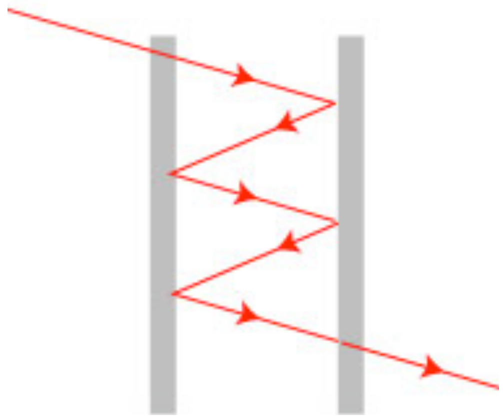
# Field quantisation in free space and in two-sided optical cavities



Almut Beige
Thomas M. Barlow
Robert Bennett

*School of Physics and Astronomy, University of Leeds, United Kingdom*

Varna, June 2015

# I

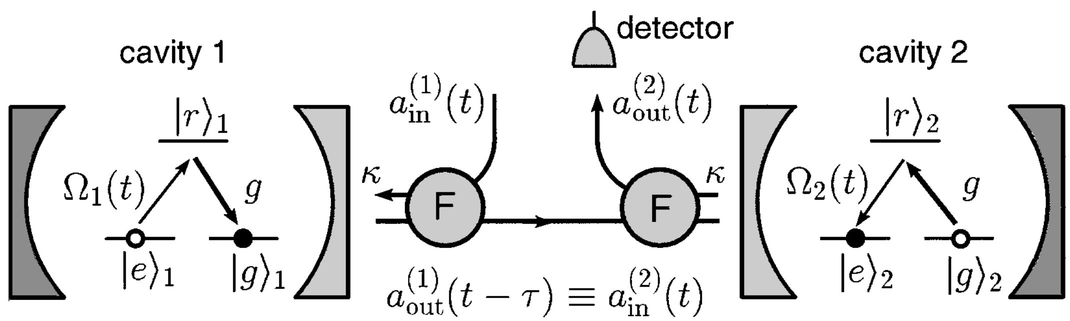
# Motivation: Coherent cavity networks [1]

[1] Kyoseva, Beige, and Kwek, New J. Phys. **14**, 023023 (2012).

# Quantum State Transfer and Entanglement Distribution among Distant Nodes in a Quantum Network

J. I. Cirac,[1,2] P. Zoller,[1,2] H. J. Kimble,[1,3] and H. Mabuchi[1,3]
[1]*Institute for Theoretical Physics, University of California at Santa Barbara, Santa Barbara, California 93106-4030*
[2]*Institut für Theoretische Physik, Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria*
[3]*Norman Bridge Laboratory of Physics 12-33, California Institute of Technology, Pasadena, California 91125*
(Received 12 November 1996)

We propose a scheme to utilize photons for ideal quantum transmission between atoms located at *spatially separated* nodes of a quantum network. The transmission protocol employs special laser pulses that excite an atom inside an optical cavity at the sending node so that its state is mapped into a *time-symmetric* photon wave packet that will enter a cavity at the receiving node and be absorbed by an atom there *with unit probability*. Implementation of our scheme would enable reliable transfer or sharing of entanglement among spatially distant atoms. [S0031-9007(97)02983-9]



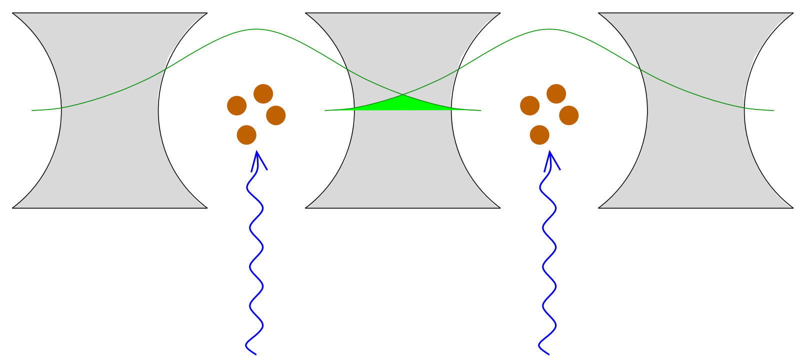
# Strong Photon Nonlinearities and Photonic Mott Insulators

