Bethe vector construction for ABCD serias

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R-matrix

$\mathfrak{gl}_2$ R-matrix

Let's start with the most basic R-matrix

$$R(u) = \begin{pmatrix} u+c & 0 & 0 & 0 \\ 0 & u & c & 0 \\ 0 & c & u & 0 \\ 0 & 0 & 0 & u+c \end{pmatrix}$$

Monodromy matrix of $Y(\mathfrak{gl}_2)$

Monodromy matrix

Monodromy matrix is 2×2 matrix with nocommutative elements

$$T(u) = \begin{pmatrix} A(u) & B(u) \\ C(u) & D(u) \end{pmatrix}$$

RTT

that satisfy RTT relation

$$R_{12}(u-v)T_1(u)T_2(v) = T_2(v)T_1(u)R_{12}(u-v),$$

where $T_1(u) = T(u) \otimes 1$ and $T_2(v) = 1 \otimes T(v)$.

Bethe vector

Vacuum

Let's suppose existence of special vector called *pseudovacuum vector* Ω, such that

$$C(u)\Omega = 0,$$
$$A(u)\Omega = \lambda_1(u)\Omega,$$
$$D(u)\Omega = \lambda_2(u)\Omega,$$

where $\lambda_i(u)$ are some scalar functions.

Standart construction

Then, usual construction for Bethe vector is

$$\mathbb{B}(\bar{u}) = B(u_1) \cdot \ldots \cdot B(u_n)\Omega$$

Eigenvector

Bethe vector becomes an eigenvector

$$t(z)\mathbb{B}(\bar{u}) = \tau(z|\bar{u})\mathbb{B}(\bar{u})$$

with eigenvalue

$$\tau(z|\bar{u}) = \lambda_1(z)\prod_{i=1}^{a}\frac{u_i - z + c}{u_i - z} + \lambda_2(z)\prod_{i=1}^{a}\frac{z - u_i + c}{z - u_i}$$

Bethe equations

if Bethe equations are satisfied

$$\frac{\lambda_1(u_i)}{\lambda_2(u_i)} = \prod_{k\neq i}\frac{u_i - u_k + c}{u_i - u_k - c}$$

Gauss decomposition of $Y(\mathfrak{gl}_2)$

Monodromy matrix

Let's consider change coordinates on monodromy matrix

$$T = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} k_1 & Fk_1 \\ k_1 E & k_2 + Fk_1 E \end{pmatrix}.$$

We call operators k_i, E, F by *semicurrent*.

This reparametrisation can be write in compact form

$$T(u)^t = \mathbb{F}(u)^t \cdot \mathbb{K}(u)^t \cdot \mathbb{E}(u)^t,$$

where t is transposition by the second diagonal, and

$$\mathbb{F} = \begin{pmatrix} 1 & F \\ 0 & 1 \end{pmatrix}, \qquad \mathbb{K} = \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}, \qquad \mathbb{E} = \begin{pmatrix} 1 & 0 \\ E & 1 \end{pmatrix}$$

Current representation

New commutation relations

We can rewrite RTT-relation for new coordinates

$$
\begin{aligned}
[k_i(u), k_j(v)] &= 0, \qquad\qquad i,j = 1,2,\\
F(u)k_1(v) &= \frac{v-u+c}{v-u}k_1(v)F(u) - \frac{c}{v-u}k_1(v)F(v),\\
F(u)k_2(v) &= \frac{v-u-c}{v-u}k_2(v)F(u) + \frac{c}{v-u}k_2(v)F(v),\\
k_1(v)E(u) &= \frac{v-u+c}{v-u}E(u)k_1(v) - \frac{c}{v-u}E(v)k_1(v),\\
k_2(v)E(u) &= \frac{v-u-c}{v-u}E(u)k_2(v) + \frac{c}{v-u}E(v)k_2(v),\\
[E(u), F(v)] &= \frac{c}{u-v}\left(\frac{k_2(u)}{k_1(u)} - \frac{k_2(v)}{k_1(v)}\right).
\end{aligned}
$$

