Cosmology and Gravitational Bags via Metric-Independent Volume-Form Dynamics

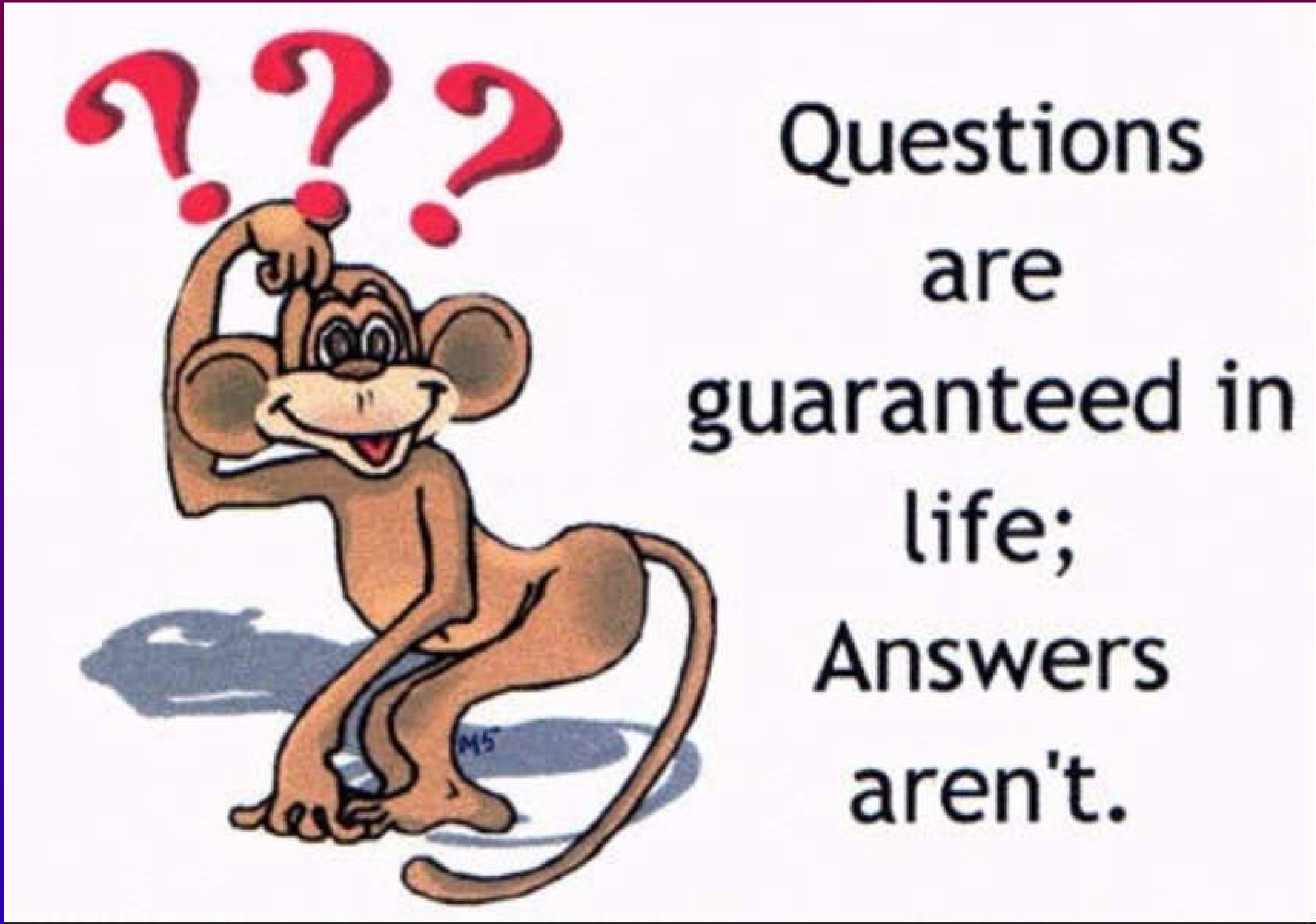
XI International Workshop "Lie Theory and Its Applications", Varna 2015

