東京大学大学院情報理工学系研究科
Graduate School of Information Science and Technology, The University of Tokyo

Boundary effects on the supersymmetric sine-Gordon model through light-cone lattice approach

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LIE THEORY AND ITS APPLICATIONS IN PHYSICS

15-21 June 2015 @Varna, Bulgaria

The University of Tokyo   Chihiro Matsui

Outline

① Correspondence between spin chains and QFTs

② Discretization of integrable QFTs

③ Nonlinear integral equations (NLIEs)

④ IR and UV analysis

⑤ Concluding remarks

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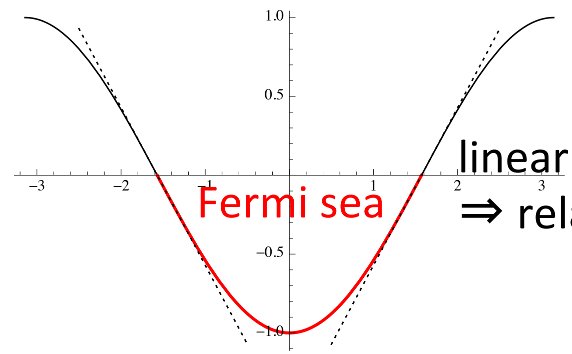
Spin chain/QFT correspondence

- Low-energy excitations of gapless spin chains are described by QFTs.

spin operators $S_j^{\pm},\ S_j^z$

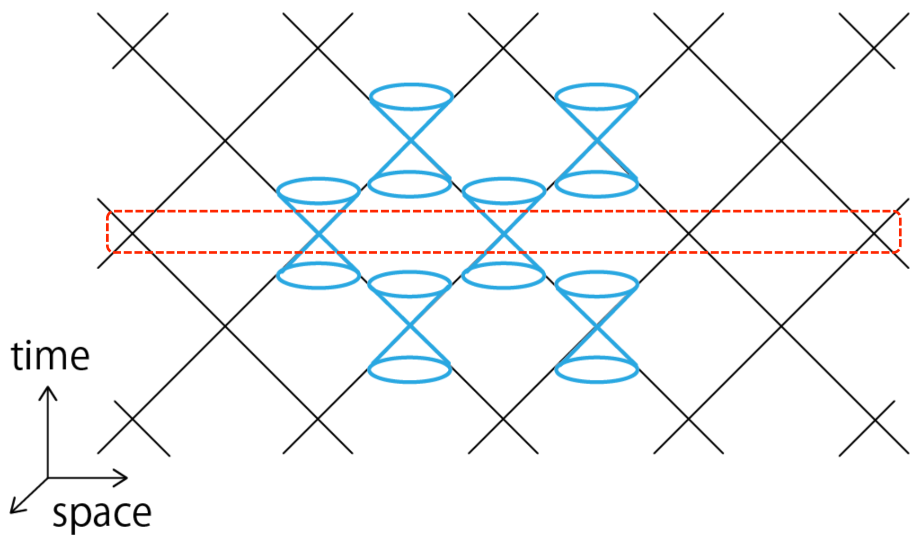
$\Big\downarrow$ J-W transf. electron-orbit.

fermions $\psi_j^{f,c},\ (\psi_j^{f,c})^\dagger$



linear disp. rel.
⇒ relativistic particles

- Inversely, time development of QFTs are given by transfer matrices of spin chains.



transfer from down to up states

History of spin chain/QFT correspondence

- From spin chains to QFTs
  - fermionic representations of spin operators
    ⇒ sigma models [Haldane 83]
  - bosonization
    ⇒ free Dirac fermion + Wess-Zumino term [Affleck 86]

- From QFTs to spin chains
  - discretization of the L-operator
    ⇒ quantum inverse scattering method (QISM) [Izergin-Korepin 82]
  - light-cone reguralization
    ⇒ nonlinear integral equations (NLIEs) [Destri-de Vega 87]

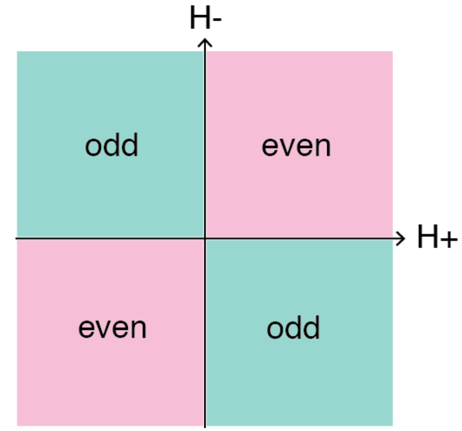
Limiting excitations from light-cone lattices

- s=1/2 XXZ ⇔ SG model

	even sector	odd sector
periodic	○	×
Dirichlet	(H-)(H+) > 0	(H-)(H+) < 0

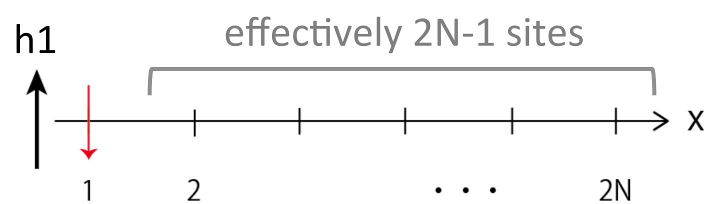
[Feverati et al. 98]

[Ahn et al. 08]



s=1/2 XXZ model with boundary magnetic fields

$$H_{\mathrm{XXZ}} = \sum_{j=1}^{N-1} \left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \cos\gamma S_j^z S_{j+1}^z \right) + \underbrace{\boxed{\sin\gamma \cot\frac{\gamma(H_1+1)}{2}}}_{h1} S_1^z + \underbrace{\boxed{\sin\gamma \cot\frac{\gamma(H_N+1)}{2}}}_{hN} S_N^z$$



arrested by boundary magnetic fields

sine-Gordon model with Dirichlet boundaries

$$A_{\mathrm{SG}} = \frac{1}{2}\int dt \int_0^L dx \left(\partial_\mu \Phi \partial^\mu \Phi + \frac{m_0^2}{\beta^2} \cos\beta\Phi \right), \quad \Phi(0,t) = \Phi_0, \ \Phi(L,t) = \Phi_L$$

boundary magnetic fields ⇔ Dirichlet boundaries

Limiting excitations from light-cone lattices

- s=1 XXZ ⇔ SSG model

	NS, even	NS, odd	R, even	R, odd	
periodic	○	×	×	×	[Hegedus et al. 07]
Dirichlet	○		??		[Ahn et al. 07, CM14]

