Correlation functions & Functional Separation of Variables

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2202.01591 N. Gromov, N. Primi, P.R

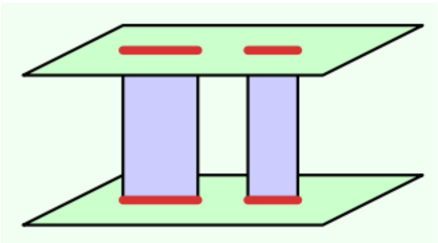
+ earlier related work with F. Levkovich-Maslyuk + D. Volin

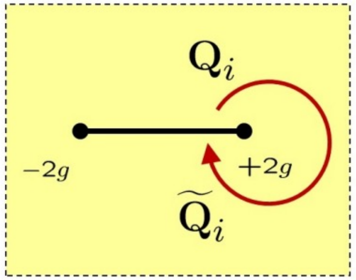
Motivation

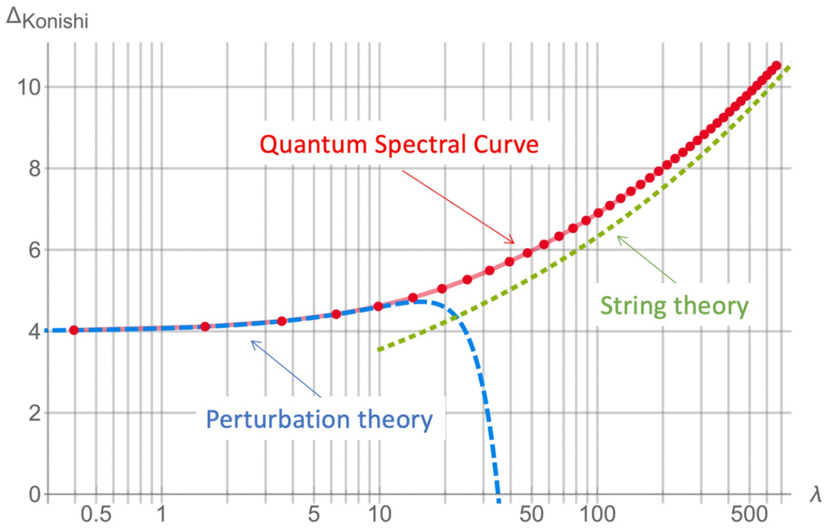
Quantum Spectral Curve

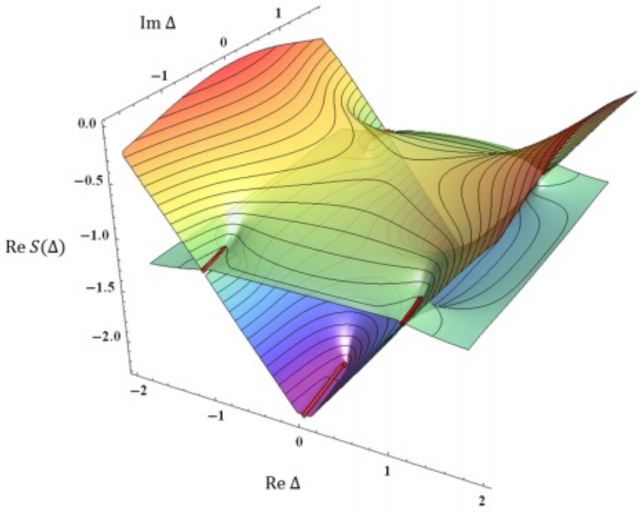
Exact solution of spectral problem of N=4 SYM

[Gromov, Kazakov, Leurent, Volin]









What about correlation functions?

Evidence of correlators in QSC

(See Alessandro's and Kolya's talks)

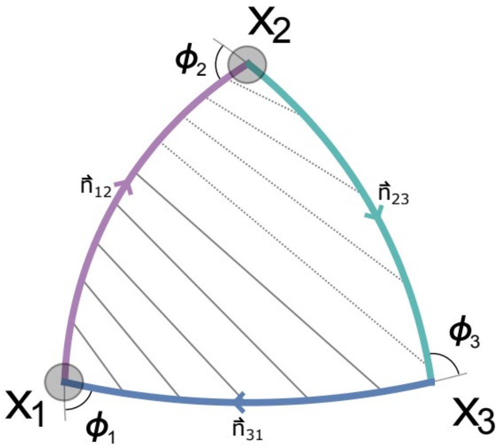
Bracket

$$\Big(f \Big) = \int_\gamma \mathrm{d}u \, \mu(u) \, f(u)$$

$$\Big(Q_A Q_B \Big) \propto \delta_{AB}$$

Cusps in ladders limit

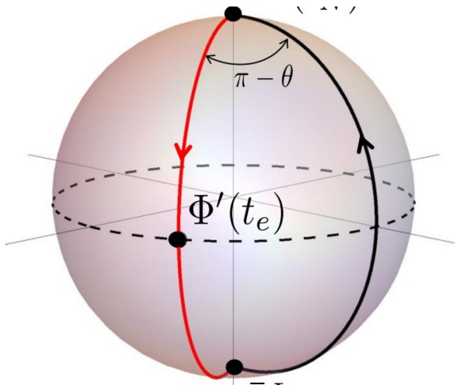
[Cavaglia, Gromov, Levkovich-Maslyuk]



