Квазинормални моди на въртящи се черни дупки

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Атестационен семинар

25 април 2016 г.

План на семинара

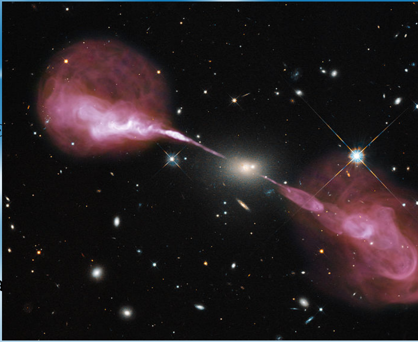
1. Квазинормални моди на въртящи се черни дупки
   - Защо квазинормални моди?
   - Уравнения на Тюколски и техните решения
2. Числени резултати
   - Двумерен алгоритъм на Мюлер
   - Резултатите при липса на въртене
   - Резултати за въртяща се черна дупка
3. Публикации

Теоретични срещу наблюдателни черни дупки

"Малко, тъмно и тежко: Но дали е черна дупка?" M.Visser et al.

BHs,GRandStrings 2008:010,2008, arXiv:0902.0346v2

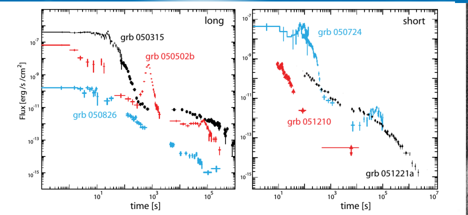
Какво казват наблюдателните данни:

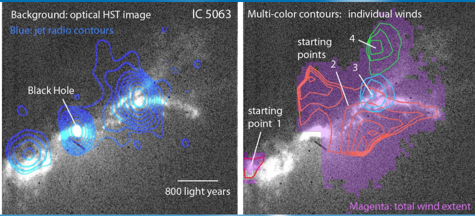


- Най-малката звездна черна дупка: $3.8 M_\odot$ XTE J1650-500 (типично M $\in (4-15) M_\odot$)
- Средни по маса черни дупки: $1.10^3 - 4.10^4 M_\odot$
- Супер-масивни черни дупки: $1.10^6 - 9.10^9 M_\odot$
- 2 двойни системи SMBH
- 5 тройни системи
- Липсващи двойни системи неутронна звезда (НЗ) – черна дупка (ЧД)
- Невъзможност към момента да се отдели НЗ от ЧД само по спектъра /случаят GRO J0422+32: от 3.6 $M_\odot$(2003) до $2.1 M_\odot$ (2012)/

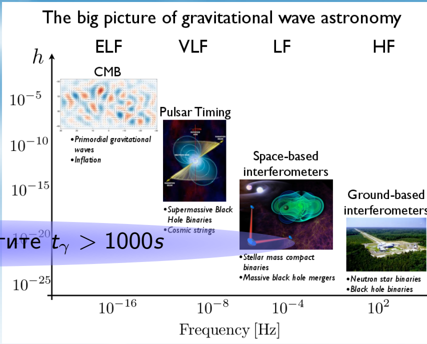
Обекти с огромна разлика в мащабите, които наричаме черни дупки.

Нерешени проблеми?





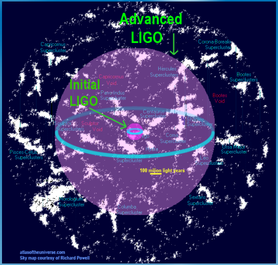
1. Липсващи гравитационни вълни от LIGO
2. Липсващи гравитационни вълни от Pulsar Timing Arrays **/Madison et al. MNRAS, arXiv:1510.08068 [astro-ph.IM]/**
3. Загадката на GRB – $E_{iso} \sim 10^{53} erg$, $t \sim sec$, $t_{flares} \sim 10^5 s$ живот на центр. двигател за ултрадългите $t_\gamma > 1000s$ **/APJ, 778:54, 2013, ApJ 766:30, 2013/**
4. Образуване на струи, M^{BH}_{min}, магн. полета – нужно $B \sim 10^{15} G$

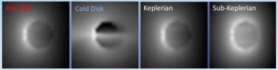


Проекти посветени на изучаването на ЧД:

- Advanced LIGO (09.2015)
- LISA Pathfinder (изстреляна на 3.12.2015!)
- Astrosat (изстрелян 28.09.2015, visible, UV, X-ray)
- Event Horizon Telescope (EHT, VLBI)
- BlackHoleCam project (финансиран от ERC, VLBI)
- Както и Chandra (X-ray), Spitzer (infrared), Fermi (gamma), Swift (transient), NuSTAR (X-ray), Hubble, Keppler etc.

