Kruskal-Penrose Formalism for Lightlike Thin-Shell Wormholes

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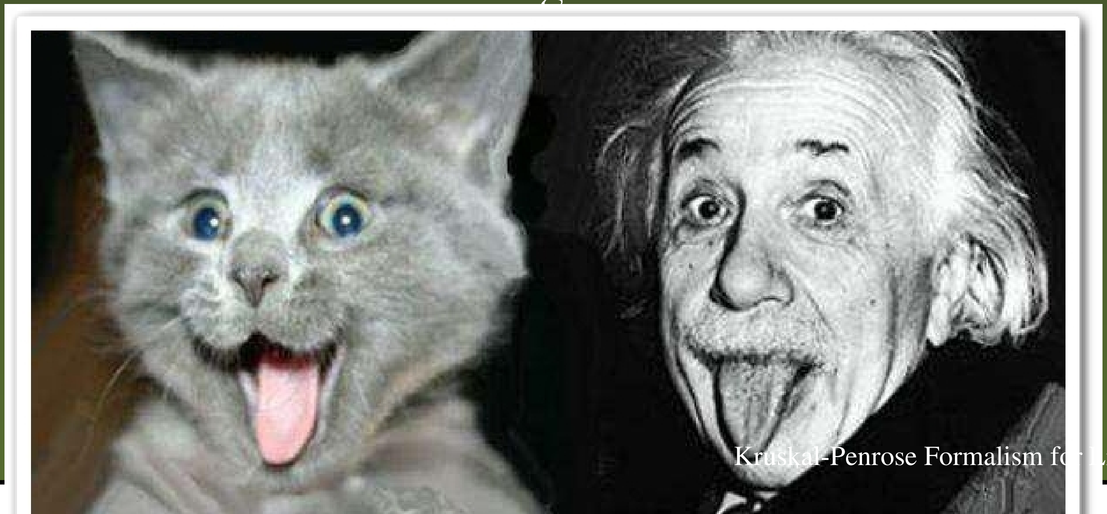

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Background material – series of papers by E. Guendelman, A. Kaganovich, E. Nissimov and S. Pacheva:

- *Phys. Lett* **B673** (2009) 288-292; **B681** (2009) 457-462;
- *Int. J. Mod. Phys.* **A25** (2010) 1405-1428; **A26** (2011) 5211-5239;
- *Gen. Rel. Grav.* **43** (2011) 1487-1513;
- in *"Seventh Mathematical Physics Meeting"*, B. Dragovic and Z. Rakic (eds.), Belgrade Inst. Phys. Press, 2013;
- *Bulg. J. Phys.* **40** (2013) 134-140;
- *Springer Proceedings in Mathematics and Statistics* **36** (2013) 169-183, ed. V. Dobrev, Springer (2013).

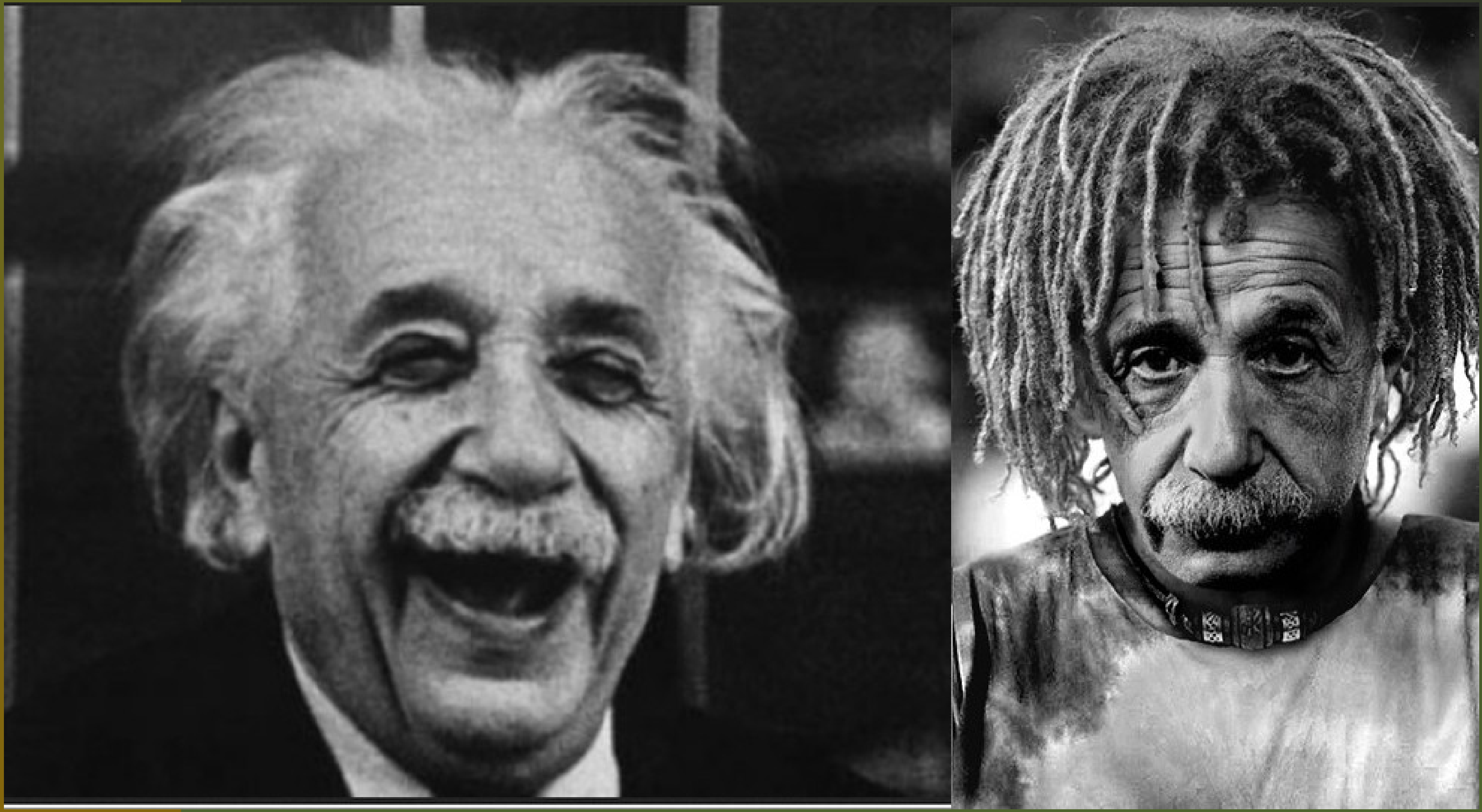
Introduction - Overview of Talk

- The original formulation of the "Einstein-Rosen bridge" as a historically first example of a static spherically-symmetric wormhole in Einstein-Rosen's original paper from 1935 is not equivalent to the concept of the dynamical and **nontraversable** "Einstein-Rosen bridge" (Schwarzschild wormhole) presented in modern textbooks on general relativity.
- As explicitly shown in some of our previous works the correct mathematical treatment of the original "Einstein-Rosen bridge" as a traversable wormhole requires the presence of a special kind of "exotic matter". This "exotic matter" is a specific lightlike brane located on the wormhole "throat" gluing the two universes - two identical copies of the external spacetime region of a Schwarzschild black hole, with a special relation between the (negative) brane tension and the Schwarzschild mass parameter.

