3,4}(u)\check{R}^{(0,0)}_{2,3}\check{L}^{(0)}_{1,2}(u)\check{L}^{(0)}_{2,3}(v)\check{L}^{(0)}_{2,3}(v)^{-1}$$
$$= \check{L}^{(0)}_{3,4}(u)\check{L}^{(0)}_{1,2}(v)\left(\check{L}^{(0)}_{2,3}(u)\check{R}^{(0,0)}_{2,3}\check{L}^{(0)}_{2,3}(v)^{-1}\right)$$

In the first order, only one equation from the two is non-trivial
and the ‘real’ unknowns are $\check{R}^{(1,1)}_{1,2,3,4}, \check{L}^{(1)}_{1,2,3}$

Generalization to long range II

$$\check{\mathcal{L}}_1^{(\ell)}(u,\lambda) = \check{\mathcal{L}}_1^{(\ell)}(u) = \check{L}_1^{(0)}(u) + \sum_{j=1}^{\ell} \lambda^j \check{L}_1^{(j)}(u)$$

$\check{L}_1^{(j)}$ range j+2

$$\check{\mathcal{R}}_1^{(\ell)}(u,v,\lambda) = \check{R}_1^{(\ell,0)}(u,v) + \sum_{j=1}^{\ell} \lambda^j \check{R}_1^{(\ell,j)}(u,v)$$

$\check{R}_1^{(j,k)}$ range 2j+2

$$\boxed{\check{\mathcal{R}}_2^{(\ell)} \check{\mathcal{L}}_1^{(\ell)}(u) \check{\mathcal{L}}_{\ell+2}^{(\ell)}(v) = \check{\mathcal{L}}_1^{(\ell)}(v) \check{\mathcal{L}}_{\ell+2}^{(\ell)}(u) \check{\mathcal{R}}_1^{(\ell)} + \mathcal{O}(\lambda^{\ell+1})}$$

$$\check{\mathcal{R}}_1^{(\ell)} \equiv \check{\mathcal{R}}_1^{(\ell)}(u,v)$$

It is l+1 equation for $\lambda^0, \lambda^1, \ldots, \lambda^\ell$

Let assume there exist $\check{\mathcal{L}}^\ell, \check{\mathcal{R}}^\ell$ and let us take the next level

$$\check{\mathcal{R}}_2^{(\ell+1)} \check{\mathcal{L}}_1^{(\ell+1)}(u) \check{\mathcal{L}}_{\ell+3}^{(\ell+1)}(v) = \check{\mathcal{L}}_1^{(\ell+1)}(v) \check{\mathcal{L}}_{\ell+3}^{(\ell+1)}(u) \check{\mathcal{R}}_1^{(\ell+1)} + \mathcal{O}(\lambda^{\ell+2})$$

$$\left.\begin{array}{ll} \lambda^0: & \check{R}_2^{(\ell+1,0)} \check{L}_1^{(0)}(u) \check{L}_{\ell+3}^{(0)}(v) = \ldots \\ \vdots & \qquad \vdots \\ \lambda^\ell: & \check{R}_2^{(\ell+1,0)} \check{L}_1^{(\ell)}(u) \check{L}_{\ell+3}^{(0)}(v) + \ldots \end{array}\right] \longrightarrow \boxed{\check{\mathcal{R}}_2^{(\ell+1)} \check{\mathcal{L}}_1^{(\ell)}(u) \check{\mathcal{L}}_{\ell+3}^{(\ell)}(v) = \check{\mathcal{L}}_1^{(\ell)}(v) \check{\mathcal{L}}_{\ell+3}^{(\ell)}(u) \check{\mathcal{R}}_1^{(\ell+1)} + \mathcal{O}(\lambda^{\ell+1})}$$

$$\lambda^{\ell+1}: \quad \check{R}_2^{(\ell+1,0)} {\color{blue}\check{L}_1^{(\ell+1)}}(u) \check{L}_{\ell+3}^{(0)}(v) + {\color{red}\check{R}_2^{(\ell+1,\ell+1)}} \check{L}_1^{(0)}(u) \check{L}_{\ell+3}^{(0)}(v) + \ldots$$

Question: Is it guarantied that these equations are compatible up to order $\mathcal{O}(\lambda^{\ell+1})$?

$$\check{\mathcal{R}}_1^{(\ell+1)}(u,v) + \mathcal{O}(\lambda^{\ell+1}) = \check{\mathcal{L}}_{\ell+2}^{(\ell)}(u) \check{\mathcal{R}}_1^{(\ell)}(u,v) \check{\mathcal{J}}_{\ell+2}^{(\ell)}(v)$$

$$\check{\mathcal{J}}_1^{(\ell)}(u) \check{\mathcal{L}}_1^{(\ell)}(u) = 1 + \mathcal{O}(\lambda^{\ell+1})$$