Michael J. Hartmann* and Martin B. Plenio

*Institute for Mathematical Sciences, Imperial College London, 53 Exhibition Road, London SW7 2PG, United Kingdom, and QOLS, Blackett Laboratory, Imperial College London, Prince Consort Road, London SW7 2BW, United Kingdom*

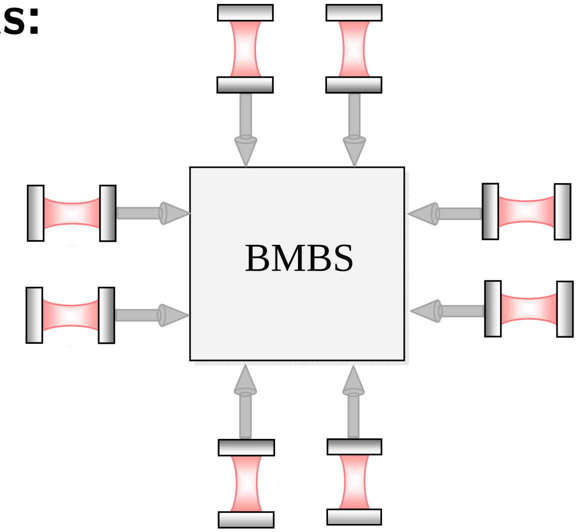
(Received 18 December 2006; published 4 September 2007)

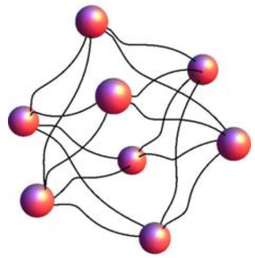
We show that photon nonlinearities in an electromagnetically induced transparency can be at least 1 order of magnitude larger than predicted in all previous approaches. As an application we demonstrate that in this regime they give rise to very strong photon-photon interactions which are strong enough to make an experimental realization of a *photonic* Mott insulator state feasible in arrays of coupled ultrahigh-$Q$ microcavities.



# But why not use medium length fibres?

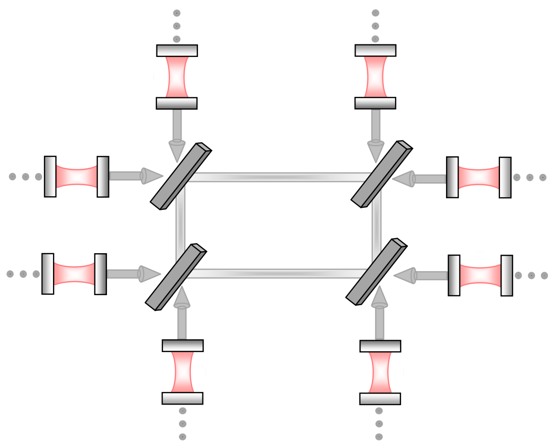
**Coherent cavity networks:**

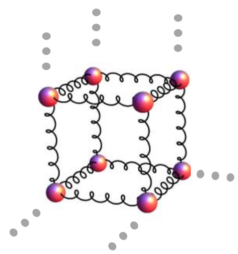


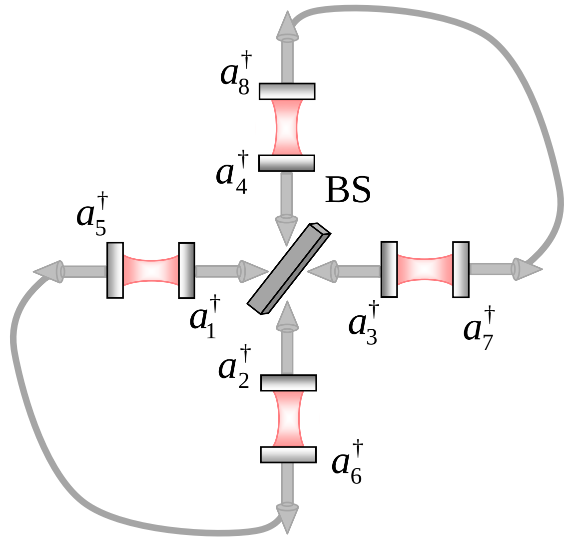



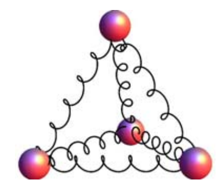
We propose to create effective cavity-cavity interactions by coupling optical cavities via linear optics elements and optical fibres.