Introduction - Overview of Talk

- The above crucial property – the presence of a lightlike thin-shell exotic matter on the wormhole "throat" has been **missed** in the original Einstein-Rosen paper (1935).
- The principal ingredient of the our result was the proposed by us qualitatively new manifestly reparametrization-invariant world-volume Lagrangian action for lightlike (null) branes where the lightlike brane dynamics is described in terms of Schwarzschild-like world-volume embedding coordinates.
- Here we continue our analysis of the mathematically consistent formulation of the original **traversable** "Einstein-Rosen bridge" by deriving the maximal analytic extension of its spacetime geometry along the lines of the well-known Kruskal-Penrose formalism.

Einstein and the Einstein-Rosen Bridge – Before and After



Schwarzschild Metric - Standard Coordinates

Schwarzschild metric – simplest static spherically symmetric black hole metric in standard coordinates (t, r, θ, φ):

$$ds^2 = -A(r)dt^2 + \frac{1}{A(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \; , \qquad (1)$$

where $A(r) = 1 - \frac{r_0}{r}$, $r_0 \equiv 2m$ (m – black hole mass) :

- (i) $r > r_0$ – exterior spacetime region; $r < r_0$ – black hole region;
- (ii) r_0 – horizon, where $A(r_0) = 0$ ($r = r_0$ – coordinate singularity; $r = 0$ – physical spacetime singularity).

Maximal analytic extension of Schwarzschild spacetime geometry: Kruskal-Szekeres coordinates – essential intermediate use of “tortoise” coordinate r^* (for light rays $t \pm r^* = \mathrm{const}$):

$$\frac{dr^*}{dr} = \frac{1}{A(r)} \longrightarrow r^* = r + r_0 \ln|r - r_0| \; . \qquad (2)$$

Schwarzschild Metric - Kruskal-Szekeres Coordinates

Kruskal-Szekeres ("light-cone") coordinates (v, w) – doubling the regions of the standard Schwarzschild geometry:

$$v = \pm \frac{1}{\sqrt{2k_h}} e^{k_h \left(t + r^*\right)} \quad , \quad w = \mp \frac{1}{\sqrt{2k_h}} e^{-k_h \left(t - r^*\right)} \quad , \tag{3}$$

where $k_h = \frac{1}{2}\partial_r A(r)\,\Big|_{r=r_0} = \frac{1}{2r_0}$ – "surface gravity" ($\frac{k_h}{2\pi} = k_B T_{\rm hawking}$).
Eqs.(3) are equivalent to: $-vw = \frac{1}{2k_h} e^{2k_h r^*} \quad , \quad -\frac{v}{w} = \frac{1}{2k_h} e^{2k_h t}$,
wherefrom r and r^* are determined as functions of vw.
The metric (1) becomes ($\widetilde{A}(vw) \equiv \frac{A(r(vw))}{k_h^2 vw}$):

$$ds^2 = \widetilde{A}(vw) dv dw + r^2(vw) \left(d\theta^2 + \sin^2\theta d\varphi^2\right) \quad , \tag{4}$$

and now there is no coordinate singularity on the horizon ($v = 0$ or $w = 0$)
$\widetilde{A}(0) = -4$ upon using Eq.(2).

Kruskal-Szekeres Diagram

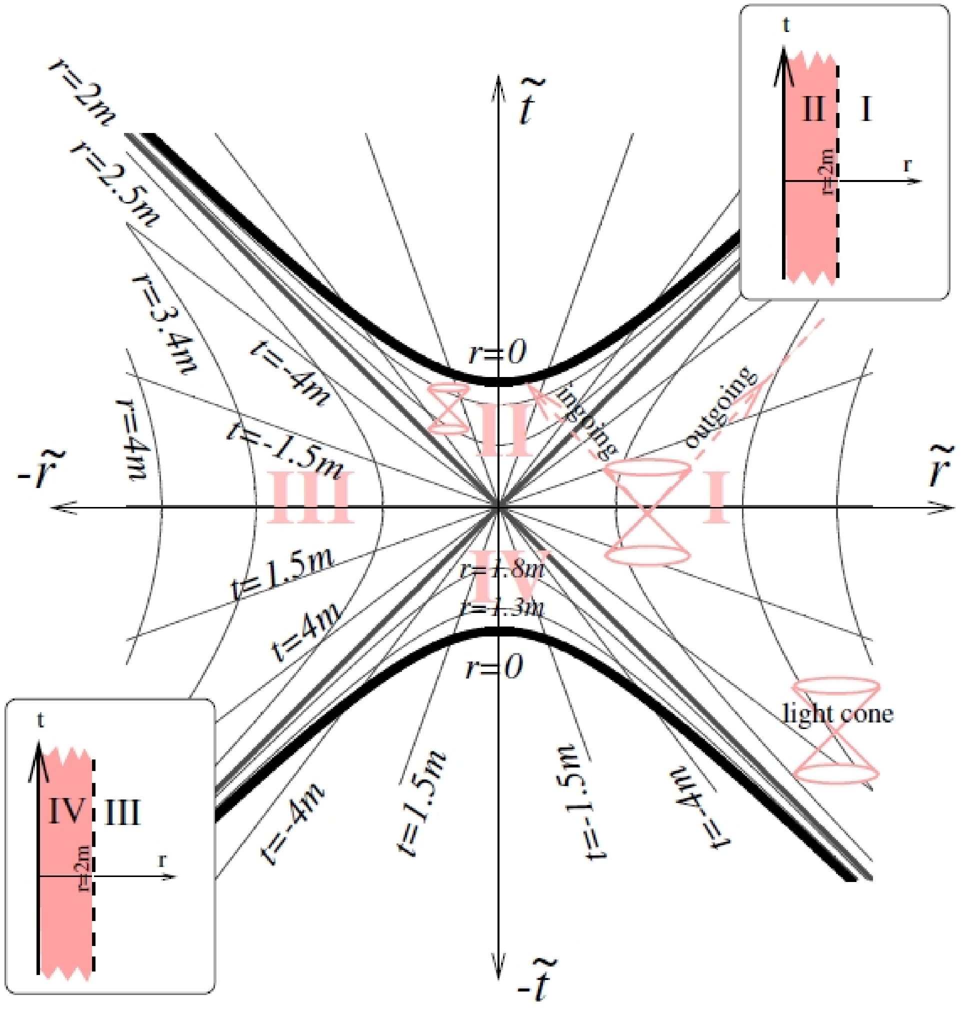


Figure 10: Kruskal-Szekeres diagram of the Schwarzschild spacetime.