Eduardo Guendelman[1], Emil Nissimov[2], Svetlana Pacheva[2]

[1] Department of Physics, Ben-Gurion University of the Negev, Beer-Sheva, Israel

[2] Institute for Nuclear Research and Nuclear Energy, Bulg. Acad. of Sciences, Sofia, Bulgaria

Cosmology and Gravitational Bags



Background material:

- E. Guendelman, E.N., S. Pacheva and M. Vasihoun, in "VIII-th Mathematical Physics Meeting", B. Dragovic and Z. Rakic (eds.), Belgrade Inst. Phys. Press, 2015 (*arxiv:1501.05518* [hep-th]);
  E. Guendelman, E.N., S. Pacheva and M. Vasihoun, *Bulg. J. Phys.* 41 (2014) 123-129 (*arxiv:1404.4733* [hep-th]).
- E. Guendelman, R. Herrera, P. Labrana, E.N. and S. Pacheva, *General Relativity and Gravitation* 47 (2015) art.10 (*arxiv:1408.5344v4* [gr-qc]).
- E. Guendelman, E.N. and S. Pacheva, *arxiv:1504.01031* [gr-qc].

Introduction - Overview of Talk

- Alternative spacetime volume-forms (generally-covariant integration measure densitites) independent on the Riemannian metric on the pertinent spacetime manifold have profound impact in (field theory) models with general coordinate reparametrization invariance – general relativity and its extensions, strings and (higher-dimensional) membranes. Although formally appearing as “pure-gauge” dynamical degrees of freedom the non-Riemannian volume-form fields trigger a number of remarkable physically important phenomena.

Introduction - Overview of Talk

Among the principal new phenomena are:

- (i) New mechanism of dynamical generation of cosmological constant;
- (ii) New mechanism of dynamical spontaneous breakdown of supersymmetry in supergravity;
- (iii) New type of "quintessential inflation" scenario in cosmology;
- (iv) Coupling of non-Riemannian volume-form gravity-matter theories to a special non-standard kind of nonlinear gauge system containing the square-root of standard Maxwell Lagrangian yields charge confinement/deconfinement phases associated with gravitational electrovacuum "bags".

Modified-Measure Theories

In a series of previous papers [E.Guendelman *et.al.*] a new class of generally-covariant (non-supersymmetric) field theory models including gravity – called “two-measure theories” (TMT) was proposed.

- TMT appear to be promising candidates for resolution of various problems in modern cosmology: the *dark energy* and *dark matter* problems, the fifth force problem, etc.
- Principal idea – employ an alternative volume form (volume element or generally-covariant integration measure) on the spacetime manifold in the pertinent Lagrangian action.

Modified-Measure Theories

- In standard generally-covariant theories (with action $S = \int d^D x \sqrt{-g}\mathcal{L}$) the Riemannian spacetime volume-form, *i.e.*, the integration measure density is given by $\sqrt{-g}$, where $g \equiv \det \|g_{\mu\nu}\|$ is the determinant of the corresponding Riemannian metric $g_{\mu\nu}$.
  $\sqrt{-g}$ transforms as scalar density under general coordinate reparametrizations.
  There is NO *a priori* any obstacle to employ insted of $\sqrt{-g}$ another alternative non-Riemannian volume element given by the following ***non-Riemannian*** integration measure density:

$$\Phi(B) \equiv \frac{1}{(D-1)!}\varepsilon^{\mu_1\ldots\mu_D}\,\partial_{\mu_1} B_{\mu_2\ldots\mu_D}\ . \qquad (1)$$