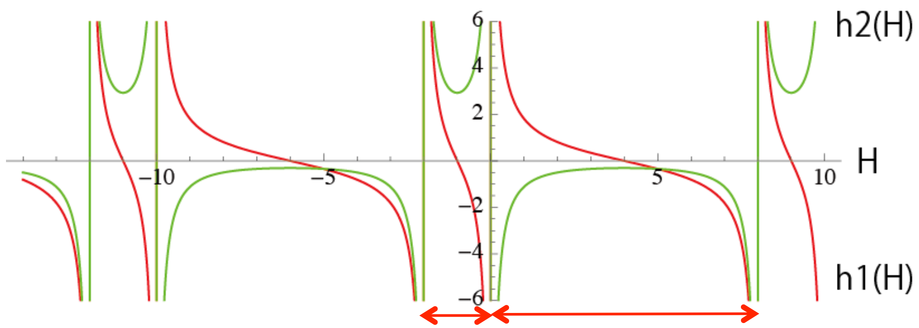
s=1 XXZ model with boundary magnetic fields

$$H_{\mathrm{XXZ}} = \sum_{j=1}^{N-1} \Big(\vec{S}_j \cdot \vec{S}_{j+1} - (\vec{S}_j \cdot \vec{S}_{j+1})^2 - 2\sin^2\gamma (\vec{S}_j \cdot \vec{S}_{j+1} + (S^z_j)^2 + (S^z_{j+1})^2 - (S^z_j S^z_{j+1})^2)$$

$$+ 4\sin^2\frac{\gamma}{2}((S^x_j S^x_{j+1} S^y_j S^y_{j+1}) S^z_j S^z_{j+1} + S^z_j S^z_{j+1}(S^x_j S^x_{j+1} S^y_j S^y_{j+1}))\Big)$$

$$+ h_1(H_+) S^z_1 + h_2(H_+)(S^z_1)^2 + h_1(H_-) S^z_N + h_2(H_-)(S^z_N)^2$$

$$h_1(H) = \tfrac{1}{2}\sin 2\gamma(\cot\tfrac{\gamma H}{2} + \cot\tfrac{\gamma(H+2)}{2}), \quad h_2(H) = \tfrac{1}{2}\sin 2\gamma(-\cot\tfrac{\gamma H}{2} + \cot\tfrac{\gamma(H+2)}{2})$$



NS/R sector separation?

Outline

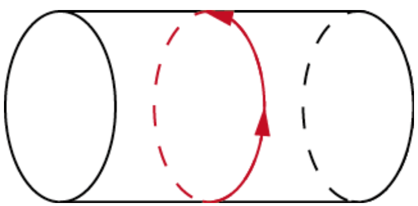
1. Correspondence between spin chains and QFTs
2. Discretization of integrable QFTs
3. Nonlinear integral equations (NLIEs)
4. IR and UV analysis
5. Concluding remarks

SSG model with Dirichlet boundaries

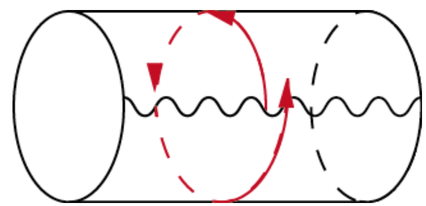
$$A_{\mathrm{SSG}} = \int dt \int_0^L dx \Big(\overbrace{\frac{1}{2}\partial^\mu \Phi \partial_\mu \Phi}^{\text{free boson}} + \overbrace{\frac{i}{2}\bar{\Psi}\gamma^\mu \partial_\mu \Psi}^{\text{free fermion}} - \overbrace{\frac{m_0}{2}\cos(\beta\Phi)\bar{\Psi}\Psi}^{\text{perturbation}} + \overbrace{\frac{m_0^2}{2\beta^2}\cos^2(\beta\Phi)}^{\text{negligible in RG}} \Big)$$

$$\Phi(x,t)\Big|_{x=0,L} = \Phi_{0,L}, \quad \Psi(x,t) - \bar{\Psi}(x,t)\Big|_{x=0,L} = 0$$

- SSG model is a CFT with c= 3/2 ← c=1 (boson) + c=1/2 (fermion)
  c=1/2 + 1/2 + 1/2 (fermions)



periodic fermion



anti-periodic fermion

SSG model with Dirichlet boundaries

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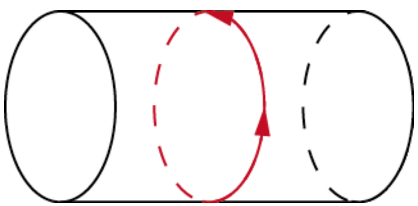
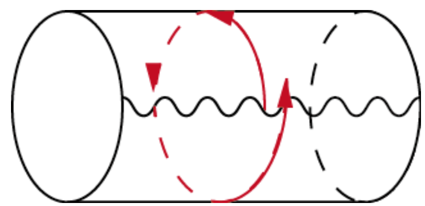


periodic fermion
⇒ NS sector



anti-periodic fermion
⇒ R sector

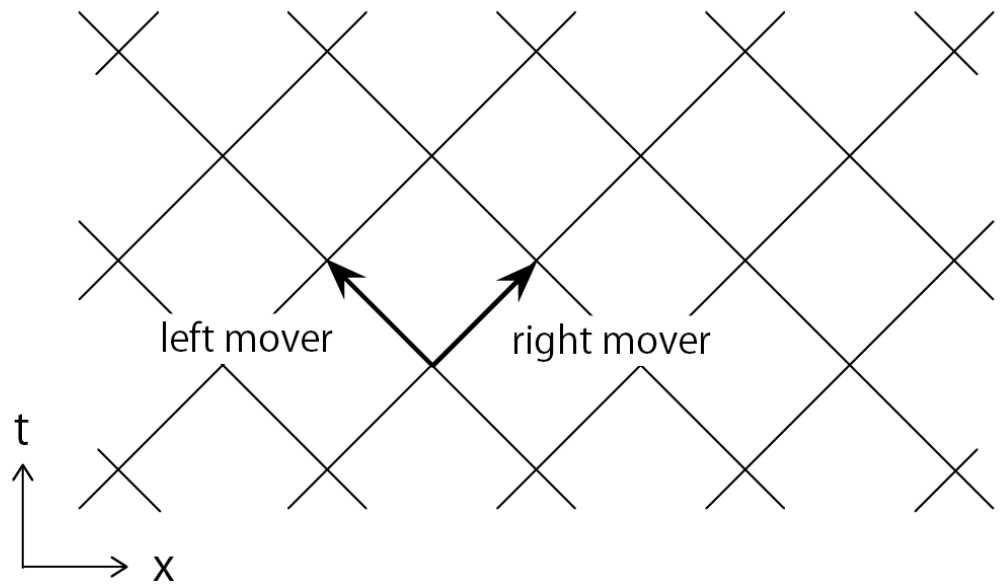
- SSG model has supersymmetry

$$\Delta_{\mathrm{BFB}} = \frac{1}{2}\left(\frac{1}{\sqrt{\pi}}(\Phi_0 - \Phi_L) + mR\right)^2$$: ♯excitation particles

$$\Delta_{\mathrm{BFF}} = 0, \frac{1}{2}, \frac{1}{16}$$: NS sector/R sector

- SSG model is an integrable system

Light-cone regularization

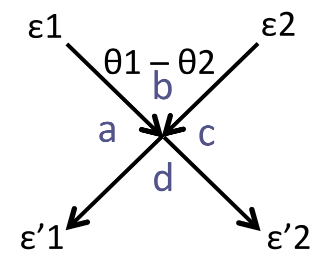


- lines = trajectories of particles
- vertices = scatterings
- integrable QFT ⇔ integrable spin chains
- Bethe ansatz is available

Scattering theory

- SSG kink $A^{\epsilon}_{ab}(\theta)$ ε: soliton charge, ε = ±
  a,b: RSOS index, a,b = 0, ±1; |a-b| = 1
  θ: rapidity