Schwarzschild Wormhole

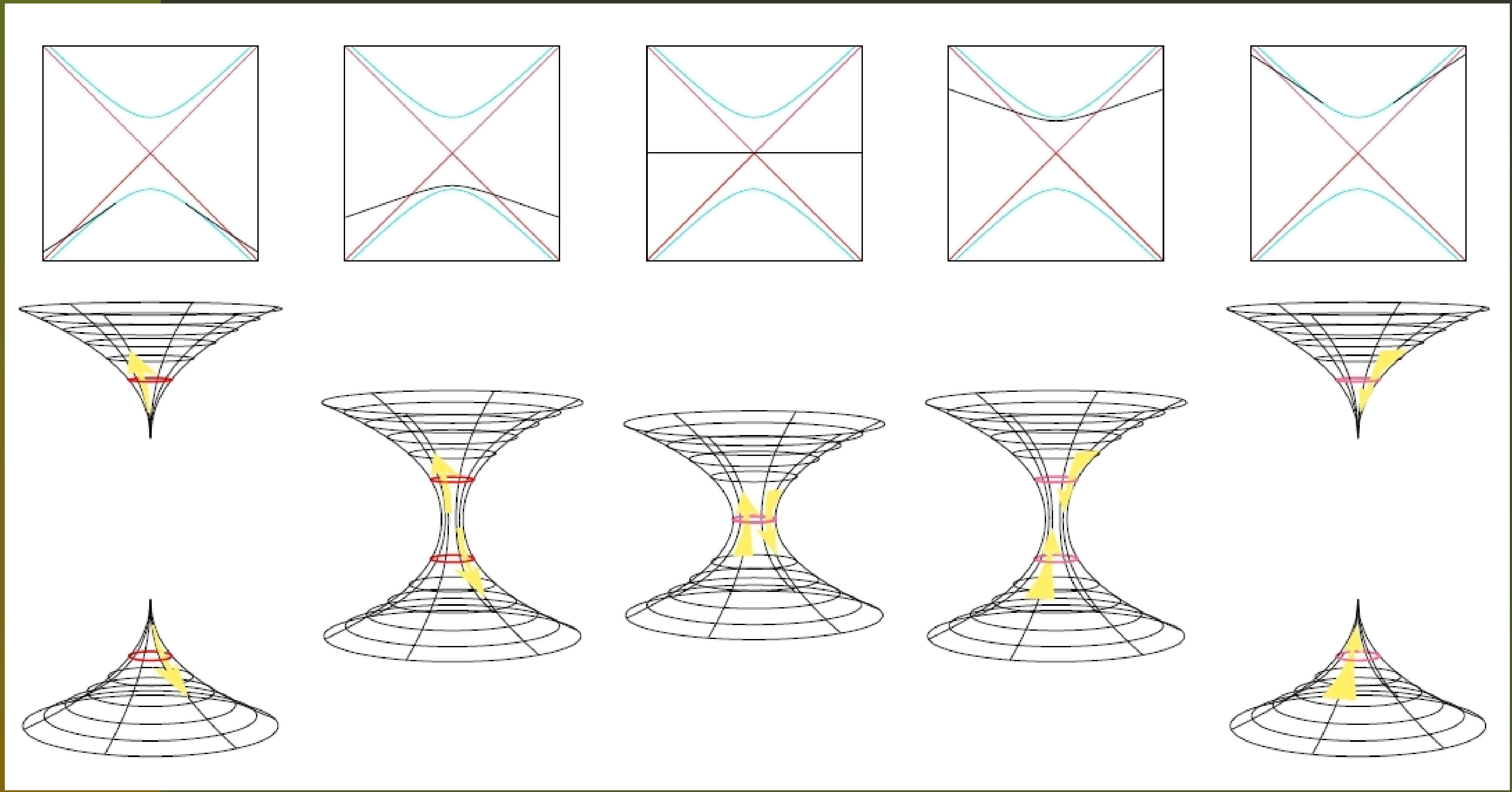


Figure 1: Schwarzschild Wormhole – "Einstein-Rosen Bridge" a'la Wheeler. It is NON-traversable!

Einstein-Rosen "Bridge" – Original Formulation (1935), Problems

In 1935 Einstein and Rosen introduced in (1) a new radial-like coordinate u via $r = r_0 + u^2$ and let $u \in (-\infty, +\infty)$:

$$ds^2 = -\frac{u^2}{u^2 + r_0} dt^2 + 4(u^2 + r_0) du^2 + (u^2 + r_0)^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right) \; . \quad (5)$$

Thus, (5) describes two identical copies of the exterior Schwarzschild spacetime region ($r \geq r_0$) for $u > 0$ and $u < 0$, which are formally glued together at the horizon $u = 0$.

Unfortunately, there are serious problems with (5):

- ER-metric has coordinate singularity at $u = 0$: $\det \| g_{\mu\nu} \|_{u=0} = 0$.
- Einstein eqs. acquire an ill-defined non-vanishing "matter" stress-energy tensor term on the r.h.s., which was overlooked in the original 1935 paper!

Einstein-Rosen “Bridge” – Correct Formulation (2009)

Indeed, from Levi-Civita idenity $R^0_0 = -\frac{1}{\sqrt{-g_{00}}}\nabla^2_{(3)}\left(\sqrt{-g_{00}}\right)$ we deduce that (5) solves vacuum Einstein eq. $R^0_0 = 0$ for all $u \neq 0$. However, since $\sqrt{-g_{00}} \sim |u|$ as $u \to 0$ and since $\frac{\partial^2}{\partial u^2}|u| = 2\delta(u)$, Levi-Civita identity tells us that:

$$R^0_0 \sim \frac{1}{|u|}\delta(u) \sim \delta(u^2) \; , \tag{6}$$

and similarly for the scalar curvature $R \sim \frac{1}{|u|}\delta(u) \sim \delta(u^2)$.

In 2009 we proposed [GKNP09] a correct reformulation of the original ER-bridge as a mathematically consistent traversable “lightlike thin-shell” wormhole via different radial-like coordinate $\eta \in (-\infty, +\infty)$, by substituting $r = r_0 + |\eta|$ in (1):

$$ds^2 = -\frac{|\eta|}{|\eta| + r_0}dt^2 + \frac{|\eta| + r_0}{|\eta|}d\eta^2 + (|\eta| + r_0)^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \; . \tag{7}$$

Einstein-Rosen Bridge as Lightlike Thin-Shell Wormhole

Eq.(7) is the correct spacetime metric for the original ER bridge:

- Eq.(7) describes two "universes" – two identical copies of the exterior Schwarzschild spacetime region for $\eta > 0$ and $\eta < 0$.
- Both "universes" are correctly glued together at their common horizon $\eta = 0$. Namely, the metric (7) solves Einstein eqs. $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T^{(brane)}_{\mu\nu}$, where $T^{(brane)}_{\mu\nu} = S_{\mu\nu}\delta(\eta)$ is the energy-momentum tensor of a special kind of **lightlike brane** located on the common horizon $\eta = 0$ – the wormhole "throat".
- The lightlike analogues of W.Israel's junction conditions on the wormhole "throat" are satisfied [GKNP09,10].
- Resulting lightlike thin-shell wormhole is **traversable**! (see below).