And two more relations

$$(u-v-c)E(u)E(v) = \\ (u-v+c)E(u)E(v) - c\ \left(E(u)^2 + E(v)^2\right),$$

$$(u-v-c)F(v)F(u) = \\ (u-v+c)F(v)F(u) - c\ \left(F(u)^2 + F(v)^2\right).$$

Semicurrent representation of Bethe vectors

In terms of semicurrents Bethe vector can be rewrite as

$$\mathbb{B}(\bar{u}) = \prod_i \lambda_1(u_i) \prod_{i<j} \frac{1}{f(u_i,u_j)}$$
$$F(u_1)\cdot F(u_2;u_1)\cdot \ldots \cdot F(u_n;u_1,\ldots,u_{n-1})\Omega,$$

where

$$F(u_k;u_1,\ldots,u_{k-1}) = F(u_n) - \sum_{i=1}^{k-1} \frac{1}{h(u_i,u_k)} \prod_{j\neq i} \frac{f(u_j,u_i)}{f(u_j,u_k)} F(u_i).$$

We used functions

$$f(u,v) = \frac{u-v+c}{u-v}, \quad h(u,v) = \frac{u-v+c}{c}.$$

Double Yangian

Two copies of RTT

Let us consider two monodromy matrices $T^{\pm}(u)$ that satisfy 4 (four!) sets RTT relations

$$R_{12}(u,v)T_1^{\mu}(u)T_2^{\nu}(v) = T_2^{\nu}(v)T_1^{\mu}(u)R_{12}(u,v), \qquad \mu,\nu=\pm.$$

Full currents

For two monodromy matrices $T^{\pm}(u)$ there are two sets of Gauss coordinates $k_i^{\pm}, E^{\pm}, F^{\pm}$. Then we define *full currents*

$$\mathcal{F}(u) = F^{+}(u) - F^{-}(u), \qquad \mathcal{E}(u) = E^{+}(u) - E^{-}(u).$$

Current construction of Bethe vector

Projection

Let's define projection P_f^+. The projection is a linear operator. On the normal ordered term the projection acts as

$$P_f^+\left(F^- ... F^- \cdot F^+ \dots F^+\right) = 0,$$
$$P_f^+\left(F^+ \dots F^+\right) = F^+ \dots F^+.$$

Normal ordering

We call term ordered if term has form $F^- ... F^- \cdot F^+ \dots F^+$. One can make normal ordering using FF-commutation relation

$$(u-v-c)F^+(v)F^-(u) = \\ (u-v+c)F^-(v)F^+(u) - c\ \left(F^-(u)^2 + F^+(v)^2\right)$$

Projection formula

$$\mathbb{B}(\bar{u}) = \prod_i \lambda_1(u_i) \prod_{i<j} \frac{1}{f(u_i, u_j)} \times P_f^+\Big(\mathcal{F}(u_1) \cdot \ldots \cdot \mathcal{F}(u_n)\Big)\Omega.$$

And dropping the prefactor we can write

$$\mathbb{B}(\bar{u}) \sim P_f^+\Big(\mathcal{F}(u_1) \cdot \ldots \cdot \mathcal{F}(u_n)\Big)\Omega.$$

RTT and RRR

Monodromy matrix

The monodromy matrix $T(u)$ is $N \times N$ matrix which satisfies bilinear relations called RTT-relation

$$R_{12}(u,v)T_1(u)T_2(v) = T_2(v)T_1(u)R_{12}(u,v),$$

where R_{12} acts in (graded) tensor product of two spaces, $T_1(u) = T(u) \otimes 1$ and $T_2(v) = 1 \otimes T(v)$.

Yang-Baxter equation

Consistency condition of this algebra is Yang-Baxter equation

$$R_{12}(u,v)R_{13}(u,w)R_{23}(v,w) = R_{23}(v,w)R_{13}(u,w)R_{12}(u,v).$$

Transfer matrix

The transfer matrix is defined as the trace of the monodromy matrix

$$t(u) = \operatorname{tr} T(u) = \sum_i T_{ii}(u).$$

It defines an integrable system, due to the relation $[t(u)\,,\, t(v)] = 0$.

Hamiltonian

Usually, Hamiltonian is one of the coefficient of series expansion of the transfer matrix $t(u)$ or some combination of them.
For example, local Hamiltonian for spin chains

$$H = t(u)^{-1} \frac{\mathrm{d}}{\mathrm{d}u} t(u) \Big|_{u=0} .$$

Other coefficients are called symmetries or higher Hamiltonians.

