Thermoelectric characteristics of $\mathbb{Z}_k$ parafermion Coulomb islands

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Support: AvH, INRNE-BAS, NCSR-BG, ESF

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Non-Abelian anyons and topological QC

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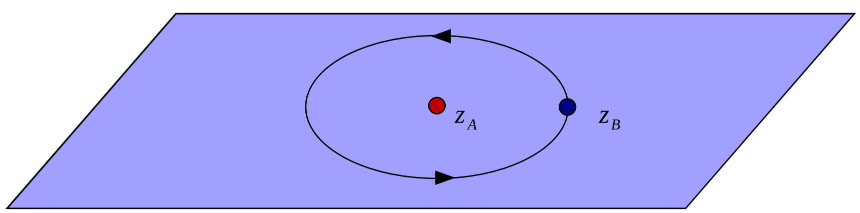
Distinguishable particles

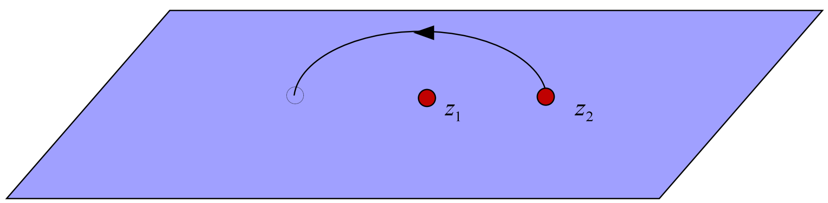
$$\langle \cdots \psi_A(z_A)\psi_B(z_B)\cdots\rangle \to e^{i\,2\theta_{AB}}\langle \cdots \psi_A(z_A)\psi_B(z_B)\cdots\rangle$$



where θ_{AB} is called the mutual statistical phase,

Indistinguishable particles: statistical angle θ_A/π

$$\langle \cdots \psi_A(z_1)\psi_A(z_2)\cdots\rangle \to \langle \cdots \psi_A(z_2)\psi_A(z_1)\cdots\rangle =$$
$$e^{i\pi(\theta_A/\pi)} \langle \cdots \psi_A(z_1)\psi_A(z_2)\cdots\rangle$$



3D: only $\theta_A/\pi = 0$ (bosons) and $\theta_A/\pi = 1$ (fermions)

2D: Laughlin anyons $\theta_L/\pi = 1/3$

“bos-ons”	“any-ons”	“fermi-ons”
$\theta_A/\pi = 0$	$0 < \theta_A/\pi < 1$	$\theta_A/\pi = 0$

$SO(3)$ has a compact simply connected covering group $SU(2) \to$ any 4π-rotation is $\simeq \mathbb{I} \Rightarrow U(2\pi) = \mathrm{e}^{-2\pi i J} = \pm\mathbb{I}$ $\Rightarrow \theta/\pi = 2J = 0$ or 1 mod 2.

$SO(2)$ does not have $\Rightarrow$ NO fermion-boson alternative.

• Construction of n-particle states:

3D: representations of $\mathcal{S}_n$ (symmetric for bosons, antisymmetric for fermions)

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2D: representations of $\mathcal{B}_n$.

Recall that $\mathcal{S}_n$ can be generated by σ_i, $1 \le i \le n-1$,

$$\sigma_i \sigma_j = \sigma_j \sigma_i , \quad |i-j| \ge 2$$

(disjoint transpositions commute) and

$$\sigma_i \, \sigma_{i+1} \, \sigma_i = \sigma_{i+1} \, \sigma_i \, \sigma_{i+1}, \quad (\sigma_i)^2 = \mathbb{I}.$$

$\mathcal{B}_n$: generated by B_i, $1 \le i \le n-1$, and their inverses ($B_i^{-1} \neq B_i$) *Artin relations*

$$B_i B_j = B_j B_i, \qquad \text{for} \quad |i-j| \geq 2$$

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states with many non-Abelian anyons **at fixed positions**:

specify the positions and q.n.s, (el. charge, single-particle energies and angular momenta)

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Fusion channels:

$\Psi_a \times \Psi_b = \sum_{c=1}^{g} {N_{ab}}^c \Psi_c$, $(N_{ab})^c$ (symmetric and associative)

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Example: Ising anyons: $\Psi_I(z) = \sigma(z) \ : \mathrm{e}^{i \frac{1}{2\sqrt{2}} \phi(z)} :$

$$\sigma \times \sigma = \mathbb{I} + \psi.$$

Information encoding

Information encoding into the fusion channels

$$|0\rangle = (\sigma,\sigma)_{\mathbb{I}} \qquad \longleftrightarrow \qquad \sigma\times\sigma\to\mathbb{I}$$
$$|1\rangle = (\sigma,\sigma)_{\psi} \qquad \longleftrightarrow \qquad \sigma\times\sigma\to\psi.$$

topological quantity–it is is independent of the fusion process details depending only on the topology

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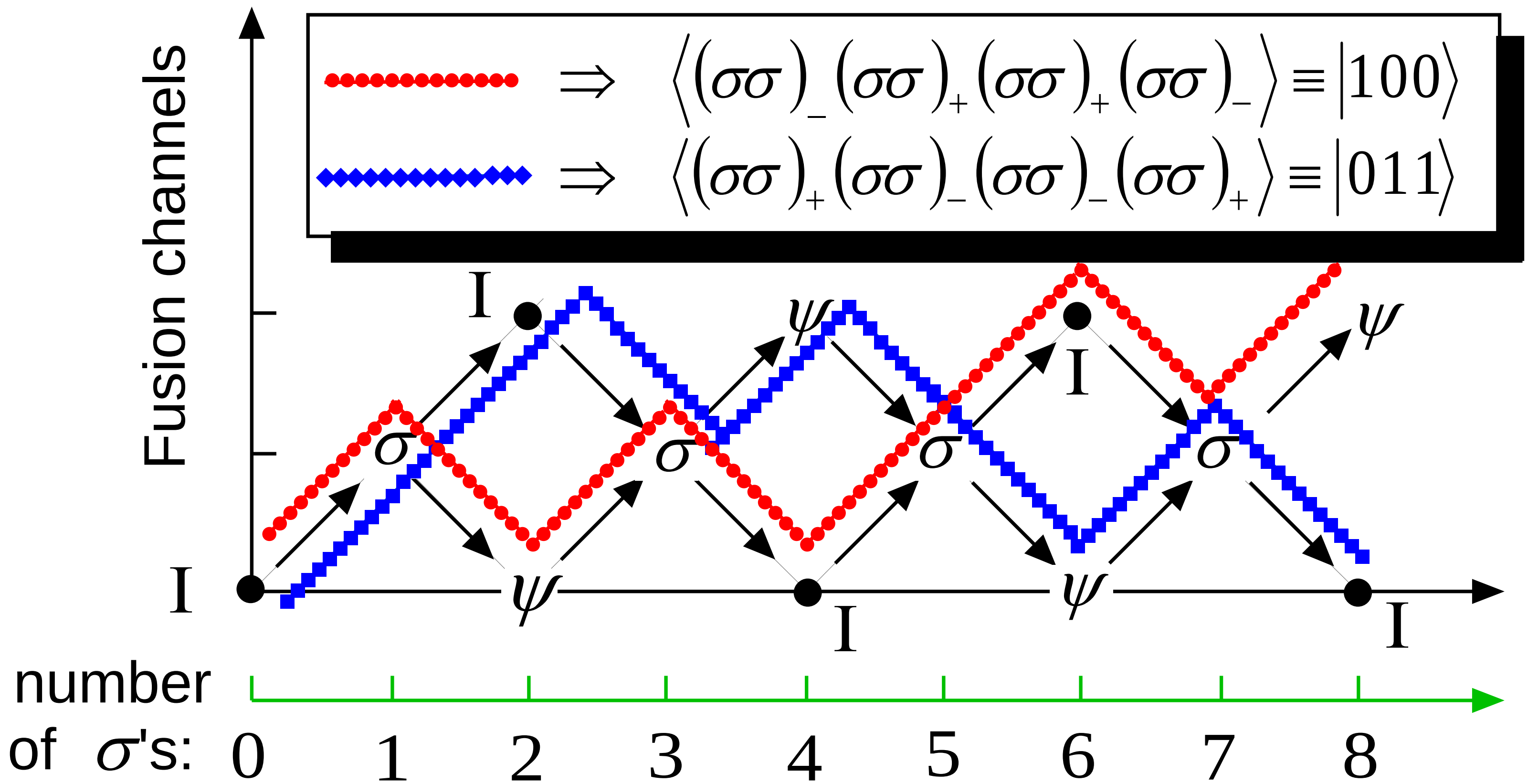
Message to be remembered:

quantum states with many non-Abelian anyons are specified/labeled by fusion paths