The matrices $\check{R}^{(\ell,j)}$ are fixed by $\check{R}^{(k,k)}$ $(j,k<\ell)$ $\longrightarrow$ Unknowns: $\check{L}^{(\ell)}, \check{R}^{(\ell,\ell)}$

Lax operators and long range Hamiltonians

The transfer matrix $\mathcal{T}_J^{(\ell)}(u) = \widehat{\mathrm{Tr}}_J \left[\check{\mathcal{L}}_J^{(\ell)}(u) \dots \check{\mathcal{L}}_1^{(\ell)}(u) \right] + \mathcal{O}(\lambda^{\ell+1})$

Truncated RLL $\longrightarrow$ $[\mathcal{T}_J^{(\ell)}(u), \mathcal{T}_J^{(\ell)}(v)] = \mathcal{O}(\lambda^{\ell+1})$

Regularity $\check{\mathcal{L}}_1^{(\ell)}(u,\lambda) = \mathbf{1} + u\mathfrak{h}_1^{(\ell)}(\lambda) + \mathcal{O}(u^2),$ $\mathfrak{h}_1^{(\ell)}(\lambda) = \sum_{k=0}^{\ell} \lambda^k h_1^{(k)}$

$\longrightarrow$ $\mathcal{T}_J^{(\ell)}(0) = \mathbf{1}$ $\frac{d}{du}\mathcal{T}_J^{(\ell)}(u)\Big|_{u=0} = \mathcal{H}_J^{(\ell)} + \mathcal{O}(\lambda^{\ell+1})$ $\mathcal{H}_J^{(\ell)} = \sum_{k=0}^{\ell} \lambda^\ell H_J^{(\ell)}$

For $J \geq \ell + 2$ (asymptotic region) $H_J^{(\ell)} = \sum_{j=1}^{J} h_j^{(\ell)}$ $\longrightarrow$ range $\ell + 2$ Hamiltonian

The regular solutions of the truncated RLL gives integrable long range Hamiltonains by definition!

Question: Is the reverse direction also true?

Are there Lax operators for every integrable long range Hamiltonains?

For general long range deformed GL(N) spin chains, there exist $\check{L}_1^0, \check{L}_1^1, \check{L}_1^2$
(for every physical parameters $\alpha_r(\lambda), \beta_{r,s}(\lambda)$)

Content

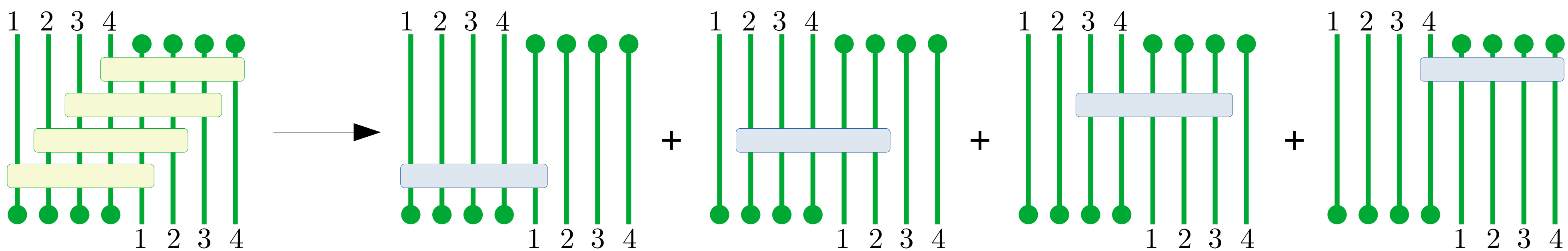
Wrapping corrections

The transfer matrix $\mathcal{T}_J^{(\ell)}(u) = \widehat{\mathrm{Tr}}_J \left[\check{\mathcal{L}}_J^{(\ell)}(u) \ldots \check{\mathcal{L}}_1^{(\ell)}(u) \right] + \mathcal{O}(\lambda^{\ell+1})$

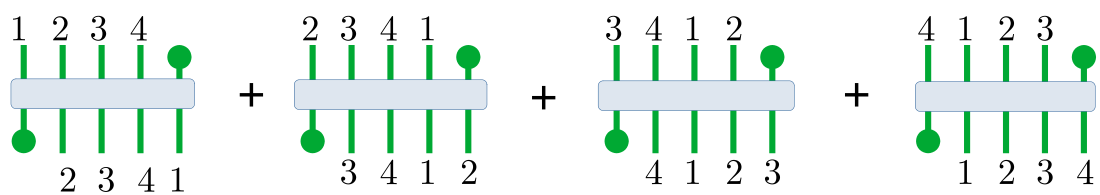
is well defined even for $J < \ell + 2$

$$\frac{d}{du} \mathcal{T}_J^{(\ell)}(u) \Big|_{u=0} = \mathcal{H}_J^{(\ell)} + \mathcal{O}(\lambda^{\ell+1})$$