$$\Big(f \Big) \sim \int_{c-i\infty}^{c+i\infty} \mathrm{d}u \, \frac{1}{2\pi i \, u} f(u), \quad c > 0$$

Near-BPS Wilson loops

[Komatsu, Giombi]



$$x + \frac{1}{x} = \frac{u}{g}$$

$$\Big(f \Big) \sim \oint \mathrm{d}u \left(x - \frac{1}{x} \right) \sinh(2\pi u) \, f(u)$$

$$\frac{\langle \Psi | \partial_\phi \left(2 \sin\phi \, \hat{D} \right) | \Psi \rangle}{\langle \Psi | \Psi \rangle} = \pm \frac{\Big(u \, Q^2 \Big)}{\Big(Q^2 \Big)}$$

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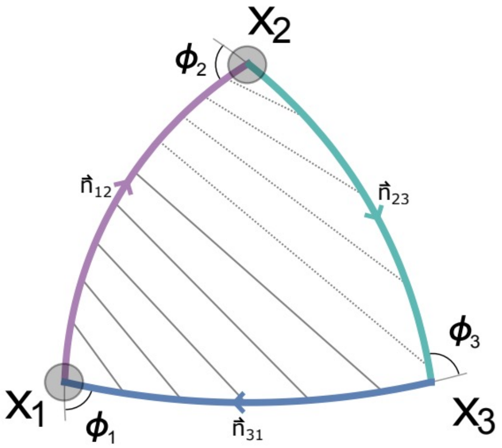
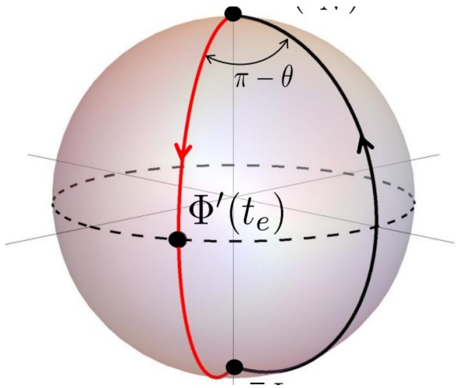


Natural object in Functional Separation of Variables

Near-BPS Wilson loops

[Komatsu, Giombi]



$$x + \frac{1}{x} = \frac{u}{g}$$

$$\left(f \right) \sim \int_{c-i\infty}^{c+i\infty} du\, \frac{1}{2\pi i\, u} f(u), \quad c > 0$$

$$\left(f \right) \sim \oint du \left(x - \frac{1}{x} \right) \sinh(2\pi u)\, f(u)$$

$$\frac{\langle \Psi | \partial_\phi \left(2 \sin\phi\, \hat{D} \right) | \Psi \rangle}{\langle \Psi | \Psi \rangle} = \pm \frac{\left(u\, Q^2 \right)}{\left(Q^2 \right)}$$

Separation of Variables (SoV)

[Sklyanin, '85]

Identify SoV basis $\langle \mathsf{x}|$

$$\Psi(\mathsf{x}) = \langle \mathsf{x}|\Psi\rangle \sim \prod_{\alpha=1}^{L} Q(\mathsf{x}_\alpha)$$

Extremely powerful in rank 1 models (SL(2))

- One-point functions [Lukyanov]
- Hexagons [Jiang, Komatsu, Kostov, Serban]
- Boundary overlaps [Caetano, Komatsu + Gombor, Pozsgay]
- Matrix elements [Smirnov]
- Basso-Dixon correlators [Derkachov, Kazakov, Olivucci]

[+ many, many others]

Only fully extended to higher-rank recently

[Cavaglia, Gromov, Levkovich-Maslyuk, Sizov + Liashyk, Slavnov + Maillet, Niccoli, Vignoli + PR, Volin + Derkachov, Olivucci]

See earlier work of Sklyanin and Smirnov

Lesson: operator-based SoV approach of Sklyanin should be supplemented with Functional SoV

Goal: Systematic study of correlators using Functional SoV

Punchline

- Studied SL(N) spin chains – finite / infinite – dim representations

- Identified special set of operators – ``Principal operators” $\mathrm{P}_{a,r}(u)$ L x (N-1) x (N+1) operators