Algebraic Bethe ansatz framework

Vacuum

Using this approach requires the existence of special vector called *pseudovacuum vector* Ω, such that

$$\begin{aligned} T_{i,j}(u)\Omega &= 0, \qquad\qquad i > j\,, \\ T_{i,i}(u)\Omega &= \lambda_i(u)\Omega, \end{aligned}$$

where $\lambda_i(u)$ are some scalar functions.

Bethe vectors belong to the space $\mathcal{H}$ in which the monodromy matrix entries act. We do not specify this space. However, we assume that it contains the *pseudovacuum vector* Ω.

Bethe vectors

Bethe vector

Typically, the Bethe vector can be represented as a polynomial in the elements of the monodromy matrix with different spectral parameters acting on the vacuum.

$$\mathbb{B}(u_1, \dots, u_n) = Pol(T_{ij}(u_1), \dots, T_{ij}(u_n))\Omega.$$

Eigenvector

The most important property of the Bethe vector is that it becomes an eigenvector

$$t(z)\mathbb{B}(\bar{u}) = \tau(z|\bar{u})\mathbb{B}(\bar{u})$$

when the spectral parameters satisfy certain constraints, called the *Bethe equations*.

R-matrix

$\mathfrak{gl}_n$ R-matrix

Let's consider the most typical R-matrix

$$R(u,v) = I + \frac{c}{u-v}P,$$

where P is the permutation operator

$$P = \sum_{i,j=1}^{n} e_{ij} \otimes e_{ji}.$$

RTT-relation

For this R-matrix one can RTT relation in components

$$[T_{ij}(u), T_{kl}(v)] = \frac{c}{u-v}\left(T_{kj}(u)T_{il}(v) - T_{kj}(v)T_{il}(u)\right).$$

Gauss decomposition

$\mathfrak{gl}_n - Yangian$

For $n \times n$ monodromy matrix the Gauss decomposition is

$$T(u)^t = \mathbb{F}(u)^t \cdot \mathbb{K}(u)^t \cdot \mathbb{E}(u)^t,$$

where t is transposition by the second diagonal, $\mathbb{F}$ is uppertriangular matrix, $\mathbb{K}$ is diagonal matrix, and $\mathbb{E}$ is lowerdiagonal matrix.

Ding-Frenkel isomorphism

Gauss decomposition describes isomorphism between RTT-algebra and *new realisation* of Yangian discovered by Drinfeld.

Ding-Frenkel isomorphism

New commutation relations

We can rewrite RTT-relation for new coordinates

$$
\begin{aligned}
[k_i(u), k_j(v)] &= 0, \qquad\qquad i,j = 1,\dots n,\\
F_i(u)k_i(v) &= \frac{v-u+c}{v-u} k_i(v)F_i(u) - \frac{c}{v-u} k_i(v)F_i(v),\\
F_i(u)k_{i+1}(v) &= \frac{v-u-c}{v-u} k_{i+1}(v)F_i(u) + \frac{c}{v-u} k_{i+1}(v)F_i(v),\\
k_i(v)E_i(u) &= \frac{v-u+c}{v-u} E_i(u)k_i(v) - \frac{c}{v-u} E_i(v)k_i(v),\\
k_{i+1}(v)E_i(u) &= \frac{v-u-c}{v-u} E_i(u)k_{i+1}(v) + \frac{c}{v-u} E_i(v)k_{i+1}(v),\\
[E_i(u), F_j(v)] &= \frac{c\,\delta_{i,j}}{u-v}\left(\frac{k_{i+1}(u)}{k_i(u)} - \frac{k_{i+1}(v)}{k_i(v)}\right).
\end{aligned}
$$

Here we use notation $F_i = F_{i+1,i}$ and $E_i = E_{i,i+1}$.