Modified-Measure Theories

- Here $B_{\mu_1\ldots\mu_{D-1}}$ is an auxiliary rank $(D-1)$ antisymmetric tensor gauge field, which will turn out to be pure-gauge degree of freedom. $\Phi(B)$ similarly transforms as scalar density under general coordinate reparametrizations.
In particular, $B_{\mu_1\ldots\mu_{D-1}}$ can also be parametrized in terms of D auxiliary scalar fields:
$B_{\mu_1\ldots\mu_{D-1}} = \frac{1}{D}\varepsilon_{IJ_1\ldots J_{D-1}}\phi^I\partial_{\mu_1}\phi^{J_1}\ldots\partial_{\mu_{D-1}}\phi^{J_{D-1}}$,
so that:
$\Phi(B) = \frac{1}{D!}\varepsilon^{\mu_1\ldots\mu_D}\,\varepsilon_{I_1\ldots I_D}\partial_{\mu_1}\phi^{I_1}\ldots\partial_{\mu_D}\phi^{I_D}$.
IMPORTANT: The non-Riemannian measure density $\Phi(B)$ becomes **on-shell** proportional to the standard Riemannian one $\sqrt{-g}$, *i.e.*, the physical meaning of $\Phi(B)$ as a measure is preserved!

Gravity-Matter Theories with Two Non-Riemannian Volume-Form

- Let us now consider modified-measure gravity-matter theories constructed in terms of two different non-Riemannian volume-forms (employing Palatini formalism, and using units where $G_{\rm Newton} = 1/16\pi$):

$$S = \int d^4x\, \Phi_1(A)\Big[R + L^{(1)}\Big] + \int d^4x\, \Phi_2(B)\Big[L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}}\Big] \; . \tag{2}$$

  - $\Phi_1(A)$ and $\Phi_2(B)$ are two independent non-Riemannian volume-forms:

$$\Phi_1(A) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu A_{\nu\kappa\lambda} \quad , \quad \Phi_2(B) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu B_{\nu\kappa\lambda} \; , \tag{3}$$

$$\Phi(H) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu H_{\nu\kappa\lambda} \; . \tag{4}$$

- $L^{(1,2)}$ denote two different Lagrangians of a single scalar matter field of the form:

$$L^{(1)} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \quad , \quad V(\varphi) = f_1 \exp\{-\alpha\varphi\} \; , \qquad (5)$$

$$L^{(2)} = -\frac{b}{2} e^{-\alpha\varphi} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + U(\varphi) \quad , \quad U(\varphi) = f_2 \exp\{-2\alpha\varphi\} \; , \qquad (6)$$

where α, f_1, f_2 are dimensionful positive parameters, whereas b is a dimensionless one.

- Global Weyl-scale invariance of the action (2):

$$g_{\mu\nu} \to \lambda g_{\mu\nu} \; , \; \Gamma^\mu_{\nu\lambda} \to \Gamma^\mu_{\nu\lambda} \; , \; \varphi \to \varphi + \frac{1}{\alpha} \ln \lambda \; ,$$
$$A_{\mu\nu\kappa} \to \lambda A_{\mu\nu\kappa} \; , \; B_{\mu\nu\kappa} \to \lambda^2 B_{\mu\nu\kappa} \; , \; H_{\mu\nu\kappa} \to H_{\mu\nu\kappa} \; .$$

Eqs. of motion w.r.t. affine connection $\Gamma^{\mu}_{\nu\lambda}$ yield a solution for the latter as a Levi-Civita connection:

$$\Gamma^{\mu}_{\nu\lambda} = \Gamma^{\mu}_{\nu\lambda}(\bar{g}) = \frac{1}{2}\bar{g}^{\mu\kappa}\left(\partial_{\nu}\bar{g}_{\lambda\kappa} + \partial_{\lambda}\bar{g}_{\nu\kappa} - \partial_{\kappa}\bar{g}_{\nu\lambda}\right) \;, \tag{7}$$

w.r.t. to the Weyl-rescaled metric $\bar{g}_{\mu\nu}$:

$$\bar{g}_{\mu\nu} = (\chi_1 + 2\epsilon\chi_2 R) g_{\mu\nu} \;\; , \;\; \chi_1 \equiv \frac{\Phi_1(A)}{\sqrt{-g}} \;\; , \;\; \chi_2 \equiv \frac{\Phi_2(B)}{\sqrt{-g}} \; . \tag{8}$$

Transition from original metric $g_{\mu\nu}$ to $\bar{g}_{\mu\nu}$: **"Einstein-frame"**, where the gravity eqs. of motion are written in the standard form of Einstein's equations: $R_{\mu\nu}(\bar{g}) - \frac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) = \frac{1}{2}T^{\rm eff}_{\mu\nu}$ with an appropriate **effective** energy-momentum tensor given in terms of an Einstein-frame scalar Lagrangian $L_{\rm eff}$ (see (11) below).