- Generate all observables + play especially nicely with SoV

- Compute all possible form-factors – determinants of Q-functions

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$$\langle\Psi_A|\Psi_B\rangle = \frac{1}{\mathcal{N}}\begin{vmatrix} \left(Q_{12}^{A-}Q_1^B\right)_1 & \left(Q_{12}^{A+}Q_1^B\right)_1 \\ \left(Q_{13}^{A-}Q_1^B\right)_2 & \left(Q_{13}^{A+}Q_1^B\right)_2 \end{vmatrix} \qquad \langle\Psi_A|\mathrm{P}_{1,1}(u)|\Psi_B\rangle = \frac{1}{\mathcal{N}}\begin{vmatrix} \left(Q_{12}^{A-}(u-w)Q_1^B\right)_1 & \left(Q_{12}^{A+}Q_1^B\right)_1 \\ \left(Q_{13}^{A-}(u-w)Q_1^B\right)_2 & \left(Q_{13}^{A+}Q_1^B\right)_2 \end{vmatrix}$$

Functional SoV

New approach to scalar products and correlation functions

Main idea:

orthogonality

$$\langle \Psi_A | \Psi^B \rangle \ \propto \ \delta_{AB}$$

States completely fixed by Q-functions.

How to reformulate orthogonality using Q-functions?

[Cavaglia, Gromov, Levkovich-Maslyuk]

Functional orthogonality

Twisted su(2) spin chain: Spectrum and Baxter equation

$$t(u) = \chi_1 u^L + \sum_{\beta=1}^{L} \hat{I}_\beta u^{\beta-1} \qquad H = \partial_u \mathrm{Log}\, t(u)|_{u=0} \qquad t(u)|\Psi\rangle = \tau(u)|\Psi\rangle$$

Transfer matrix - generating function of integrals of motion

Like N=4 SYM, spectrum of XXX spin chain determined by Q-functions

$$\boxed{\tau(u)Q(u) = Q_\theta^+(u)Q^{--}(u) + \chi_2 Q_\theta^-(u)Q^{++}(u)\,.}$$

$$Q(u) = \lambda_1^{iu} \times \text{polynomial} \qquad Q_\theta(u) = \prod_{\alpha=1}^{L}(u-\theta_\alpha)$$

Inhomogeneity

$$f^\pm(u) = f(u \pm \frac{i}{2})$$

$$f^{\pm\pm}(u) = f(u \pm i)$$

Functional orthogonality

Recast Baxter equation as a finite-difference operator

[Cavaglia, Gromov, Levkovich-Maslyuk]

$$\mathcal{O} = Q_\theta^+(u)D^{-2} - \tau(u) + \chi_2 Q_\theta^-(u)D^{+2} \qquad \mathcal{O}Q(u) = 0$$

Introduce the following bracket $\langle f(u)\rangle_\alpha = \oint \mathrm{d}u\, \mu_\alpha(u)\, f(u)$

Self-adjointness

$$\langle \chi_2^{-iu}\, f\, \mathcal{O}\, g\rangle_\alpha = \langle \chi_2^{-iu}\, g\, \mathcal{O}\, f\rangle_\alpha$$

$$\mu_\alpha(u) = \frac{\prod_{\beta\neq\alpha} 1 - e^{2\pi(u-\theta_\beta+\frac{i}{2})}}{Q_\theta^+(u)Q_\theta^-(u)}$$

$$\langle f\, \mathcal{O}\, Q\rangle_\alpha = 0 \qquad \langle \bar{Q}(u)\, \mathcal{O}\, f\rangle_\alpha = 0 \qquad \bar{Q}(u) = \chi_2^{-iu} Q(u)$$

Functional orthogonality

Consider two distinct states $|\Psi_A\rangle \qquad |\Psi_B\rangle$

Corresponding Q-functions $Q_A \qquad Q_B$

$$\boxed{\langle \bar{Q}_B(\mathcal{O}_A - \mathcal{O}_B)Q_A\rangle_\alpha = 0}$$

Expand $\mathcal{O}_A = Q_\theta^+(u)D^{-2} - \tau_A(u) + \chi_2 Q_\theta^-(u)D^{+2}$

$$\tau_A(u) = \chi_1 u^L + \sum_{\beta=1}^{L} u^{\beta-1} I_\beta^A$$

$$\sum_{\beta=1}^{L} \langle \bar{Q}_B u^{\beta-1} Q_A\rangle_\alpha (I_\beta^A - I_\beta^B) = 0$$

Must be non-zero

$$\boxed{\langle \Psi_B|\Psi_A\rangle = \frac{1}{\mathcal{N}} \det_{1\leq\alpha,\beta\leq L} \langle \bar{Q}_B u^{\beta-1} Q_A\rangle_\alpha}$$

[Cavaglia, Gromov, Levkovich-Maslyuk, PR, Volin]

Observables

Functional orthogonality leads to scalar product

What other observables can we extract?