Fusion channels
$\Rightarrow \langle (\sigma\sigma)_- (\sigma\sigma)_+ (\sigma\sigma)_+ (\sigma\sigma)_- \rangle \equiv |100\rangle$
$\Rightarrow \langle (\sigma\sigma)_+ (\sigma\sigma)_- (\sigma\sigma)_- (\sigma\sigma)_+ \rangle \equiv |011\rangle$
I
σ
ψ
number of σ's:
0 1 2 3 4 5 6 7 8

non-Abelian statistics (illustration): 8 Ising anyons

$$
\begin{aligned}
|000\rangle &\equiv \langle [\sigma(\eta_1)\sigma(\eta_2)]_+ [\sigma(\eta_3)\sigma(\eta_4)]_+ [\sigma(\eta_5)\sigma(\eta_6)]_+ [\sigma(\eta_7)\sigma(\eta_8)]_+ \rangle \\
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\end{aligned}
$$

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☞ L.G., J. Phys. A: Math. Theor. 42 (2009) 225203

$$\left(B_6^{(8,+)}\right)^2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

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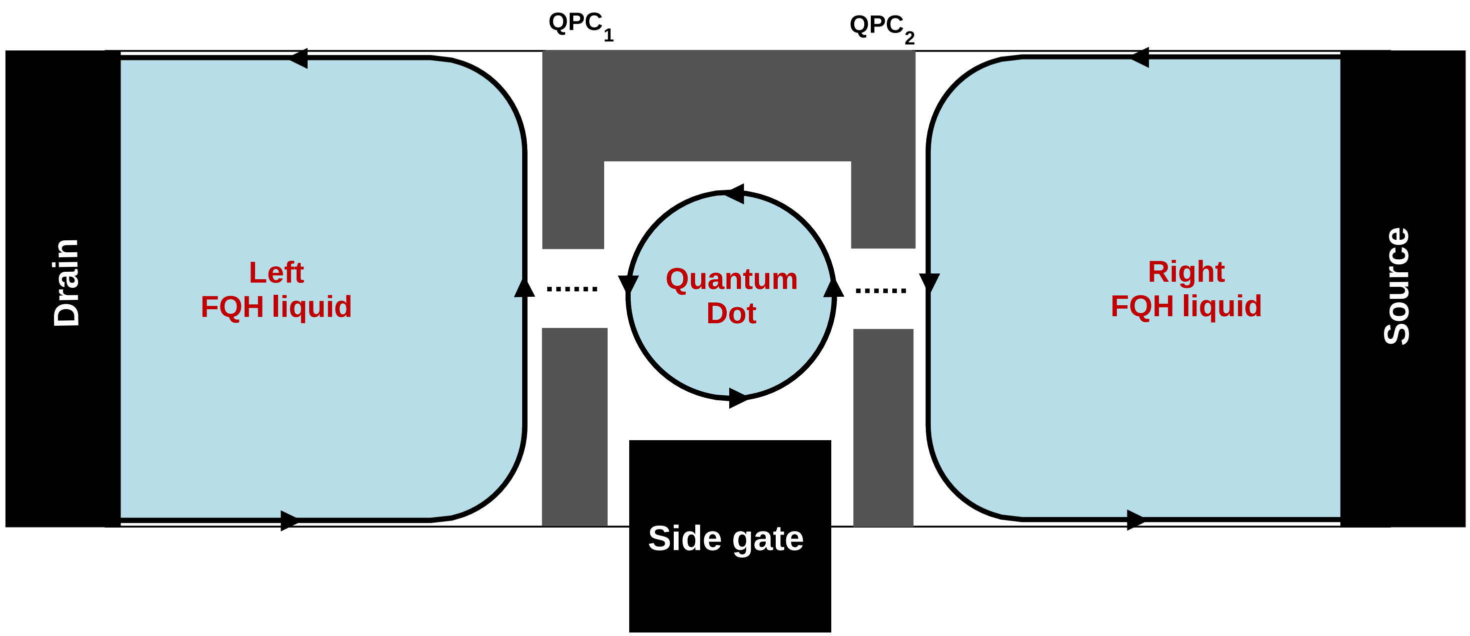
Coulomb-blockade conductance spectrometry

Coulomb-blockaded island: QD+gates=SET



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- **Electron number** ☞ [L.G., EPL 91 (2010) 41001]

$$\begin{aligned}\langle N_{\rm el}(\phi)\rangle_{\beta,\mu_N} &= -\frac{\partial \Omega_\phi(\beta,\mu_N)}{\partial\phi} + \nu_H\phi + \nu_H\left(\frac{\mu_N}{\Delta\epsilon}\right)\\ &= \nu_H\left(\phi + \frac{\mu_N}{\Delta\epsilon}\right) + \frac{1}{2\pi^2}\left(\frac{T}{T_0}\right)\frac{\partial}{\partial\phi}\ln Z_\phi(T,\mu_N)\end{aligned}$$

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- **Edge conductance** ☞ [L.G., EPL 91 (2010) 41001]

$$G_{\mathrm{is}}(\phi) = \frac{e^2}{h}\left(\nu_H + \frac{1}{2\pi^2}\left(\frac{T}{T_0}\right)\frac{\partial^2}{\partial\phi^2}\ln Z_\phi(T,0)\right).$$

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- **Seebeck coefficient:** potential difference V between the leads of the SET when $\Delta T = T_R - T_L \ll T_L$, under the condition that $I = 0$

Usually $S = G_T/G$, where G and G_T are electric and thermal conductances, respectively. However,

$$S \equiv - \lim_{\Delta T \to 0} \frac{V}{\Delta T}\bigg|_{I=0} = -\frac{\langle \varepsilon \rangle}{eT}, \quad \langle \varepsilon \rangle = \mathrm{av.\ tun.\ energy}$$

is better for SET as $G_T \to 0$ and $G \to 0$ in the CB valleys.

- **CFT approach:** partition function for the FQH edge of QD

$$\langle \varepsilon \rangle^{\phi}_{\beta,\mu_N} = E^{\beta,\mu_{N+1}}_{\mathrm{QD}}(\phi) - E^{\beta,\mu_N}_{\mathrm{QD}}(\phi)$$

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- **Total QD energy** (Grand canonical ensemble)

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$\langle \cdots \rangle_{\beta,\mu}$ is the Grand canonical average of H_{CFT} on the edge

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- **The thermoelectric power factor $\mathcal{P}_T$:**

electric power P generated by ΔT

$$P = V^2/R = \mathcal{P}_T(\Delta T)^2, \quad \mathcal{P}_T = S^2 G,$$

where $R = 1/G$ is the electric resistance of the CB island.

Average tunneling energy: ☞ L.G., Nucl. Phys. B 894 (2015) pp. 284–306

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Edge energy average:

$$\langle H_{\rm CFT}(\phi) \rangle_{\beta,\mu_N} = \Omega_\phi(T,\mu_N) - T\frac{\partial \Omega_\phi(T,\mu_N)}{\partial T} - \mu_N \frac{\partial \Omega_\phi(T,\mu_N)}{\partial \mu}$$

where $\Omega_\phi(T,\mu_N) = -k_B T \ln Z_\phi(T,\mu)$ is the Grand potential in presence of AB flux ϕ.

$\mathbb{Z}_k$ parafermion quantum Hall islands

- **CFT:**

$$\left(\widehat{u(1)} \otimes \frac{\widehat{su(k)}_1 \oplus \widehat{su(k)}_1}{\widehat{su(k)}_2} \right)^{\mathbb{Z}_k}$$

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- **Total disk partition function:** ☞ A. Cappelli, L.G., I.T. Todorov, Nucl. Phys. B 599 (2001), pp. 499

$$\chi_{l,\rho}(\tau,\zeta) = \sum_{s=0}^{k-1} K_{l+s(k+2)}(\tau, k\zeta; k(k+2)) \mathrm{ch}(\Lambda_{l-\rho+s} + \Lambda_{\rho+s})(\tau')$$

Labels: $l \bmod k+2$, $\rho \bmod k : l - \rho \leq \rho \bmod k$.