And few more relations

$$(u-v-c)E_i(u)E_i(v) = \\ (u-v+c)E_i(u)E_i(v) - c\ \left(E_i(u)^2 + E_i(v)^2\right),$$

$$(u-v+c)E_i(v)E_{i+1}(u) = (u-v)E_{i+1}(u)E_i(v) + \\ c\ \left(E_{i+1}(u)E_i(u) + E_{i,i+2}(u) - E_{i,i+2}(v)\right),$$

$$(u-v-c)F_i(v)F_i(u) = \\ (u-v+c)F_i(v)F_i(u) - c\ \left(F_i(u)^2 + F_i(v)^2\right),$$

$$(u-v+c)F_{i+1}(u)F_i(v) = (u-v)F_i(v)F_{i+1}(u) + \\ c\ \left(F_i(u)F_{i+1}(u) + F_{i+2,i}(u) - F_{i+2,i}(v)\right).$$

Projection formula

$$\mathbb{B}(\bar{u}^1,\ldots,\bar{u}^{n-1}) \sim P_f^+\Big(\mathcal{F}_1(u_1^1)\cdot\ldots\cdot\mathcal{F}_1(u_{r_1}^1)\cdot\ldots\cdot \mathcal{F}_{n-1}(u_1^{n-1})\cdot\ldots\cdot\mathcal{F}_{n-1}(u_{r_{n-1}}^{n-1})\Big)\Omega.$$

Here we also use notation for *full current* $\mathcal{F}_i = F^+_{i,i+1} - F^-_{i.i+1}$.

$\mathfrak{gl}_n$-eigenvector

Eigenvector

And this Bethe vector becomes eigenvector of the transfer matrix

$$t(z)\mathbb{B}(\bar{u}) = \tau(z|\bar{u})\mathbb{B}(\bar{u})$$

with eigenvalue

$$\tau(z|\bar{u}) = \sum_{i=1}^{n} \lambda_i(z) f(z, \bar{t}^{i-1}) f(\bar{t}^i, z),$$

where $\bar{t}^0 = \bar{t}^n = \emptyset$,

when Bethe equations are satisfied

$$\frac{\lambda_i(t^i_k)}{\lambda_{i+1}(t^i_k)} = \frac{f(t^i_k, \bar{t}^i_k)}{f(\bar{t}^i_k, t^i_k)} \frac{f(\bar{t}^{i+1}, t^i_k)}{f(t^i_k, \bar{t}^{i-1})}.$$

Notation

we use notation of rational function f

$$f(x,y) = \frac{x-y+c}{x-y}.$$

Subindex means no element

$$\bar{u}_k = \bar{u} \setminus \{u_k\}.$$

We also used shorthand notation for product over set

$$f(z,\bar{u}\,) = \prod_{u_i \in \bar{u}} f(z,u_i), \qquad f(\bar{u}_k,\, z) = \prod_{u_i \in \bar{u}, i \neq k} f(u_i,z).$$

R-matrix

$\mathfrak{so}_{2n+1}, \mathfrak{so}_{2n}, \mathfrak{sp}_{2n}$ R-matrices

Let's consider the most typical R-matrix

$$R(u,v) = I + \frac{c}{u-v}P - \frac{c}{u-v+c\kappa}Q,$$

where P is the permutation operator

$$P = \sum_{i,j=1}^{n} e_{ij} \otimes e_{ji}, \qquad Q = P^{t_1} = \sum_{i,j=1}^{n} \epsilon_i \epsilon_j \; e_{j'i'} \otimes e_{ji},$$

where $i' = N + 1 - i$, $\epsilon_i = 1$ for $\mathfrak{so}_N$ and $\epsilon_i = \mathrm{sign}(i - n + 1/2)$ for $\mathfrak{sp}_{2n}$

Center

Quantum orthogonality condition

Considering the residue at the point of the equation, we can prove the existence of the center

$$z(u) = T(u + c\kappa)^t T(u)$$

Constrains

This center implies constraints

$$F_i(u) = -F_{i'}(u + c(\kappa - i)), \quad E_i(u) = -E_{i'}(u + c(\kappa - i)),$$

$$\frac{k_i(u)}{k_{i+1}(u)} = \frac{k_{i'-1}(u + c(\kappa - 1))}{k_{i'}(u + c(\kappa - 1))}.$$