# Examples [1]







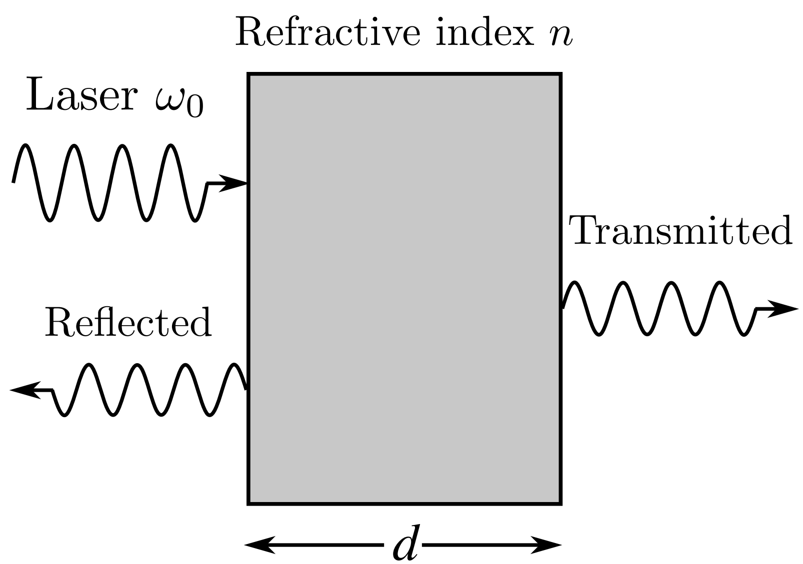
[1] Kyoseva, Beige, and Kwek, New J. Phys. **14**, 023023 (2012).

# II

# Standard models of the e.m. field between two mirrors

# A two-sided laser-driven optical cavity

For simplicity, we consider an experimental setup with no absorption in the cavity mirrors:



# Photons and the Fourier series

Any real-valued function $f$ with arguments $x \in (0, d)$ can be expanded in a series of exponentials with complex coefficients $c_m$ with $c_m = c^*_{-m}$,

$$f(x) \;=\; \sum_{m=-\infty}^{\infty} c_m \exp\left(\mathrm{i}m\,\frac{2\pi x}{d}\right)\,.$$

When quantising the em field in a finite volume, the above $c_m$ and $c^*_m$ are usually replaced by operators $c_m$ and $c^\dagger_m$ to yield

$$H_{\rm cav} \;=\; \sum_{m=1}^{\infty} \hbar\omega_m\, c^\dagger_m c_m\,.$$

## A quantum optics perspective

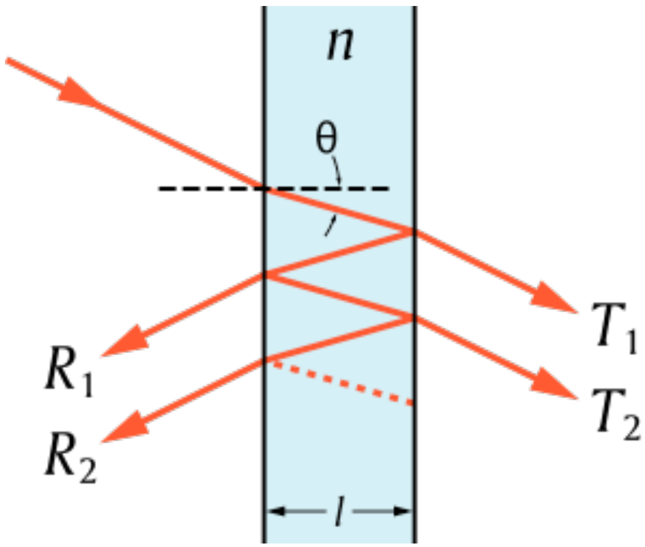
Suppose a cavity supports only standing wave photon modes.
Then a laser field only excites these modes!
Such a model would predict that

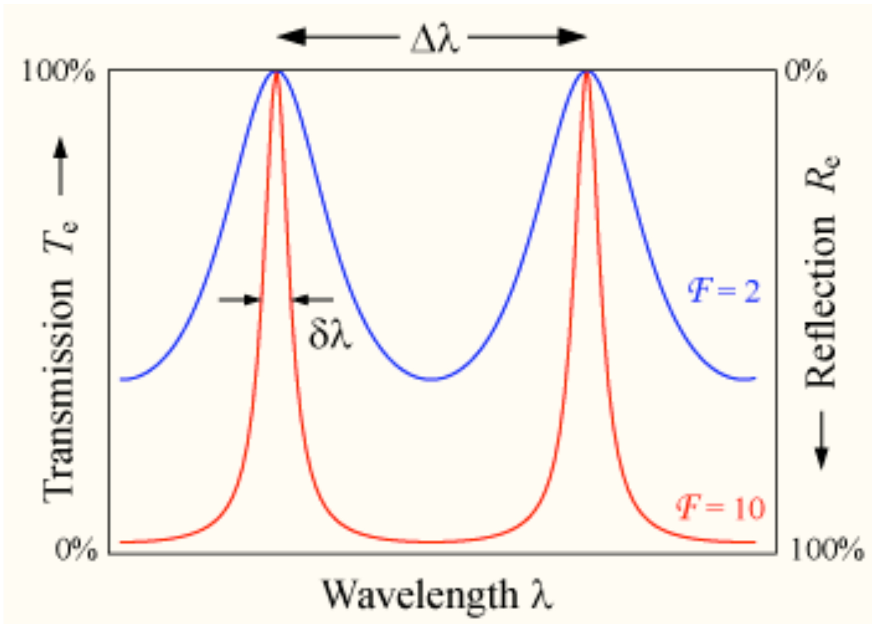
$$T_{\rm cav}(\omega_0) \;\; = \;\; R_{\rm cav}(\omega_0) \, .$$

This contradicts classical electrodynamics!

# Predictions of Maxwell's equations

**Fabry-Perot cavity:**



# Different quantum models for light scattering through two-sided optical cavities

- **Scattering theory:**
  This approach answers only certain questions and does not provide a Hamiltonian.

- **Input-output formalism:**
  This model imposes boundary conditions, thereby restricting the Hilbert space on which the standard Hamiltonian acts.

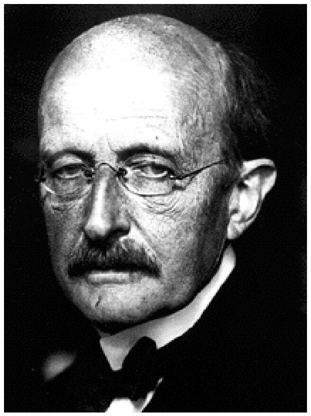
- **Universe-mode models:**
  Single photons of frequency $\omega$ are energy eigenstates of the cavity Hamiltonian with energy $\hbar\omega$.

# III

# A physically-motivated quantisation of the e.m. field in free space [1]

[1] Bennett, Barlow, and Beige, arXiv:1506.03305 (2015).

# What are photons?



As early as 1900, Planck introduced the idea of so-called basic energy elements into which the radiation field of a black body could be divided. Only much later, these energy elements became known as photons.

Nowadays, quantum opticians often answer the question *"What is a photon?"* by stating that a photon is what causes a click at a detector. Others say that photons are the basic energy quanta of the e.m. field.

# A harmonic oscillator field Hamiltonian

In the following, we describe the free field by a continuum of annihilation operators $a_{\rm L}(\omega)$ and $a_{\rm R}(\omega)$ with

$$[a_{\rm A}(\omega), a^\dagger_{\rm A'}(\omega')] = \delta_{\rm A,A'}\,\delta(\omega-\omega')\,,$$

where $\rm A, A' = L, R$ and $\omega$ is positive.

Assuming that a photon of frequency $\omega$ has the energy $\hbar\omega$, the electromagnetic field Hamiltonian $H_{\rm field}$ can now be written as

$$H_{\rm field} = \sum_{\rm A=L,R} \int_0^\infty {\rm d}\omega\, \hbar\omega\, a^\dagger_{\rm A}(\omega) a_{\rm A}(\omega)\,.$$

# Electric and magnetic field observables

We assume that the electric and magnetic field observables $E(x)$ and $B(x)$ can be written as

$$E(x) = \sum_{\mathrm{A=L,R}} \int_0^\infty \mathrm{d}\omega \, f_{\mathrm{A}}(x,\omega) \, a_{\mathrm{A}}(\omega) + \mathrm{H.c.} \, ,$$

$$B(x) = \sum_{\mathrm{A=L,R}} \int_0^\infty \mathrm{d}\omega \, g_{\mathrm{A}}(x,\omega) \, a_{\mathrm{A}}(\omega) + \mathrm{H.c.}$$

The $f_{\mathrm{A}}(x,\omega)$ and $g_{\mathrm{A}}(x,\omega)$ should be defined such that $\langle E(x) \rangle$ and $\langle B(x) \rangle$ evolve according to Maxwell's equations and such that $a_{\mathrm{L}}$ and $a_{\mathrm{R}}$ describe left and right traveling photon modes, respectively.

