Gravitational building blocks and ABJM at finite N

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Based on..

- Main conjecture - 2111.06903, implication - 2204.02992
- General rotating BPS black holes in AdS_4 - *[KH, Katmadas, Toldo'18-19]*
- Gravitational building blocks - *[Hosseini, KH, Zaffaroni'19]*
- Higher derivative asymptotically AdS_4 backgrounds - *[Bobev, Charles, KH, Reys'20-21 + Gang'20]*
- Supergravity localization - *[KH, Lodato, Reys'18-19], [KH, Reys'21]*

Main message

- Structure of supersymmetric observables in 4d $\mathcal{N} = 2$ supergravity in precise analogy with the one in 4d $\mathcal{N} = 2$ field theory.

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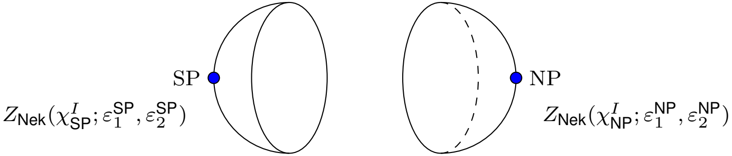
- Nekrasov partition function $\to$ gravitational Nekrasov-like partition function as a basic building block.
- Very detailed agreement with holographically dual results for ABJM theory.

Field theory localization



- Ω-deformation: exact evaluation of the partition function on $\mathbb{C}^2$, Z_{Nek} - $\varepsilon_{1,2}$ deformation parameters, χ^I Coulomb branch parameters, *[Nekrasov'02]*. $\varepsilon_1\varepsilon_2 \log Z_{\mathsf{Nek}}$ - expansion in $\varepsilon_{1,2}$.

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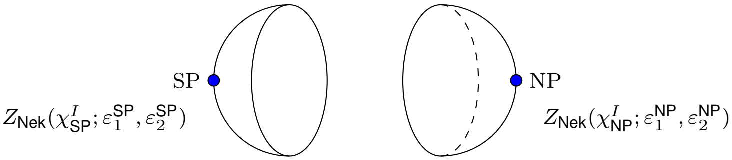

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- "Gluing" copies of Z_{Nek} on fixed points σ to reproduce many localization results, *[Nekrasov'03], [Pestun'07]*.

$$Z = \int \prod_I \mathrm{d}\chi^I \prod_\sigma Z_{\mathsf{Nek}}(\chi^I_\sigma; \varepsilon_1^\sigma, \varepsilon_2^\sigma)$$

Conjecture for supergravity backgrounds

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- On-shell action $\mathcal{F}(M_4) = -\log Z(M_4)$ localizes on the fixed point set of ξ_{M_4}. Works for AlAdS$_4$ examples in 2∂ minimal gauged sugra *[Genolini, Ipiña, Sparks'19]*, and 2∂ matter-coupled black holes *[Hosseini, KH, Zaffaroni'19]*.

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- Near a fixed point,

$$\xi = \varepsilon_1 \partial_{\varphi_1} + \varepsilon_2 \partial_{\varphi_2} \ , \qquad \varepsilon_2/\varepsilon_1 \equiv \omega \ ,$$

only the ratio ω is physical in sugra (difference with rigid susy).

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- Conjecture: extend to higher derivative $\mathcal{N} = 2$ sugra with $U(1)$ vector multiplets, build intuition with more examples.

Plan of the talk

- Introduction ✓
- Higher derivative supergravity formalism
- Formulation of the conjecture
- BPS black holes in Minkowski
- AdS_4 space = ABJM on S^3_b
- Static/rotating BPS black holes in AdS_4 = ABJM on $\text{S}^1 \times \text{S}^2_\omega$
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Higher derivative supergravity

- The formalism of $4d\ \mathcal{N}=2$ superconformal gravity *[de Wit, van Proeyen et al'80-84]* allows for the construction of large classes of HD terms with $\geq 4\partial$.

- F-terms from (anti-)chiral superspace integrals, correcting the 2∂ *prepotential*, D-terms from full superspace integrals, correcting the 2∂ Kähler potential.