Как да видим черна дупка?
– радио-телескопи /планирана разделителна способност за SgrA* - $4r_{Sh}$/
– гравитационни вълни
– multimessenger approach





Звъненето на черните дупки /Teukolsky (1972)/

Линейна пертурбация на метриката на Кер за $\Psi = e^{i(\omega t + m\phi)} S(\theta) R(r)$

$$\left(\left(1-u^2\right) S_{lm,u} \right)_{,u} + \left((a\omega u)^2 + 2a\omega s u + {}_sE_{lm} - s^2 - \frac{(m+su)^2}{1-u^2} \right) S_{lm} = 0, \quad (1)$$

и Радиално уравнение(TRE):

$$\frac{d^2 R_{\omega,E,m}}{dr^2} + (1+s)\left(\frac{1}{r-r_+} + \frac{1}{r-r_-} \right) \frac{dR_{\omega,E,m}}{dr} + + \left(\frac{K^2}{(r-r_+)(r-r_-)} - is\left(\frac{1}{r-r_+} + \frac{1}{r-r_-} \right) K - -\lambda - 4\, is\omega r \right) \frac{R_{\omega,E,m}}{(r-r_+)(r-r_-)} = 0 \quad (2)$$

където $\Delta = r^2 - 2Mr + a^2 = (r-r_-)(r-r_+)$, $K = -\omega(r^2+a^2) - ma$, $\lambda = E - s(s+1) + a^2\omega^2 + 2am\omega$ и $u = \cos(\theta)$.

За ЕМ пертурбации: $s = -1$. Двата хоризонта са: $r_\pm = M \pm \sqrt{M^2 - a^2}$.

Решения в термини на конфлуентни функции на Хойн

Решението на TAE:

$$S_{1,2}(\theta) = e^{\alpha_{1,2} z_{1,2}} z_{1,2}^{\beta_{1,2}/2} z_{2,1}^{\gamma_{1,2}/2} \mathrm{HeunC}(\alpha_{1,2}, \beta_{1,2}, \gamma_{1,2}, \delta_{1,2}, \eta_{1,2}, z_{1,2}) \quad (3)$$

където $z_1 = \cos(\theta/2)^2$, $z_2 = \sin(\theta/2)^2$, а параметрите са:

За случаят $m = 0$:

$$\alpha_1 = -\alpha_2 = 4\,a\omega,$$
$$\beta_1 = \beta_2 = 1,$$
$$\gamma_1 = -\gamma_2 = -1,$$
$$\delta_1 = -\delta_2 = 4\,a\omega,$$
$$\eta_1(\omega) = \eta_2(-\omega) = 1/2 - E - 2\,a\omega - a^2\omega^2$$

За случаят $m = 1$:

$$\alpha_1 = \alpha_2 = -4\,a\omega,$$
$$\beta_1 = \gamma_2 = 2,$$
$$\gamma_1 = \beta_2 = 0,$$
$$\delta_1 = -\delta_2 = 4\,a\omega,$$
$$\eta_1(\omega) = \eta_2(-\omega) = 1 - E - 2\,a\omega - a^2\omega^2$$

Където $HeunC(\alpha, \beta, \gamma, \delta, \eta, z)$ е решението на:

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2} H(z) + (\alpha + \frac{\beta+1}{z} + \frac{\gamma+1}{z-1}) \frac{\mathrm{d}}{\mathrm{d}z} H(z) + (\frac{\mu}{z} + \frac{\nu}{z-1}) H(z) = 0$$

и $\delta = \mu + \nu - \alpha(\beta+\gamma+2)/2, \eta = \alpha(\beta+1)/2 - \mu - (\beta+\gamma+\beta\gamma)/2$