Experimental indication:

Different Fermi velocities of the charged and neutral modes:

$$v_n < v_c, \quad r = \frac{v_n}{v_c}, \quad \tau = i\frac{\hbar v_c}{k_B L T}, \quad \tau' = r\tau,$$

- **The Luttinger liquid partition function:** $(R_c = 1/m)$

$$K_l(\tau, \zeta; m) = \frac{\mathrm{CZ}}{\eta(\tau)} \sum_{n=-\infty}^{\infty} q^{\frac{m}{2}\left(n+\frac{l}{m}\right)^2} \mathrm{e}^{2\pi i \zeta \left(n+\frac{l}{m}\right)}$$

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$$q = \mathrm{e}^{-\beta\Delta\varepsilon} = \mathrm{e}^{2\pi i \tau}, \quad \Delta\varepsilon = \hbar\frac{2\pi v_F}{L}$$

Support: AvH, INRNE-BAS, NCSR-BG, ESF

- **Dedekind function and Cappelli–Zemba factors:**

$$\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n), \quad \mathrm{CZ} = \mathrm{e}^{-\pi \nu_H \frac{(\mathrm{Im}\, \zeta)^2}{\mathrm{Im}\, \tau}}$$

- **Neutral partition function:** $\mathrm{ch}(\Lambda_\mu + \Lambda_\rho)(\tau)$

$$\mathrm{ch}_{\sigma,Q}(\tau) = q^{\Delta(\sigma)-\frac{c}{24}} \sum_{\substack{m_1,m_2,\ldots,m_{k-1}=0 \\ \sum_{i=1}^{k-1} i\, m_i \equiv Q \mod k}}^{\infty} \frac{q^{\underline{m}.C^{-1}.(\underline{m}-\Lambda_\sigma)}}{(q)_{m_1}\cdots(q)_{m_{k-1}}},$$

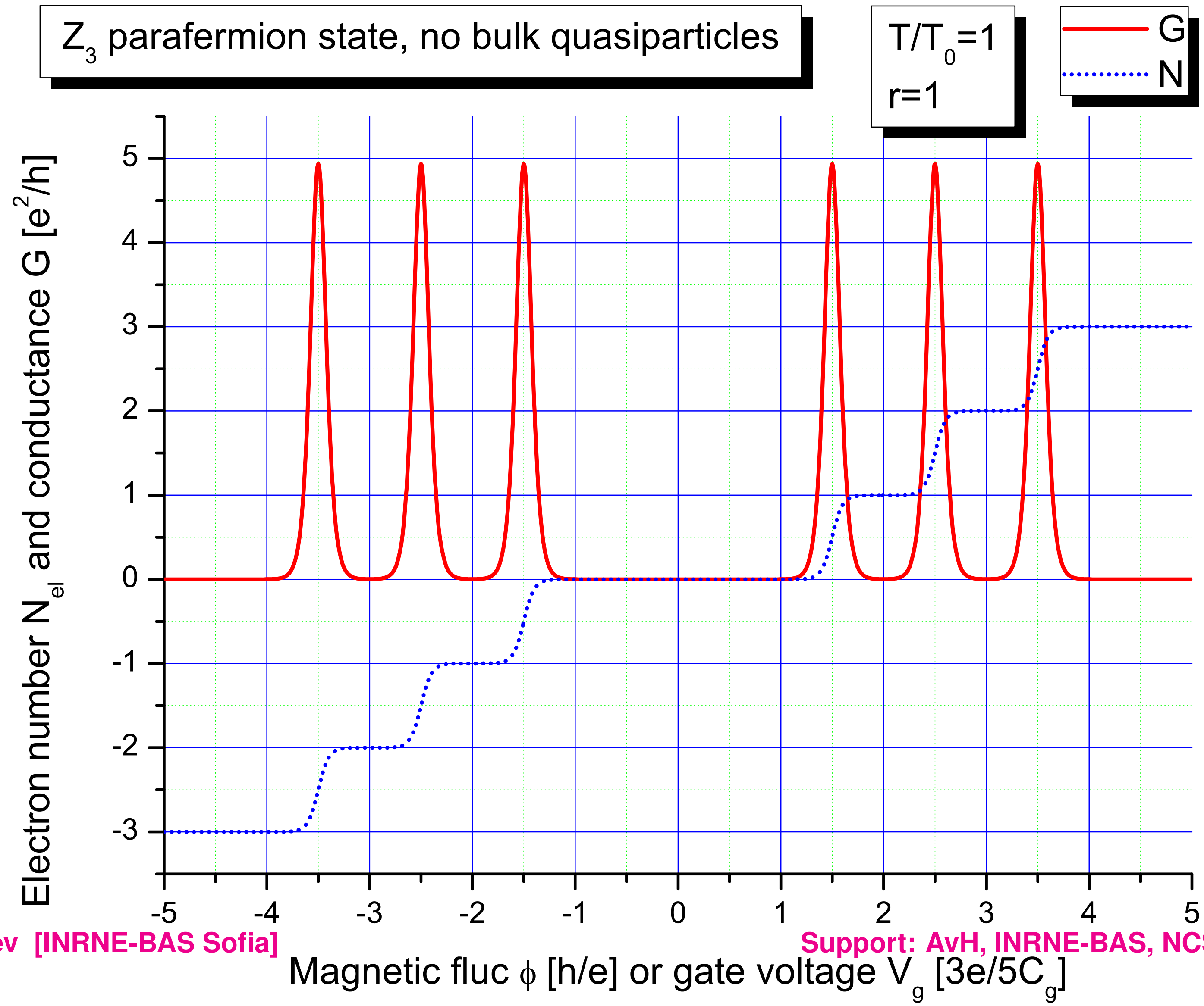
$$(q)_n = \prod_{j=1}^{n}(1-q^j), \quad \Delta(\sigma) = \frac{\sigma(k-\sigma)}{2k(k+2)}, \quad c = \frac{2(k-1)}{k+2}$$

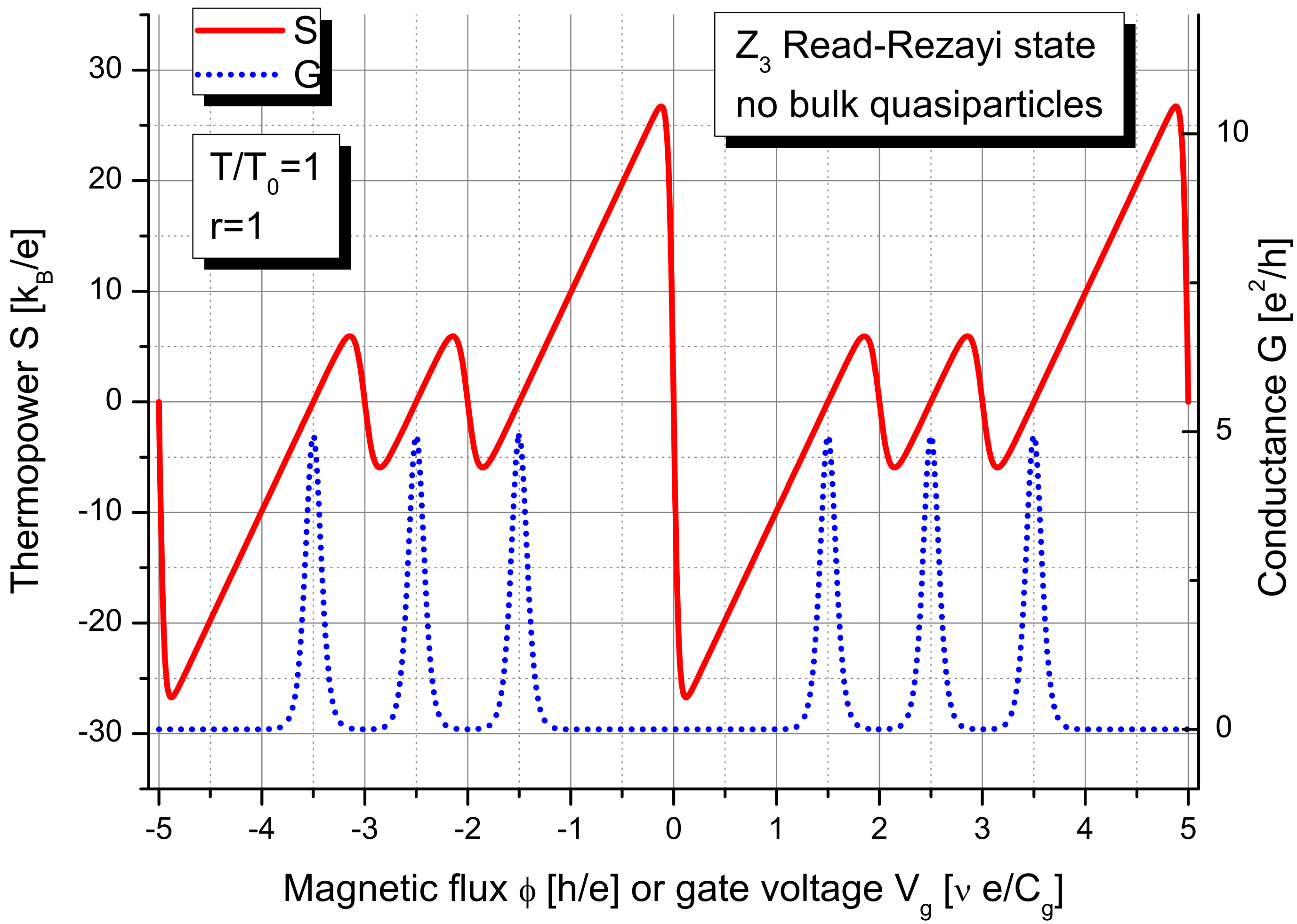
$\underline{m} = (m_1, \ldots, m_{k-1}), \quad 0 \le \sigma \le Q \le k-1.$