- Variation of the action (2) w.r.t. auxiliary tensor gauge fields $A_{\mu\nu\lambda}$, $B_{\mu\nu\lambda}$ and $H_{\mu\nu\lambda}$ yields the equations:

$$\partial_\mu \Big[R + L^{(1)}\Big] = 0 \; , \; \partial_\mu \Big[L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}}\Big] = 0 \; , \; \partial_\mu \Big(\frac{\Phi_2(B)}{\sqrt{-g}}\Big) = 0 \; , \tag{9}$$

whose solutions read:

$$\frac{\Phi_2(B)}{\sqrt{-g}} \equiv \chi_2 = \text{const} \;\; , \;\; R + L^{(1)} = -M_1 = \text{const} \; ,$$
$$L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}} = -M_2 = \text{const} \; . \tag{10}$$

Here M_1 and M_2 are arbitrary dimensionful and χ_2 arbitrary dimensionless integration constants.

The first integration constant χ_2 in (10) preserves global Weyl-scale invariance whereas the appearance of the second and third integration constants $M_1,\ M_2$ signifies *dynamical spontaneous breakdown* of global Weyl-scale invariance due to the scale non-invariant solutions (second and third ones) in (10).

It is very instructive to elucidate the physical meaning of the three arbitrary integration constants $M_1,\ M_2,\ \chi_2$ from the point of view of the canonical Hamiltonian formalism: $M_1,\ M_2,\ \chi_2$ are identified as conserved Dirac-constrained canonical momenta conjugated to (certain components of) the auxiliary maximal rank antisymmetric tensor gauge fields $A_{\mu\nu\lambda}, B_{\mu\nu\lambda}, H_{\mu\nu\lambda}$ entering the original non-Riemannian volume-form action (2).

- Performing transition to the Einstein frame yields the following effective scalar Lagrangian of non-canonical "k-essence" (kinetic quintessence) type ($X \equiv -\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$ – scalar kinetic term):

$$L_{\rm eff} = A(\varphi)X + B(\varphi)X^2 - U_{\rm eff}(\varphi) \;, \tag{11}$$

where (recall $V = f_1 e^{-\alpha\varphi}$ and $U = f_2 e^{-2\alpha\varphi}$):

$$A(\varphi) \equiv 1 + \Big[\frac{1}{2}be^{-\alpha\varphi} - \epsilon(V - M_1)\Big]\frac{V - M_1}{U + M_2 + \epsilon(V - M_1)^2} \;, \tag{12}$$

$$B(\varphi) \equiv \chi_2 \frac{\epsilon\Big[U + M_2 + (V - M_1)be^{-\alpha\varphi}\Big] - \frac{1}{4}b^2 e^{-2\alpha\varphi}}{U + M_2 + \epsilon(V - M_1)^2} \;, \tag{13}$$

$$U_{\rm eff}(\varphi) \equiv \frac{(V - M_1)^2}{4\chi_2\Big[U + M_2 + \epsilon(V - M_1)^2\Big]} \;. \tag{14}$$