Решенията на TRE:

$$R(r) = C_1 R_1(r) + C_2 R_2(r), \text{за} \qquad (4)$$
$$R_1(r) = e^{\frac{\alpha z}{2}} (r - r_+)^{\frac{\beta+1}{2}} (r - r_-)^{\frac{\gamma+1}{2}} \text{HeunC}(\alpha, \beta, \gamma, \delta, \eta, z)$$
$$R_2(r) = e^{\frac{\alpha z}{2}} (r - r_+)^{\frac{-\beta+1}{2}} (r - r_-)^{\frac{\gamma+1}{2}} \text{HeunC}(\alpha, -\beta, \gamma, \delta, \eta, z),$$

където $z = -\frac{r - r_+}{r_+ - r_-}$, а параметрите са:

$$\alpha = -2\,i\left(r_+ - r_-\right)\omega, \beta = -\frac{2\,i(\omega\left(a^2 + {r_+}^2\right) + am)}{r_+ - r_-} - 1, \gamma = \frac{2\,i(\omega\left(a^2 + {r_-}^2\right) + am)}{r_+ - r_-} - 1,$$
$$\delta = -2i(r_+ - r_-)\omega\left(1 - i\left(r_- + r_+\right)\omega\right),$$
$$\eta = \frac{1}{2}\frac{1}{\left(r_+ - r_-\right)^2}\Big[4\omega^2 {r_+}^4 + 4\left(i\omega - 2\omega^2 r_-\right){r_+}^3 + \left(1 - 4a\omega\, m - 2\omega^2 a^2 - 2E\right) \times$$
$$({r_+}^2 + {r_-}^2) + 4\left(i\omega\, r_- - 2i\omega\, r_+ + E - \omega^2 a^2 - \frac{1}{2}\right) r_-\, r_+ - 4a^2\left(m + \omega\, a\right)^2\Big].$$

Гранични условия: /Fiziev(2009), Staicova and Fiziev (2010, 2015)/

За TAE изискваме регулярност на сферата. Т.е. Вронскианът на 2 решения $S_1(\theta)$ and $S_2(\theta)$, да е $W[S_1(\theta), S_2(\theta)] = 0$, или:

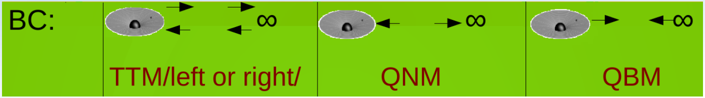
$$W[S_1, S_2] = \frac{\mathrm{HeunC}'(\alpha_1, \beta_1, \gamma_1, \delta_1, \eta_1, (\cos(\pi/6))^2)}{\mathrm{HeunC}(\alpha_1, \beta_1, \gamma_1, \delta_1, \eta_1, (\cos(\pi/6))^2)} + \frac{\mathrm{HeunC}'(\alpha_2, \beta_2, \gamma_2, \delta_2, \eta_2, (\sin(\pi/6))^2)}{\mathrm{HeunC}(\alpha_2, \beta_2, \gamma_2, \delta_2, \eta_2, (\sin(\pi/6))^2)} + p = 0 \qquad (5)$$

За TRE:

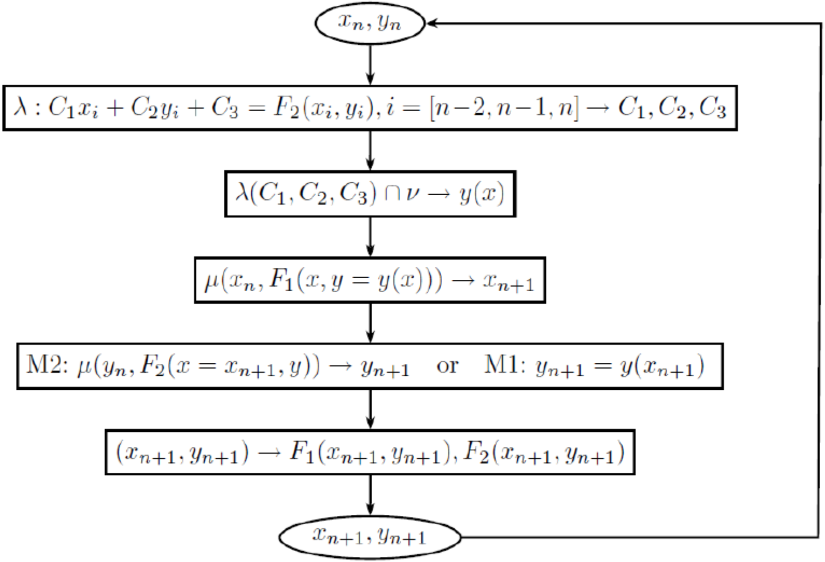
– (black hole boundary conditions) BHBC: R_2 е валидно за $\Re(\omega) \notin (-\frac{ma}{2Mr_+}, 0)$ и $\sin(\arg(\omega) + \arg(r)) < 0$ (DSD).