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- A number of different auxiliary multiplets allow for different off-shell formulations.
- Assume (physical) hypermultiplets are decoupled - consider only extra abelian vector multiplets.
- Argue that D-terms vanish on susy backgrounds, consider only F-terms

Bosonic field content and HD invariants

- Weyl multiplet: vielbein $e_\mu{}^a$, auxiliary $\mathrm{U}(1) \times \mathrm{SU}(2)$ R-symmetry gauge fields $A_\mu, \mathcal{V}_\mu{}^{ij}$, auxiliary tensor $T^{\pm}_{ab}$, auxiliary scalar D.

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- Aux. hypermultiplet: four real scalars ${A_i}^\alpha$, gauging of a $\mathrm{U}(1)$ subgroup of the $\mathrm{SU}(2)_R$ via the combination $g_I W^I_\mu$, constant FI parameters g_I. Limit to ungauged sugra: $g_I = 0$.

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- Two different 4∂ F-terms: the Weyl^2 *[Bergshoeff, de Roo, de Wit'81]* and the T-$\log$ *[Butter, de Wit, Kuzenko, Lodato'13]* invariants. Defined via composite chiral multiplets with lowest components $A_{\mathbb{W}}$ and $A_{\mathbb{T}}$.

HD Lagrangian

- HD invariants encoded in the holomorphic prepotential

$$F(X^I; A_{\mathbb{W}}, A_{\mathbb{T}}) := \sum_{m,n=0}^{\infty} F^{(m,n)}(X^I)\,(A_{\mathbb{W}})^m\,(A_{\mathbb{T}})^n \; . \qquad (1)$$

- Lagrangian specified by the choice for $F(X^I; A_{\mathbb{W}}, A_{\mathbb{T}})$ and gauging g_I - 4∂ theory *off-shell*, an infinite derivative expansion *on-shell* ($A_{\mathbb{W},\mathbb{T}} \sim 2\partial$).

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- Higher order terms $F^{(m,n)}(X^I)$: homogeneous of degree $2(1-m-n)$ ($A_{\mathbb{W},\mathbb{T}}$ of weight 2), giving rise to terms with $2(1+m+n)$ derivatives.

Conjecture, part I: the on-shell action

- On-shell action,

$$\begin{aligned} \mathcal{F}(M_4, \chi^I, \omega) &= \sum_{\sigma \in M_4} s_{(\sigma)}\, \mathcal{B}(\kappa^{-1}\, X^I_{(\sigma)}(\chi^I, \omega), \omega_{(\sigma)}(\omega)) \;, \\ \mathcal{B}(X^I, \omega) &:= \frac{4i\pi^2\, F(X^I; (1-\omega)^2, (1+\omega)^2)}{\omega} \;, \end{aligned} \tag{2}$$

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- Additional constraint $\lambda^{M_4}(g_I, \chi^I, \omega) = 0$, restoring the correct number of Coulomb branch parameters (one aux. v.m.).

Conjecture, part I: the entropy function

- For black hole solutions, χ^I conjugate to q_I, ω conjugate to $\mathcal{J}$:

$$\mathcal{I}(M_4, \chi^I, \omega, q_I, \mathcal{J}) = -\mathcal{F}(M_4, \chi^I, \omega) - \frac{8i\pi^2}{\kappa^2}(\chi^I q_I - \omega \mathcal{J}) \ , \quad (3)$$

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- Recover the BH entropy via extremization,

$$S_{\text{BH}}(M_4, q_I, \mathcal{J}) = \mathcal{I}(M_4, \chi^I\Big|_{\text{crit.}}, \omega\Big|_{\text{crit.}}, q_I, \mathcal{J}) \in \mathbb{R} \,, \quad (4)$$

with a resulting constraint

$$\hat{\lambda}^{M_4}(g_I, q_I, \mathcal{J}) := \text{Im}\left(\mathcal{I}(M_4, \chi^I\Big|_{\text{crit.}}, \omega\Big|_{\text{crit.}}, q_I, \mathcal{J})\right) = 0 \,.$$