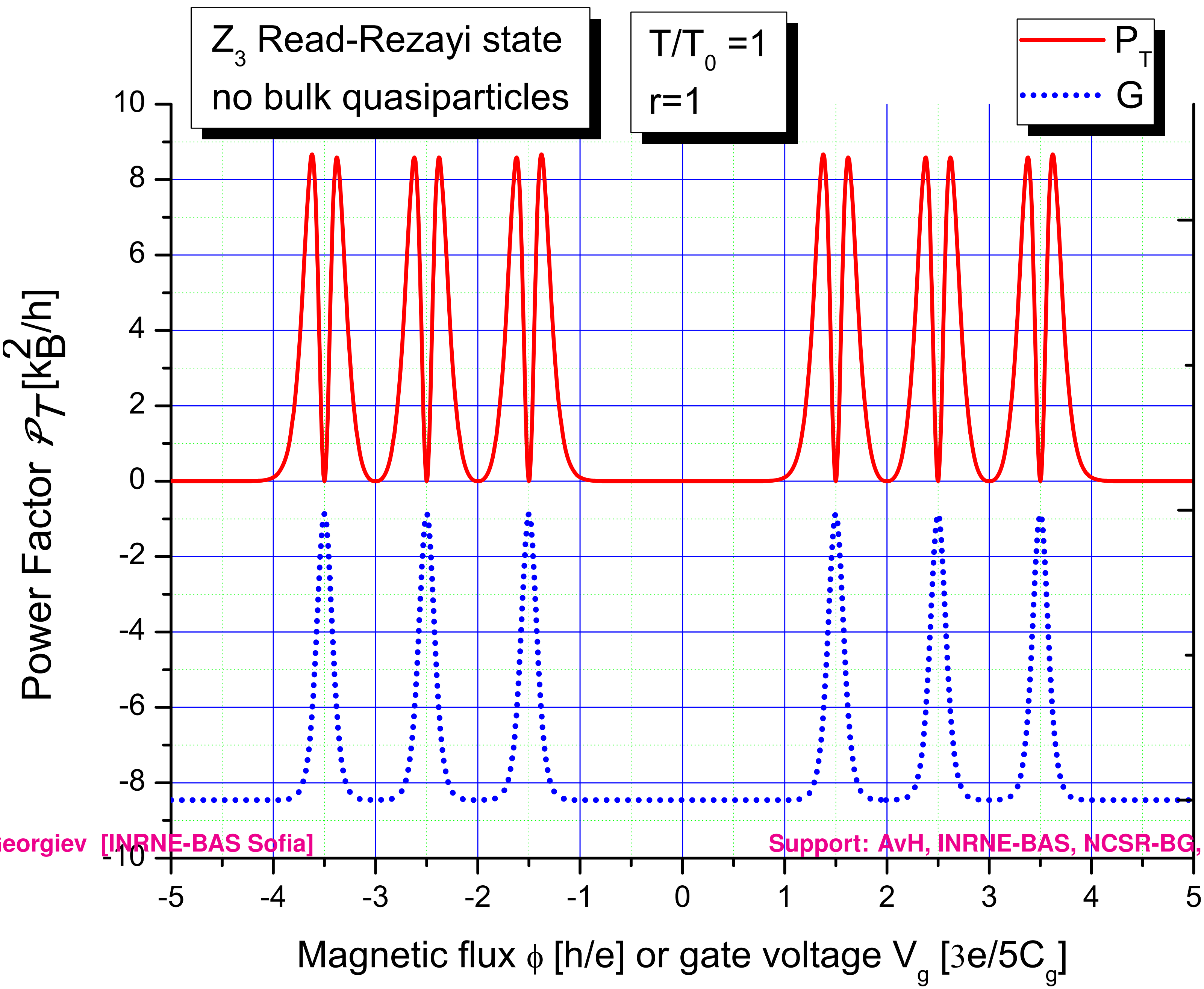
(Lepowsky, Primc, Schilling)

Relation: $\mu = Q - \sigma, \quad \rho = Q$

Support: AvH, INRNE-BAS, NCSR-BG, ESF


Z3 parafermion state, no bulk quasiparticles
T/T0=1
r=1
G
N
Electron number Nel and conductance G [e2/h]
Magnetic fluc ϕ [h/e] or gate voltage Vg [3e/5Cg]
5
4
3
2
1
0
-1
-2
-3
-5
-4
-3
-2
-1
0
1
2
3
4
5


S
G
T/T0=1
r=1
Z3 Read-Rezayi state
no bulk quasiparticles
Thermopower S [kB/e]
Conductance G [e2/h]
Magnetic flux ϕ [h/e] or gate voltage Vg [ν e/Cg]
30
20
10
0
-10
-20
-30
10
5
0
-5
-4
-3
-2
-1
0
1
2
3
4
5



Support: AvH, INRNE-BAS, NCSR-BG, ESF

Comparison with the experiment at $\nu_H = 2/3$

- **The experiment: “A new Hope”**

I. Gurman, R. Sabo, M. Heiblum, V. Umansky, D. Mahalu, *Extracting net current from an upstream neutral mode in the fractional quantum Hall regime*, **Nature Communications 3 (2012) 1289.**