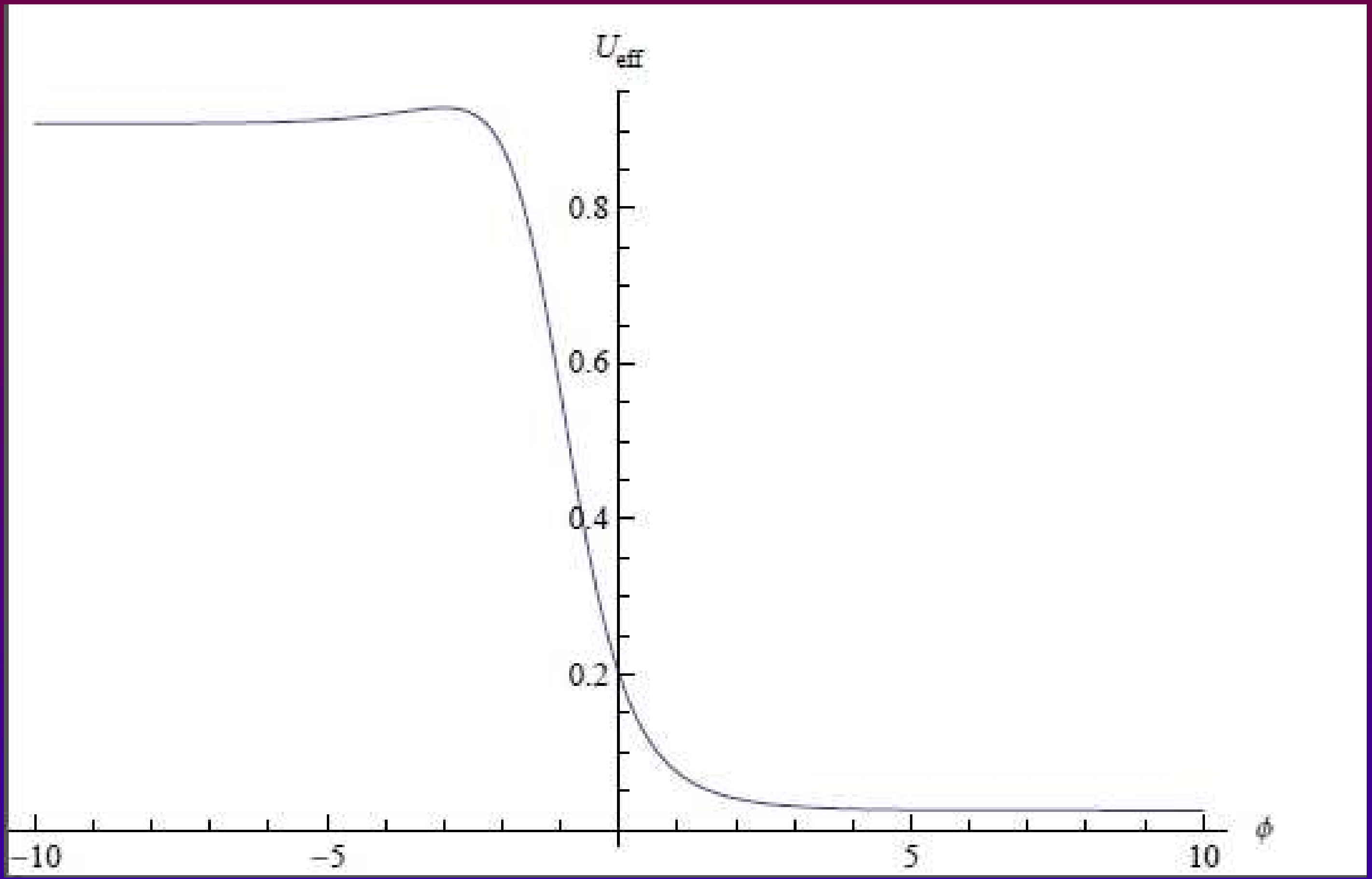
Most remarkable feature of the effective scalar potential $U_{\rm eff}(\varphi)$ (14) – two **infinitely large flat regions**:

- **(-) flat region** – for large negative values of φ:

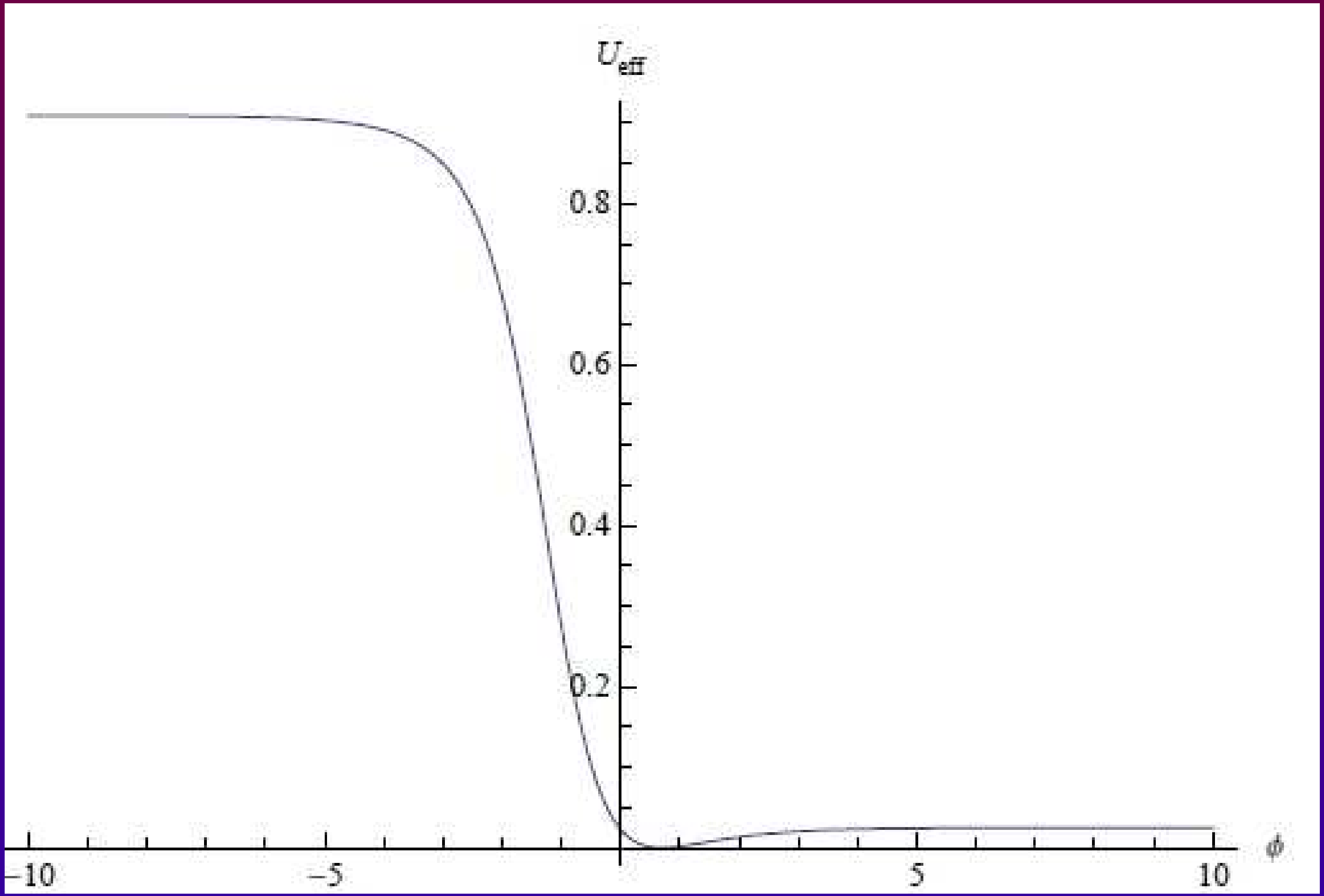
$$U_{\rm eff}(\varphi) \simeq U_{(-)} \equiv \frac{f_1^2/f_2}{4\chi_2(1+\epsilon f_1^2/f_2)} \;, \qquad (15)$$

- **(+) flat region –** for large positive values of φ:

$$U_{\rm eff}(\varphi) \simeq U_{(+)} \equiv \frac{M_1^2/M_2}{4\chi_2(1+\epsilon M_1^2/M_2)} \;, \qquad (16)$$



Qualitative shape of the effective scalar potential $U_{\rm eff}$ (14) as function of φ for $M_1 < 0$.



Qualitative shape of the effective scalar potential $U_{\rm eff}$ (14) as function of φ for $M_1 > 0$.