Gauss decomposition

We consider Gauss decomposition

$$T(u)^t = \mathbb{F}(u)^t \cdot \mathbb{K}(u)^t \cdot \mathbb{E}(u)^t.$$

$\mathfrak{so}(5)$ case

$$\mathbb{F}(u) = \begin{pmatrix} 1 & F_1(u) & * & * & * \\ 0 & 1 & F_2(u) & * & * \\ 0 & 0 & 1 & -F_2(u+c/2) & * \\ 0 & 0 & 0 & 1 & -F_1(u-c/2) \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$(u-v+c)F_1(v)F_1(u) = (u-v-c)F_1(v)F_1(u) + \dots,$$
$$(u-v+c/2)F_2(v)F_2(u) = (u-v-c/2)F_2(v)F_2(u) + \dots,$$
$$(u-v+c)F_1(u)F_2(v) = (u-v)F_2(v)F_1(u) + \dots.$$

$\mathfrak{sp}(4)$ case

$$\mathbb{F}(u) = \begin{pmatrix} 1 & F_1(u) & * & * \\ 0 & 1 & F_2(u) & * \\ 0 & 0 & 1 & -F_1(u+c) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$(u-v+c)F_1(v)F_1(u) = (u-v-c)F_1(v)F_1(u) + \dots,$$

$$(u-v+2c)F_2(v)F_2(u) = (u-v-2c)F_2(v)F_2(u) + \dots,$$

$$(u-v+2c)F_1(u)F_2(v) = (u-v)F_2(v)F_1(u) + \dots.$$

$\mathfrak{so}(4)$ case

$$\mathbb{F}(u) = \begin{pmatrix} 1 & F_1(u) & F_2(u) & * \\ 0 & 1 & 0 & -F_2(u) \\ 0 & 0 & 1 & -F_1(u) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$(u - v + c)F_1(v)F_1(u) = (u - v - c)F_1(v)F_1(u) + \dots,$$
$$(u - v + c)F_2(v)F_2(u) = (u - v - c)F_2(v)F_2(u) + \dots,$$
$$[F_1(u), F_2(v)] = 0.$$

Projection formula

For all the cases $\mathfrak{so}_{2n+1}, \mathfrak{so}_{2n}, \mathfrak{sp}_{2n}$ construction of Bethe vector is

$$\mathbb{B}(\bar{u}^1, \dots, \bar{u}^n) \sim P_f^+ \Big(\mathcal{F}_1(u_1^1) \cdot \ldots \cdot \mathcal{F}_1(u_{r_1}^1) \cdot \ldots \cdot \mathcal{F}_n(u_1^n) \cdot \ldots \cdot \mathcal{F}_n(u_{r_n}^n) \Big) \Omega.$$

Eigenvalues of the transfer matrices

$\mathfrak{so}(n+1)$ case

Eigenvalue is

$$\tau(z|\bar{u}) = \lambda_{n+1} f(\bar{u}^n, z) f(z_n, \bar{u}^n) + \sum_{s=1}^{n} \Big(\lambda_s(z) f(\bar{u}^s, z) f(z, \bar{u}^{s-1}) + \lambda_{s'}(z) f(\bar{u}^s, z_s) f(z_{s-1}, \bar{u}^{s-1}) \Big)$$

where $z_i = z + c(i - 1/2)$.

$\mathfrak{sp}(2n)$ case

Eigenvalue is

$$\tau(z|\bar{u}) = \lambda_n(z)\mathfrak{f}(\bar{u}^n, z)\mathfrak{f}(z, \bar{u}^{n-1}) + \lambda_{n+1}(z)\mathfrak{f}(z, \bar{u}^n)\mathfrak{f}(\bar{u}^{n-1}, z_1) + \sum_{s=1}^{n-1} \big(\lambda_s(z) f(\bar{u}^s, z) f(z, \bar{u}^{s-1}) + \lambda_{s'}(z) f(\bar{u}^s, z_s) f(z_{s-1}, \bar{u}^{s-1})\big)$$

where $z_i = z + c(i+1)$ and

$$\mathfrak{f}(x, y) = \frac{x - y + 2c}{x - y}.$$

$\mathfrak{so}(n)$ case

Eigenvalue is

$$\tau(z|\bar{u}) = \lambda_{n-1}(z) f(\bar{u}^{n-1}, z) f(\bar{u}^{n}, z) f(z, \bar{u}^{n-2}) + \lambda_{n}(z) f(z, \bar{u}^{n-1}) f(\bar{u}^{n}, z) + \lambda_{n+1}(z) f(\bar{u}^{n-1}, z) f(z, \bar{u}^{n}) + \lambda_{n+2}(z) f(z, \bar{u}^{n-1}) f(z, \bar{u}^{n}) f(\bar{u}^{n-2}, z_2) + \sum_{s=1}^{n-2} \left(\lambda_s(z) f(\bar{u}^{s}, z) f(z, \bar{u}^{s-1}) + \lambda_{s'}(z) f(\bar{u}^{s}, z_s) f(z_{s-1}, \bar{u}^{s-1}) \right)$$

where $z_i = z + c(i-1)$.