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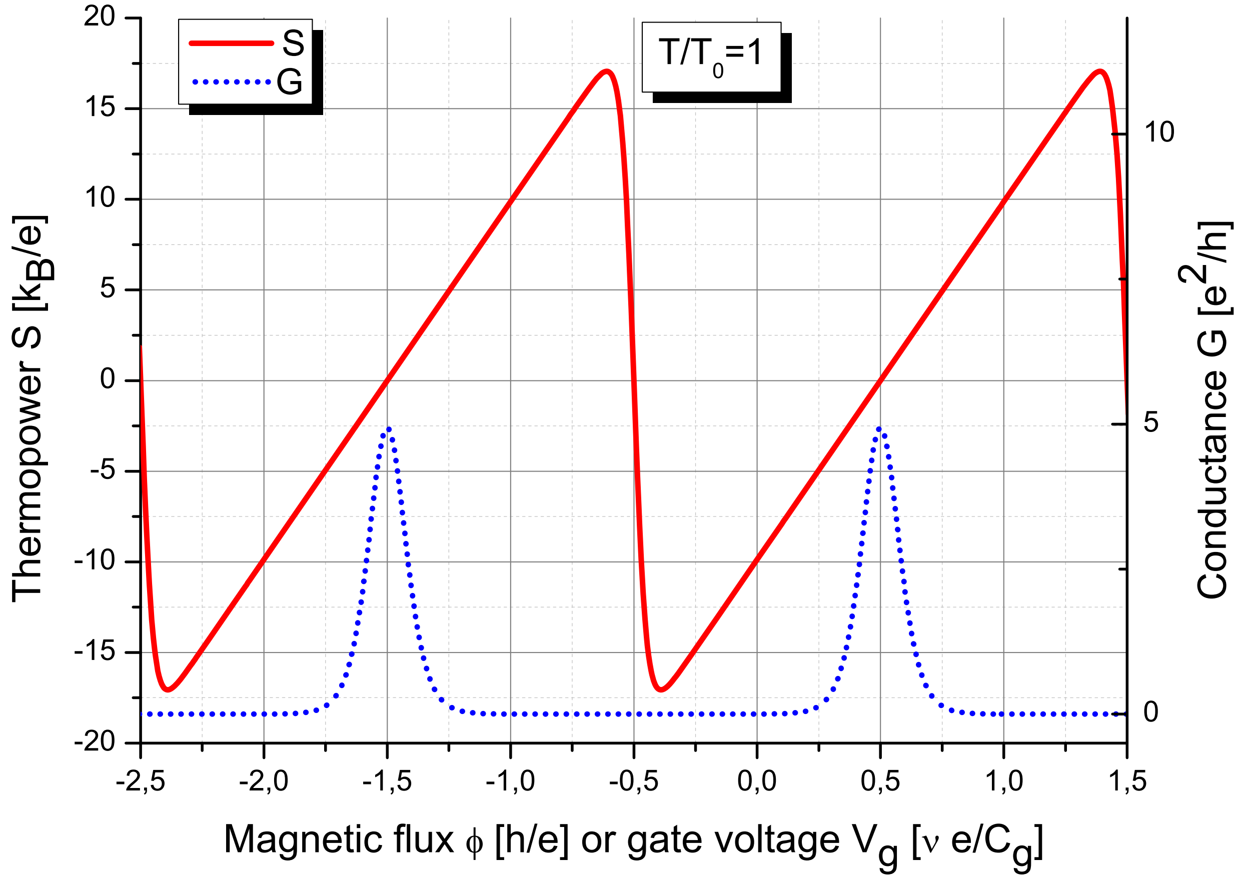
• **(Different) model to compare with: $\mathbb{Z}_2$ parafermion a.k.a (Moore–Read) Pfaffian state:**
Thermopower for odd number of bulk quasiparticles - the same as for the Abelian $\nu = 1/2$ Luttinger liquid ($R_c = 1/2$).

Theory: (Moore–Read) Pfaffian, odd



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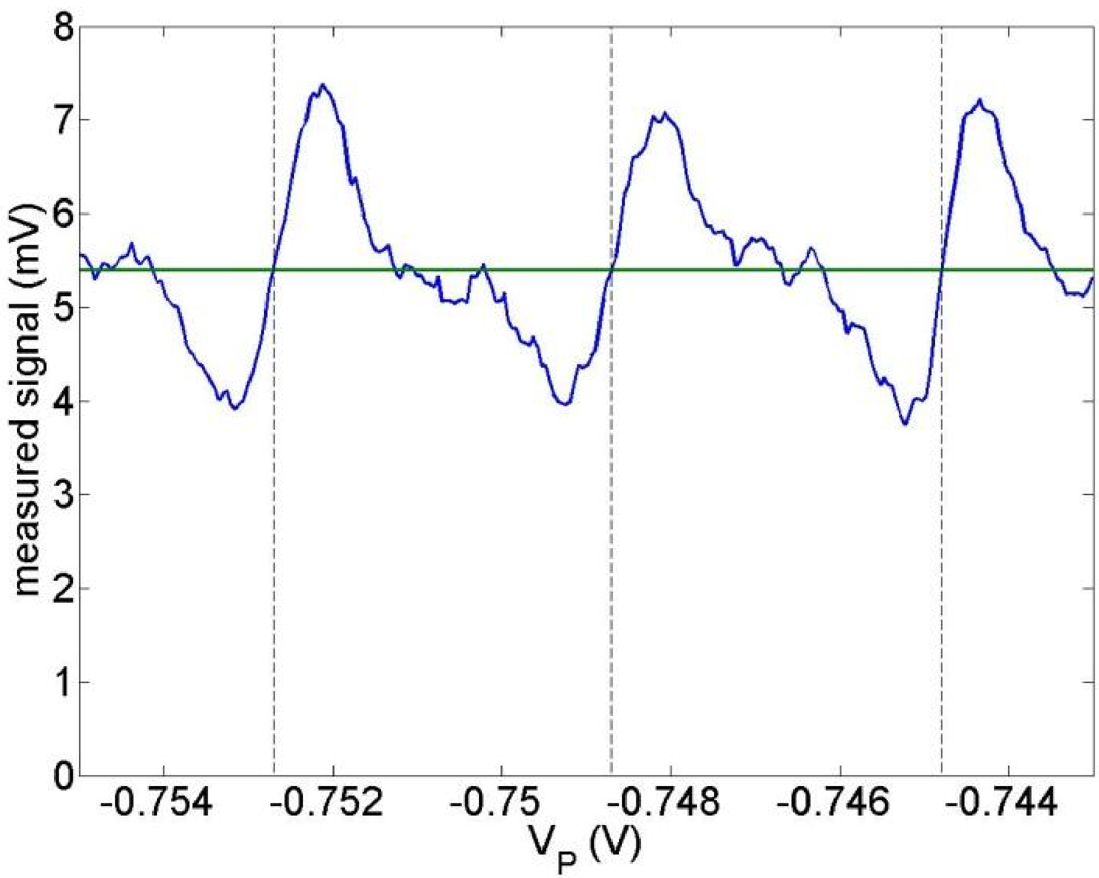
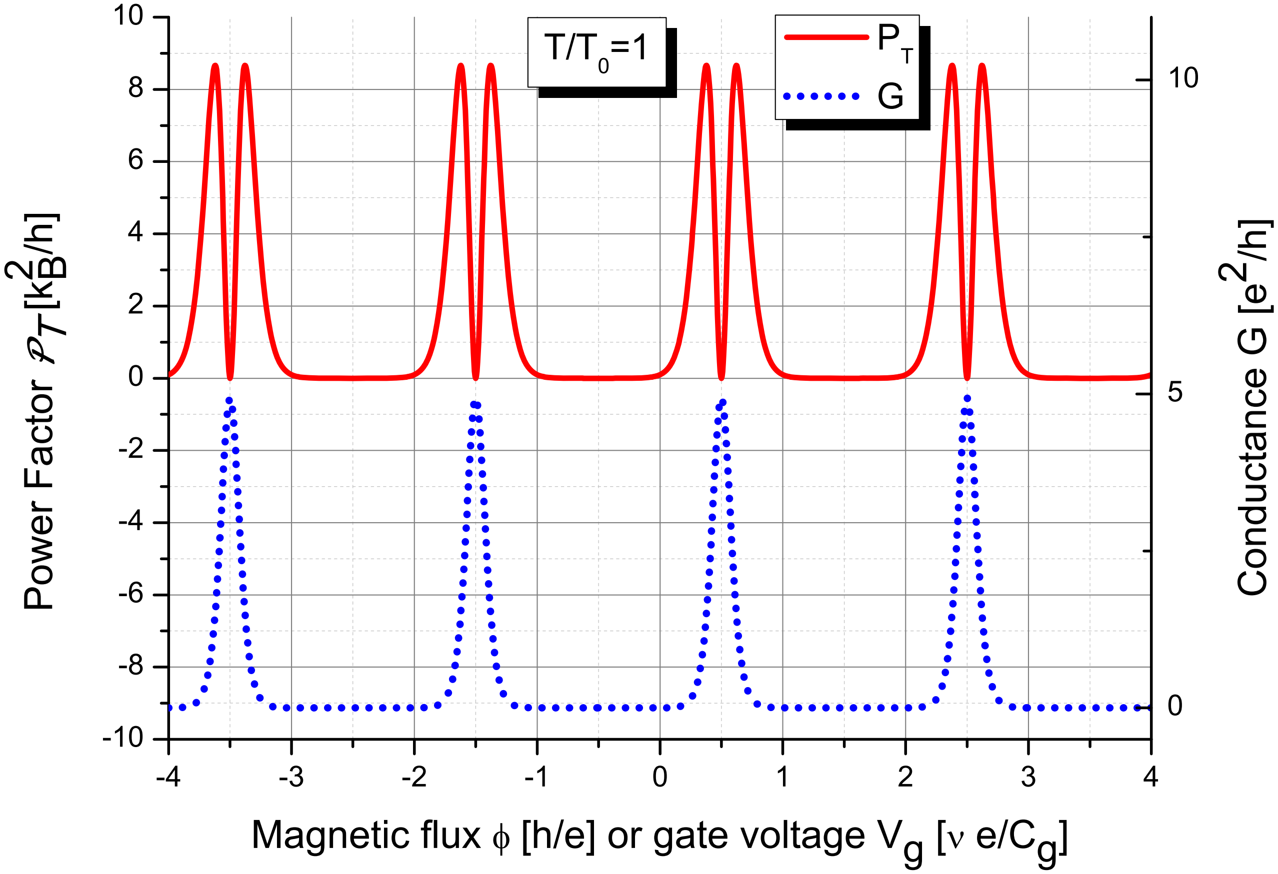


Power factor: theory (Moore–Read) Pfaffian state, odd



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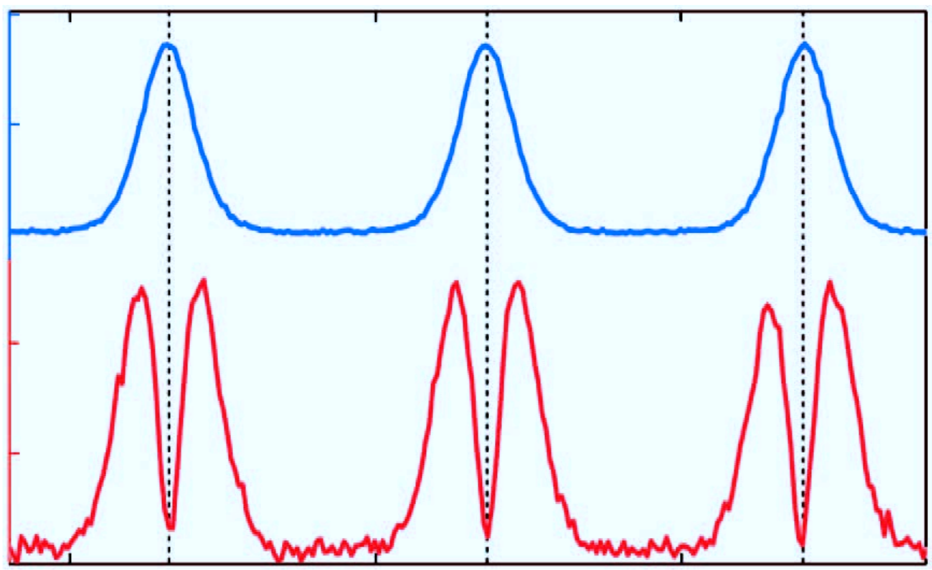

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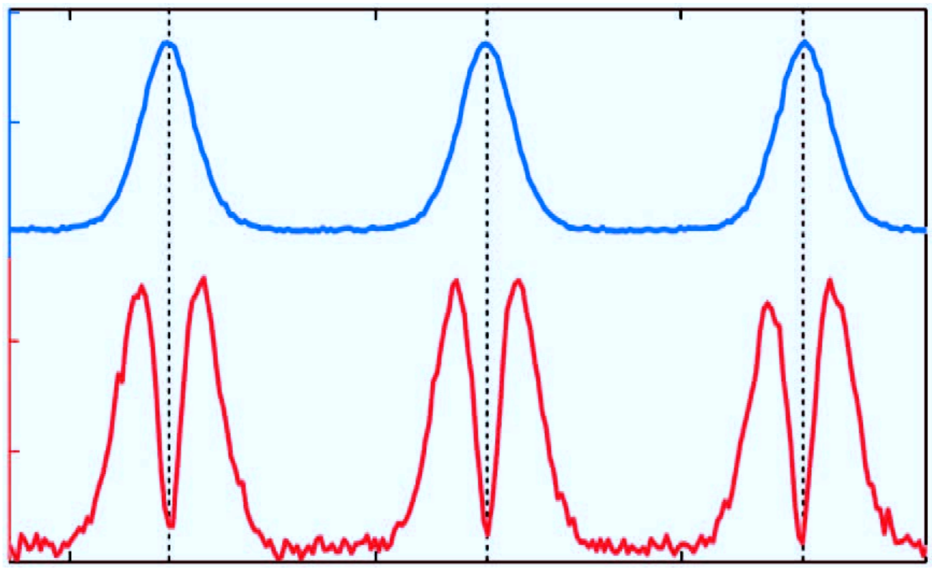
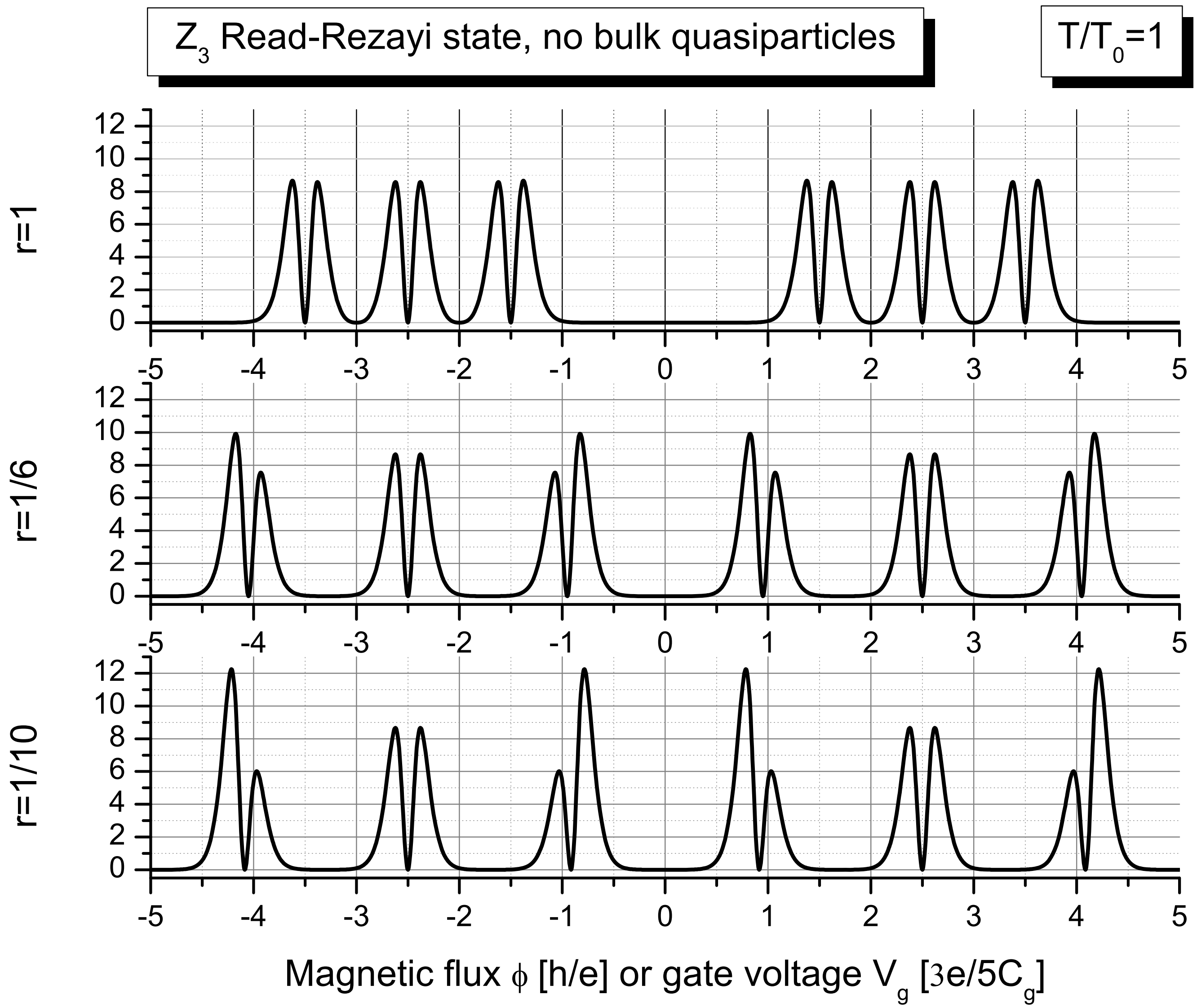
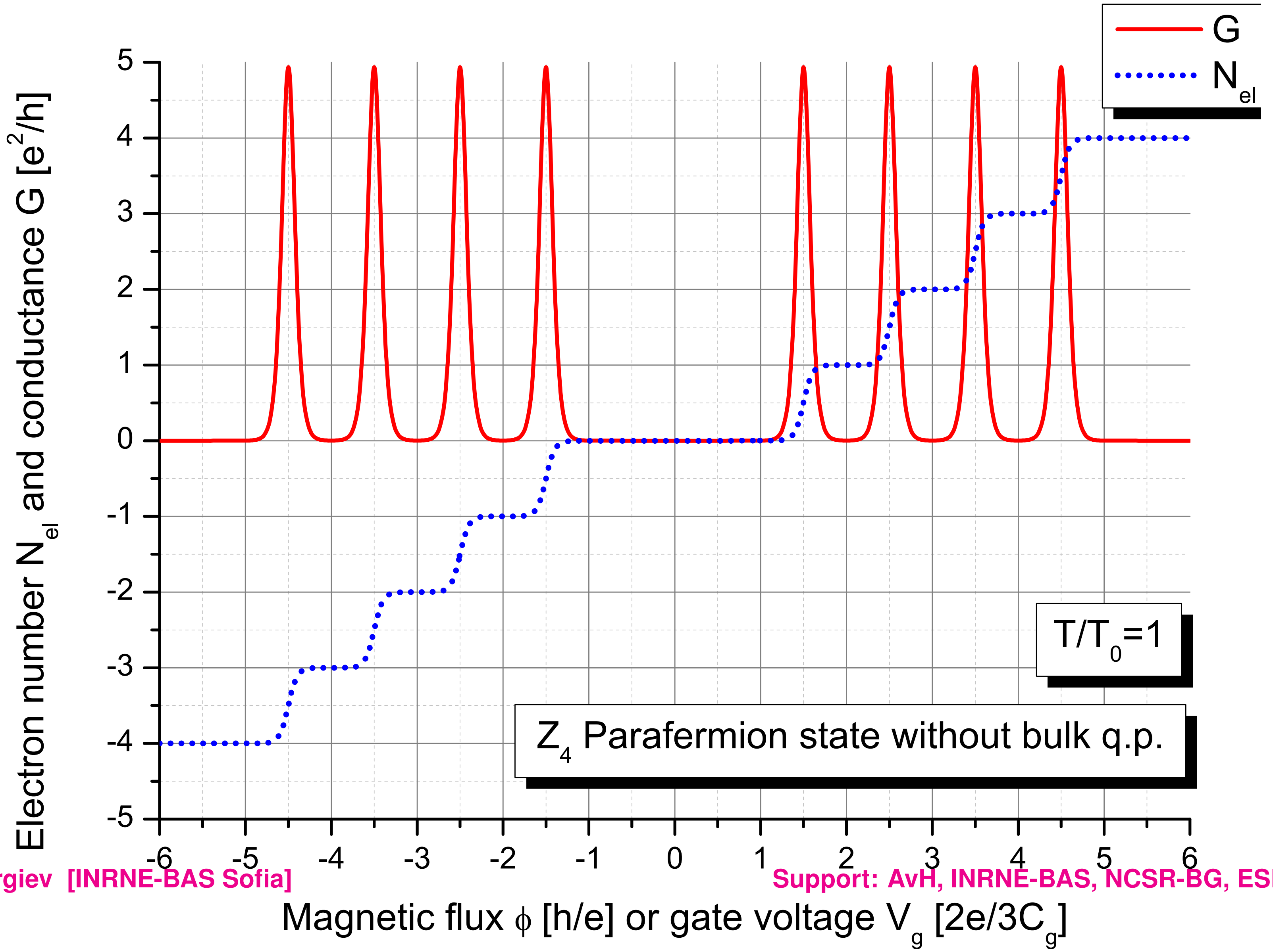
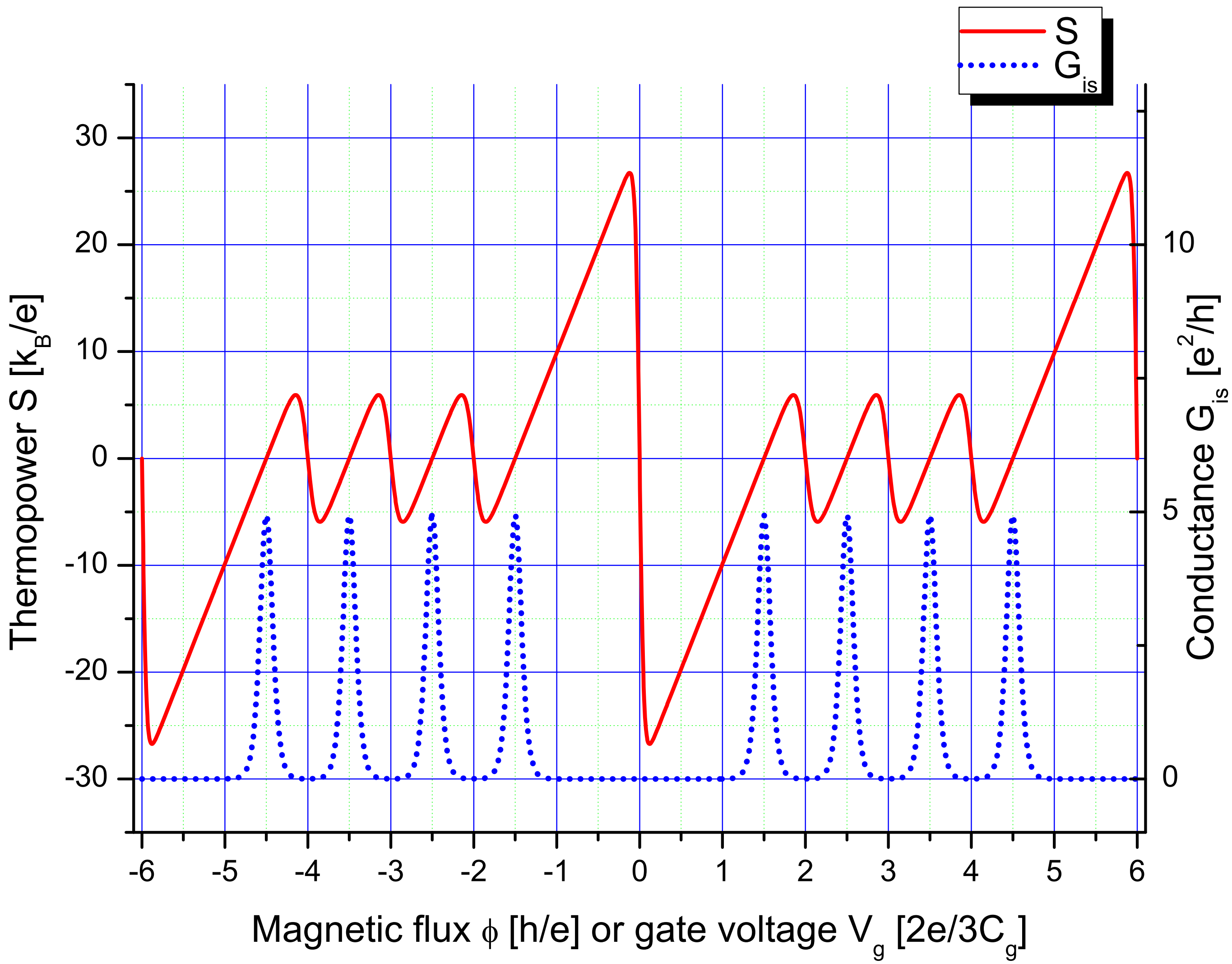


Power-factor profiles–the tool



Z3 Read-Rezayi state, no bulk quasiparticles
T/T0=1
r=1
r=1/6
r=1/10
12
10
8
6
4
2
0
-5
-4
-3
-2
-1
0
1
2
3
4
5
Magnetic flux ϕ [h/e] or gate voltage Vg [3e/5Cg]


G
N_{el}
T/T_0=1
Z_4 Parafermion state without bulk q.p.
Electron number N_{el} and conductance G [e^2/h]
Magnetic flux ϕ [h/e] or gate voltage V_g [2e/3C_g]
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
6