Recall Hellmann-Feynman theorem

[Cavaglia, Gromov, Levkovich-Maslyuk, PR]

$$H|\Psi\rangle = E|\Psi\rangle$$

$$\langle\Psi|H = E\langle\Psi|$$

$$\langle\Psi|\partial_p H|\Psi\rangle = \partial_p E\langle\Psi|\Psi\rangle$$

Functional orthogonality allows to compute at fixed value of p!

Diagonal Form-factors

Functional orthogonality leads to not only scalar product but a whole host of diagonal form-factors!

p some parameter of the model; twist, inhomogeneity, …

$$p \to p + \delta_p \qquad \langle \bar{Q}(\mathcal{O} + \delta\mathcal{O})(Q + \delta Q)\rangle_\alpha = 0$$

$$\boxed{\langle \bar{Q}\,\delta\mathcal{O}\,Q\rangle_\alpha = 0}$$

$$\mathcal{O} = Y - \sum_{\beta=1}^{L} I_\beta u^{\beta-1}$$

Inhomogeneous linear system for $\partial_p I_\beta$

Solvable by Cramer's rule!

$$\partial_p I_\beta = \frac{[\beta \to \langle \bar{Q}\,\partial_p Y\,Q\rangle]}{\det\limits_{1\le\alpha,\beta\le L}\langle \bar{Q} u^{\beta-1} Q\rangle_\alpha}$$

$$\boxed{\langle\Psi|\partial_p \hat{I}_\beta|\Psi\rangle = \frac{1}{\mathcal{N}}[\beta \to \langle \bar{Q}\,\partial_p Y\,Q\rangle]}$$

$\boxed{\text{Can we extend to off-diagonal correlators?}}$

Yes! For appropriate choice of p – twist eigenvalues $\lambda_1,\, \lambda_2$

$$\langle\Psi|\partial_p \hat{I}_\beta|\Psi\rangle = \frac{1}{\mathcal{N}}[\beta \to \langle\bar{Q}\,\partial_p Y\,Q\rangle]$$

Survives a number of upgrades!

$$\boxed{\langle\Psi_A|\partial_{\lambda_1}^{n_1}\partial_{\lambda_2}^{n_2}\hat{I}_\beta|\Psi_B\rangle = \frac{1}{\mathcal{N}}[\beta \to \langle\bar{Q}_A\,\partial_{\lambda_1}^{n_1}\partial_{\lambda_2}^{n_2}Y\,Q_B\rangle]}$$

Can do even better! $\langle\Psi_A|$ and $|\Psi_B\rangle$ need not even be Hamiltonian eigenstates!

More general class of states - ``factorizable" states

Incl. off-shell Bethe states

Can we extend to off-diagonal correlators?

Yes! For appropriate choice of p – twist eigenvalues λ_1, λ_2

$$\langle\Psi|\partial_p \hat{I}_\beta|\Psi\rangle = \frac{1}{\mathcal{N}}[\beta \to \langle \bar{Q}\, \partial_p Y\, Q\rangle]$$

Survives a number of upgrades!

Generate full algebra of observables!

Easy to extract SoV matrix elements!

$$\boxed{\langle\Psi_A|\partial^{n_1}_{\lambda_1}\partial^{n_2}_{\lambda_2}\hat{I}_\beta|\Psi_B\rangle = \frac{1}{\mathcal{N}}[\beta \to \langle \bar{Q}_A\, \partial^{n_1}_{\lambda_1}\partial^{n_2}_{\lambda_2} Y\, Q_B\rangle]}$$

Can do even better! $\langle\Psi_A|$ and $|\Psi_B\rangle$ need not even be Hamiltonian eigenstates!

More general class of states - ``factorizable" states

Incl. off-shell Bethe states

Functional can only take us so far…

… we need some operatorial input!

Spin chain – algebraic construction

$$E_{12} = S^+, \quad E_{21} = S^-$$

$$E_{11} = \frac{1}{2} + S_z, \quad E_{22} = \frac{1}{2} - S_z$$

Lax operator $\mathcal{L}_{ij}(u) = u\,\delta_{ij} + iE_{ji}$

Monodromy matrix $T_{ij}(u) = \sum_{i_1 \ldots i_{L-1}} \mathcal{L}_{ii_1}(u-\theta_1) \otimes \cdots \otimes \mathcal{L}_{i_{L-1}j}(u-\theta_L)$

Yangian algebra $-i(u-v)[T_{jk}(u), T_{lm}(v)] = T_{lk}(v)T_{jm}(u) - T_{lk}(u)T_{jm}(v)$

Transfer matrix $t(u) = \sum_{ij} T_{ij}(u) G_{ji}$

Generates integrals of motion $[t(u), t(v)] = 0$

What's a good twist?