## Consistency with classical electrodynamics

**Requirements:**

$$\partial_x f_{\mathrm{L,R}}(x,\omega) = \mp \mathrm{i}\omega\, g_{\mathrm{L,R}}(x,\omega)\,,$$

$$\frac{1}{\mu}\partial_x g_{\mathrm{L,R}}(x,\omega) = \mp \mathrm{i}\varepsilon\omega\, f_{\mathrm{L,R}}(x,\omega)\,,$$

$$H_{\mathrm{field}} = \int_{I\!R^3} \mathrm{d}^3\mathbf{r}\,\frac{1}{2}\left(\epsilon\mathbf{E}(\mathbf{r})^2 + \frac{1}{\mu}\mathbf{B}(\mathbf{r})^2\right)$$

The above equations yield the usual field operators:

$$\mathbf{E}(\mathbf{r}) = \frac{\mathrm{i}}{(2\pi)^{3/2}} \sum_{\lambda=1,2} \int \mathrm{d}^3\mathbf{k}\,\sqrt{\frac{\hbar\omega_k}{2\epsilon}}\,\mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{r}}\,a_{\mathbf{k}\lambda}\,\mathbf{e}_{\mathbf{k}\lambda} + \mathrm{H.c.}\,,$$

$$\mathbf{B}(\mathbf{r}) = -\frac{\mathrm{i}}{(2\pi)^{3/2}}\sqrt{\epsilon\mu} \sum_{\lambda=1,2} \int \mathrm{d}^3\mathbf{k}\,\sqrt{\frac{\hbar\omega_k}{2\epsilon}}\,\mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{r}}\,a_{\mathbf{k}\lambda}\left(\hat{\mathbf{k}}\times\mathbf{e}_{\mathbf{k}\lambda}\right) + \mathrm{H.c.}$$

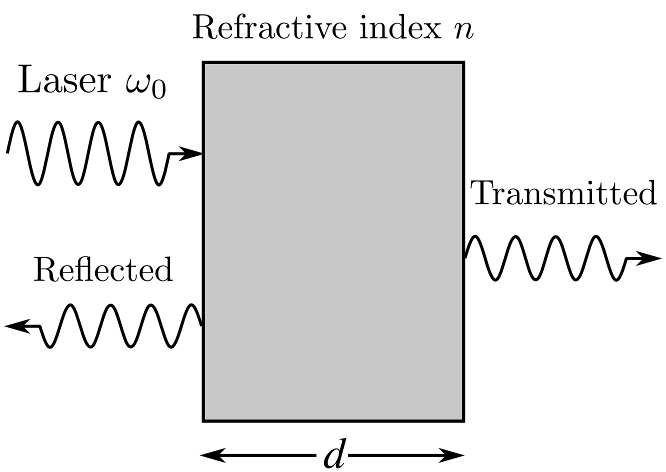
# IV

# An alternative model of the e.m. field between two mirrors [1]

[1] Barlow, Bennett, and Beige, J. Mod. Opt. **62**, S11 (2015).

# The relevant Hilbert space

We use the same notion of photons as when modelling the e.m. field in free space.




- We consider a continuum of field modes, since photons do not change their frequency when traveling through the resonator.
- We distinguish photons traveling left and photons traveling right so that we can assign different decay channels to different directions.

# The cavity Hamiltonian

**Cavity Hamiltonian:**

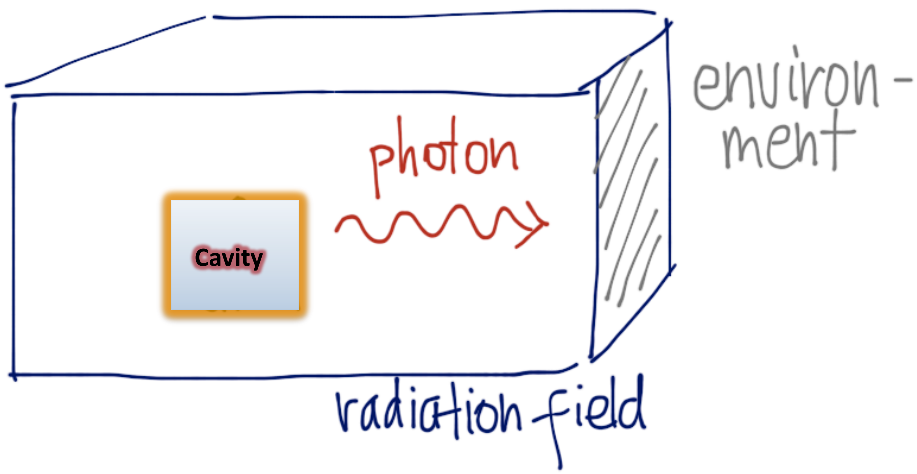
$$
\begin{aligned}
H_{\mathrm{cav}} &= \int_0^\infty \mathrm{d}\omega\, \hbar\omega \left( a_{\mathrm{L}}^\dagger(\omega) a_{\mathrm{L}}(\omega) + a_{\mathrm{R}}^\dagger(\omega) a_{\mathrm{R}}(\omega) \right) \\
&\quad + \frac{1}{2} \int_0^\infty \mathrm{d}\omega\, \hbar J(\omega) \left( a_{\mathrm{L}}^\dagger(\omega) a_{\mathrm{R}}(\omega) + \mathrm{H.c.} \right)
\end{aligned}
$$

$a_{\mathrm{A}}(\omega)$: photon annihilation operators

$\omega$: corresponding frequency

$J(\omega)$: photon coupling rate

## Spontaneous photon emission




- single quantum system (cavity)
- free radiation field
- photon-absorbing environment

# Effect of a photon-absorbing environment

- **Without the cavity:** $|\psi\rangle_{\rm F} \longrightarrow |0\rangle_{\rm F}$

  Photons are rapidly absorbed by detectors or the walls of the lab.
  The vacuum state is a pointer state of system and environment. [1]

- **With an atomic system:** $\rho_{\rm SF} \longrightarrow {\rm Tr}_{\rm F}\left(\rho_{\rm SF}\right) \otimes |0\rangle_{\rm FF}\langle 0|$

  The result of the interaction with the environment is the same as the effect of photon-absorbing measurements on a coarse grained time scale $\Delta t$. [2]

[1] Zurek, Rev. Mod. Phys. **75**, 715 (2003).
[2] Hegerfeldt, Phys. Rev. A **47**, 449 (1993) and others.

# Derivation of master equations [1]

- **Initial state:** $\rho_{\mathrm{S}}(t) \otimes |0\rangle_{\mathrm{FF}}\langle 0|$
- **Time evolution between resetting:**

$$\rho_{\mathrm{S}}(t+\Delta t) = \mathrm{Tr}_{\mathrm{F}} \left[ U_{\mathrm{I}}(t+\Delta t, t)\, \rho_{\mathrm{S}}(t) \otimes |0\rangle_{\mathrm{FF}}\langle 0|\, U_{\mathrm{I}}^{\dagger}(t+\Delta t, t) \right]$$

- **The same evolution is given by the master equation:**

$$\dot{\rho}_{\mathrm{S}} \;\; = \;\; -\frac{\mathrm{i}}{\hbar}\,[H, \rho_{\mathrm{S}}] + \text{decay terms}$$

[1] Stokes, Kurcz, Spiller, and Beige, Phys. Rev. A **85**, 053805 (2012).

# An effective two-mode description

**Traveling-wave description:**

$$
\begin{aligned}
H &= \hbar\omega_0 \left(a_{\rm L}^\dagger a_{\rm L} + a_{\rm R}^\dagger a_{\rm R}\right) + \frac{1}{2}\hbar J \left(a_{\rm R} a_{\rm L}^\dagger + a_{\rm L} a_{\rm R}^\dagger\right) \\
&\quad + \hbar\Omega \left(a_{\rm R}\, {\rm e}^{{\rm i}\omega_0 t} + {\rm H.c.}\right) \\
\dot\rho_{\rm S} &= -\frac{\rm i}{\hbar}\left[H, \rho_{\rm S}\right] + \sum_{\rm A=L,R} \kappa \left(a_{\rm A}\rho_{\rm S} a_{\rm A}^\dagger - \frac{1}{2}\rho_{\rm S} a_{\rm A}^\dagger a_{\rm A} - \frac{1}{2} a_{\rm A}^\dagger a_{\rm A}\rho_{\rm S}\right)
\end{aligned}
$$

- The laser only excites photons travelling to the right.
- Photons in the $a_{\rm L,R}$ mode are converted into $a_{\rm R,L}$ photons at a rate $J$.
- We assign different decay channels to photons with different directions.