Conjecture, part II: the partition function

- A gravitational Nekrasov-like partition function

$$Z^{\mathsf{sugra}}_{\mathsf{Nek}}(X^I, \omega) := \exp\left(-\frac{4i\pi^2\, F(\kappa^{-1}\, X^I; (1-\omega)^2, (1+\omega)^2)}{\omega}\right),$$

corrected in a UV complete theory

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- Grand-canonical partition function via gluing rules

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- Microcanonical partition function / Quantum entropy function

$$Z(M_4, q_I, \mathcal{J}) := \int \mathrm{d}\chi^I \mathrm{d}\omega\, \delta(\lambda(g_I, \chi^I, \omega))\, e^{-\frac{8i\pi^2}{\kappa^2}(\chi^I q_I - \omega \mathcal{J})}\, Z(M_4, \chi^I, \omega) \,.$$

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- 2∂ ungauged supergravity,

$$F_{2\partial} = -\frac{1}{6}\, c_{ijk}\, \frac{X^i X^j X^k}{X^0}\ , \qquad g_I = 0\ , \quad I = \{0, i\}\ .$$

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- Half-BPS flow between asymptotic Minkowski and $AdS_2 \times S^2$ near-horizon (NH) geometry,

$$\mathrm{d}s^2 = v_1\,\mathrm{d}s^2_{AdS_2} + v_2\,\mathrm{d}s^2_{S^2}\ .$$

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- Fixed points of the canonical isometry: centre of AdS_2 and SP/NP of the sphere

$$\xi = -\partial_\tau + \partial_\varphi\ ,$$

$$\Rightarrow \omega_{\text{SP}} = \omega_{\text{NP}} = \omega = -1\ , \qquad s_{\text{SP}} = -s_{\text{NP}} = 1\ .$$

BPS black holes in Minkowski: attractor mechanism

- Scalars fixed at the horizon, *[Ferrara, Kallosh'96]*

$$\frac{1}{2}\left(e^{i\alpha}X^I + e^{-i\alpha}\bar{X}^I\right) = p^I\ , \qquad \frac{1}{2}\left(e^{i\alpha}F_I + e^{-i\alpha}\bar{F}_I\right) = q_I\ .$$

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- In a mixed ensemble, *[Ooguri, Strominger, Vafa'04]*,

$$e^{i\alpha}X^I = p^I + \frac{i}{\pi}\,\phi^I \ ,$$

ϕ^I conjugate to q_I,

$$\mathcal{F}_{\text{OSV}}(\phi^I, p^I) = \frac{i\pi}{2\,G_N^{(4)}}\left(F_{2\partial}(p^I + \frac{i}{\pi}\,\phi^I) - F_{2\partial}(p^I - \frac{i}{\pi}\,\phi^I)\right) \ ,$$

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Attractor mechanism from gluing

- Gluing rule: 2 fixed points with constraint $\omega = -1$, $s_{(1,2)} = \pm 1$,

$$\omega_{(1)} = \omega\ , X^I_{(1)} = \chi^I - \omega p^I\ , \quad \omega_{(2)} = \omega\ , X^I_{(2)} = \chi^I + \omega p^I .$$

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- Resulting on-shell action/entropy function

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- Precise match with OSV form upon $\phi^I = -i\pi\, \chi^I$. Confirmed by *BPS thermodynamics* on Euclidean saddles, *[KH'22]*.

Higher derivative generalization

- HD version of the Bertotti-Robinson, *[Cardoso, de Wit, Mohaupt'98-99]*. Full HD on-shell action,

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- Relation with the (unrefined) topological string (OSV conjecture) - infer the explicit form of the $\mathbb{W}$ tower $F^{(m,0)}$:

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- Part II of the conjecture - agreement with *[Denef, Moore'07]* and sugra localization *[Dabholkar, Gomes, Murthy'10-11]*.:

$$Z(p^I, q_I) := \int \left(\prod_{I=0}^{n_V} \mathrm{d}\chi^I \right) e^{-\mathcal{F}(\chi^I, p^I) - \frac{8i\pi^2}{\kappa^2}\, \chi^I q_I}\, Z^{\text{UV}}(\chi^I, p^I) \ .$$