Natural guess - diagonal $$g = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

GL(2) covariance lets us choose any twist we like with the same eigenvalues

Much more convenient $$G = \begin{pmatrix} \chi_1 & -\chi_2 \\ 1 & 0 \end{pmatrix}$$

$$\operatorname{tr} G = \chi_1 = \lambda_1 + \lambda_2$$

$$\det G = \chi_2 = \lambda_1 \lambda_2$$

$$t(u) = T_{12}(u) + \chi_1 T_{11}(u) - \chi_2 T_{21}(u)$$

SoV bases independent of twist!

[Ryan, Volin]

Serve to factorise wave functions of different Hamiltonians!

[Gromov, Levkovich-Maslyuk, Ryan]

Principal operators

$$t(u) = T_{12}(u) + \chi_1 T_{11}(u) - \chi_2 T_{21}(u)$$

$$t(u) = \chi_1 u^L + \sum_{\beta=1}^{L} \hat{I}_\beta u^{\beta-1}$$

Now integrals of motion admit character expansion

$$\hat{I}_\beta \longrightarrow \hat{I}_\beta^{(0)} + \chi_1 \hat{I}_\beta^{(1)} + \chi_2 \hat{I}_\beta^{(2)}$$

$$t(u) \longrightarrow \mathbb{P}_0(u) + \chi_1 \mathbb{P}_1(u) + \chi_2 \mathbb{P}_2(u)$$

$\mathbb{P}_r(u)$ - Principal operators

Principal operators

$$t(u) = T_{12}(u) + \chi_1 T_{11}(u) - \chi_2 T_{21}(u)$$

$$t(u) = \chi_1 u^L + \sum_{\beta=1}^{L} \hat{I}_\beta u^{\beta-1}$$

Generate remaining operator $T_{22}(u)$

Now integrals of motion admit character expansion

$$\hat{I}_\beta \longrightarrow \hat{I}_\beta^{(0)} + \chi_1 \hat{I}_\beta^{(1)} + \chi_2 \hat{I}_\beta^{(2)}$$

$$t(u) \longrightarrow \mathbb{P}_0(u) + \chi_1 \mathbb{P}_1(u) + \chi_2 \mathbb{P}_2(u)$$

$\mathbb{P}_r(u)$ - Principal operators

We will compute

$$\langle \Psi_A | \mathbb{P}_r(u) | \Psi_B \rangle$$

Starting point is the scalar product

$$\langle\Psi_B|\Psi_A\rangle = \frac{1}{\mathcal{N}} \det_{1\le\alpha,\beta\le L} \langle \bar{Q}_B u^{\beta-1} Q_A\rangle_\alpha$$

Now consider the trivial equality $$\frac{1}{\mathcal{N}}[\beta' \to \langle \bar{Q}_B \mathcal{O}_A Q_A\rangle] = 0$$

and expand $$\langle\Psi_B|\Psi_A\rangle I^A_{\beta'} = -\frac{1}{\mathcal{N}}[\beta' \to \langle \bar{Q}_B Y Q_A\rangle]$$

$$\hat{I}_{\beta'}|\Psi_A\rangle = I^A_{\beta'}|\Psi_A\rangle$$

$$\boxed{\langle\Psi_B|\hat{I}_{\beta'}|\Psi_A\rangle = -\frac{1}{\mathcal{N}}[\beta' \to \langle \bar{Q}_B Y Q_A\rangle]}$$

$$\langle\Psi_B|\hat{I}_{\beta'}|\Psi_A\rangle = -\frac{1}{\mathcal{N}}[\beta' \to \langle\bar{Q}_B Y Q_A\rangle]$$

$$\hat{I}_\beta \longrightarrow \hat{I}_\beta^{(0)} + \chi_1 \hat{I}_\beta^{(1)} + \chi_2 \hat{I}_\beta^{(2)}$$

$$Y = Q_\theta^+ D^{-2} - \chi_1\, u^L + \chi_2\, Q_\theta^- D^2$$

$$Y = \mathcal{O}_{(1)} + \chi_1\, \mathcal{O}_{(1)} + \chi_2\, \mathcal{O}_{(2)}$$

Now lhs and rhs linear in characters

Character projection!

$$\boxed{\langle\Psi_B|\hat{I}_{\beta'}^{(r)}|\Psi_A\rangle = -\frac{1}{\mathcal{N}}[\beta' \to \langle\bar{Q}_B \mathcal{O}_{(r)} Q_A\rangle]}$$

Is character projection a legit thing to do?