Caution: The above lightlike brane is a specific example of an "exotic" matter violating the null-energy condition (typical property in wormholes).

Lightlike Branes

Lightlike Branes (LL-branes, for short) are of particular interest in general relativity primarily due to their role in the effective treatment of many important cosmological and astrophysical effects:

- (i) impulsive lightlike signals arising in cataclysmic astrophysical events – description of thin shells of ejected ultrarelativistic matter;
- (ii) the "membrane" paradigm theory of black hole physics;
- (iii) thin-wall description of domain walls coupled to gravity.

More recently LL-branes became significant also in the context of modern non-perturbative string theory.

Lightlike Branes

In a series of papers [GKNP] we have proposed a systematic Lagrangian action description of LL-branes from first principles and found a series of physically interesting phenomena triggered by LL-brane dynamics:

- large class of spherically symmetric and rotating thin-shell wormholes;
- “mass inflation” effect around black hole and cosmological (de Sitter) horizons;
- creating regular black holes (no spacetime singularities – de Sitter interior geometry);
- triggering spontaneous compactification and decompactification of spacetime;
- charge-hiding and charge-confining via “tube-like” wormholes (gravitational analog of QCD quark confinement), etc.

Lightlike Branes – World-Volume Action

The energy-momentum tensor of LL-branes $T^{(brane)}_{\mu\nu}$ is self-consistently derived as $T^{(brane)}_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{\rm LL}}{\delta g^{\mu\nu}}$ from the following manifestly reparametrization invariant world-volume Polyakov-type LL-brane action [GKNP] (for $D = (p+1)+1$):

$$S_{\rm LL} = -\frac{1}{2}\int d^{p+1}\sigma\, T b_0^{\frac{p-1}{2}}\sqrt{-\gamma}\left[\gamma^{ab}\bar{g}_{ab} - b_0(p-1)\right] \; , \qquad (8)$$

$$\bar{g}_{ab} \equiv g_{ab} - \frac{1}{T^2}\left(\partial_a u + q\mathcal{A}_a\right)\left(\partial_b u + \mathcal{A}_b\right) \;\; , \;\; \mathcal{A}_a \equiv \partial_a X^\mu A_\mu \; . \qquad (9)$$

Here and below the following notations are used:

- γ_{ab} is the *intrinsic* Riemannian metric on the world-volume with $\gamma = \det\|\gamma_{ab}\|$; b_0 is a positive constant measuring the world-volume “cosmological constant”; $(\sigma) \equiv (\sigma^a)$ with $a = 0, 1, \ldots, p$; $\partial_a \equiv \frac{\partial}{\partial\sigma^a}$.

Lightlike Branes – World-Volume Action

- $X^\mu(\sigma)$ are the p-brane embedding coordinates in the bulk D-dimensional spacetime with Riemannian metric $g_{\mu\nu}(x)$ $(\mu, \nu = 0, 1, \ldots, D-1)$. A_μ is a spacetime electromagnetic field.
- $g_{ab} \equiv \partial_a X^\mu g_{\mu\nu}(X) \partial_b X^\nu$ is the *induced* metric on the world-volume which becomes *singular* on-shell – manifestation of the lightlike nature of the brane.
- u is auxiliary world-volume scalar field defining the lightlike direction of the induced metric and it is a non-propagating degree of freedom.
- T is *dynamical (variable)* brane tension (also a non-propagating degree of freedom).
- Coupling parameter q is the surface charge density of the LL-brane.

Lightlike Branes – Fundamental Properties

- **"Horizon straddling"**: Consistency of LL-brane dynamics given by the action (8) requires the Riemannian metric $g_{\mu\nu}$ of embedding spacetime to possess a "horizon", which is automatically occupied by the LL-brane world-volume! In case of (7) this is $\eta = 0$.
- **Lightlike junction conditions**.
  When LL-brane is moving in the ER-bridge embedding spacetime (7), the latter imply the following relation between the LL-brane parameters and the ER-bridge "mass" ($r_0 = 2m$):

  $$-T = \frac{1}{8\pi m} \quad , \quad b_0 = \frac{1}{4} \; , \tag{10}$$

  i.e., LL-brane dynamical tension T becomes **negative** on-shell – manifestation of "exotic matter".

Traversability – Particle Dynamics in Einstein-Rosen Wormhole

Motion of test-particle ("observer") of mass m_0 in a gravitational background is given by the reparametrization-invariant world-line action:

$$S_{\text{particle}} = \frac{1}{2}\int d\lambda \left[\frac{1}{e} g_{\mu\nu}\,\dot{x}^\mu \dot{x}^\nu - e m_0^2\right] \,, \tag{11}$$

where $\dot{x}^\mu \equiv \frac{dx^\mu}{d\lambda}$, e is the world-line "einbein" and in the present case $(x^\mu) = (t, \eta, \theta, \varphi)$. For a static spherically symmetric background such as (7) there are conserved Noether "charges" – energy $\mathcal{E}$ and angular momentum $\mathcal{J}$. In what follows we will consider purely "radial" motion ($\mathcal{J} = 0$) so, upon taking into account the "mass-shell" constraint (eq.motion w.r.t. e) and introducing the world-line proper-time parameter τ ($\frac{d\tau}{d\lambda} = e m_0$), the timelike geodesic eqs. read:

$$\left(\frac{d\eta}{d\tau}\right)^2 = \frac{\mathcal{E}^2}{m_0^2} - A(\eta) \quad , \quad \frac{dt}{d\tau} = \frac{\mathcal{E}}{m_0 A(\eta)} \quad , \quad A(\eta) \equiv \frac{|\eta|}{|\eta| + r_0} \,. \tag{12}$$

Traversability of Original Einstein-Rosen Wormhole

For a test-particle starting for $\tau = 0$ at initial position in "our" (right) universe $\eta_0 = \eta(0)$, $t_0 = t(0)$ and **infalling** towards the "throat":

$$\frac{\mathcal{E}}{2k_h m_0} \int_{2k_h\eta(\tau)}^{2k_h\eta_0} dy \sqrt{(1+|y|)\Big[(1+(1-\frac{m_0^2}{\mathcal{E}^2})|y|\Big]^{-1}} = \tau \; , \qquad (13)$$

$$\frac{1}{2k_h} \int_{2k_h\eta(\tau)}^{2k_h\eta_0} dy \frac{1}{|y|} \sqrt{(1+|y|)\Big[(1+(1-\frac{m_0^2}{\mathcal{E}^2})|y|\Big]} = t(\tau) - t_0 \; . \qquad (14)$$

- The particle will cross the wormhole "throat" ($\eta = 0$) for a finite proper-time $\tau_0 > 0$ ($\tau_0 =$ integral in (13) with zero lower limit).
- It will continue into the second (left) universe and reach any point $\eta_1 < 0$ within another **finite** proper-time $\tau_1 > \tau_0$.

Traversability of Original Einstein-Rosen Wormhole

- On the other hand from (14) it follows that $t(\tau_0 - 0) = +\infty$, *i.e.*, from the point of view of static observer in “our” (right) universe it will take infinite “laboratory” time for the particle to reach the “throat” – the latter appears to the static observer as a future black hole horizon.
- Eq.(14) also implies $t(\tau_0 + 0) = -\infty$, which means that from the point of view of static observer in the second (right) universe, upon crossing the “throat”, the particle starts its motion in the second (right) universe from infinite past, so that it will take an infinite amount of “laboratory” time to reach the point $\eta_1 < 0$ – *i.e.* the “throat” now appears as a past black hole horizon.

Einstein-Rosen "Tortoise" Coordinate

In analogy with the usual "tortoise" coordinate r^* let us introduce ER-bridge "tortoise" coordinate η^* (recall $r_0 = \frac{1}{2k_h}$):

$$\frac{d\eta^*}{d\eta} = \frac{|\eta| + r_0}{|\eta|} \quad \longrightarrow \quad \eta^* = \eta + \mathrm{sign}(\eta) r_0 \ln|\eta| \,. \tag{15}$$

For infalling/outgoing massless particles (light rays) Eqs.(13)-(14) imply:

$$t \pm \eta^* = \mathrm{const} \,. \tag{16}$$

For infalling massive particles towards the "throat" ($\eta = 0$) starting at $\eta_0^+ > 0$ in "our" (right) universe or starting in the second (right) universe at some $\eta_0^- < 0$ we have correspondingly:

$$\big[t \pm \eta^*\big](\eta) = \frac{\pm 1}{2k_h} \int_{2k_h\eta}^{2k_h\eta_0^\pm} dy \Big(1 + \frac{1}{|y|}\Big) \left[\sqrt{(1+|y|)\Big[\big(1 + \big(1 - \frac{m_0^2}{\mathcal{E}^2}\big)|y|\Big]^{-1}} - 1\right] . \tag{17}$$

Kruskal-Like Coordinates for Original ER Bridge

We define the maximal analytic extension of original Einstein-Rosen wormhole geometry via introducing Kruskal-like cooridnates (v, w):

$$v = \pm \frac{1}{\sqrt{2k_h}} e^{\pm k_h (t+\eta^*)} \quad , \quad w = \mp \frac{1}{\sqrt{2k_h}} e^{\mp k_h (t-\eta^*)} \; , \tag{18}$$

and accordingly:

$$-vw = \frac{1}{2k_h} e^{\pm 2k_h \eta^*} \quad , \quad -\frac{v}{w} = e^{\pm 2k_h t} \; . \tag{19}$$

- Upper signs in (18)-(19) correspond to region I $(v > 0, w < 0)$ describing "our" (right) universe $\eta > 0$.
- Lower signs in (18)-(19) correspond to region II $(v < 0, w > 0)$ describing the second (left) universe $\eta < 0$.

Kruskal-Like Coordinates for Original ER Bridge

Metric of ER-bridge in Kruskal-like coordinates:

$$ds^2 = \widetilde{A}(vw)dvdw + \widetilde{r}^2(vw)\Big(d\theta^2 + \sin^2\theta d\varphi^2\Big) \; , \tag{20}$$

$$\widetilde{r}(vw) = r_0 + |\eta(vw)| \quad (r_0 \equiv \frac{1}{2k_h}) \; ,$$

$$\widetilde{A}(vw) = \frac{A\big(\eta(vw)\big)}{k_h^2 vw} = -\frac{4e^{-2k_h|\eta(vw)|}}{1 + 2k_h|\eta(vw)|} \; , \tag{21}$$

where $\eta(vw)$ is determined from (19) and (15) as:

$$-vw = \frac{|\eta|}{2k_h} e^{2k_h|\eta|} \quad \longrightarrow \quad |\eta(vw)| = \frac{1}{2k_h}\mathcal{W}(-4k_h^2 vw) \; , \tag{22}$$

$\mathcal{W}(z)$ being the Lambert (product-logarithm) function ($z = \mathcal{W}(z)e^{W(z)}$).

Kruskal-Like Coordinates for Original ER Bridge

Using the explicit expression (15) for η^* in (19) we find:

- "Throats" (horizons) – at $v = 0$ or $w = 0$;
- In region I the "throat" $(v > 0, w = 0)$ is a future horizon $(\eta = 0\,,\, t \to +\infty)$, whereas the "throat" $(v = 0, w < 0)$ is a past horizon $(\eta = 0\,,\, t \to -\infty)$.
- In region II the "throat" $(v = 0, w > 0)$ is a future horizon $(\eta = 0\,,\, t \to +\infty)$, whereas the "throat" $(v < 0, w = 0)$ is a past horizon $(\eta = 0\,,\, t \to -\infty)$.

It is customary to replace Kruskal-like coordinates (v, w) (18) with compactified Penrose-like coordinates $(\bar{v}, \bar{w})$:

$$\bar{v} = \arctan(\sqrt{2k_h}v) \quad , \quad \bar{w} = \arctan(\sqrt{2k_h}w) \; , \tag{23}$$

mapping various "throats" (horizons) and infinities to finite lines/points:

Kruskal-Penrose Diagram for Original ER Bridge

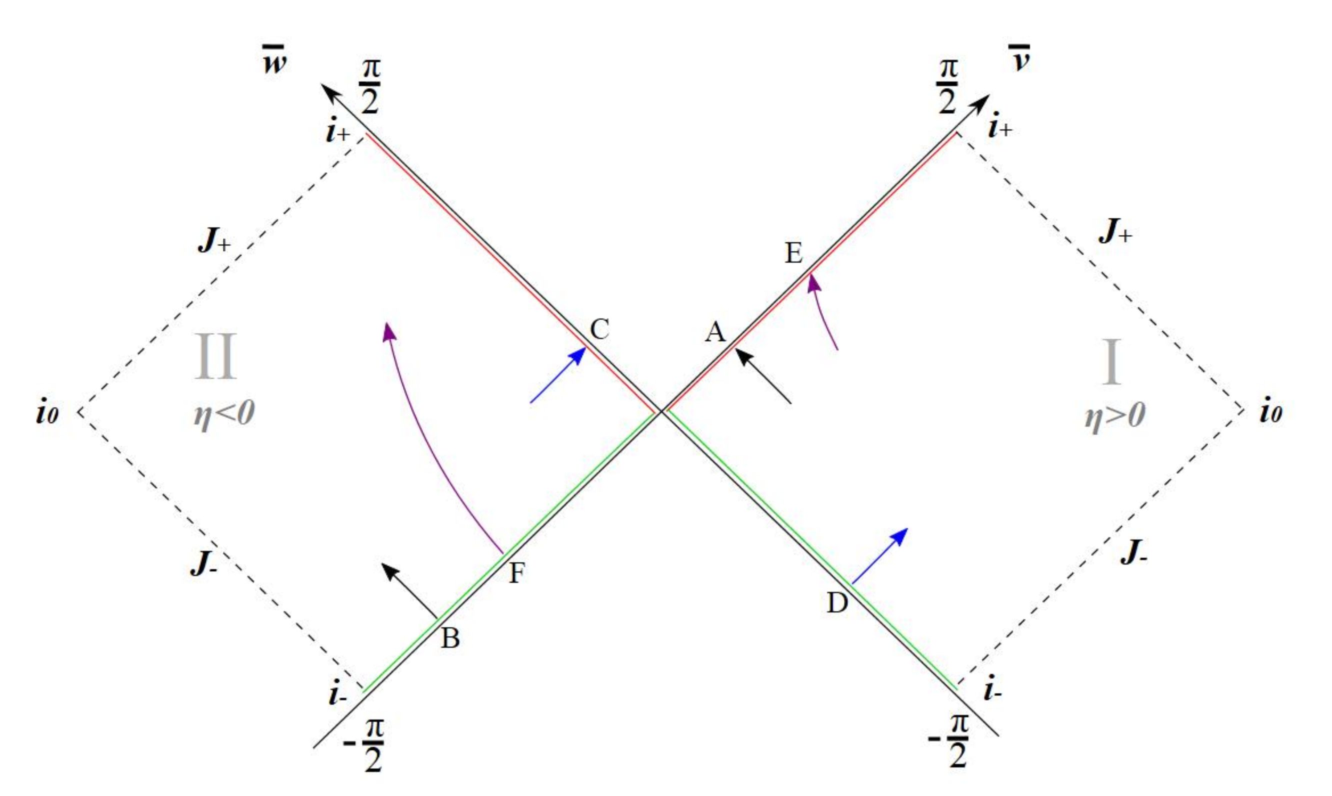
- In region I: future horizon – $(0 < \bar{v} < \frac{\pi}{2}, \bar{w} = 0)$;
  past horizon – $(\bar{v} = 0, -\frac{\pi}{2} < \bar{w} < 0)$.
- In region II: future horizon – $(\bar{v} = 0, 0 < \bar{w} < \frac{\pi}{2})$;
  past horizon – $(-\frac{\pi}{2} < \bar{v} < 0, \bar{w} = 0)$.
- i_0 – spacelike infinity ($t = \text{fixed}, \eta \to \pm\infty$):
  $i_0 = (\frac{\pi}{2}, -\frac{\pi}{2})$ in region I;
  $i_0 = (-\frac{\pi}{2}, \frac{\pi}{2})$ in region II.
- $i_\pm$ – future/past timelike infinity ($t \to \pm\infty, \eta = \text{fixed}$):
  $i_+ = (\frac{\pi}{2}, 0),\ i_- = (0, -\frac{\pi}{2})$ in region I;
  $i_+ = (0, \frac{\pi}{2}),\ i_- = (-\frac{\pi}{2}, 0)$ in region II.

Kruskal-Penrose Diagram for Original ER Bridge

- J_+ – future lightlike infinity ($t \to +\infty, \eta \to \pm\infty,\ t \mp \eta^* = \text{fixed}$):
  $J_+ = (\bar{v} = \frac{\pi}{2}, -\frac{\pi}{2} < \bar{w} < 0)$ in region I;
  $J_+ = (-\frac{\pi}{2} < \bar{v} < 0, \bar{w} = \frac{\pi}{2})$ in region II.
- J_- – past lightlike infinity ($t \to -\infty, \eta \to \pm\infty),\ t \pm \eta^* = \text{fixed}$):
  $J_- = (0 < \bar{v} < \frac{\pi}{2}, \bar{w} = -\frac{\pi}{2})$ in region I:
  $J_- = (\bar{v} = -\frac{\pi}{2}, 0 < \bar{w} < \frac{\pi}{2})$ in region II.

Inserting Eqs.(15)–(17) into the definitions of Kruskal-like (18) and Penrose-like (23) coordinates we obtain the following visual representation of the original ER-bridge Kruskal-Penrose diagram:

Visual Representation of ER-Bridge Kruskal-Penrose Diagram



- Future horizon in I (red) identified with past horizon in II (green): $(\bar{v}, 0) \sim (\bar{v} - \frac{\pi}{2}, 0)$ – infalling light rays from I into II ($A \sim B$);
- Future horizon in II (red) identified with past horizon in I (green): $(0, \bar{w}) \sim (0, \bar{w} - \frac{\pi}{2})$ – infalling light rays from II into I ($C \sim D$).

Visual Representation of ER-Bridge Kruskal-Penrose Diagram

- For infalling light rays starting in region I and crossing into region II we have the lightlike geodesic $t + \eta^* = c_1 \equiv \text{const}$. Thus, according to (18) we must identify the crossing point A on the future horizon of region I with Kruskal-like coordinates $\left(v = \frac{1}{\sqrt{2k_h}} e^{k_h c_1}, 0\right)$ with the point B on the past horizon of region II where the light rays enters into region II whose Kruskal-like coordinates are $\left(v = -\frac{1}{\sqrt{2k_h}} e^{-k_h c_1}, 0\right)$.
- Similarly, for infalling light rays starting in region II and crossing into region I we have $t - \eta^* = c_2 \equiv \text{const}$. Therefore, the crossing point C on the future horizon of region II with Kruskal-like coordinates $\left(0, w = \frac{1}{\sqrt{2k_h}} e^{k_h c_2}\right)$ must be identified with the exit point D $\left(0, w = -\frac{1}{\sqrt{2k_h}} e^{-k_h c_2}\right)$ on the past horizon of region I.

Lightlike Thin-Shell Wormholes with Two “Throats”

In [IJMPA26(2011)5211] we have found an interesting example of a two-throat lightlike thin-shell wormhole – **charge-confining “tube-like” wormhole**. the full wormhole spacetime consists of three “universes” glued pairwise via two oppositely charge LL-branes located on their common horizons:

- Left-most noncompact electrically neutral “universe” – exterior region beyond the Schwarzschild horizon of a Schwarzschild-de Sitter blackhole;
- Middle “tube-like” “universe” of Levi-Civita-Bertotti-Robinson type with finite radial-like spacial extend and compactified transverse spacial dimensions;