– (quasibound boundary conditions) QBBC: R_1 е валидно за $\Re(\omega) \notin (-\frac{ma}{2Mr_+}, 0)$ и $\sin(\arg(\omega) + \arg(r)) > 0$.

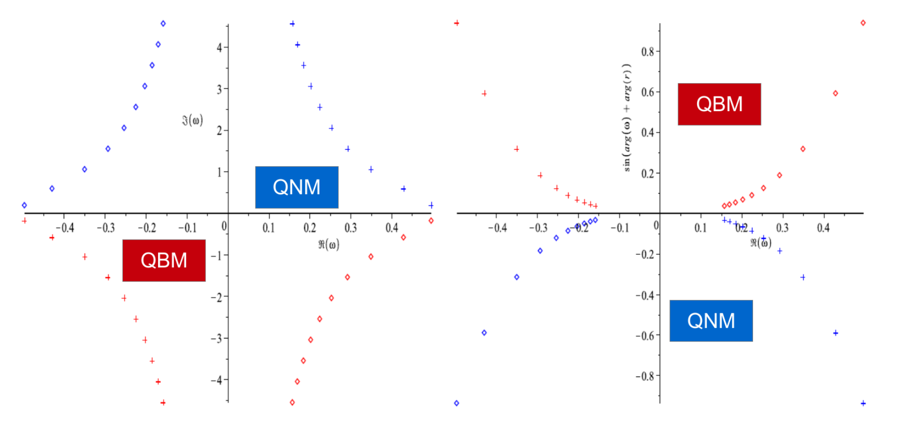
TTM моди липсват в EM случаят, ϵ-методът в най-обща форма е $r = |r| e^{i \arg(r)}$.



Двумерен алгоритъм на Мюлер /P.Fiziev and D.S. AJCM 02:02 (2012)/

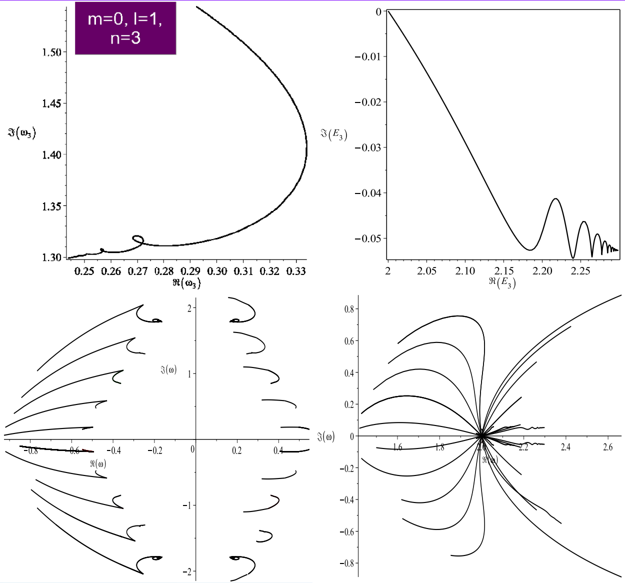


Спектърът при a=0 /D.S. and Fiziev (2015)/



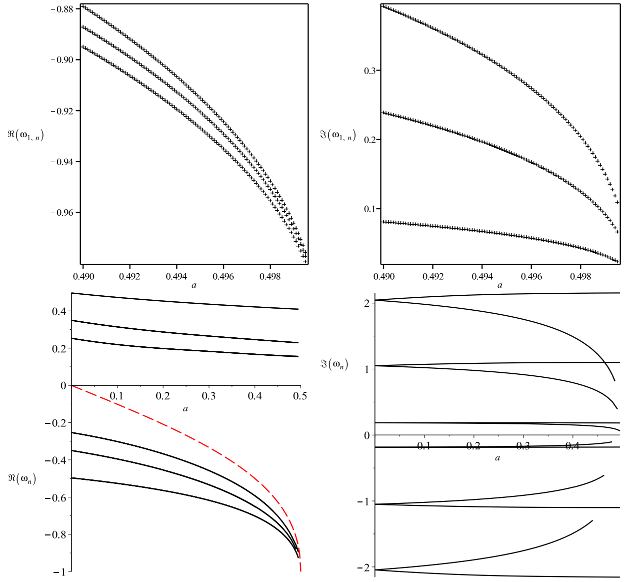
Фигура: a) QNM и QBM моди за $m = 0, l = 1$ б) Граничното условие за тях: $sin(arg(\omega) + arg(r))0$