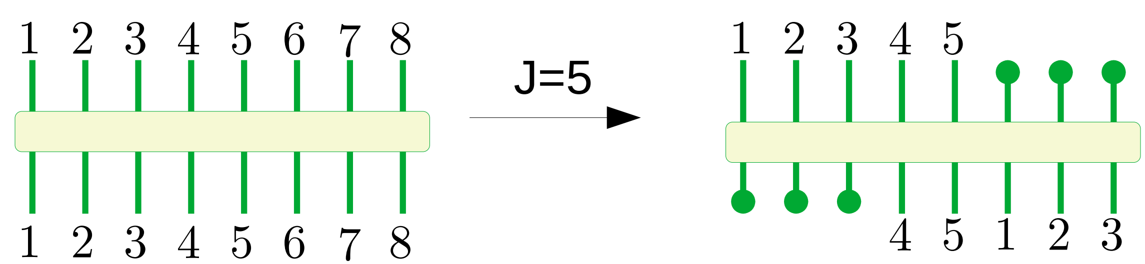
$\ell = 3, J = 4$



$$H_4^{(3)} = \sum_{j=1}^{4} \tilde{h}_j^{(4,3)} =$$



More generally: $H_J^{(\ell)} = \sum_{j=1}^{J} \tilde{h}_j^{(J,\ell)}$ $\qquad \tilde{h}_1^{(J,\ell)} = \widehat{\mathrm{Tr}}_J h_1^{(\ell)}$



Inozemtsev spin chain

Let us take a finite volume long range spin chain $\mathcal{H}_J$ for every length J

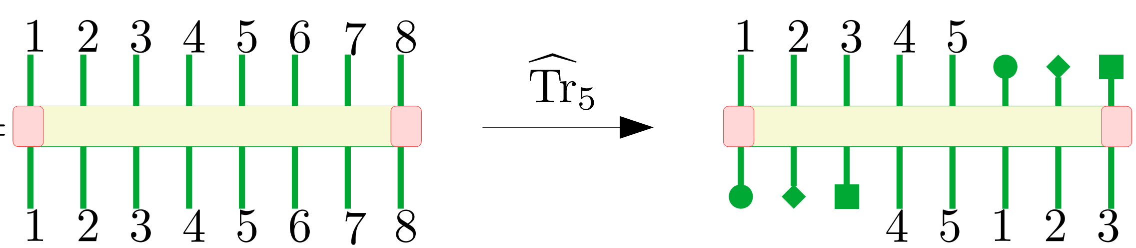
It defines the asymptotic Hamiltonian $\mathcal{H} = \lim_{J\to\infty} \mathcal{H}_J$

“Physical” consistency: Does out method give the initial finite volume Hamiltonians $\mathcal{H}_J$?

The Inozemtsev’s spin chain is $$\mathcal{H}_J = \sum_{1\leq j,k\leq J} \left(\wp(k-j) + \frac{2}{\omega}\zeta(\frac{\omega}{2})\right) P_{j,k} \qquad \omega = i\frac{\pi}{\kappa}$$

The asymptotic limit is $$\mathcal{H} = \sum_{-\infty<j<k<\infty} V(k-j)P_{j,k} \qquad V(j) = \left(\frac{\kappa}{\sinh(j\kappa)}\right)^2$$

The transformation $\tilde{h}^{(J,\ell)} = \widehat{\mathrm{Tr}}_J h^{(\ell)}$

$P_{1,8} =$  $= P_{1,3}$

acts on the permutation as $$\widehat{\mathrm{Tr}}_J P_{1,k} = \begin{cases} N, & \mathrm{Mod}(k,J) = 1, \\ P_{1,\mathrm{Mod}(k,J)}, & \mathrm{Mod}(k,J) = 2,\ldots,J. \end{cases}$$

$$\mathcal{H}_J = const. + \sum_{1\leq j<k\leq J} P_{j,k} \sum_{-\infty<l<\infty} V(k-j+lJ) \qquad \sum_{-\infty<l<\infty} V(k+lJ) = \left(\wp(k) + \frac{2}{\omega}\zeta(\frac{\omega}{2})\right)$$

Content

Motivation (long range spin chains in AdS/CFT, wrapping corrections)

Preliminaries (Definition of asymptotic long range spin chains)

Construction of the transfer matrix for long range spin chains

Finite size long range Hamiltonians

Requirements from AdS/CFT duality

Argument 0

The anomalous dimensions of single trace operators = Energy levels of a finite volume 1+1d field theory