# Cavity transmission and reflection rates

The calculation of the stationary state photon emission rates is relatively straightforward:

$$\frac{I_R^{\rm ss}}{I_{\rm Tot}^{\rm ss}} = \frac{1}{1+\frac{J^2}{\kappa^2}} \qquad \sim \qquad T_{\rm cav}(\omega_0) = \frac{1}{1+F\sin^2(k_0 n d)}\,,$$

$$\frac{I_L^{\rm ss}}{I_{\rm Tot}^{\rm ss}} = \frac{\frac{J^2}{\kappa^2}}{1+\frac{J^2}{\kappa^2}} \qquad \sim \qquad R_{\rm cav}(\omega_0) = \frac{F\sin^2(k_0 n d)}{1+F\sin^2(k_0 n d)}\,,$$

where $F$ : Fresnel coefficient

# Consistency with Maxwell's equations

The predictions of this model are consistent with the predictions of Maxwell's equations, if we choose:

$$\kappa = -\frac{2c}{nd}\ln r$$

$$J(\omega_0) = \frac{4c}{nd}\cdot\frac{r\ln r}{1-r^2}\sin(k_0 nd)$$

$$\text{with } r = \frac{n-1}{n+1}$$

## Special cases

- **Very long cavity:** $J = 0$ and $\kappa = 0$ for $d \to \infty$
- **Resonant cavity:** $J = 0$ and $\kappa \neq 0$
- **Near resonant laser driving:** $J = -2\Delta = -2(\omega_{\rm cav} - \omega_0)$

In this case, the mean total photon number $n_{\rm L} + n_{\rm R}$ evolves as predicted by the standard model of a laser-driven optical cavity.

**No contradiction of actual experiments!** [1]

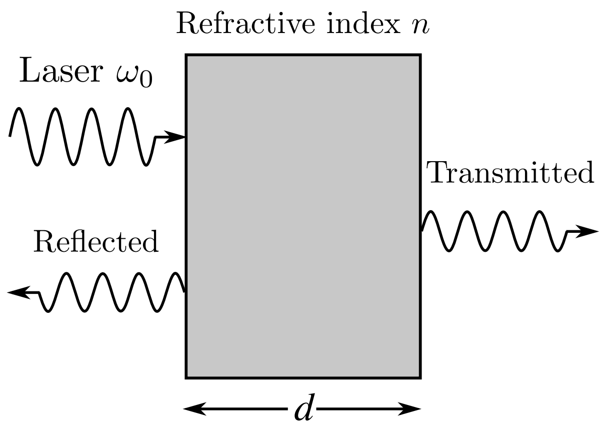
[1] Barlow, Bennett, and Beige, J. Mod. Opt. **62**, S11 (2015).

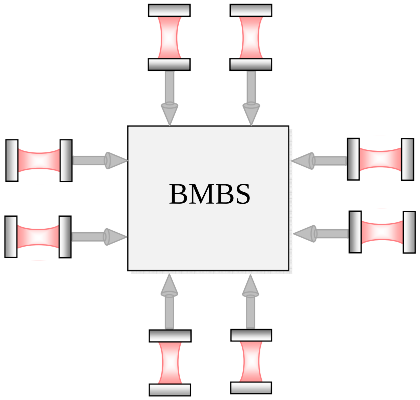
# V

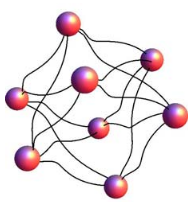
# Conclusions

# Conclusions

We discussed the quantisation of the electromagnetic field in free space and in a two-sided optical cavity. [1,2]



The proposed cavity Hamiltonian can be used to design cavity-fibre networks with complete connectivity. [3]

[1] Bennett, Barlow, and Beige, arXiv:1506.03305 (2015).
[2] Barlow, Bennett, and Beige, J. Mod. Opt. **62**, S11 (2015).
[3] Kyoseva, Beige, and Kwek, New J. Phys. **14**, 023023 (2012).