Спектърът при $a \in [0, M]$ /Fiziev and D.S. (2015)/



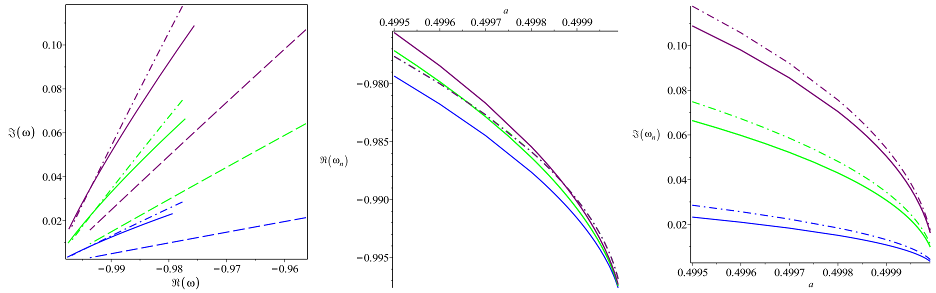
Фигура: $\omega_{m,n}(a)$ и $E_{m,n}(a)$ за $a = [0, M)$, $m = 0, 1, l = 1$ $n = 0..4$

Спектърът при $a \to M$ /D.S. and Fiziev (2015)/



Фигура: $\Re(\omega_{1,n})(a)$ и $\Im(\omega_{1,n})(a)$ за $a = [0.49, 0.4995]$ за моди $n = 0, 1, 2,$

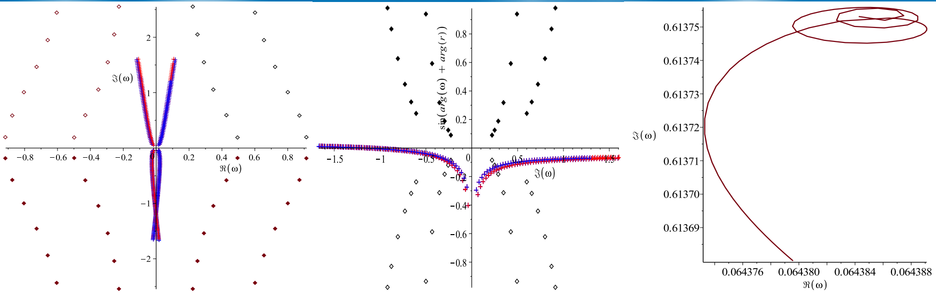
Аналитично приближение на спектъра при $a \to M$ /D.S. and Fiziev (2015)/



Фигура: ($\omega^1(a)$ (пунктир), $\omega^2(a)$ (точки-пунктир), и нашите числени резултати за ω_n for n=0..2, m=1, l=1

Ако $\Omega = \frac{a}{r_+^2 - a^2}$, $r_\pm = M \pm \sqrt{M^2 - a^2}$, $A = 4\pi(r_+ + a^2)$ и $T_{BH} = \frac{r_+ - r_-}{A}$.
Hod (2008) предлага 2 аналитични формули за ω близо до екстремалния режим:
$\omega^1 = m\Omega - i2\pi T_{BH}(n + 1/2)$ и $\omega^2 = m\Omega - i2\pi T_{BH}(n + 1/2 + i\delta)$.
Нашите числени резултати се описват по-добре с ω^2 с $\delta = -1/2 + i/6$, а за $m = 2$ с $\delta = -1/2$. Отклонението от теоретичната формула е $< 5\%$ for $a \to M$.