“Quintessential” Inflation Scenario

From the expression for $U_{\rm eff}(\varphi)$ (14) and the figures 1 and 2 we deduce that we have an **explicit realization of quintessential inflation scenario** (continuously connecting an inflationary phase to a slowly accelerating “present-day” universe through the evolution of a single scalar field).

The flat regions (15) and (16) correspond to the evolution of the **early** and the **late** universe, respectively, provided we choose the ratio of the coupling constants in the original scalar potentials versus the ratio of the scale-symmetry breaking integration constants to obey:

$$\frac{f_1^2/f_2}{1+\epsilon f_1^2/f_2} \gg \frac{M_1^2/M_2}{1+\epsilon M_1^2/M_2}\,, \qquad (17)$$

which makes the **vacuum energy density of the early universe** $U_{(-)}$ **much bigger than that of the late universe** $U_{(+)}$.

“Quintessential” Inflation Scenario

The inequality (17) is equivalent to the requirements:

$$\frac{f_1^2}{f_2} \gg \frac{M_1^2}{M_2} \quad , \quad |\epsilon| \frac{M_1^2}{M_2} \ll 1 \; . \tag{18}$$

If we choose the scales $|M_1| \sim M_{EW}^4$ and $M_2 \sim M_{Pl}^4$, where M_{EW}, M_{Pl} are the electroweak and Plank scales, respectively, we are then naturally led to a very small vacuum energy density:

$$U_{(+)} \sim M_{EW}^8 / M_{Pl}^4 \sim 10^{-120} M_{Pl}^4 \; , \tag{19}$$

which is the right order of magnitude for the present epoche’s vacuum energy density.

“Quintessential” Inflation Scenario

- On the other hand, if we take the order of magnitude of the coupling constants in the effective potential $f_1 \sim f_2 \sim (10^{-2} M_{Pl})^4$, then the order of magnitude of the vacuum energy density of the early universe becomes:

$$U_{(-)} \sim f_1^2/f_2 \sim 10^{-8} M_{Pl}^4 \, , \tag{20}$$

which conforms to the Planck Collaboration data (also BICEP2) implying the energy scale of inflation of order $10^{-2} M_{Pl}$.

"Emergent universe"

- There exists explicit cosmological solution of the Einstein-frame system (11)-(14) describing an epoch of a non-singular creation of the universe – "emergent universe", preceding the inflationary phase. The starting point are the Friedman eqs.:

$$\frac{\ddot{a}}{a} = -\frac{1}{12}(\rho + 3p) \quad , \quad H^2 + \frac{K}{a^2} = \frac{1}{6}\rho \quad , \quad H \equiv \frac{\dot{a}}{a} \, , \tag{21}$$

describing the universe' evolution. Here:

$$\rho = \frac{1}{2}A(\varphi)\,\dot{\varphi}^2 + \frac{3}{4}B(\varphi)\,\dot{\varphi}^4 + U_{\rm eff}(\varphi) \, , \tag{22}$$

$$p = \frac{1}{2}A(\varphi)\,\dot{\varphi}^2 + \frac{1}{4}B(\varphi)\,\dot{\varphi}^4 - U_{\rm eff}(\varphi) \tag{23}$$

are the energy density and pressure of the scalar field $\varphi = \varphi(t)$.

“Emergent universe”

“Emergent universe” is defined as a solution of the Friedman eqs.(21) subject to the condition on the Hubble parameter H:

$$H = 0 \;\;\to\;\; a(t) = a_0 = \text{const}\,, \;\; \rho + 3p = 0 \;\;, \;\; \frac{K}{a_0^2} = \frac{1}{6}\rho\,(= \text{const})\;, \tag{24}$$

with ρ and p as in (22)-(23). Here $K = 1$ (“Einstein universe”). The “emergent universe” condition (24) implies that the φ-velocity $\dot\varphi \equiv \dot\varphi_0$ is time-independent and satisfies the bi-quadratic algebraic equation:

$$\frac{3}{2} B_{(-)}\, \dot\varphi_0^{\,4} + 2A_{(-)}\, \dot\varphi_0^{\,2} - 2U_{(-)} = 0\;, \tag{25}$$

where $A_{(-)}$, $B_{(-)}$, $U_{(-)}$ are the limiting values on the $(-)$ flat region of $A(\varphi)$, $B(\varphi)$, $U_{\rm eff}(\varphi)$ (12)-(14).