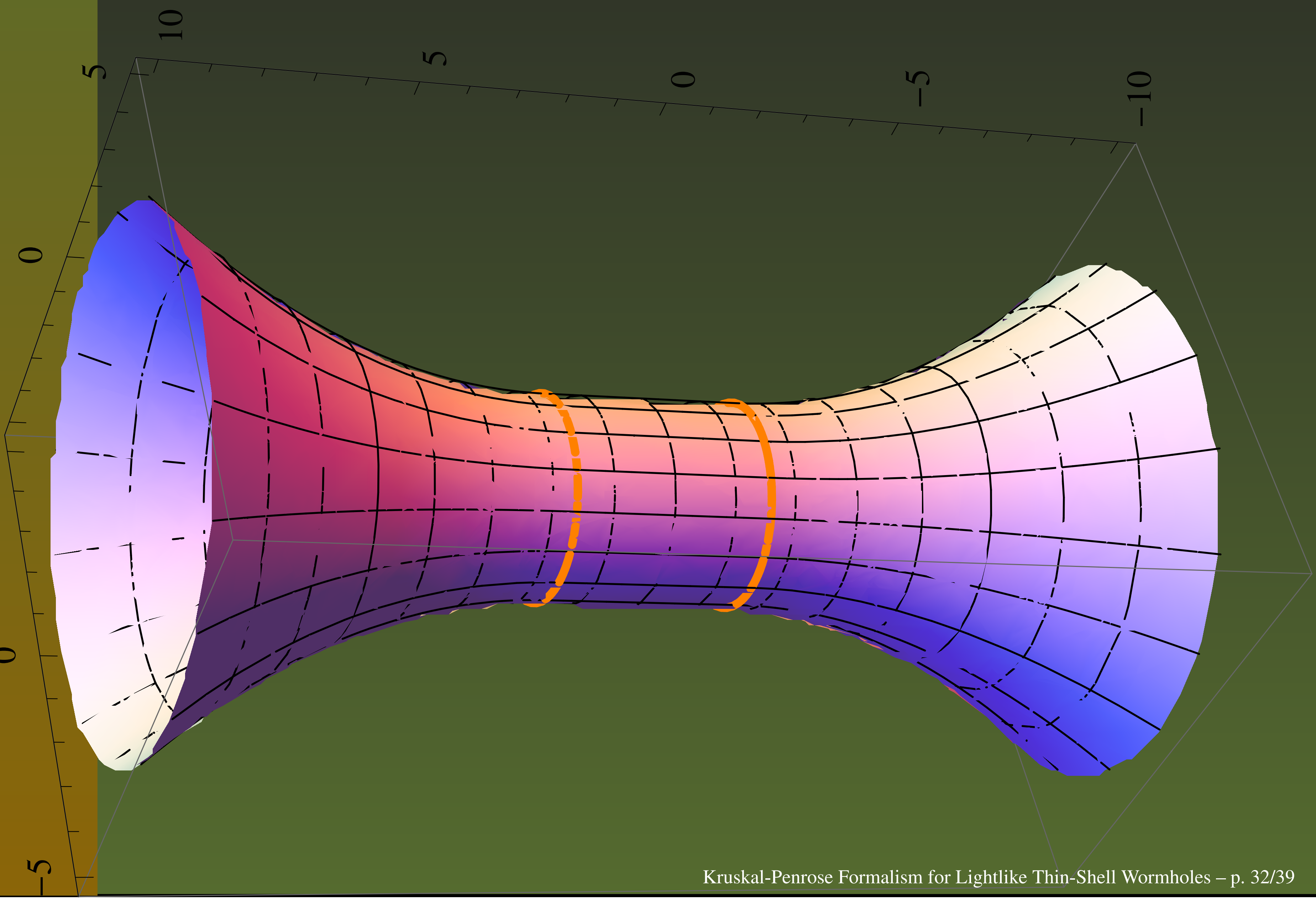
Traversable Lightlike Thin-Shell Wormholes with Two “Throats”

- Right-most noncompact electrically neutral “universe” – exterior region beyond the Schwarzschild horizon of a Schwarzschild-de Sitter blackhole, mirror copy of the left-most “universe”.
- Most remarkable property: the whole electric flux generated by the two oppositely charge LL-branes on the two “throats” is completely confined within the finite-spacial-size middle “tube-like” universe – analog of QCD quark confinement!

Visual representation.

Shape of $t = const$ and $\theta = \frac{\pi}{2}$ slice of charge-confining wormhole geometry – electric flux is confined within the middle cylindric tube:

Charge-Confining Tube-like Wormhole



Traversable Lightlike Thin-Shell Wormholes with Two “Throats”

Generically the metric of a spherically symmetric traversable lightlike thin-shell wormhole with two “throats” reads:

$$ds^2 = -A(\eta)dt^2 + \frac{d\eta^2}{A(\eta)} + r^2(\eta)\left(d\theta^2 + \sin^2\theta d\varphi^2\right) ,$$

$$A(\eta_1) = 0 \, , \; A(\eta_2) = 0 \, , \; a^{(1)}_{(\pm)} = \pm \frac{\partial}{\partial\eta} A \,\Big|_{\eta_1 \pm 0} > 0 \, , \; a^{(2)}_{(\pm)} = \pm \frac{\partial}{\partial\eta} A \,\Big|_{\eta_2 \pm 0} > 0 \, .$$

Accordingly, for the wormhole “tortoise” coordinate η^* we have:

$$\eta^* = \mathrm{sign}(\eta - \eta_1) a^{(1)}_{(\pm)} \ln|\eta - \eta_1| + O\big((\eta - \eta_1)^2\big) , \tag{24}$$

$$\eta^* = \mathrm{sign}(\eta - \eta_2) a^{(2)}_{(\pm)} \ln|\eta - \eta_2| + O\big((\eta - \eta_2)^2\big) . \tag{25}$$

Now we can introduce the Kruskal-like and the compactified Kruskal-Penrose coordinates for the maximal analytic extension of the two-throat LL thin-shell wormhole generalizing formulas (18) and (23):

Traversable Lightlike Thin-Shell Wormholes with Two "Throats"

Kruskal-Penrose coordinates $(\bar{v}, \bar{w})$:

- In region I (left-most universe – $(+\infty > \eta > \eta_1)$):

$$\bar{v}, \bar{w} = \pm \frac{\pi}{2\sqrt{a^{(1)}_{(-)}}} \pm \frac{1}{\sqrt{a^{(1)}_{(+)}}} \arctan\left(e^{\frac{1}{2}a^{(1)}_{(+)}(\eta^* \pm t)}\right) \tag{26}$$