AdS_4 space

- 2∂ gauged supergravity, from 11d on S^7

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- Fully BPS (Euclidean) AdS$_4$ vacuum, choose round S^3 boundary,

$$X^I = \frac{1}{4}\ , \forall I\ .$$

AdS$_4$ space

- 2∂ gauged supergravity, from 11d on S^7

$$F_{2\partial} = -2i\sqrt{X^0 X^1 X^2 X^3}\,, \qquad g_I = 1\,, \forall I.$$

- Fully BPS (Euclidean) AdS$_4$ vacuum, choose round S^3 boundary,

$$X^I = \frac{1}{4}\,, \forall I\,.$$

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- Single fixed point: centre of AdS$_4$

$$\xi = \partial_\tau + \partial_\varphi\,, \quad \Rightarrow \quad \omega = 1\,, \quad s = 1\,.$$

Holographic (squashed) sphere

- On-shell action from “gluing”, $X^I = 2\chi^I$

$$\mathcal{F}(S^3, \chi^I, \omega = 1) = 4i\pi^2\, F(2\kappa^{-1}\, \chi^I; 0, 4)\ .$$

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- Agreement with 4∂ minimal sugra results in *[Bobev, Charles, KH, Reys'20-21 + Gang'20]*.

Holographic bootstrap

- Exact results for round/squashed sphere of ABJM theory from susy localization - *[Fuji et al'11], [Marino, Putrov'11], [Nosaka'15], [Hatsuda'16], [Chester et al'21]*.

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- Exact results for round/squashed sphere of ABJM theory from susy localization - *[Fuji et al'11], [Marino, Putrov'11], [Nosaka'15], [Hatsuda'16], [Chester et al'21]*.
- Match with supergravity conjecture fixes the higher derivative prepotential *uniquely*:

$$F = -2i\sqrt{X^0X^1X^2X^3}\,\sum_{n=0}^{\infty} f_n \left(\frac{k_{\mathbb{W}}(X)A_{\mathbb{W}} + k_{\mathbb{T}}(X)A_{\mathbb{T}}}{64\,X^0X^1X^2X^3}\right)^n ,$$

$$k_{\mathbb{W}}(X) = -2\sum_{I<J} X^I X^J , \quad k_{\mathbb{T}}(X) = \sum_I (X^I)^2 + \ldots,$$

$$\frac{2\pi f_n}{(8\pi G_N)^{2(1-n)}} = \left(\frac{(2n-5)!!\,3}{n!\,(6k)^n}\right)\,\frac{\sqrt{2k}}{3}\,(N - \frac{k}{24})^{3/2-n}.$$

Airy function

- Supergravity prediction generalizes available matrix model results, complete perturbative answer (also conjectured in *[Bobev, Hong, Reys'22]*):

$$Z_{S^3}(b;\Delta_i) \simeq \exp\left(-\frac{2}{3}\, C_{S^3}^{-1/2}(N - \frac{k}{24} - B_{S^3})^{3/2}\right) ,$$

$$C_{S^3} = \frac{2(b+b^{-1})^{-4}}{\pi^2 k \prod_i \Delta_i} \,, B_{S^3} = \frac{1}{48k \prod_i \Delta_i} \left(k_{\mathbb{T}}(\Delta) + \frac{(b-b^{-1})^2}{(b+b^{-1})^2}\, k_{\mathbb{W}}(\Delta) \right) ,$$

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- Consistent with the expansion of the Airy function,

$$\text{Ai/Bi}(z)\sim\frac{e^{\mp 2/3\,z^{3/2}}}{2\sqrt{\pi}z^{1/4}}\Big[\sum_{n=0}^{\infty}\frac{(\mp1)^n 3^n\Gamma(n+\frac{5}{6})\Gamma(n+\frac{1}{6})}{2\pi n!4^n z^{3n/2}}\Big]\,.$$

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- Complete (upto N^0 and non-perturbative) answer:

$$Z_{S^3}(b;\Delta_i) \simeq \text{Ai}\left(C_{S^3}^{-1/3}(N - \frac{k}{24} - B_{S^3})\right) .$$

Black holes in AdS_4: twisted branch

- Rotating black holes, susy with a twist, NH: $\mathrm{AdS}_2 \times_w \mathrm{S}^2$, *[Cacciatori, Klemm'09], [KH, Katmadas, Toldo'18]*. 2 fixed points, *[Hosseini, KH, Zaffaroni'19]*, $g_I \chi^I = 1$, $\sum_I p^I = -1$,

$$\omega_{(1)} = \omega \, , X^I_{(1)} = \chi^I - \omega p^I \ , \quad \omega_{(2)} = -\omega \, , X^I_{(2)} = \chi^I + \omega p^I$$

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- Prediction for the partition function (topologically twisted index):

$$Z_{\mathsf{TTI}}(\mathfrak{n}_i, \omega, \Delta_i) \simeq \mathsf{Ai}[C_+^{-1/3}(N - B^0_+)] \times \mathsf{Bi}[C_-^{-1/3}(N - B^1_-)] \ ,$$

$$C_\pm = \frac{2\,\omega^2}{\pi^2 k \prod_i (\Delta_i \pm \omega \mathfrak{n}_i)} \ ,$$

$$B^s_\pm = \frac{k}{24} + \frac{(\omega + (-1)^s)^2 k_{\mathbb{T}}(\Delta \pm \omega \mathfrak{n}_i) + (\omega - (-1)^s)^2 k_{\mathbb{W}}(\Delta \pm \omega \mathfrak{n}_i)}{48 k \prod_i (\Delta_i \pm \omega \mathfrak{n}_i)} \ .$$

Black holes in AdS$_4$: static limit of twisted branch

- Admit static/unrefined limit $\omega = 0$:

$$-\log Z_{\text{TTI}}^{\text{unref}} \simeq \frac{\pi\sqrt{2k\prod_i \Delta_i}}{3}\left(\sum_i \frac{\mathfrak{n}_i}{\Delta_i}(N_{k,\Delta} - k_i)\right) N_{k,\Delta}^{1/2} + \frac{1}{2}\log N_{k,\Delta}\,,$$

$$N_{k,\Delta} := N - \frac{k}{24} + \frac{\sum_i (\Delta_i)^{-1}}{12k}\,, \quad k_i := \frac{(2-\Delta_i)\prod_{j\neq i}(\Delta_i + \Delta_j)}{8k\Delta_1\Delta_2\Delta_3\Delta_4}\,.$$

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- Precise agreement with numerical matrix model result in *[Bobev, Hong, Reys'22]*.

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- Very strong test for part I of the conjecture, weaker for part II.

Black holes in AdS$_4$: non-twisted branch

- Kerr-Newman-like black holes, no twist, NH: AdS$_2 \times_w$ S^2, *[KH, Katmadas, Toldo'19]*. 2 fixed points, *[Hosseini, KH, Zaffaroni'19]*, $\sum_I \chi^I = 1 + \omega$, $\sum_I p^I = 0$,

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- Prediction for the partition function (superconformal index):

$$Z_{\mathsf{SCI}}(\mathfrak{n}_i, \omega, \Delta_i) \simeq \mathsf{Ai}[C_+^{-1/3}(N - B_+^0)] \times \mathsf{Ai}[C_-^{-1/3}(N - B_-^0)] \ .$$

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- Admit Cardy limit $\omega \to 0$ (subleading magnetic charges):

$$-\log Z_{SCI}^{\text{Cardy}} \simeq \frac{2\pi\sqrt{2k\Delta_1\Delta_2\Delta_3\Delta_4}}{3\omega} N_{k,\Delta}^{3/2} + \frac{1}{2}\log N_{k,\Delta} \ .$$

Summary

- Sugra observables closely follow from the structure of susy field theory observables.
- A very general conjecture predicting full perturbative expansion of the on-shell action/entropy function based on 2∂ gluing rules.
- A general proposal for the UV completed form of supersymmetric partition functions.
- A number of sugra predictions testable via holography at finite N.

Many open questions

- Precise nature of gluing rules? Build up intuition with more examples - lenses, spindles, regular and irregular punctures, fixed two-submanifolds...
- Understand better all possible HD terms - prove the conjecture (part I) in supergravity?
- Derive the explicit HD form of the prepotential from string compactifications?
- Extend to non-perturbative corrections, charged hypers, other holographic examples, other dimensions...
- Use the conjecture to prove AdS/CFT for supersymmetric observables?

Thank you!