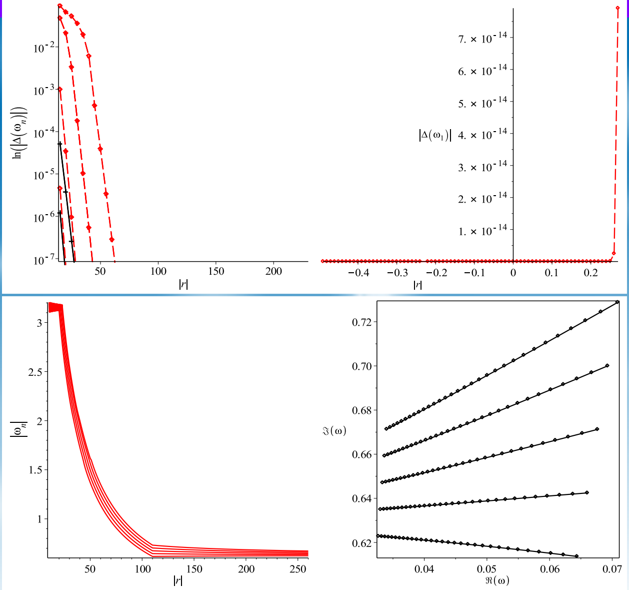
Spurious modes /D.S. and Fiziev (2015)/



Фигура: (a) Нефизични моди (кръстчета) с КНМ и КБМ (ромбчета) за $a = 0$, $m = 0, l = 1, 2$. (b) Граничното условие за тях $\sin(\arg(\omega) + \arg(r))$ (c) модата $n = 3$ за $a \in [0, M)$

Допълнителните нефизични моди бяха изследвани числено с епсилон-метода, за да се намери критерий за тяхното отсяване.

Числена стабилност на различните моди /D.S. and Fiziev (2015)/



Фигура: Горе: QNM Долу: Spurious

Статии

1. “Numerical stability of the electromagnetic quasinormal and quasibound modes of Kerr black holes” Denitsa R. Staicova , Plamen P. Fiziev, Bulg. Astr. J., 23 (2015), issn: 1313-2709, (arXiv:1511.09081)
2. “New results for electromagnetic quasinormal and quasibound modes of Kerr black holes ” Denitsa R. Staicova, Plamen P. Fiziev Astrophysics and Space Science, June 2015, 358:10, (arXiv:1412.4111), SJR=0.760, H=50
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Статии и доклади от конференции

7. The Spectrum of Electromagnetic Jets from Kerr Black Holes and Naked Singularities in the Teukolsky Perturbation Theory, D. Staicova, P. Fiziev, Astrophys Space Sci (2011) 332: 385-401 , SJR=0.760, H=50
8. Toward a New Model of the Central Engine of GRB, P. Fiziev, D. Staicova, Bulgarian Astronomical Journal, 11, pp. 13-21, 2009
9. A new model of the Central Engine of GRB and the Cosmic Jets, P. Fiziev, D. Staicova, Bulgarian Astronomical Journal, 11, pp. 3-11, 2009

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10. P. Fiziev, and D. Staicova, "Towards New Paradigms: Proceeding of the Spanish Relativity Meeting 2011", Ed. by I.B. Jimenez,J.S.R. Cembrano s, A. Dobado, et. Al, Book Series: AIP Conference Proceedings, Vol 1458, pp. 395-398, (2012) , SJR=0.152, H=53

Цитати:

Забелязани цитати:

- (9): 1 независим цитат (2015)
- (7): 1 независим цитат (2011)
- (6): 3 независими цитата (2011, 2012, 2013)
- (4): 7 независими цитати (2012, 2013а,б, 2014, 2015а,б,в):
- (5): 9 независими цитати (2012, 2013а,б,в,г), 2014(а,б), 2015(а,б):
- (10): 2 независими цитати (2013а,б)

Общо: 23 независими цитати
От тях цитати през 2013: 9
От тях цитати през 2014: 3
От тях цитати през 2015: 6

