Large volume supersymmetry breaking without decompactification problem

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Outline

Introduction

We want a theory : Realistic and analytically under control

Realistic

- Gauge and gravitational interactions + matter content
  $\Rightarrow$ String theory
- In this talk, no cosmological issue $\Rightarrow$ 4D flat backgrounds + 6 dimensional internal space.
- Non supersymmetric: $\mathcal{N} = 1$ susy is spontaneously broken at a low scale (1 to 10 TeV) to solve hierarchy problem (Higgs mass $\ll M_{GUT}$).

Analytic control

In this talk, we want perturbation theory to be valid.

- The 2D CFT on the worldsheet must be known enough to compute quantum corrections.
- In particular, the spontaneous $\mathcal{N} = 1 \to \mathcal{N} = 0$ susy breaking must be introduced at the string level. (Non perturbative gaugino condensation could be considered but only at the level of the effective field theory.)

  The susy breaking scale is given by a characteristic size of the internal space. For a single compact direction of radius R,

$$M_{\rm susy} = \frac{M_{\rm Planck}}{R} = \mathcal{O}(10 \text{ TeV}) \quad \Longrightarrow \quad R \sim 10^{15}$$

[R. Rhom (84); C. Kounnas, Porrati, Ferrara, Zwirner (88), C. Kounnas, B. Rostand (90), I. Antoniadis (91); ...]

Problem

A full tower of light Kaluza-Klein states of masses n/R are charged under the gauge group and contribute to the running gauge couplings. In general, at 1–loop,

$$\frac{16\,\pi^2}{g^2_{\rm YM}(\mu)} = k\,\frac{16\,\pi^2}{g^2_{\rm string}} + b\log\frac{M^2_{\rm Planck}}{\mu^2} + b\left(\frac{\pi}{3}R^2 - \log R^2 + \mathcal{O}(1)\right)$$

- $b > 0 \quad \Longrightarrow \quad g_{\rm YM}(\mu) \to 0$: The theory is free.
- $b < 0 \quad \Longrightarrow \quad g_{\rm YM}(\mu) \to \infty$: The theory is non-perturbative.

This is the **"decompactification problem"**: low susy breaking scale AND perturbation theory are hard to reconcile (for gauge, gravitational, Yukawa couplings).

The problem is ubiquitous when one wants to interpret the internal CFT as a geometrical space. This space must be "large" compared to the Planck scale. E.g. Calabi-Yau compactifications, which lead to $\mathcal{N} = 2$ or $\mathcal{N} = 1$ susy models.

Idea

At 1–loop, the massive corrections to the gauge couplings are

$$\Delta = \begin{cases} 0 & \text{for } \mathcal{N} = 4 \\ b\left(\frac{\pi}{3}R^2 - \log R^2 + \mathcal{O}(1)\right) & \text{for } \mathcal{N} = 2\,. \end{cases}$$

Exception: When $\mathcal{N} = 2$ is realized as a spontaneous breaking of $\mathcal{N} = 4$.

$M_{\text{susy}} \sim M_{\text{Planck}}/R$ where R **is a scalar with flat potential** *i.e.* arbitrary (modulus). For large R, $\mathcal{N} = 4$ is recovered and Δ is expected to vanish. In fact,

$$\Delta = b\left(\qquad - \log R^2 + \mathcal{O}(1)\right) \quad \text{for } \mathcal{N} = 4 \to \mathcal{N} = 2\,.$$

[E. Kiritsis, C. Kounnas, P.M. Petropoulos, Rizos (96)]

(This is non zero when charged states have masses cM_{susy}, with $c < 1$.)

Outline

The class of models

Heterotic string

A closed string theory, is defined by a choice of SCFT living on the worldsheet. In heterotic string:

- The holomorphic part is superconformal $\Rightarrow$ 10 bosonic + 10 fermionic degrees of freedom.
- The antiholomorphic part is conformal $\Rightarrow$ 10+16 bosonic degrees of freedom.

This leads to 4D Minkowski spacetime + 6D internal space, and non-Abelian degrees of freedom.

Free fermionic construction

In order to compute the 1–loop corrections to the gauge couplings, we need the 1–loop partition function. In free fermionic construction :

- Free SCFT on the worldsheet.
- The bosonic d.o.f. are replaced by 2 Majorana-Weyl fermions.
- A model is defined by a discrete choice of boundary conditions for these worldsheet fermions along the closed string.
- $\Longrightarrow$ Enormous number of models : Compact in any D, susy or not.
- But discrete : In bosonic language, the radii of compactification take fixed values $\mathcal{O}(1)$, e.g. $R = \sqrt{2}$.

[I. Antoniadis, C. Bachas, C. Kounnas, P. Windey (86);
A.E. Faraggi, C. Kounnas, C. Rizos (04); ...]

Moduli deformation

However, we need $M_{\text{susy}} \propto \frac{M_{\text{Planck}}}{R}$ with $R \sim 10^{15}$:

- The SCFT admits marginal deformations.
- We add operators on the worldsheet, whose effect is to deform the continuous parameters that define the 6D compact space : Metric G_{ij}, antisymmetric tensor B_{ij} and Wilson lines Y_i^a.
- These operators preserve the quadratic nature of the worldsheet action in bosonic language : The partition function is exact.

Compactification on torus $T^6 \implies \mathcal{N} = 4$

Compactification on orbifold $T^2 \times \dfrac{T^4}{\mathbb{Z}_2} \implies \mathcal{N} = 2$

$X^i \equiv X^i + 2\pi R_i \quad \text{for} \quad i = 4, 5, 6, 7, 8, 9$

and

$(X^6, X^7, X^8, X^9) \equiv (-X^6, -X^7, -X^8, -X^9)$

- The oscillating modes of the string living on this orbifold must be invariant under the transformation $X^{6,7,8,9} \to -X^{6,7,8,9}$. This reduces the number of d.o.f. from $\mathcal{N} = 4$ to $\mathcal{N} = 2$ multiplets.
- The two ends of a closed string can also be identified up to this transformation !
$$X^i(\text{one string end}) = -X^i(\text{second string end})$$
- $\implies$ Untwisted sector ($H = 0$) and Twisted sector ($H = 1$).
- Same thing for the quantum loop :
Closed in the usual sense ($G = 0$) or up to the twist ($G = 1$).

[L.J. Dixon, J.A. Harvey, C. Vafa, E. Witten (85)]

Compactification on orbifold $\frac{T^6}{\mathbb{Z}_2 \times \mathbb{Z}_2} \Longrightarrow \mathcal{N} = 1$

$$\mathbb{Z}_2^{(1)} : \ (X^4, X^5, X^6, X^7, X^8, X^9) \longrightarrow (X^4, X^5, -X^6, -X^7, -X^8, -X^9)$$
$$\mathbb{Z}_2^{(2)} : \ (X^4, X^5, X^6, X^7, X^8, X^9) \longrightarrow (-X^4, -X^5, X^6, X^7, -X^8, -X^9)$$

The product fixes the third T^2 :

$$(X^4, X^5, X^6, X^7, X^8, X^9) \longrightarrow (-X^4, -X^5, -X^6, -X^7, X^8, X^9)$$

Three $\mathcal{N} = 2$ sectors :

- $H_2 = G_2 = 0$ (with H_1, G_1 arbitrary) $\Rightarrow \Delta$ large when 1^{st} T^2 is large.
- Same thing for $H_1 = G_1 = 0$, with the 2^{d} T^2.
- Same thing with $H_1 + H_2 = G_1 + G_2 = 0$, with the 3^{rd} T^2.

With a spontaneous breaking $\mathcal{N}=4 \to \mathcal{N}=2$

Changing the action of $\mathbb{Z}_2^{(1)}$ for the breaking $\mathcal{N}=4 \to \mathcal{N}=2$ to be spontaneous, the 1st 2-torus large will be allowed to be large.

This is done by making free the action of $\mathbb{Z}_2^{(1)}$:

$$(X^4, X^5, X^6, X^7, X^8, X^9) \longrightarrow (X^4, X^5 + \pi R_5, -X^6, -X^7, -X^8, -X^9)$$

- In the non-freely acting case, 2 gravitini remain massless and 2 are projected out.
- In the freely acting case, 2 gravitini remain massless, 2 are projected out and 2 new arise from the twisted sector $H_1 = 1$, with masses $M_{3/2} = \frac{M_{\text{Planck}}}{R_5}$.
- $R_5 \to \infty \Longrightarrow \mathcal{N}=4$ recovered.
  $R_5 \to 0 \Longrightarrow$ the 2 new gravitini decouple : the non-free case is recovered.

Spontaneous breaking $\mathcal{N} = 1 \to \mathcal{N} = 0$

Implemented via a Scherk-Schwarz mechanism upgraded to string theory. In field theory :

- In 4D + 1 circle, a space-time field

$$\phi(x^\mu, x^4) = \sum_m \phi_m(x^\mu)\, e^{im\frac{x^4}{R_4}} \Longrightarrow \Box_5 \equiv \Box_4 + \partial_4^2 = \Box_4 + \left(\frac{m}{R_4}\right)^2$$

  leads to a massless state in 4D ($m = 0$) + a tower of massive Kaluza-Klein states ($m \neq 0$).
- If fermionic, we are free to impose instead antiperiodicity : $m + \frac{1}{2}$. All 4D modes have now masses $\geq \frac{M_{\text{Planck}}}{2R_4}$.
- Susy is broken, $M_{\text{susy}} = \mathcal{O}\big(\frac{M_{\text{Planck}}}{R_4}\big)$, where R_4 is arbitrary (it is a scalar with flat potential).
- The models with spontaneous breaking $\mathcal{N} = 1 \to \mathcal{N} = 0$ and M_{susy} arbitrary are no-scale models. $\mathcal{N} = 1$ is recovered when $R_4 \to \infty$ and we want $R_4 \sim 10^{15}$. [E. Cremmer, S. Ferrara, C. Kounnas, D.V. Nanopoulos (83); J. Ellis, A.B. Lahanas, K. Tamvakis (84); ...]

Partition function

For $a = 0$ (bosons) and $a = 1$ (fermions), the KK tower contributes as a dressing

$$\sum_m e^{-\frac{\ell}{2}\left(\frac{m+\frac{a}{2}}{R_4}\right)} = 2R_4\left(\frac{\pi}{\ell}\right)^{\frac{1}{2}} \sum_{\tilde{m}} e^{-\frac{(2\pi R_4)^2}{\ell}\tilde{m}^2} (-1)^{a\tilde{m}}$$

In string theory, the boundary condition $(-1)^{a\tilde{m}}$ becomes $(-1)^{a\tilde{m}+bn+\tilde{m}n}$, where

- n is the winding number (the string can wrap the periodic direction 4). $b = 0, 1$ implements a "GSO projection" needed for consistency (e.g. spin/statistics).
- It is convenient to define $n = 2N + h$ and $\tilde{m} = 2\tilde{M} + g$, where h and h are 0 or 1

$$\Longrightarrow \quad (-1)^{a\tilde{m}+bn+\tilde{m}n} = (-1)^{ag+bh+gh}$$

  The sector $h = g = 0$ is supersymmetric.
- We can use (anti-)periodic B.C. up to any symmetry : Use the conserved charge $a + Q$ *i.e.* fermionic number + any other charge.

String partition function

Essentially, its structure is:

$$Z = \frac{1}{2}\sum_{H_1,G_1} \quad \frac{1}{2}\sum_{H_2,G_2} \quad \frac{1}{2}\sum_{h,g} \quad (-1)^{ag+bh+gh}$$

$$\frac{1}{2}\sum_{a,b}(-1)^{a+b+ab}\,\frac{\theta[^a_b]}{\eta}\,\frac{\theta[^{a+H_2}_{b+G_2}]}{\eta}\,\frac{\theta[^{a+H_1}_{b+G_1}]}{\eta}\,\frac{\theta[^{a-H_1-H_2}_{b-G_1-G_2}]}{\eta}$$

$$Z_{\text{Minkow}}\; Z_{4,5}\left[\begin{matrix} h, H_1 \\ g, G_1 \end{matrix}\middle|\begin{matrix} H_2 \\ G_2 \end{matrix}\right] Z_{6,7}\left[\begin{matrix} H_1 \\ G_1 \end{matrix}\right] Z_{8,9}\left[\begin{matrix} H_1+H_2 \\ G_1+G_2 \end{matrix}\right] Z_{\text{gauge}}\left[\begin{matrix} h, H_1, H_2 \\ g, G_1, G_2 \end{matrix}\right],$$

- $H_1, G_1 \Longrightarrow \mathcal{N}=4 \to \mathcal{N}=2$
- $H_2, G_2 \Longrightarrow \mathcal{N}=1$
- $h, g \qquad \Longrightarrow \mathcal{N}=1 \to \mathcal{N}=0$
- 2^{d} line : Organizes the states in $\mathcal{N}=1$ multiplets ($a=0$ for bosons, $a=1$ for fermions).

$$Z = \frac{1}{2}\sum_{H_1,G_1} \quad \frac{1}{2}\sum_{H_2,G_2} \quad \frac{1}{2}\sum_{h,g} \quad (-1)^{ag+bh+gh}$$

$$\frac{1}{2}\sum_{a,b}(-1)^{a+b+ab}\,\frac{\theta[^a_b]}{\eta}\,\frac{\theta[^{a+H_2}_{b+G_2}]}{\eta}\,\frac{\theta[^{a+H_1}_{b+G_1}]}{\eta}\,\frac{\theta[^{a-H_1-H_2}_{b-G_1-G_2}]}{\eta}$$

$$Z_{\text{Minkow}}\; Z_{4,5}\left[\begin{matrix} h, H_1 \\ g, G_1 \end{matrix}\middle|\begin{matrix} H_2 \\ G_2 \end{matrix}\right]\; Z_{6,7}\left[\begin{matrix} H_1 \\ G_1 \end{matrix}\right]\; Z_{8,9}\left[\begin{matrix} H_1+H_2 \\ G_1+G_2 \end{matrix}\right]\; Z_{\text{gauge}}\left[\begin{matrix} h, H_1, H_2 \\ g, G_1, G_2 \end{matrix}\right],$$

- 3^{rd} line : spacetime bosons + gauge d.o.f. + 3 twisted T^2's. The first one also used to implement the spontaneous breaking $\mathcal{N}=4 \to \mathcal{N}=2$ (direction 5) and $\mathcal{N}=1 \to \mathcal{N}=0$ (direction 4).
- $Z_{4,5}, Z_{6,7}, Z_{8,9}$ depend on moduli of each tori.
- Redefine moduli T and U for the large 2-torus (the 1^{st}) :

$$G_{ij} = \frac{\operatorname{Im} T}{\operatorname{Im} U}\begin{pmatrix} 1 & \operatorname{Re} U \\ \operatorname{Re} U & |U|^2 \end{pmatrix}, \quad B_{ij} = \operatorname{Re} T\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad i,j = 4,5$$

$\operatorname{Im} T$ is the large volume of the 2-torus and U parametrizes its shape.

Outline

Gauge coupling + Effective potential

1–loop gauge couplings

For a gauge group factor G, $$\frac{16\,\pi^2}{g^2_{\rm YM}(\mu)} = k\,\frac{16\,\pi^2}{g^2_{\rm string}} + b\log\frac{M^2_{\rm Planck}}{\mu^2} + \Delta$$

$$\Delta = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2}\left(\mathcal{Q}(v)\left(\mathcal{P}^2(\bar{w}) - \frac{k}{4\pi\tau_2}\right)\tau_2\, Z(v,\bar{w}) - b\right)\Bigg|_{v=\bar{w}=0} + b\log\frac{2\,e^{1-\gamma}}{\pi\sqrt{27}}$$

- $Z(v,\bar{w})$ is the refined partition function, on which
- $\mathcal{Q}(v)$ (helicity operator) acts as a derivative operator on the holomorphic part.
- $\mathcal{P}(\bar{w})$ (the charge operator of G) acts on the antiholomorphic part.

[Kaplunovsky (88); Dixon, Louis (91); Antoniadis, Narain, Taylor (91); Kiritsis, Kounnas (95), ...]

1–loop effective potential

$$\mathcal{V} = -\frac{1}{(2\pi)^4}\int_{\mathcal{F}} \frac{d^2\tau}{2\tau_2^2}\; Z|_{v=\bar{w}=0}$$

Sector $(H_2, G_2) = (0, 0)$

Only the first $\mathbb{Z}_2$ ($\mathcal{N} = 4 \to \mathcal{N} = 2$) and spontaneous breaking to $\mathcal{N} = 0$.

Sector A: $(H_1, G_1) = (0, 0)$, $(h, g) = (0, 0)$

$\mathcal{N} = 4$ susy is preserved $\implies$ $\Delta_A = \mathcal{V}_A = 0$.

Sector B: $(H_1, G_1) = (0, 0)$, $(h, g) \neq (0, 0)$

We have $\mathcal{N} = 4 \to \mathcal{N} = 0$

- $$\Delta_B = -\frac{b_B}{4} \log\left(\frac{\pi^2}{4} \left|\theta_2(U)\right|^4 \operatorname{Im} T \operatorname{Im} U\right) + \mathcal{O}\left(\frac{1}{\sqrt{\operatorname{Im} T}}\right)$$

  The KK states along the large 2-torus dominate. The modes of masses $\mathcal{O}(M_{\text{Planck}})$ are exponentially suppressed ($\sqrt{\operatorname{Im} T} \sim 10^{15}$).

- $$\mathcal{V}_B = -\frac{n_{\text{bosons}} - n_{\text{fermions}}}{64\pi^7} \frac{1}{(\operatorname{Im} T)^2} E(U|\,3) + \mathcal{O}\left(e^{-c\sqrt{\operatorname{Im} T}}\right)$$

  where $E(U|\,s) = \sum_{\tilde{m}_1, \tilde{m}_2} \frac{(\operatorname{Im} U)^s}{|\tilde{m}_1 + \frac{1}{2} + \tilde{m}_2 U|^{2s}}$ is a "shifted real analytic Eisenstein series".

Sector C: $(H_1, G_1) \neq (0,0)$, $(h, g) = (0,0)$

We have $\mathcal{N} = 4 \to \mathcal{N} = 2 \implies \mathcal{V}_C = 0$

$$\Delta_C = -\frac{b_C}{4} \log\left(\frac{\pi^2}{4} \left|\theta_4(U)\right|^4 \mathrm{Im}\, T\, \mathrm{Im}\, U \right) + \mathcal{O}\left(\frac{1}{\sqrt{\mathrm{Im}\, T}} \right)$$

Sector D: $(H_1, G_1) \neq (0,0)$, $(h, g) = (H_1, G_1)$

In this sector, another $\mathcal{N} = 2'$ supersymmetry is realized :
$\mathcal{N} = 4 \to \mathcal{N} = 2' \implies \mathcal{V}_D = 0$

$$\Delta_D = -\frac{b_D}{4} \log\left(\frac{\pi^2}{4} \left|\theta_3(U)\right|^4 \mathrm{Im}\, T\, \mathrm{Im}\, U \right) + \mathcal{O}\left(\frac{1}{\sqrt{\mathrm{Im}\, T}} \right)$$

Sector E: $\begin{vmatrix} h & H_1 \\ g & G_1 \end{vmatrix} \neq 0$

In this sector, $\mathcal{N} = 2 \to \mathcal{N} = 0$ and $\mathcal{N} = 2' \to \mathcal{N} = 0'$.

- $(h, H_1) \neq (0,0) \implies$ the winding numbers around the large 2-torus are non-zero ($n_4 = 2N_4 + h, n_5 = 2N_5 + H_1$).
- Super massive strings $\implies$ exponentially suppressed contributions.

Sectors $(H_2, G_2) \neq (0,0)$

$$2^{\text{d}}\ \text{generator}: \ (X^4, X^5, X^6, X^7, X^8, X^9) \longrightarrow (-X^4, -X^5, X^6, X^7, -X^8, -X^9)$$
$$1^{\text{st}} \times 2^{\text{d}}: \ (X^4, X^5, X^6, X^7, X^8, X^9) \longrightarrow (-X^4, -X^5, -X^6, -X^7, X^8, X^9)$$

- The first (and large) 2-torus is twisted.
- In these sectors, there is no dependance in the moduli T, U.

$$\partial X^{4,5} = \text{constant mode} + \text{oscillating modes}$$

  where the constant mode contains the dependance on the shape and volume. There are none here.
- $\Longrightarrow$ At tree level, these sectors are independent of M_{susy} and are supersymmetric. Susy is broken by gauge and gravitational interactions with the non susy states in quantum loops.
- However, these sectors depend either on T_2, U_2 or T_3, U_3. These moduli must be close to 1 to not introduce the decompactification problem back.

- For these sectors $I = 2, 3$:

$$\Delta_I = \frac{b_I}{2}\,\Delta(T_I, U_I) - \frac{k}{2}\,Y(T_I, U_I)$$

where

$$\Delta(T_I, U_I) = -\log\Big(4\pi^2 \big|\eta(T_I)\big|^4 \, \big|\eta(U_I)\big|^4 \, \mathrm{Im}\, T_I \, \mathrm{Im}\, U_I\Big)\,,$$

$$Y(T_I, U_I) = \frac{1}{12}\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2}\,\Gamma_{2,2}(T_I, U_I)\Big[\Big(\bar{E}_2 - \frac{3}{\pi\tau_2}\Big)\frac{\bar{E}_4\bar{E}_6}{\bar{\eta}^{24}} - \bar{\jmath} + 1008\Big]$$

Outline

Summary

Effective potential

In total, we have

- one $\mathcal{N} = 4 \rightarrow \mathcal{N} = 0$ sector B, which contributes
- 4 susy sectors C, D, $I = 2, 3$
- all other sectors are exponentially suppressed, $\mathcal{O}(e^{-c\sqrt{\operatorname{Im} T}})$.

$$\mathcal{V} = \mathcal{V}_B \propto \frac{1}{(\operatorname{Im} T)^2} \propto M_{\text{susy}}^4$$

- No M_{Planck}^4 correction : The cosmological constant is not of order the Planck scale. Because $\mathcal{N} = 1 \rightarrow \mathcal{N} = 0$.
- No $M_{\text{susy}}^2 M_{\text{Planck}}^2$ because with one freely acting $\mathbb{Z}_2$, the $\mathcal{N} = 2 \rightarrow \mathcal{N} = 0$ sectors are exponentially suppressed.

Gauge couplings

Define moduli-dependent scales appearing in the Δ's

$$\frac{M_B}{M_{\text{Planck}}} = \left(\left|\theta_2(U)\right|^4 \operatorname{Im} T \operatorname{Im} U\right)^{-\frac{1}{2}} \sim 10^{-15}$$

$$\frac{M_C}{M_{\text{Planck}}} = \left(\left|\theta_4(U)\right|^4 \operatorname{Im} T \operatorname{Im} U\right)^{-\frac{1}{2}} \sim 10^{-15}$$

$$\frac{M_D}{M_{\text{Planck}}} = \left(\left|\theta_3(U)\right|^4 \operatorname{Im} T \operatorname{Im} U\right)^{-\frac{1}{2}} \sim 10^{-15}$$

$$\frac{M_I}{M_{\text{Planck}}} = \left(16 \left|\eta(T_I)\right|^4 \left|\eta(U_I)\right|^4 \operatorname{Im} T_I \operatorname{Im} U_I\right)^{-\frac{1}{2}} \sim 1\,, \quad I = 2,3$$

$$\frac{16\,\pi^2}{g^2_{\rm YM}(\mu)} = k\left(\frac{16\,\pi^2}{g^2_{\rm string}} - \frac{1}{2}Y(T_2,U_2) - \frac{1}{2}Y(T_3,U_3)\right)$$
$$- \frac{b_B}{4}\log\left(\frac{\mu^2}{\mu^2+M_B^2}\right) - \frac{b_C}{4}\log\left(\frac{\mu^2}{\mu^2+M_C^2}\right) - \frac{b_D}{4}\log\left(\frac{\mu^2}{\mu^2+M_D^2}\right)$$
$$- \frac{b_2}{2}\log\left(\frac{\mu^2}{M_2^2}\right) - \frac{b_3}{2}\log\left(\frac{\mu^2}{M_3^2}\right) \qquad + \mathcal{O}\left(\frac{1}{\sqrt{\operatorname{Im} T}}\right)$$

- Written this way, for $\mu \geq M_{B,C,D}$ the sector B, C, D decouples.
- This expression is valid up to the Planck scale, $\mu \leq M_{\rm Planck}$.
- It is universal, up to k and the β-function coeff.in each sectors

$$b_B = -\frac{8}{3}\left\{C(\mathcal{A}_B) - C(\mathcal{R}_B)\right\} \qquad b_2 = -2\left\{C(\mathcal{A}_2) - C(\mathcal{R}_2)\right\}$$
$$b_C = -2\left\{C(\mathcal{A}_C) - C(\mathcal{R}_C)\right\} \qquad b_3 = -2\left\{C(\mathcal{A}_3) - C(\mathcal{R}_3)\right\}$$
$$b_D = -2\left\{C(\mathcal{A}_D) - C(\mathcal{R}_D)\right\}$$

where $C(\mathcal{R})\delta^{ab} = \operatorname{Tr}(T^a T^b)$