Always have the relation $$\langle\Psi_A|\hat{I}^{(r)}_{\beta'}|\Psi_B\rangle = \sum_{\mathsf{x}}\langle\Psi_A|\mathsf{x}\rangle\langle\mathsf{x}|\hat{I}^{(r)}_{\beta'}|\mathsf{x}\rangle\langle\mathsf{x}|\Psi_B\rangle$$

Can be shown: $$-\frac{1}{\mathcal{N}}[\beta' \to \langle\bar{Q}_B\mathcal{O}_{(r)}Q_A\rangle] = \sum_{\mathsf{x}}\langle\Psi_A|\mathsf{x}\rangle\mathcal{M}^{(r)}_{\beta'\mathsf{x}}\langle\mathsf{x}|\Psi_B\rangle$$

Independent of twist!

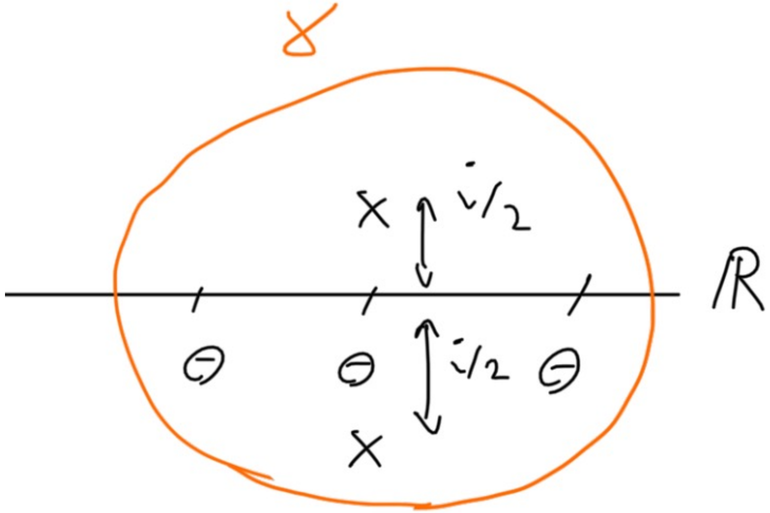
$$\sum_{\mathsf{x}}\langle\Psi_A|\mathsf{x}\rangle\langle\mathsf{x}|\Psi_B\rangle\sum_{r=0}^{2}\chi_r(\mathcal{M}^{(r)}_{\beta'\mathsf{x}} - \langle\mathsf{x}|\hat{I}^{(r)}_{\beta'}|\mathsf{x}\rangle) = 0$$

Completeness of transfer matrix eigenstates: $$\sum_{r=0}^{2}\chi_r(\mathcal{M}^{(r)}_{\beta'} - \langle\mathsf{x}|\hat{I}^{(r)}_{\beta'}|\mathsf{x}\rangle) = 0$$

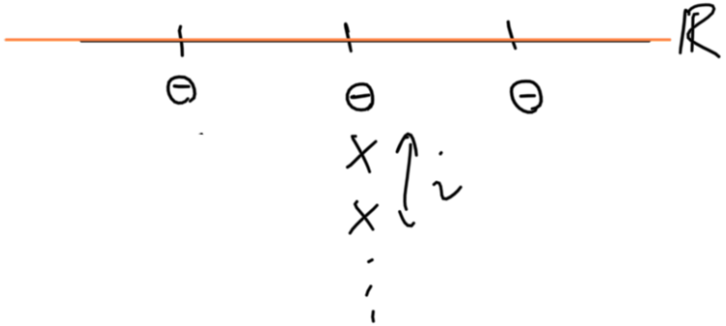
$$\boxed{\mathcal{M}^{(r)}_{\beta'\mathsf{x}} = \langle\mathsf{x}|\hat{I}^{(r)}_{\beta'}|\mathsf{x}\rangle}$$

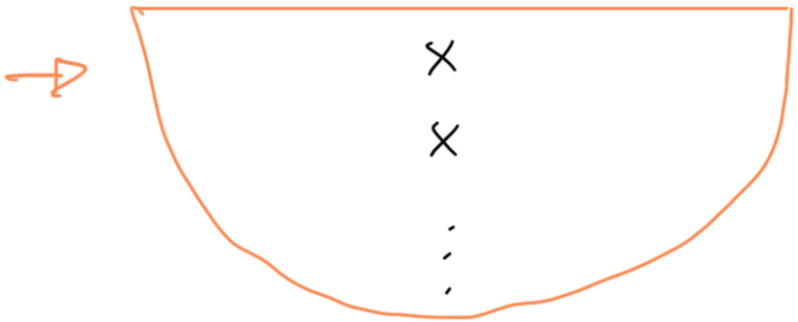
Upgrading: non-compact spin -s

$$\langle f(u)\rangle_\alpha = \oint \mathrm{d}u\, \mu_\alpha(u)\, f(u) \qquad \mu_\alpha(u) = \frac{\prod_{\beta\neq\alpha} 1 - e^{2\pi(u-\theta_\beta+\frac{i}{2})}}{Q_\theta^+(u)Q_\theta^-(u)}$$



$$\langle f(u)\rangle_\alpha = \int_{-\infty}^{+\infty} \mathrm{d}u\, \mu_\alpha(u)\, f(u) \qquad \mu_\alpha = \frac{\varepsilon}{1-e^{2\pi(u-\theta_\alpha - i\mathbf{s})}} \qquad \varepsilon = \prod_{\beta=1}^{L} \frac{\Gamma(\mathbf{s}-i(u-\theta_\beta))}{\Gamma(1-\mathbf{s}-i(u-\theta_\beta))}$$





Higher-rank SU(N) / SL(N) spin chains

Our approach extends to higher rank SU(N) and SL(N) with minimal effort

Just need to extend range of indices!!

Integrals of motion $\hat{I}_\beta \longrightarrow \hat{I}_{b,\beta}$

$$a, b \in \{1, 2, \ldots, N-1\}$$

Dual Q-functions $\bar{Q}(u) \longrightarrow \bar{Q}^a(u)$

Scalar product

$$\langle \Psi_A | \Psi_B \rangle = \frac{1}{\mathcal{N}} \det_{(a,\alpha),(b,\beta)} \langle Q_B u^{\beta-1} D^{N-2b} \bar{Q}^a_A \rangle_\alpha$$

Higher-rank SU(N) / SL(N) spin chains

Characters $\chi_1,\,\chi_2 \longrightarrow \chi_1,\,\ldots,\,\chi_N$

Transfer matrices $t_1(u) \longrightarrow t_a(u)$

Principal operators $\mathbb{P}_r(u) \longrightarrow \mathbb{P}_{a,r}(u)$

$$\mathbb{P}_{a,r}(u) = \delta_{ar} u^L + \sum_{\beta=1}^{L} \hat{I}^{(r)}_{a,\beta} u^{\beta-1}$$

Baxter equation

$$\sum_{\beta,b} \hat{I}_{a,\beta} u^{\beta-1} D^{N-2b} = -\sum_{r=0}^{N} \chi_r\, \mathcal{O}_{(r)}$$

$$\boxed{\langle \Psi_A | \hat{I}^{(r)}_{b,\beta} | \Psi_B \rangle = \frac{1}{\mathcal{N}} [(b,\beta) \to \langle Q_B \mathcal{O}_{(r)} \bar{Q}^a_A \rangle]}$$

$$\langle \Psi_A | \hat{I}^{(r)}_{b,\beta} | \Psi_B \rangle = \frac{1}{\mathcal{N}} [(b,\beta) \to \langle Q_B \mathcal{O}_{(r)} \bar{Q}^a_A \rangle]$$

Unlike SU(2) spin chain - need not one but two SoV bases! [See also Martin, Smirnov '15]

$$\Psi(\mathsf{x}) = \langle \mathsf{x} | \Psi \rangle = \prod_{\alpha=1}^{L} \prod_{a=1}^{N-1} Q_1(\mathsf{x}_{\alpha,a})$$

$$\Psi(\mathsf{y}) = \langle \Psi | \mathsf{y} \rangle = \prod_{\alpha=1}^{L} \det_{1 \le a,b \le N-1} \bar{Q}^a(\mathsf{y}_{\alpha,b})$$

[Gromov, Levkovich-Maslyuk, Ryan, Volin]

Determinant for form-factor can be expressed as a sum over wave functions

$$\langle \Psi_A | \hat{I}^{(r)}_{b,\beta} | \Psi_B \rangle = \sum_{\mathsf{x},\mathsf{y}} \Psi_A(\mathsf{y}) \langle \mathsf{y} | \hat{I}^{(r)}_{b,\beta} | \mathsf{x} \rangle \Psi_B(\mathsf{x})$$

Can extract matrix elements in SoV basis, hence can compute correlators with any number of insertions, only needing Q-functions of the two external states.