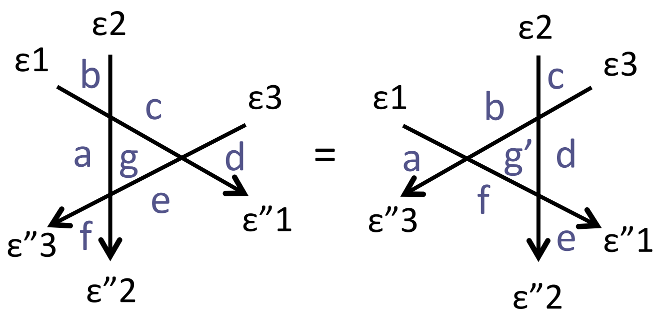
$$A^{\epsilon_1}_{ab}(\theta_1)A^{\epsilon_2}_{bc}(\theta_2) = \sum_{\epsilon'_1,\epsilon'_2}\sum_{d} S^{\epsilon_1\epsilon_2}_{\epsilon'_1\epsilon'_2}\Big|^{ac}_{bd}(\theta_1-\theta_2)A^{\epsilon'_2}_{ad}(\theta_2)A^{\epsilon'_1}_{dc}(\theta_1)$$



$$S^{\epsilon_1\epsilon_2}_{\epsilon'_1\epsilon'_2}\Big|^{ac}_{bd}(\theta) = \boxed{S^{\epsilon_1\epsilon_2}_{\epsilon'_1\epsilon'_2}(\theta)}\times\boxed{S^{ac}_{bd}(\theta)}$$

SG part SUSY part

- Yang-Baxter eq.



$$S^{\epsilon_1\epsilon_2}_{\epsilon'_1\epsilon'_2}(\theta_1-\theta_2)S^{\epsilon'_1\epsilon_3}_{\epsilon''_1\epsilon'_3}(\theta_1-\theta_3)S^{\epsilon'_2\epsilon'_3}_{\epsilon''_2\epsilon''_3}(\theta_2-\theta_3) = S^{\epsilon_2\epsilon_3}_{\epsilon'_2\epsilon'_3}(\theta_2-\theta_3)S^{\epsilon_1\epsilon'_3}_{\epsilon'_1\epsilon''_3}(\theta_1-\theta_3)S^{\epsilon'_1\epsilon'_2}_{\epsilon''_1\epsilon''_2}(\theta_1-\theta_2),$$

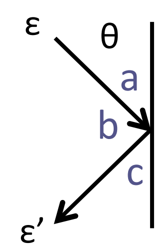
$$S^{ac}_{bg}(\theta_1-\theta_2)S^{gd}_{ce}(\theta_1-\theta_3)S^{ae}_{gf}(\theta_2-\theta_3) = S^{bd}_{cg'}(\theta_2-\theta_3)S^{ag'}_{bf}(\theta_1-\theta_3)S^{fd}_{g'e}(\theta_1-\theta_2)$$

Scattering theory

- SSG kink $A^{\epsilon}_{ab}(\theta)$

ε: soliton charge, ε = ±
a,b: RSOS index, a,b = 0, ±1; |a-b| = 1
θ: rapidity

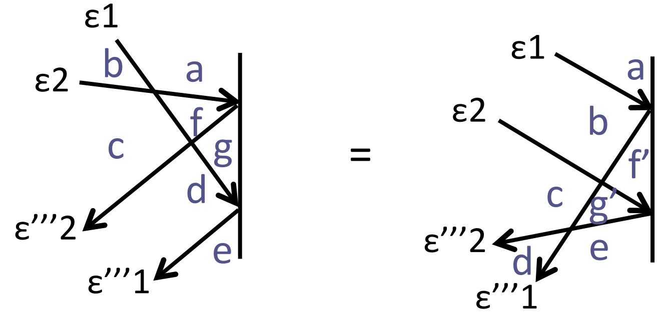
$$A^{\epsilon}_{ab}(\theta)B = \sum_{c}\sum_{\epsilon'} R^{\epsilon}_{\epsilon'}|^{b}_{ac} A^{\epsilon'}_{bc}(-\theta)B$$



$$R^{\epsilon}_{\epsilon'}|^{c}_{ab}(\theta) = \boxed{R^{\epsilon}_{\epsilon'}(\theta)} \times \boxed{R^{c}_{ab}(\theta)}$$

SG part SUSY part

- reflection relations



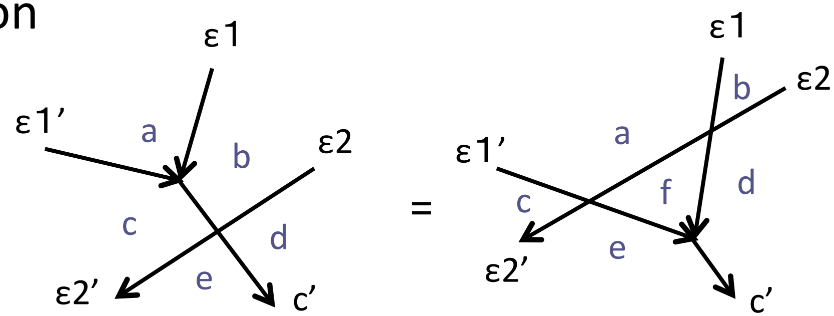
$$S^{\epsilon_1\epsilon_2}_{\epsilon'_2\epsilon'_1}(\theta_1-\theta_2)R^{\epsilon'_2}_{\epsilon''_2}(\theta_2)S^{\epsilon''_2\epsilon'_1}_{\epsilon''_1\epsilon'''_2}(\theta_1+\theta_2)R^{\epsilon''_1}_{\epsilon'''_1}(\theta_1) = R^{\epsilon_1}_{\epsilon'_1}(\theta_1)S^{\epsilon'_1\epsilon_2}_{\epsilon''_2\epsilon'_1}(-\theta_1-\theta_2)R^{\epsilon'_2}_{\epsilon''_2}(\theta_2)S^{\epsilon'_2\epsilon''_1}_{\epsilon'''_1\epsilon'''_2}(-\theta_1+\theta_2),$$

$$S^{ac}_{bf}(\theta_1-\theta_2)R^{f}_{ag}(\theta_2)S^{gc}_{fd}(\theta_1+\theta_2)R^{d}_{ge}(\theta_1) = R^{b}_{af'}(\theta_1)S^{f'c}_{bg'}(-\theta_1-\theta_2)R^{g'}_{f'e}(\theta_2)S^{ec}_{g'd}(-\theta_1+\theta_2)$$

different amplitudes for the NS/R sector

Scattering theory

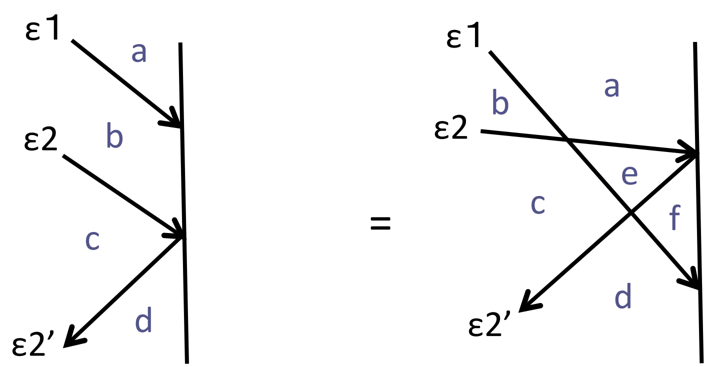
- bootstrap relation



$$f^{c}_{\epsilon_1\epsilon_1'} S^{\epsilon_2 c}_{c'\epsilon_2'}(\theta) = f^{c'}_{\epsilon_1''\epsilon_1'''} S^{\epsilon_2\epsilon_1'}_{\epsilon_1''\epsilon_2''}(\theta+iu_1) S^{\epsilon_2''\epsilon_1'}_{\epsilon_1'''\epsilon_2'}(\theta+iu_2)$$

$$f^{a}_{bc} S^{dc}_{be}(\theta) = f^{f}_{de} S^{da}_{bf}(\theta+iu_1) S^{fc}_{ae}(\theta+iu_2)$$

- boundary bootstrap relation



$$g^{\epsilon_1} R^{\epsilon_2}_{\epsilon_2'}(\theta) = g^{\epsilon_2''} S^{\epsilon_1\epsilon_2}_{\epsilon_2''\epsilon_1'}(\theta-iu_1) R^{\epsilon_2''}_{\epsilon_2'''}(\theta) S^{\epsilon_2'''\epsilon_1'}_{\epsilon_1''\epsilon_2'}(\theta+iu_2)$$

$$g^{a}_{b} R^{c}_{bd}(\theta) = g^{f}_{d} S^{ac}_{bc}(\theta-iu_1) R^{e}_{af}(\theta) S^{fc}_{ed}(\theta+iu_2)$$

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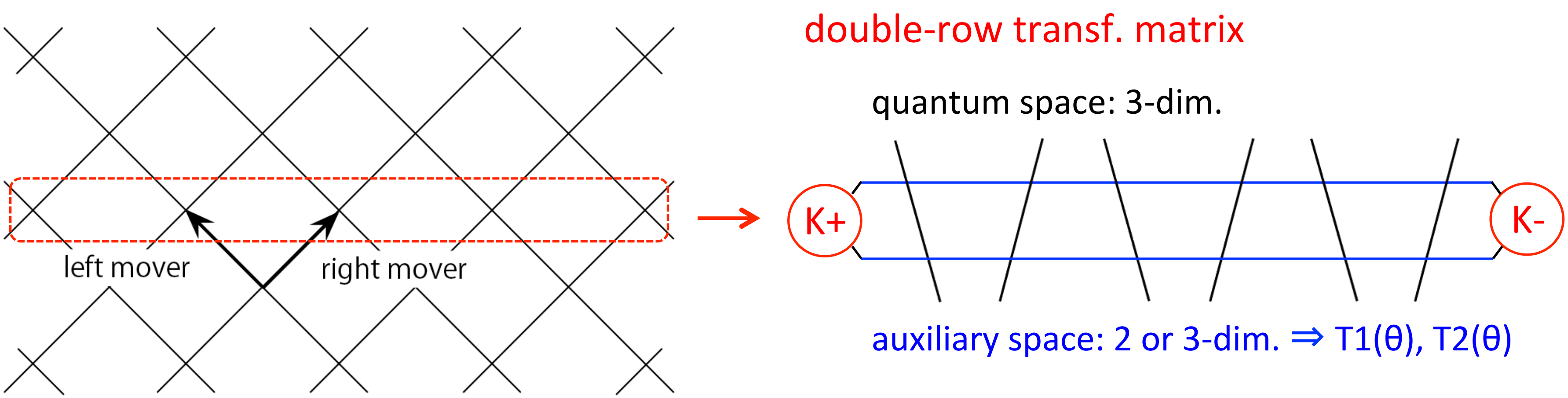
Transfer matrix of LSSG model



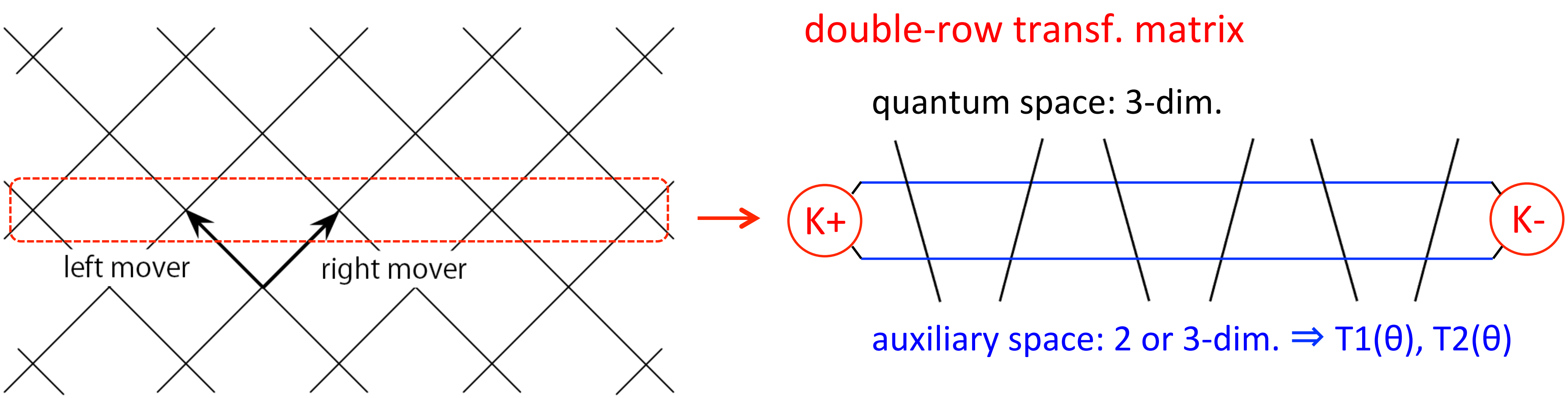
- momentum

$$P = \frac{1}{2ia} \ln T_2(\Theta) + \frac{1}{2ia} \ln T_2(-\Theta)$$

- energy

$$E = \frac{1}{4ia} \left(\frac{d}{d\theta} \ln T_2(\theta) \Big|_{\theta=\Theta+\frac{i\pi}{2}} - \frac{d}{d\theta} \ln T_2(\theta) \Big|_{\theta=\Theta-\frac{i\pi}{2}} \right)$$

Transfer matrix of LSSG model



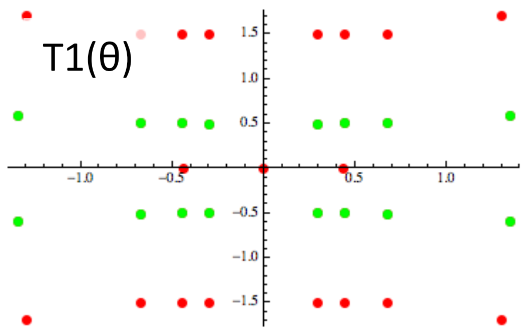
$$T_1(\theta) = \sinh\tfrac{\gamma}{\pi}(2\theta + i\pi) B_+(\theta)\phi(\theta + i\pi)\frac{Q(\theta - i\pi)}{Q(\theta)} + \sinh\tfrac{\gamma}{\pi}(2\theta - i\pi) B_-(\theta)\phi(\theta - i\pi)\frac{Q(\theta + i\pi)}{Q(\theta)}$$

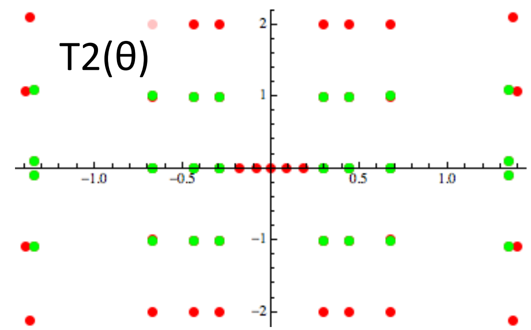
$$\begin{aligned} T_2(\theta) = {} & \sinh\tfrac{\gamma}{\pi}(2\theta - 2i\pi) B_-(\theta - \tfrac{i\pi}{2}) B_-(\theta + \tfrac{i\pi}{2})\phi(\theta - \tfrac{3i\pi}{2})\phi(\theta - \tfrac{i\pi}{2})\frac{Q(\theta + \frac{3i\pi}{2})}{Q(\theta - \frac{i\pi}{2})} \\ & + \sinh\tfrac{\gamma}{\pi}(2\theta) B_+(\theta - \tfrac{i\pi}{2}) B_-(\theta + \tfrac{i\pi}{2})\phi(\theta - \tfrac{i\pi}{2})\phi(\theta + \tfrac{i\pi}{2})\frac{Q(\theta + \frac{3i\pi}{2})Q(\theta - \frac{3i\pi}{2})}{Q(\theta - \frac{i\pi}{2})Q(\theta + \frac{i\pi}{2})} \\ & + \sinh\tfrac{\gamma}{\pi}(2\theta + 2i\pi) B_+(\theta - \tfrac{i\pi}{2}) B_+(\theta + \tfrac{i\pi}{2})\phi(\theta + \tfrac{3i\pi}{2})\phi(\theta + \tfrac{i\pi}{2})\frac{Q(\theta - \frac{3i\pi}{2})}{Q(\theta + \frac{i\pi}{2})} \end{aligned}$$

Analyticity structure of T-functions

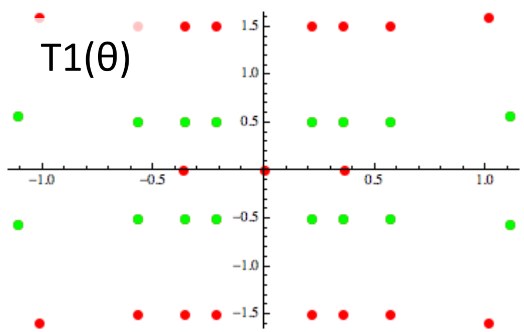
green dots: roots
red dots : holes

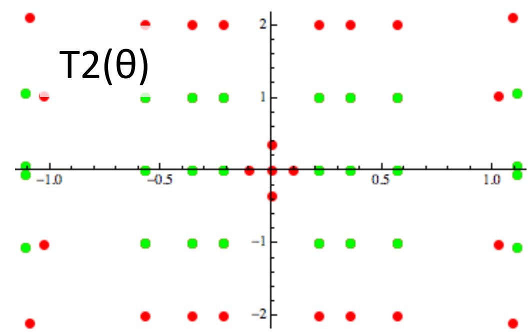
H+ = 1.5, H- = 2.2,
γ = 0, N = n = 8



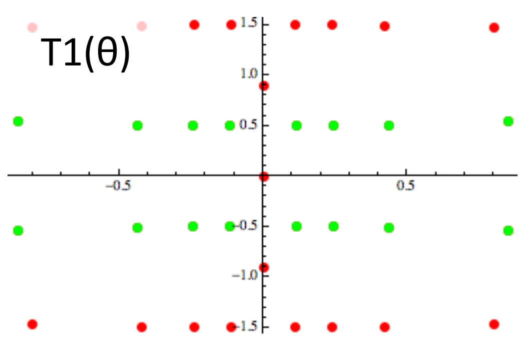


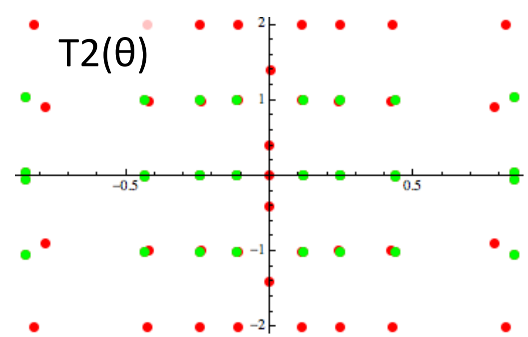
H+ = 1.5, H- = 0.3,
γ = 0, N = n = 8





H+ = 1.5, H- = -1.8,
γ = 0, N = n = 8





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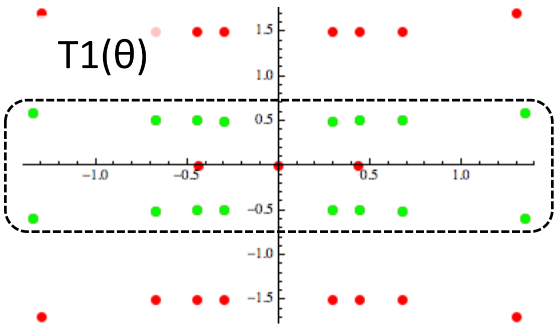

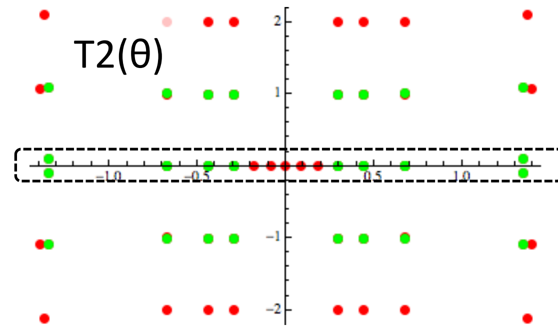

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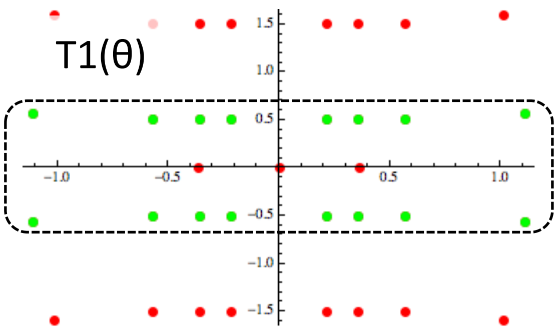

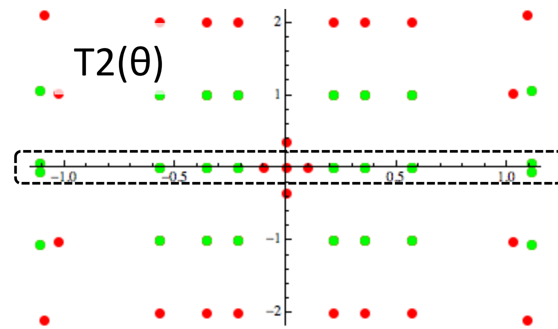

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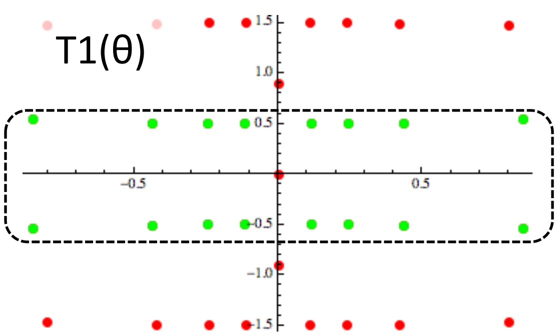
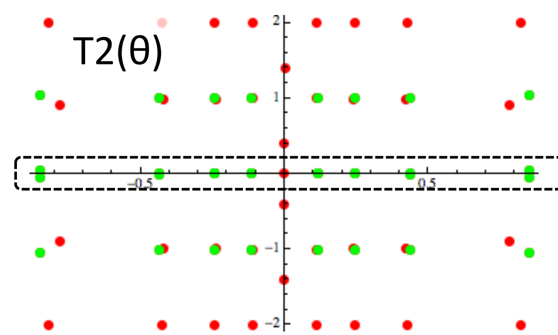




Nonlinear integral equations

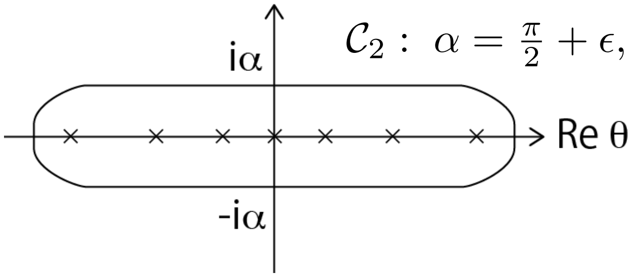
- Cauchy theorem

$$\oint_{\mathcal{C}_1} d\theta\, e^{ik\theta}[\ln T_2(\theta)]'' = \frac{2\pi k}{1-e^{-\pi k}}\Big(1+\sum_{h_j\in\mathbb{R}} e^{ikh_j}\Big)$$

$$\oint_{\mathcal{C}_2} d\theta\, e^{ik\theta}[\ln T_1(\theta)]'' = \frac{2\pi k}{1-e^{-\pi k}}\Big(1+\sum_{\mathrm{Im} h_j^{(1)}\in[-\frac{\pi}{2},\frac{\pi}{2})} e^{ikh_j^{(1)}}\Big)$$

$\mathcal{C}_1 : \alpha = \epsilon,$

$\mathcal{C}_2 : \alpha = \frac{\pi}{2} + \epsilon,$



- T-systems

$$T_1(\theta - \tfrac{i\pi}{2})T_1(\theta + \tfrac{i\pi}{2}) = f(\theta) + T_0(\theta)T_2(\theta),$$
$$T_0(\theta) = \sinh \tfrac{\gamma}{\pi}(2\theta)$$

- T-Y relations (⇒ Y-systems)

$$y(\theta) = \frac{T_0(\theta)T_2(\theta)}{f(\theta)}, \quad T_1(\theta - \tfrac{i\pi}{2})T_1(\theta + \tfrac{i\pi}{2}) = f(\theta)Y(\theta),$$
$$Y(\theta) = 1 + y(\theta), \quad f(\theta) = l_2(\theta - \tfrac{i\pi}{2})l_1(\theta + \tfrac{i\pi}{2})$$

Nonlinear integral equations

$$\ln b(\theta) = \int_{-\infty}^{\infty} d\theta' \; G(\theta - \theta' - i\epsilon) \ln B(\theta' + i\epsilon) - \int_{-\infty}^{\infty} d\theta' \; G(\theta - \theta' + i\epsilon) \ln \bar{B}(\theta' - i\epsilon)$$
$$+ \int_{-\infty}^{\infty} d\theta' \; G_K(\theta - \theta' - \tfrac{i\pi}{2} + i\epsilon) \ln Y(\theta' - i\epsilon) + iD_{\text{bulk}}(\theta) + iD_{\text{B}}(\theta) + iD(\theta)$$
$$+ C_b$$

$$\ln y(\theta) = \int_{-\infty}^{\infty} d\theta' \; G_K(\theta - \theta' + \tfrac{i\pi}{2} - i\epsilon) \ln B(\theta' + i\epsilon) + \int_{-\infty}^{\infty} d\theta' \; G_K(\theta - \theta' - \tfrac{i\pi}{2} + i\epsilon) \ln \bar{B}(\theta' - i\epsilon)$$
$$+ iD_{\text{SB}}(\theta) + iD_K(\theta) + C_y$$

Nonlinear integral equations

scattering amp.

$$\ln b(\theta) = \int_{-\infty}^{\infty} d\theta' \boxed{G(\theta-\theta'-i\epsilon)} \ln B(\theta'+i\epsilon) - \int_{-\infty}^{\infty} d\theta' \boxed{G(\theta-\theta'+i\epsilon)} \ln \bar{B}(\theta'-i\epsilon)$$
$$+ \int_{-\infty}^{\infty} d\theta' \boxed{G_K(\theta-\theta'-\tfrac{i\pi}{2}+i\epsilon)} \ln Y(\theta'-i\epsilon) + \boxed{iD_{\text{bulk}}(\theta)} + \boxed{iD_{\text{B}}(\theta)} + \boxed{iD(\theta)}$$
$$+ C_b$$

bulk phase shift

particle sources

$$\ln y(\theta) = \int_{-\infty}^{\infty} d\theta' \, G_K(\theta-\theta'+\tfrac{i\pi}{2}-i\epsilon) \ln B(\theta'+i\epsilon) + \int_{-\infty}^{\infty} d\theta' \, G_K(\theta-\theta'-\tfrac{i\pi}{2}+i\epsilon) \ln \bar{B}(\theta'-i\epsilon)$$
$$+ \boxed{iD_{\text{SB}}(\theta)} + \boxed{iD_K(\theta)} + C_y$$

reflection amp.

$\Rightarrow$ coming from boundary terms

Boundary-dependent terms

$$D_{\mathrm{B}}(\theta) = F(\theta; H_+) + F(\theta; H_-) + J(\theta)$$
$$D_{\mathrm{SB}}(\theta) = F_y(\theta; H_+) + F_y(\theta; H_-) + J_K(\theta)$$

regime (a)
$$F(\theta; H) = \int_0^\infty d\theta' \int_{-\infty}^\infty dk\ e^{-ik\theta'} \frac{\sinh(\frac{\pi}{\gamma} - H)\frac{\pi k}{2}}{2\cosh\frac{\pi k}{2}\sinh(\frac{\pi}{\gamma} - 2)\frac{\pi k}{2}},$$
$$F_y(\theta; H) = 0$$

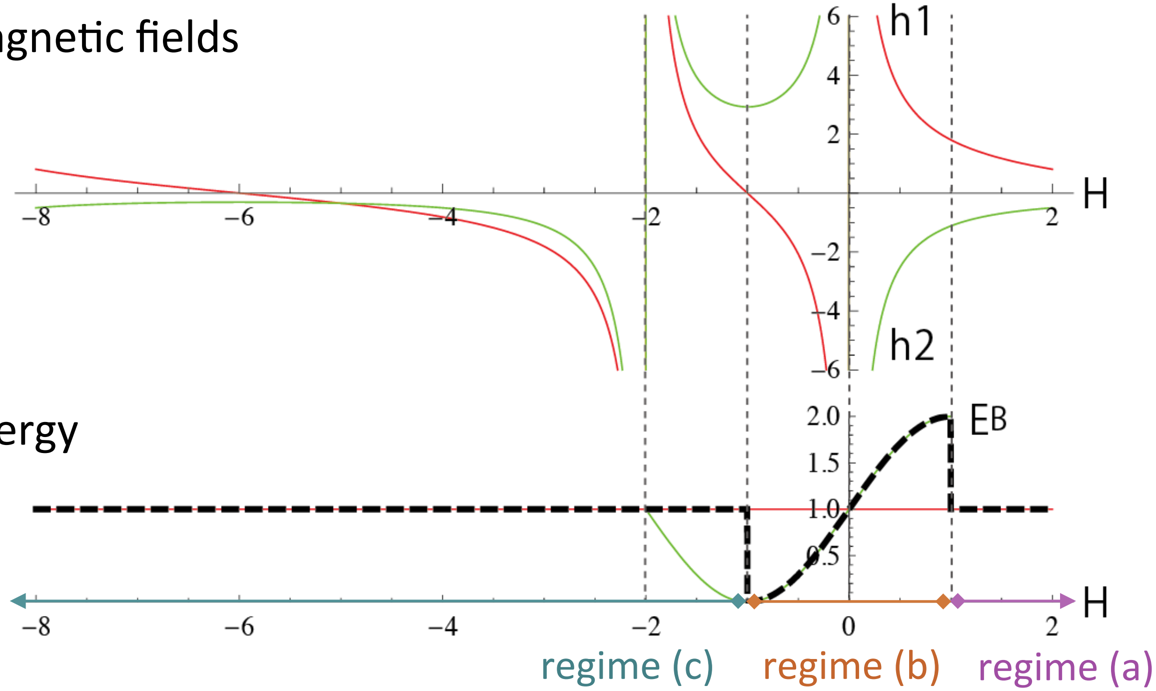
regime (b)
$$F(\theta; H) = -\int_0^\infty d\theta' \int_{-\infty}^\infty dk\ e^{-ik\theta'} \frac{\sinh(\frac{\pi}{\gamma} + \pi H - 2)\frac{\pi k}{2}}{2\cosh\frac{\pi k}{2}\sinh(\frac{\pi}{\gamma} - 2)\frac{\pi k}{2}},$$
$$F_y(\theta; H) = \widetilde{g}_K(\theta - \tfrac{i\pi(1-H)}{2}) + \widetilde{g}_K(\theta + \tfrac{i\pi(1-H)}{2})$$

regime (c)
$$F(\theta; H) = -\int_0^\infty d\theta' \int_{-\infty}^\infty \frac{dk}{2\pi}\ e^{-ik\theta'} \frac{\sinh(\frac{\pi}{\gamma} + H)\frac{\pi k}{2}}{2\cosh\frac{\pi k}{2}\sinh(\frac{\pi}{\gamma} - 2)\frac{\pi k}{2}},$$
$$F_y(\theta; H) = 0$$

Boundary energy

boundary magnetic fields

boundary energy



Outline

1. Correspondence between spin chains and QFTs
2. Discretization of integrable QFTs
3. Nonlinear integral equations (NLIEs)
4. IR and UV analysis
5. Concluding remarks

Infrared limit

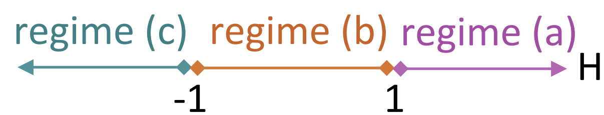
- The IR lim. is the large-volume lim. where the g.s. roots exactly form the two-strings.
- Rapidity of b.b.s. approaches to the singular point of BAE.
- Compare the quantization conditions.

$$\left[\begin{array}{l} e^{2im_0 L \sinh\theta_j} R(\theta_j;\xi_+) \cdot \prod_{\substack{l=1 \\ l\neq j}}^{n} S(\theta_j-\theta_l) S(\theta_j+\theta_l) \cdot R(\theta_j;\xi_-) = 1 \\ b(h_j) = -1 \end{array}\right.$$

← phase shifts

← NLIEs

- reflection amplitudes



$$iF^{(a)}(\theta;H) = \ln R(\theta)$$

holes

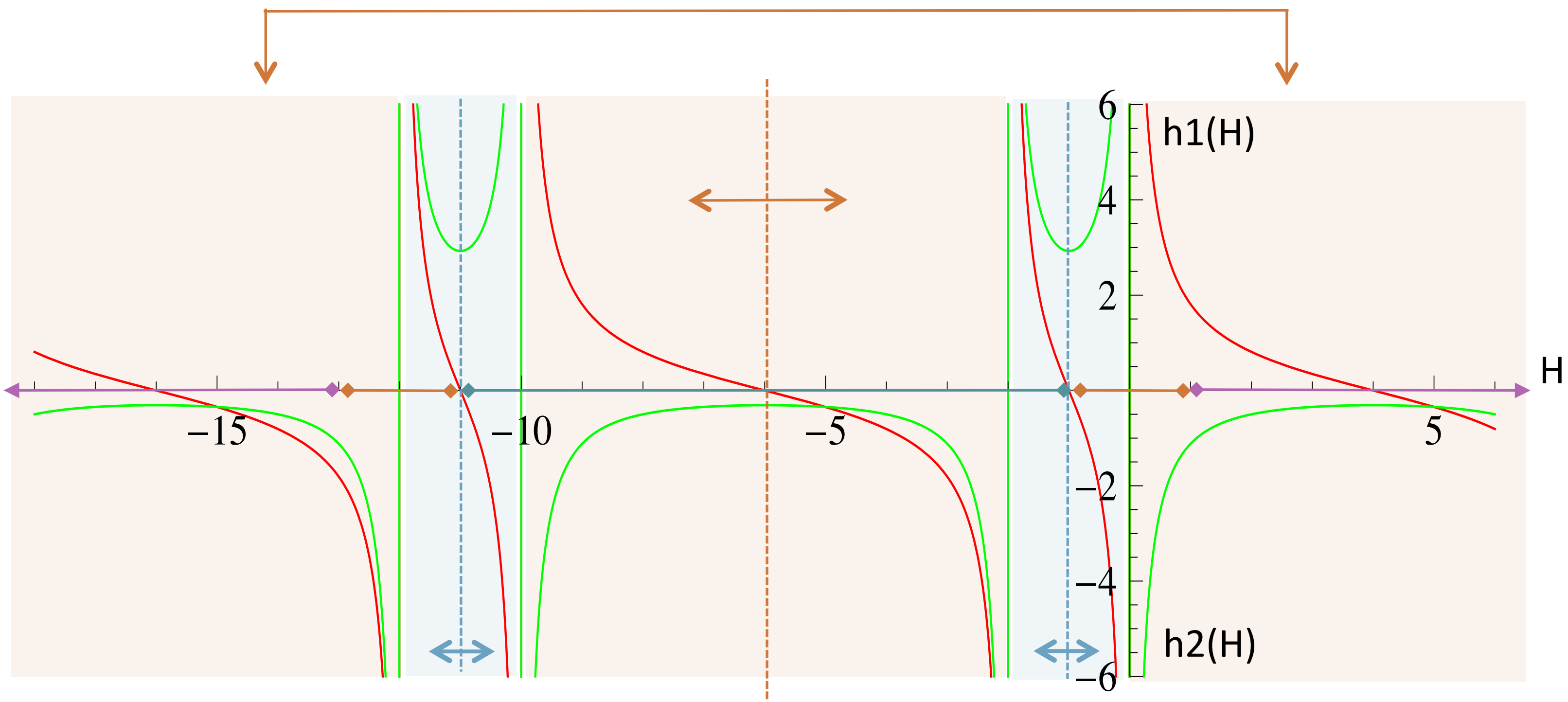
$$iF^{(b)}(\theta;H) = \ln R(\theta) + ig(\theta \boxed{-\frac{i\pi(1-H)}{2}}) + ig(\theta \boxed{+\frac{i\pi(1-H)}{2}})$$

$$iF^{(c)}(\theta;H) = \ln R(\theta) + ig_{\mathrm{II}}(\theta \boxed{-\frac{i\pi(1-H)}{2}}) + ig_{\mathrm{II}}(\theta \boxed{+\frac{i\pi(1-H)}{2}})$$

boundary bootstrap relations

Infrared limit

soliton-antisoliton sym.



soliton-antisoliton sym.