Други

Конференции с доклади: 2013-2015:
IberiCOS 2015 Xth Iberian Cosmology Meeting /доклад: Minimal Dilatonic Gravity from cosmology to compact massive objects/
XI. International Workshop Lie Theoryand Itsapplications In Physics /19. 06.2015 доклад: The Heun functions and theirapplications in astrophysics.applications in astrophysic/
10-та научна конференция на Съюза на астрономите в България Белоградчик, 2 – 5 юли 2015 г./с доклад Minimal Dilatonic Gravity from cosmology to compact massive objects/
Командировки:
15.09.2014-15.10.2014 – Университетът в Аликанте, Испания
20.10.2014-21.11.2014 – Университетът Гьоте във Франкфурт, Германия /по COST Action: MP1304/
Училища /в рамките на Cost Action NewCompStar/:
"The many faces of compact stars": Barcelona (Spain), September 22 – 26, 2014
"Dense matter in compact stars: Experimental and observational signatures, 21.-25-09.15 , Bucharest, Romania

Семинари и други

Семинари:

„The Heun functions and their applications in astrophysics.“ (Семинар в ИЯИЯЕ, БАН, 19.12.2013)

„Compact stars in minimal dilatonic gravity“. (Семинар в Astro-Coffee, ITP, Frankfurt, Germany, 18.11.2014)

„Compact static stars in Minimal Dilatonic Gravity”. (Семинар в ИЯИЯЕ, БАН, 12.2014)

„Minimal Dilatonic Gravity from cosmology to compact massive stars“”' (Семинар в Института по Астрономия, БАН, 12.03.2015)

Статии в годишника на ИЯИЯЕ:

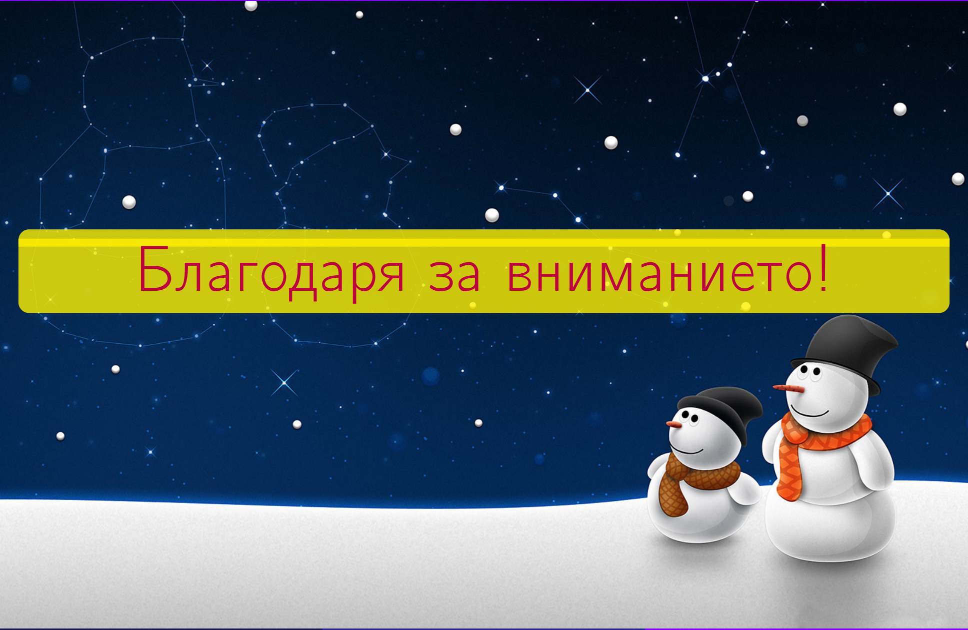
Electromagnetic Spectra of Rotating And Non-Rotating Black Holes, Denitsa Staicova, Plamen Fiziev, 2013, p. 86

Minimal dilatonic gravity in compact relativistic stars, Denitsa Staicova, Plamen Fiziev, 2014

Постери:

Постер за 43 годишнината на ИЯИЯЕ (43 years 1972 – 2015 INRNE),
“Electromagnetic quasi-normal modes of rotating black holes”, Denitsa Staicova and Plamen Fiziev

Това е!



Благодаря за вниманието!

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(10): 1 независим цитат (2015)
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(8): 1 независим цитат (2011)
D. Pugliese, H. Quevedo, and R. Ruffini, “Equatorial circular motion in Kerr spacetime”, Phys. Rev. D 84, 044030 (2011)
(7): 3 независими цитата (2011, 2012, 2013)
M. Cadoni, P. Pani, “Holography of charged dilatonic black branes at finite temperature” - JHEP 1104:049, (2011)
Pani, P. "Applications of black hole perturbation theory."The European Physical Journal Plus 127.6 : 1-52, (2012)
Neville I. