The multi-measure cosmological model and its peculiar effective potential

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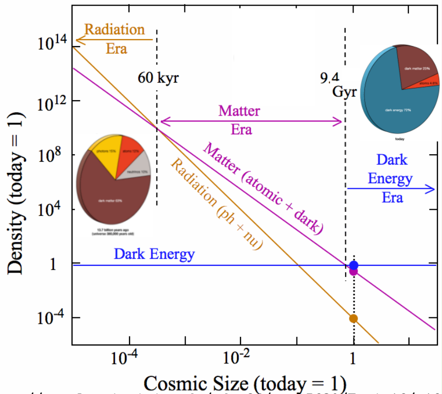
The current state of cosmology

What we know :

- The Universe is isotropic and homogeneous
- The Universe is expanding in an accelerated rate
- The Universe is flat (within error of $\sim 0.4\%$ by WMAP)
- The age of the Universe is 13.798 ± 0.037 bln. years
- The Universe contains $4.82 \pm 0.05\%$ ordinary matter, $25.8 \pm 0.4\%$ dark matter and $69 \pm 1\%$ dark energy.

Data from Planck, SNe Ia, the HST key project, the Sloan Digital Sky Survey, WMAP, COBE, large-scale galaxy

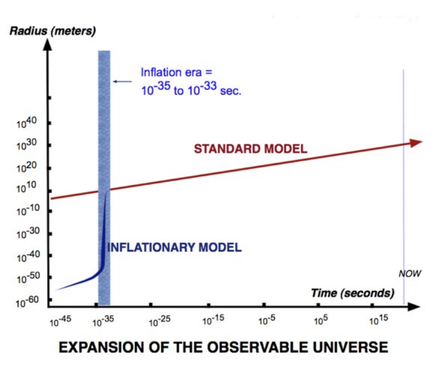


Credit:http://people.virginia.edu/~dmw8f/astr5630/Topic16/t16_dens_evol.html

To extend Λ-CDM, we need theory with inflation and with DE/DM!

Problems

– The horizon problem, the flatness problem, the missing monopols problem and the large-structures formation problem
– Λ-tension, H_0-tension, σ_8 tension,



Credit:https://www.physicsoftheuniverse.com/

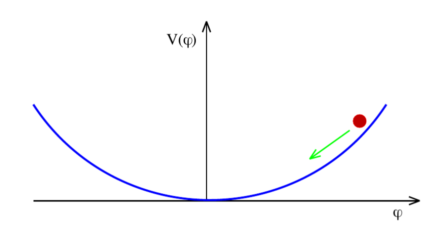
So one introduces inflation

$\Lambda - CDM$: $H = \frac{\dot{a}}{a} = H_0\sqrt{\Omega_m a^{-3} + \Omega_{rad} a^{-4} + \Omega_\Lambda}$
Inflation:

$$H^2 = \frac{8\pi}{3m_{Pl}^2}(V(\phi) + \frac{1}{2}\dot{\phi}^2)$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

Inflation occurs when $\ddot{a}(t) > 0 \leftrightarrow \dot{\phi}^2 < V(\phi)$



Credit:https://arxiv.org/pdf/astro-ph/9901124.pdf

The multimeasure model

- Model developped by Guendelman, Nissimov and Pacheva 2014, 2016
- Aimed to produce a model that describes early inflation and a smooth exit to modern times.
- The action of the model: $S = S_{darkon} + S_{inflaton}$ is :

$$S_{darkon} = \int d^4x(\sqrt{-g} + \Phi(C))L(u,Y)$$

$$S_{inflaton} = \int d^4x\Phi_1(A)(R + L^{(1)}) + \int d^4x\Phi_2(B)\left(L^{(2)} + \frac{\Phi(H)}{\sqrt{-g}}\right)$$

where $\Phi_i(X) = \frac{1}{3}\epsilon^{\mu\nu\kappa\lambda}\partial_\mu X_{\nu\kappa\lambda}$, and

$$L(u,X) = -\frac{1}{2}g^{\mu\nu}\partial_\mu u\partial_\nu u - W(u)$$

$$L^{(1)} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi),\ V(\phi) = f_1 e^{-\alpha\phi}$$

$$L^{(2)} = -\frac{b}{2}e^{-\alpha\phi}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + U(\phi),\ U(\phi) = f_2 e^{-2\alpha\phi}$$

In Einstein frame

There is a Weyl-rescaled meric $\tilde{g}$ for which

$$S^{(eff)} = \int d^4x \sqrt{-\tilde{g}}(\tilde{R} + L^{(eff)}) \tag{1}$$

with

$$L^{(eff)} = \tilde{X} - \tilde{Y}(V + M_1 - \chi_2 b e^{-\alpha\phi}\tilde{X}) + \tilde{Y}^2(\chi_2(U + M_2) - 2M_0).$$

satisfies the Einstein equations:

$$\tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{R} = \frac{1}{2}T^{(eff)}_{\mu\nu} \tag{2}$$

Non-linear with respect to both scalar fields kinetic terms, thus of the generalized k-essence type.

The effective potential of the model is: $U_{eff}(\phi) = \frac{1}{4}\frac{(f_1 e^{-\alpha\phi} + M_1)^2}{\chi_2(f_2 e^{-2\alpha\phi} + M_2) - 2M_0}$.

Details on moving from Jordan to Einstein frame

– There are 4 dynamically generated integration constants:

$$L^{(0)} = -2M_0,$$
$$R + L^{(1)} = M_1,$$
$$L^{(2)} + \frac{\Phi(\mathcal{H})}{\sqrt{-g}} = -M_2,$$
$$\frac{\Phi(\mathcal{B})}{\sqrt{-g}} = \chi_2$$

– The transformation to Einstein frame:

$$\tilde{g}_{\mu\nu} = \chi_1 g_{\mu\nu}, \text{for } \chi_1 = \frac{\Phi(\mathcal{A})}{\sqrt{-g}}.$$
$$u \to \tilde{u} : \frac{\partial \tilde{u}}{\partial u} = (W - 2M_0)^{-\frac{1}{2}}$$
$$\tilde{Y} = -\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu \tilde{u}\partial_\nu \tilde{u}, \tilde{X} = -\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu \phi \partial_\nu \phi$$

In FLRW metric:

The action becomes:

$$S^{(eff)} = \int dt\; a(t)^3 \Big(-6\, \frac{\dot{a}(t)^2}{a(t)^2} + \frac{\dot{\phi}^2}{2} - \frac{v^2}{2}\Big(V + M_1 - \chi_2 b e^{-\alpha\phi} \dot{\phi}^2/2 \Big) + \frac{v^4}{4}\left(\chi_2 (U + M_2) - 2M_0\right) \Big).$$

And the equations of motion are:

$$v^3 + 3\mathbf{a}v + 2\mathbf{b} = 0 \text{ for } \mathbf{a} = \frac{-1}{3}\,\frac{V(\phi) + M_1 - \frac{1}{2}\chi_2 b e^{-\alpha\phi}\dot{\phi}^2}{\chi_2(U(\phi) + M_2) - 2M_0}, \mathbf{b} = \frac{-p_u}{2a(t)^3(\chi_2(U(\phi) + M_2) - 2M_0)} \tag{3}$$

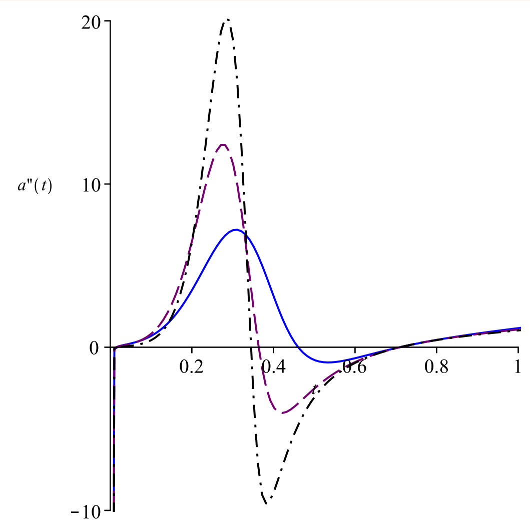
$$\dot{a}(t) = \sqrt{\frac{\rho}{6}}\, a(t), \quad \rho = \frac{1}{2}\dot{\phi}^2 (1 + \frac{3}{4}\chi_2 b e^{-\alpha\phi} v^2) + \frac{v^2}{4}(V + M_1) + \frac{3 p_u v}{4 a(t)^3} \tag{4}$$

$$\frac{d}{dt}\Big(a(t)^3 \dot{\phi} (1 + \frac{\chi_2}{2} b e^{-\alpha\phi} v^2) \Big) + a(t)^3 (\alpha \frac{\dot{\phi}^2}{4}\chi_2 b e^{-\alpha\phi} v^2 + \frac{1}{2} V_\phi v^2 - \chi_2 U_\phi \frac{v^4}{4}) = 0 \tag{5}$$

The parameters of this system are 12:

4 free parameters $\{\alpha, b_0, f_1, f_2\}$, 5 integration constants $\{M_0, M_1, M_2, \chi_2, p_u\}$ and 3 initial conditions $\{a(0), \phi(0), \dot{\phi}(0)\}$

The process



The initial conditions and normalization are:

$$a(0) = 10^{-15}, \phi(0) = \phi_0, \dot{\phi}(0) = 0 \text{ and } a(1) = 1, \ddot{a}(0.71) = 0 \quad (6)$$

Numerical solutions, two cases, 3 periods of evolution

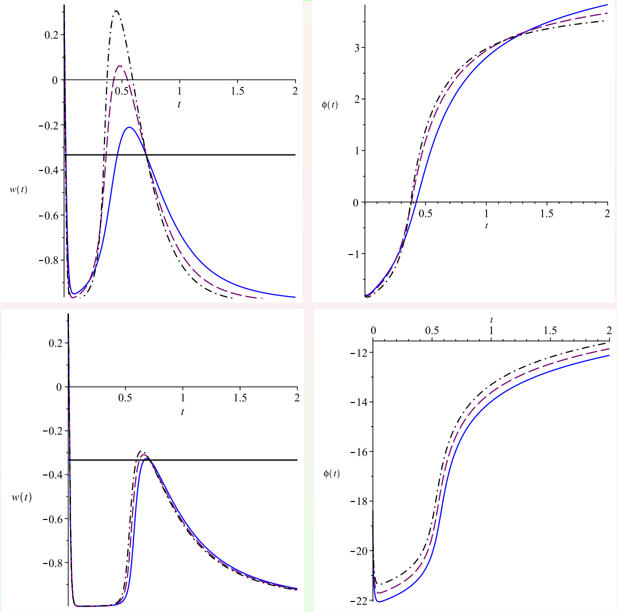
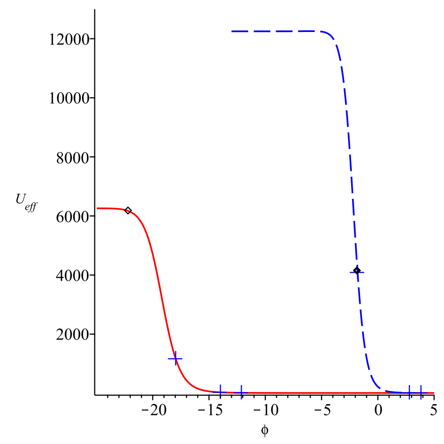


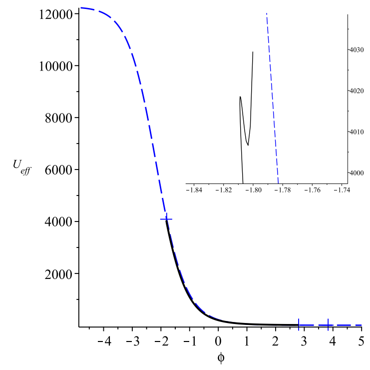
Figure: The Universe evolution for Case 1 (top): $\chi_2 \sim 1$, $M_0 \sim -0.04$, $0 < M_2 << |M_0|$. and Case 2 (bottom): $\chi_2 << 1$, $M_0 \sim -0.01$, $M_2 = 4$ $(\gg |M_0|)$. $(w = p/\rho)$

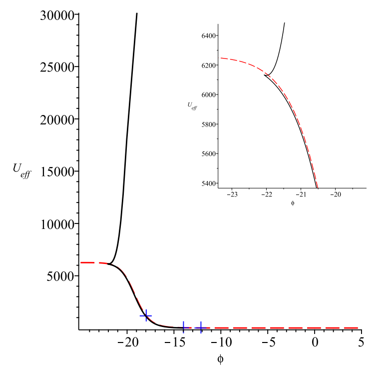
Climbing up the slope? arXiv1801.07133, arXiv:1806.08199, arXiv:1808.08890



Due to the strong friction term, we are unable to start from the left plateau.
But then, why it seems that the inflaton climbs up the slope?

If we integrate the inflaton equation





The **inflaton equation** (with $\left(A(t) = \frac{b_0 \chi_2}{2e^{\alpha\phi(t)}}\right)$

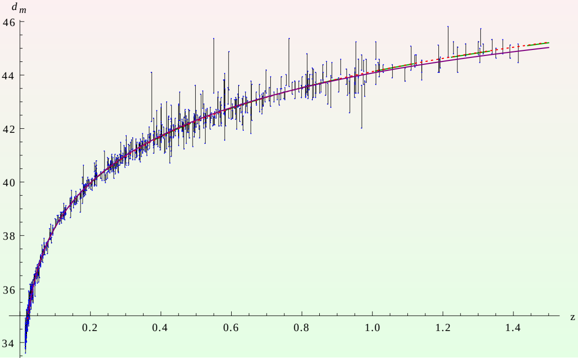
$$\left(v(t)^2 A(t)+1\right)\ddot{\phi}(t) - \frac{1}{2} v(t)^2 \alpha A(t) \dot{\phi}(t)^2 + \left(3v(t)^2 A(t) H + 2v(t) A(t) \dot{v}(t) + 3H\right) \dot{\phi}(t) - \frac{v(t)^2 f_1 \alpha}{2e^{\alpha\phi(t)}} + \frac{\chi_2 f_2 \alpha v(t)^4}{2e^{2\alpha\phi(t))}} = 0, \qquad (7)$$

The effective potential describes extremely well the numerical potential except for a small moment in beginning of the integration.

Conclusions and questions:

1. We have a model that qualitatively produces a "realistic" Universe if evolution starts from the slope of the effective potential.
2. There is friction term which stops the evolution of the inflaton
3. The effective potential is a good approximation to the actual potential term only after certain moment.
4. Questions: is there a better way to study the parameter space?

– MCMC integration vs. Machine learning?
– Minimal set of point needed for ML?
– From the QA, how to introduce the model in the ML?



Thank you for you attention!

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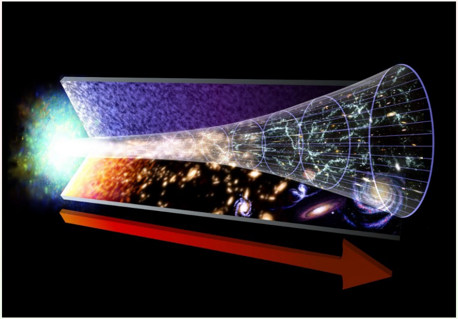

For more details: arXiv:1610.08368, 1801.07133, 1806.08199, 1808.08890