Ultraviolet limit

- The UV limit is the small volume limit where the conformal structure shows up.
- Conformal dimensions are read off from the system-size dependence of energy.

$$E_{\mathrm{CFT}}(L) = E(L) - (E_{\mathrm{bulk}} + E_{\mathrm{B}}) = E_{\mathrm{ex}}(L) + E_C(L),$$

$$E_{\mathrm{ex}}(L) = \frac{1}{2L}\sum_{j=1}^{N_H^+} e^{\hat{h}_j} - \frac{1}{2L}\sum_{j=1}^{M_C^+} e^{\hat{c}_j},$$

$$E_C(L) = \frac{1}{2\pi L}\mathrm{Im}\int_{-\infty}^{\infty} d\hat{\theta}\, e^{\hat{\theta}} \ln \bar{B}^+(\hat{\theta})$$

evaluated from NLIEs

Evaluation of energy

- NLIEs in vector forms

$$\boldsymbol{lb}^{+}(\hat{\theta}) = \mathcal{G} * \boldsymbol{lB}^{+}(\hat{\theta}) + i\boldsymbol{g}(\hat{\theta})$$

- energy

Rogers' dilogarithm

$$E_{\mathrm{CFT}} = \frac{1}{8\pi^2}\boxed{\int_{-\infty}^{\infty} d\hat{\theta}\Big(\boldsymbol{lb}^{+'}(\hat{\theta}) \cdot \boldsymbol{lB}^{+}(\hat{\theta}) - \boldsymbol{lb}^{+}(\hat{\theta}) \cdot \boldsymbol{lB}^{+'}(\hat{\theta})\Big)}$$
$$+ \frac{1}{2}\Big(\frac{\Phi_{+} - \Phi_{-}}{\sqrt{\pi}} + mR + \frac{n}{R}\Big)^2 + \mathrm{const.}$$

Evaluation of energy

- NLIEs in vector forms

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$$+ \frac{1}{2}\Big(\frac{\Phi_+ - \Phi_-}{\sqrt{\pi}} + mR + \frac{n}{R} \Big)^2 + \mathrm{const.}$$

- Rogers' dilogarithm

$$L(x) = -\frac{1}{2}\int_0^x dy \Big(\frac{\ln(1-y)}{y} + \frac{\ln y}{1-y} \Big), \quad L_+(x) = L\Big(\frac{x}{1+x}\Big)$$

$$L(0) = 0, \quad L(\tfrac{1}{2}) = \tfrac{\pi^2}{12}, \quad L(-\infty) = -\tfrac{\pi^2}{6}, \quad L(1) = L(x) + L(1-x) = \tfrac{\pi^2}{6} \quad x \in [0,1]$$

$$2L(1) = 2L(\tfrac{1}{n+1}) + \sum_{j=0}^{n-1} L(\tfrac{1}{(1+j)^2}) \quad L(1)\tfrac{3n}{n+2} = \sum_{j=0}^{n-1} L(\tfrac{\sin^2 \frac{\pi}{n+2}}{\sin^2 \frac{\pi(j+1)}{n+2}}) \quad n \in \mathbb{Z}_{\geq 0}$$

Evaluation of energy

- NLIEs in vector forms

$$\boldsymbol{l}\boldsymbol{b}^{+}(\hat{\theta}) = \mathcal{G} * \boldsymbol{l}\boldsymbol{B}^{+}(\hat{\theta}) + i\boldsymbol{g}(\hat{\theta})$$

- energy

$$\begin{aligned} E_{\mathrm{CFT}} = & \frac{1}{4\pi^2}\Big(L_{+}(b^{+}(\infty)) - L_{+}(b^{+}(-\infty)) + L_{+}(\bar{b}^{+}(\infty)) - L_{+}(\bar{b}^{+}(-\infty)) \\ & + L_{+}(y^{+}(\infty)) - \boxed{L_{+}(y^{+}(-\infty))}\Big) \\ & + \frac{1}{2}\Big(\frac{\Phi_{+} - \Phi_{-}}{\sqrt{\pi}} + mR + \frac{n}{R}\Big)^{2} + \mathrm{const.} \end{aligned}$$

determines the NS/R sector

Evaluation of energy

- NLIEs in vector forms

$$\boldsymbol{lb}^+(\hat{\theta}) = \mathcal{G} * \boldsymbol{lB}^+(\hat{\theta}) + i\boldsymbol{g}(\hat{\theta})$$

- energy

$$E_{\mathrm{CFT}} = \frac{1}{4\pi^2}\Big(L_+(b^+(\infty)) - L_+(b^+(-\infty)) + L_+(\bar{b}^+(\infty)) - L_+(\bar{b}^+(-\infty)) + L_+(y^+(\infty)) - \boxed{L_+(y^+(-\infty))}\Big) + \frac{1}{2}\Big(\frac{\Phi_+ - \Phi_-}{\sqrt{\pi}} + mR + \frac{n}{R}\Big)^2 + \mathrm{const.}$$

determines the NS/R sector

$$L_+(y^+(-\infty)) = \begin{cases} \frac{\pi^2}{12} & |1-H_+||1+H_+| \times |1-H_-||1+H_-| > 0 \\ -\frac{\pi^2}{6} & |1-H_+||1+H_+| \times |1-H_-||1+H_-| < 0 \end{cases}$$

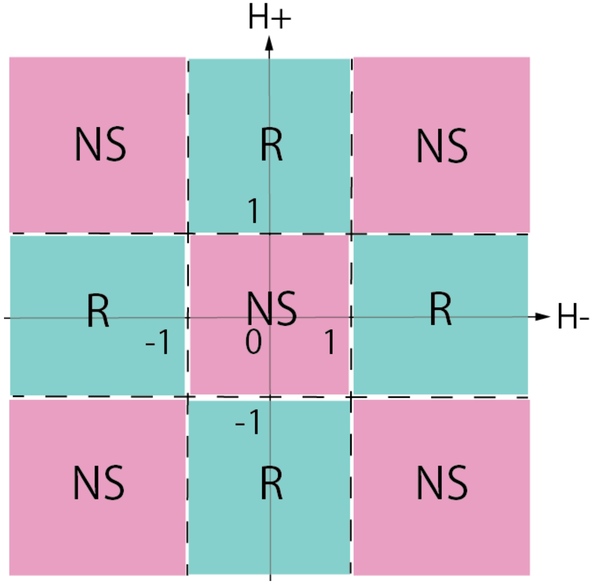
Phase separation of NS/R sectors

- system−size dependence of energy

$$E_{\mathrm{CFT}}(L) = -\frac{\pi}{24L}(c - 24\Delta), \qquad c = \frac{3}{2}$$

- conformal dimension

$$\Delta = \frac{1}{2}\left(\frac{\Phi_+ - \Phi_-}{\sqrt{\pi}} + mR + \frac{n}{R}\right)^2$$
$$+\frac{1}{16}\left(\frac{1}{2}(\mathrm{sgn}(1-H_+) + \mathrm{sgn}(1-H_-) + \mathrm{sgn}(1+H_+) + \mathrm{sgn}(1+H_-))\right)_{\mathrm{mod}\,2}$$



Both of NS/R sector is realized under Dirichlet boundaries.

Restriction on the winding number
⇒ NS: m = integer
R : m = half-integer

Summary

- We showed the complete analysis of the correspondence between the spin-1 chain and the SSG model with boundary magnetic fields.
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- We analyzed only in the IR and the UV limits.
  ⇒ intermediate-volume analysis?
- How R sector can be obtained from a periodic lattice?
  ⇒ “supercharge”, which connects even and odd systems.

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Thank you.