Field theory description: Asymptotic data $e(p), \mathbb{S}(p_1, p_2)$ $\longrightarrow$ The finite volume spectrum

Spin chain description: Asymptotic Hamiltonian $\mathcal{H}$ $\longrightarrow$ Finite size Hamiltonians $\mathcal{H}_J$

Our method provides unique finite size Hamiltonians

Argument 1

unfixed parameters coming from the free choice of the renormalization scheme in the dilation operator

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$\epsilon_k(\lambda)$ parameters of the asymptotic Hamiltonian

These are unphysical parameters which disappear from the spectrum

Requirement: the parameters $\epsilon_k(\lambda)$ should disappear from the spectrum of the finite size Hamiltonians

$$\tilde{\mathcal{H}} = \exp(\mathcal{U})\mathcal{H}\exp(-\mathcal{U})$$

$$\mathcal{U} = \sum_k \epsilon_k(\lambda) X^{(k)}$$

It can be proved

$$\tilde{\mathcal{H}}_J = \exp(\mathcal{U}_J)\mathcal{H}_J\exp(-\mathcal{U}_J)$$

$$\mathcal{U}_J = \sum_k \epsilon_k(\lambda) X_J^{(k)}$$

Argument 2

Let us consider three asymptotic Hamiltonians $\mathcal{H}^{\mathcal{N}=4}, \mathcal{H}^{SU(N)}, \mathcal{H}^{SU(2)}$

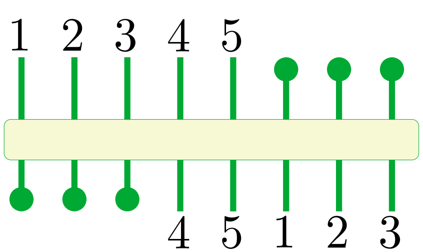
for which $$\mathcal{H}^{\mathcal{N}=4}\Big|_{SU(2)} = \mathcal{H}^{SU(N)}\Big|_{SU(2)} = \mathcal{H}^{SU(2)}$$

In the AdS/CFT, the wrapping corrections of the SU(2) sector includes contributions from the full spectrum

Z. Bajnok, R. Janik ‘08

The finite volume Hamiltonian in a closed sector depends on the full asymptotic Hamiltonian $$\mathcal{H}_J^{\mathcal{N}=4}\Big|_{SU(2)} \neq \mathcal{H}_J^{SU(2)}$$

Our definition for the wrapping region contains sums for the full spectrum



Our finite volume Hamiltonian in a closed sector depends also on the full asymptotic Hamiltonian

$$\boxed{\mathcal{H}_J^{SU(N)}\Big|_{SU(2)} \neq \mathcal{H}_J^{SU(2)}}$$

Argument 3

The four-loop asymptotic dilatation operator in the SU(2) sector contains: $\zeta(3)$

The four-loop length 4 dilatation operator in the SU(2) sector contains: $\zeta(3), \zeta(5)$

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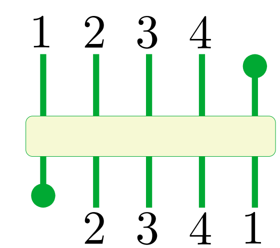
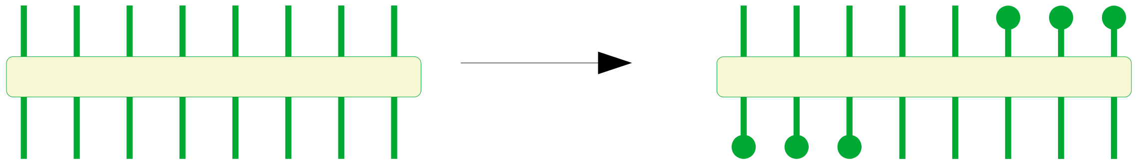
$\longrightarrow$ $\boxed{\text{The finite volume Hamiltonian could contain extra transcendental numbers}}$

Our definition for the wrapping region contains sums for the full spectrum



For infinite dimensional local Hilbert space, we have an infinite sums which may result in extra transcendental numbers.

Conclusions

The main message: 

- I showed a perturbative construction for transfer matrices which give perturbative long range commuting charges.
- I checked that this construction gives all the SU(N) symmetric long range integrable spin chain up to λ^2
- This transfer matrix is well defined even when the interaction range is smaller than the spin chain length therefore it defines long range Hamiltonian with finite length.
- We saw that these finite volume Hamiltonians satisfy some properties of AdS/CFT wrapping corrections.

  1) The finite volume spectrum is independent from the parameters $\epsilon_k(\lambda)$
  2) The wrapping corrections of the closed sectors depends on the full spectrum
  3) Extra transcendental numbers can appear in the wrapping corrections