Completeness of Principal operators

From RTT:

$$T_{ij}(u) = \delta_{ij}\mathrm{P}_{1,1}(u) + (-1)^{j-1}[\mathcal{E}_{j1}, \mathrm{P}_{1,i}(u)]$$

Global symmetry generators

Not principal…

$$T_{ij}(u) = u^L\delta_{ij} + u^{L-1}\left(i\mathcal{E}_{ji} - \delta_{ij}\Theta\right) + \mathcal{O}\left(u^{L-2}\right), \quad \Theta := \sum_{\alpha=1}^{L}\theta_\alpha$$

$$\mathcal{E}^- := \mathcal{E}_{21} + \mathcal{E}_{32}$$ Principal $$\mathrm{P}_{1,0}(u) = \mathcal{E}^-\, u^{L-1} + \ldots$$

For SL(3)

$$\mathcal{E}_{21} = [\mathcal{E}^-, \mathcal{E}_{11}]$$

$$\mathcal{E}_{31} = [\mathcal{E}^-, \mathcal{E}_{21}]$$

We generate full Yangian;
Complete algebra of observables

Comparison

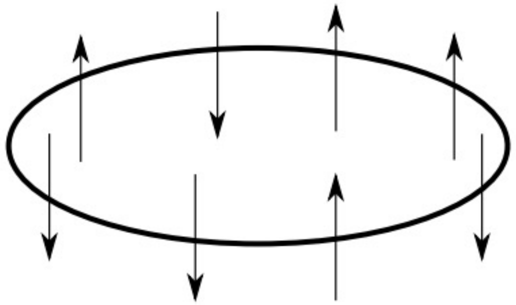
Traditional methods

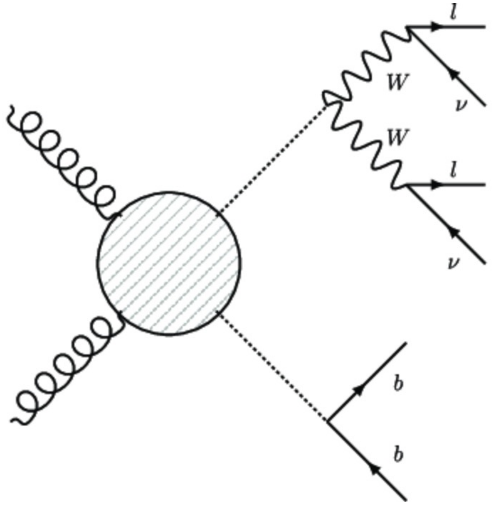
- Bethe roots
- Requires highest-weight state
- Okay for SL(2), much harder for higher rank
- Multiple insertions very difficult, need to know Bethe roots for every state

SoV methods

- Q-functions
- Doesn’t care
- All SL(N) on same footing
- Simple in SoV basis

Spin chains are nice…





… but what about 4D QFT?

4D conformal fishnet theory

Still holds: $$\langle \Psi_A | \Psi_B \rangle = \frac{1}{\mathcal{N}} \det_{(a,\alpha),(b,\beta)} \langle Q_B u^{\beta-1} D^{N-2b} \bar{Q}^a_A \rangle_\alpha$$

Main difference – Q-functions have poles

Not necessarily clear which combinations to choose

Already interesting results for diagonal form factors [Cavaglia, Gromov, Levkovich-Maslyuk]

$$\boxed{\partial_{\xi^2} \Delta = \frac{2i \int_{|} du \, \frac{p_a^{\uparrow} q_a^{\downarrow}}{(u-\theta)}}{\int_{|} du \, p_a^{\uparrow} \left(\mathcal{L}_1 q_a^{\downarrow ++} + \mathcal{L}_2 q_a^{\downarrow --} + \mathcal{L}_4 \frac{u \, q_a^{\downarrow}}{(u-\theta)} \right)} \,,}$$

N=4 supersymmetric Yang-Mills

Still have the key relation: [Cavaglia, Gromov, Levkovich-Maslyuk]

$$\boxed{\langle \bar{Q}_B(\mathcal{O}_A - \mathcal{O}_B)Q_A\rangle_\alpha = 0}$$

Main difference vs fishnets: N=4 SYM dual to sigma model

infinite number independent of integrals of motion!

Determinants of infinite size – should reduce to finite size for each fixed order in perturbation theory

Transfer matrix no longer polynomial – how to find a good basis of integrals of motion?

Summary

New approach to correlation functions in high-rank integrable systems

Based on Functional separation of variables (FSoV)

Identifies distinguished set of operators (principal) which generate all observables

Can compute Hamiltonian + SoV matrix elements – allows to consider correlators with any number of insertions!

Approach trivially extends to any rank!

Outlook

- Supersymmetry SL(M|N) – Hubbard model, AdS3

- Other algebras (see Simon's talk)

- Interpretation of principal operators in fishnets

- Comparison with recent hexagon proposal for short operators (see Alessandro's talk)

- Boundary overlaps, crosscaps

Thank you!