"Emergent universe"

The solution of Eq.(25) reads:

$$\dot{\varphi}_0^2 = -\frac{2}{3B_{(-)}}\left[A_{(-)} \mp \sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}}\right] . \qquad (26)$$

and, thus, the "emergent universe" is characterized with **finite initial** Friedman factor and density:

$$a_0^2 = \frac{6K}{\rho_0} \quad , \quad \rho_0 = \frac{1}{2}A_{(-)}\,\dot{\varphi}_0^2 + \frac{3}{4}B_{(-)}\,\dot{\varphi}_0^4 + U_{(-)} \, , \qquad (27)$$

with $\dot{\varphi}_0^2$ as in (26).

“Emergent universe”

- Analysis of stability of the “emergent universe” solution (27) yields a harmonic oscillator type equation for the perturbation of the Friedman factor δa:

$$\delta\,\ddot{a} + \omega^2 \delta a = 0 \quad , \quad \omega^2 \equiv \frac{2}{3}\rho_0 \frac{\sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}}}{A_{(-)} - 2\sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}}} \; . \tag{28}$$

Thus stability condition $\omega^2 > 0$ yields the following constraint on the coupling parameters:

$$\max\Big\{-2\, ,\; -8\big(1+3\epsilon f_1^2/f_2\big)\Big[1-\sqrt{1-\frac{1}{4\big(1+3\epsilon f_1^2/f_2\big)}}\Big]\Big\} < b\frac{f_1}{f_2} < -1 \; . \tag{29}$$

“Emergent universe”

Since the ratio $\frac{f_1^2}{f_2}$ proportional to the height of the $(-)$ flat region of the effective scalar potential, *i.e.*, the vacuum energy density in the early universe, must be large (cf. (17)), we find that the lower end of the interval in (29) is very close to the upper end, *i.e.*, $b\frac{f_1}{f_2} \simeq -1$.

From Eqs.(26)-(27) we obtain an inequality satisfied by the initial energy density ρ_0 in the emergent universe: $U_{(-)} < \rho_0 < 2U_{(-)}$, which together with the estimate of the order of magnitude for $U_{(-)}$ (20) implies order of magnitude for $a_0^2 \sim 10^{-8} K M_{Pl}^{-2}$, where K is the Gaussian curvature of the spacial section.

Conclusions

- Non-Riemannian volume-form formalism in gravity/matter theories (*i.e.*, employing alternative non-Riemannian reparametrization covariant integration measure densities on the spacetime manifold) naturally generates a ***dynamical cosmological constant*** as an arbitrary dimensionful integration constant.
- Within non-Riemannian-modified-measure minimal $N = 1$ supergravity the dynamically generated cosmological constant triggers spontaneous supersymmetry breaking and mass generation for the gravitino (supersymmetric Brout-Englert-Higgs effect).

Conclusions

- Within modified-measure anti-de Sitter supergravity we can fine-tune the dynamically generated cosmological integration constant in order to achieve simultaneously a very small physical observable cosmological constant and a very large physical observable gravitino mass – a paradigm of modern cosmological scenarios for slowly expanding universe of today.
- Employing two different non-Riemannian volume-forms leads to the construction of a new class of gravity-matter models, which produce an effective scalar potential with two infinitely large flat regions. This allows for a unified description of both early universe inflation as well as of present dark energy epoch.

Conclusions

- For a definite parameter range the above model with the two different non-Riemannian volume-forms possesses a *non-singular "emergent universe"* solution which describes an initial phase of evolution that precedes the inflationary phase. For a reasonable choice of the parameters this model conforms to the Planck Collaboration data.
- Adding interaction with a special nonlinear ("square-root" Maxwell) gauge field (known to describe charge confinement in flat spacetime) produces various phases with different strength of confinement and/or with deconfinement, as well as gravitational electrovacuum "bags" partially mimicking the properties of MIT bags and solitonic constituent quark models.