S
G_is
30
20
10
0
-10
-20
-30
Thermopower S [k_B/e]
10
5
0
Conductance G_is [e^2/h]
-6
-5
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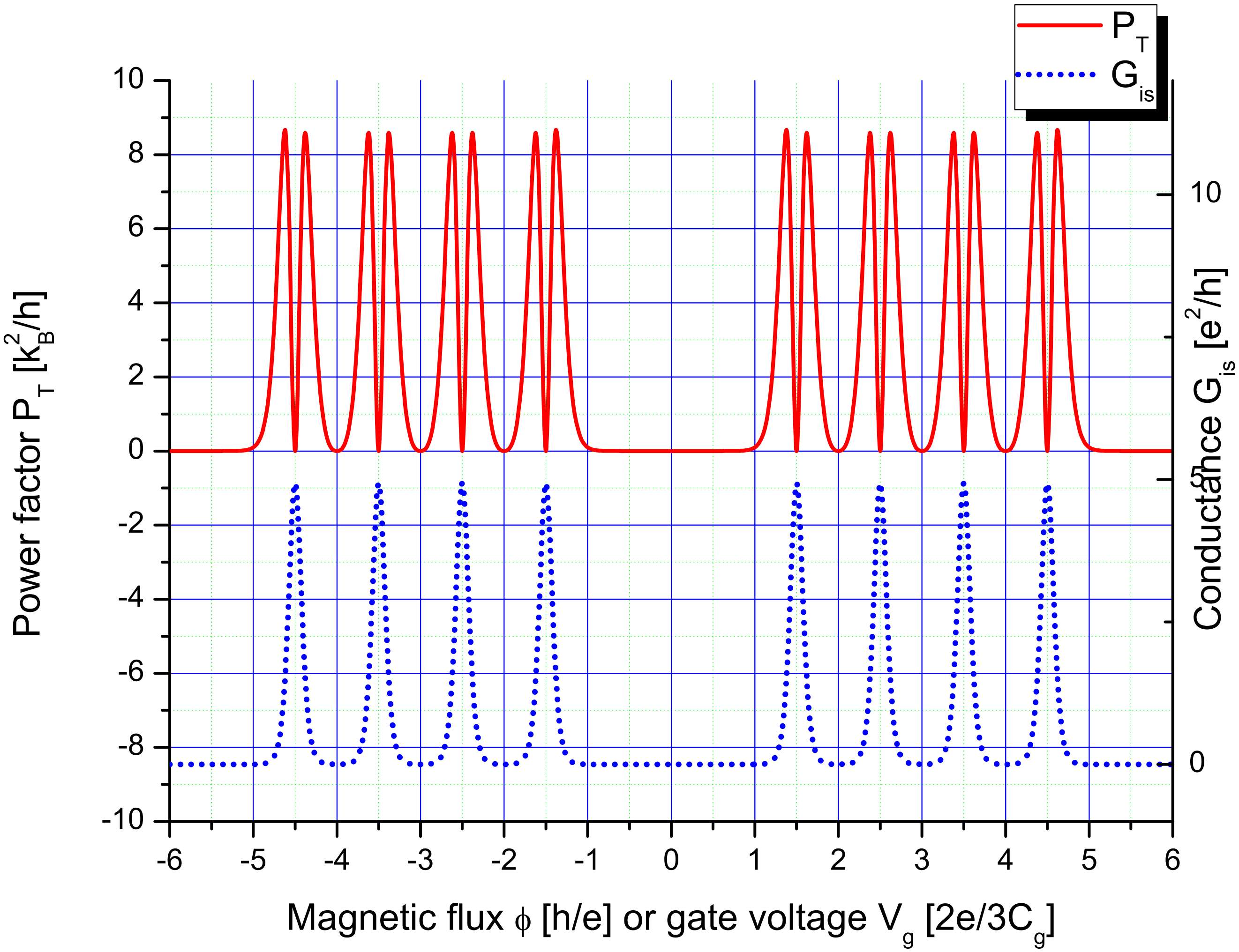
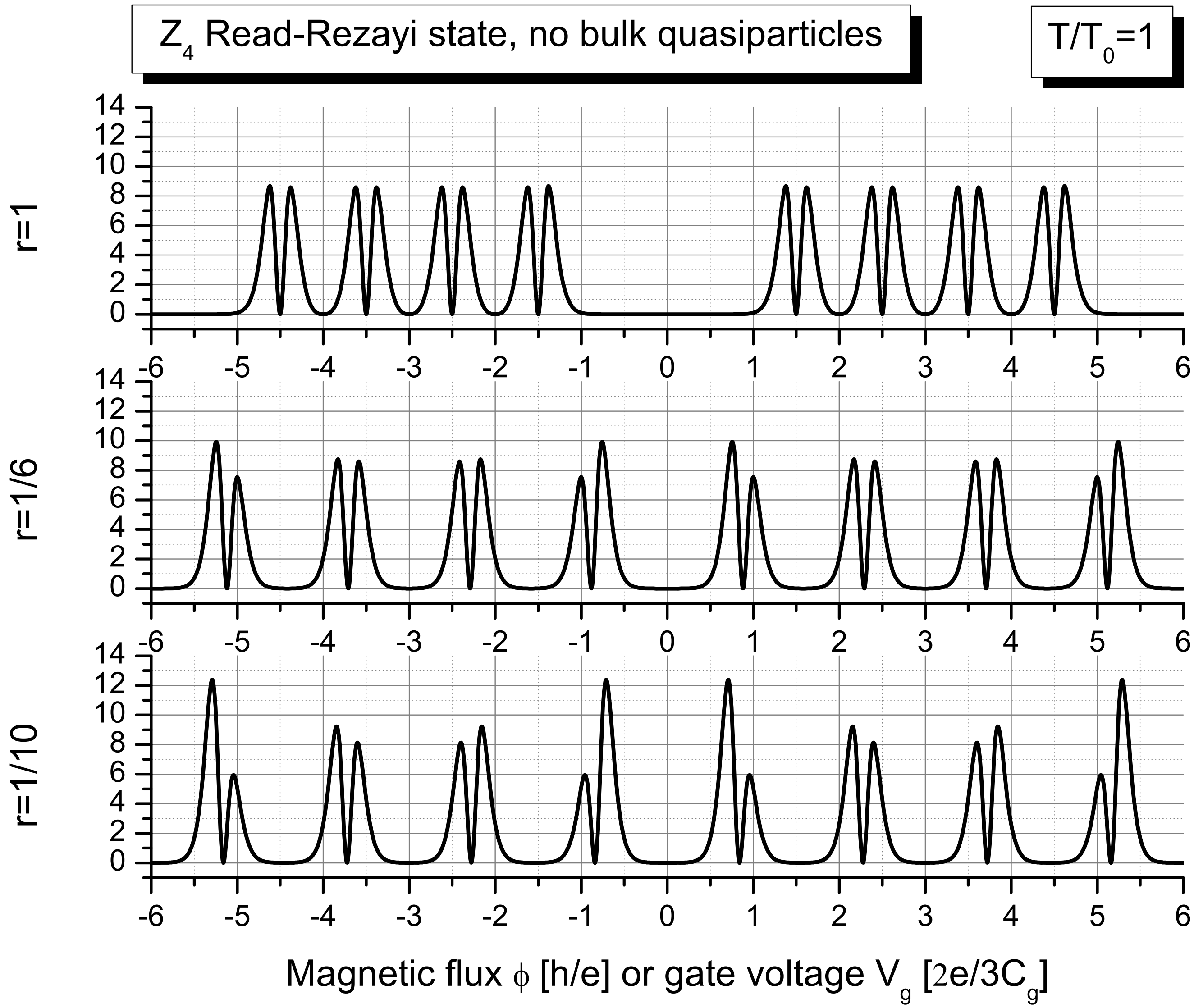
1
2
3
4
5
6
Magnetic flux ϕ [h/e] or gate voltage V_g [2e/3C_g]


P_T
G_is
Power factor P_T [k_B^2/h]
Conductance G_is [e^2/h]
10
8
6
4
2
0
-2
-4
-6
-8
-10
10
0
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
Magnetic flux ϕ [h/e] or gate voltage V_g [2e/3C_g]



Z_4 Read-Rezayi state, no bulk quasiparticles
$T/T_0=1$
r=1
r=1/6
r=1/10
14
12
10
8
6
4
2
0
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
Magnetic flux ϕ [h/e] or gate voltage V_g [2e/3C_g]

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