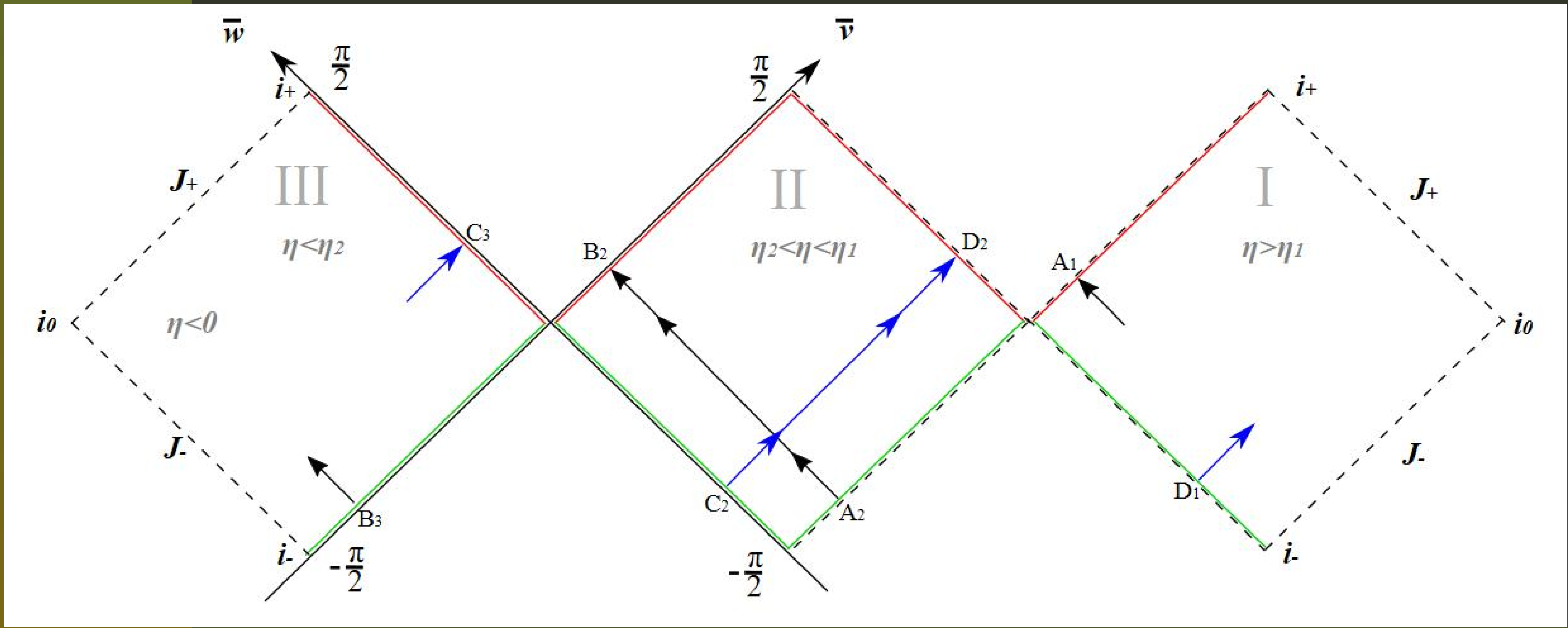
- In region II (middle universe – $(\eta_1 > \eta > \eta_2)$; here $a^{(1)}_{(-)} = a^{(2)}_{(+)}$):

$$\bar{v}, \bar{w} = \pm \frac{1}{\sqrt{a^{(1)}_{(-)}}} \arctan\left(e^{\frac{1}{2}a^{(1)}_{(-)}(\eta^* \pm t)}\right) . \tag{27}$$

- In region III (right-most universe – $(\eta_2 > \eta > -\infty)$):

$$\bar{v}, \bar{w} = \mp \frac{\pi}{2\sqrt{a^{(2)}_{(-)}}} \pm \frac{1}{\sqrt{a^{(2)}_{(-)}}} \arctan\left(e^{\frac{1}{2}a^{(2)}_{(-)}(\eta^* \pm t)}\right) . \tag{28}$$

Kruskal-Penrose Diagram for Two-Throat LL-Wormhole



Arrowed black lines: infalling light ray geodesics starting in Region I and crossing from Region I, traversing Region II and arriving in Region III within finite world-line ("proper") time interval.

Arrowed blue lines: infalling light ray geodesics starting in Region III and arriving in Region I within finite "proper" time interval.

Conclusions

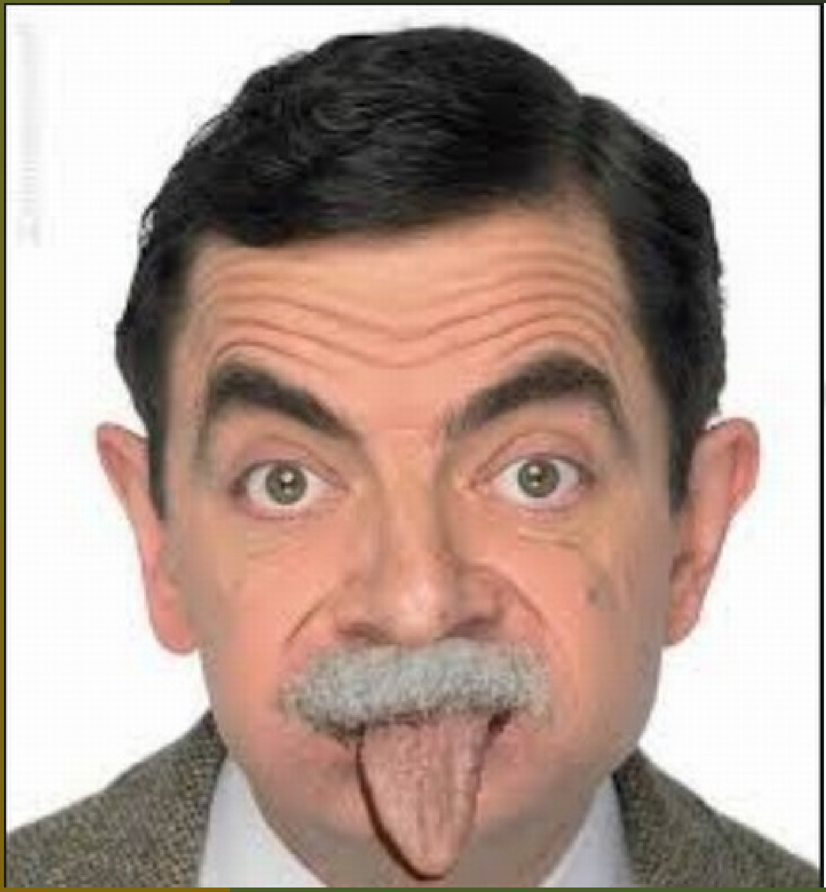
- The present mathematically correct reformulation of original Einstein-Rosen "bridge" construction shows that it is the simplest example in the class of static spherically symmetric *traversable* lightlike thin-shell wormhole solutions in general relativity.
- Consistency of Einstein-Rosen "bridge" as a traversable wormhole solution is guaranteed by the remarkable special properties ("horizon straddling", consistent lightlike W.Israel junction conditions) of **lightlike** branes serving as "exotic" thin-shell matter (and charge) sources of gravity.

Conclusions

We have explicitly derived the Kruskal-like extension and the associated Kruskal-Penrose diagram representation of the original Einstein-Rosen "bridge" with the following significant differences w.r.t. Kruskal-Penrose extension of the standard Schwarzschild black hole and the corresponding "textbook" (Wheeler's) version of Einstein-Rosen "bridge":

- The Kruskal-Penrose diagram has only two regions corresponding to "our" (left) and the second (right) universes unlike the four regions in the standard Schwarzschild case (no black/white hole regions).
- The correctly formulated original Einstein-Rosen "bridge" is *traversable* static spherically symmetric wormhole unlike the non-traversable non-static "textbook" version. Traversability is equivalent to the pairwise specific identifications of future with past horizons of the neighbouring Kruskal regions.

Einstein-Rosen Bridge – The Ultimate Verdict

Conclusions

THANK YOU – Merci beaucoup

Vielen Dank – Tack – Multumesc

Gracias – Obrigado – Grazie

хвала – hvala – спасибо – благодаря
спасибі – дзякуй – dziękuję

ありがとう – شكرا – תודה לך – ευχαριστώ