Sum formula in $\mathfrak{gl}(N+1)$ case

Let $\mathbb{B}(\bar{t})$ be a generic Bethe vector and $\mathbb{C}(\bar{s})$ be a generic dual Bethe vector such that $\#\bar{t}^k = \#\bar{s}^k = r_k$, $k = 1, \dots, N$. Then their scalar product $S(\bar{s}|\bar{t}) = \mathbb{C}(\bar{s})\mathbb{B}(\bar{t})$ is given by

$$S(\bar{s}|\bar{t}) = \sum \frac{Z(\bar{s}_{\mathrm{I}}|\bar{t}_{\mathrm{I}})\ Z(\bar{t}_{\mathrm{II}}|\bar{s}_{\mathrm{II}}) \prod_{k=1}^{N} \alpha_k(\bar{s}^k_{\mathrm{I}})\alpha_k(\bar{t}^k_{\mathrm{II}}) f(\bar{s}^k_{\mathrm{II}}, \bar{s}^k_{\mathrm{I}}) f(\bar{t}^k_{\mathrm{I}}, \bar{t}^k_{\mathrm{II}})}{\prod_{j=1}^{N-1} f(\bar{s}^{j+1}_{\mathrm{II}}, \bar{s}^j_{\mathrm{I}}) f(\bar{t}^{j+1}_{\mathrm{I}}, \bar{t}^j_{\mathrm{II}})}.$$

Here all the sets of the Bethe parameters $\bar{t}^k$ and $\bar{s}^k$ are divided into two subsets $\bar{t}^k \Rightarrow \{\bar{t}^k_{\mathrm{I}}, \bar{t}^k_{\mathrm{II}}\}$ and $\bar{s}^k \Rightarrow \{\bar{s}^k_{\mathrm{I}}, \bar{s}^k_{\mathrm{II}}\}$, such that $\#\bar{t}^k_{\mathrm{I}} = \#\bar{s}^k_{\mathrm{I}}$. The sum is taken over all possible partitions of this type.

Co-product formula

Co-product on algebra

We can divide spin-chain in two pieces

$$T(z) = T^{(2)}(z)T^{(1)}(z),$$

or in terms of the monodromy matrix entries

$$T_{ij}(u) = \sum_k T_{ki}(u) \otimes T_{jk}(u).$$

Co-product on Bethe vector

This division of spinchain implies division of Bethe vector (Co-product formula)

$$\mathbb{B}(\bar{t}) = \sum \frac{\prod_{\nu=1}^{N} \alpha_\nu^{(2)}(\bar{t}_{\rm i}^\nu) f(\bar{t}_{\rm ii}^\nu, \bar{t}_{\rm i}^\nu)}{\prod_{\nu=1}^{N-1} f(\bar{t}_{\rm ii}^{\nu+1}, \bar{t}_{\rm i}^\nu)} \mathbb{B}^{(1)}(\bar{t}_{\rm i}) \otimes \mathbb{B}^{(2)}(\bar{t}_{\rm ii}).$$

Projection as integral

Projection formula

Bethe vector is

$$\mathbb{B}(\bar{u}^1, \dots, \bar{u}^{n-1}) \sim P_f^+\Big(\mathcal{F}_{1,\dots,n}(\bar{u})\Big)\Omega.$$

Integral form

And it can be rewrited as integral

$$P_f^+\Big(\mathcal{F}_{1,\dots,n}(\bar{u})\Big) = \int d\bar{w}\ Z(\bar{u}|\bar{w})\ \mathcal{F}_{1,\dots,n}(\bar{w}),$$

where integral kernel Z coincides with the highest coefficient.

Thank you for your attention!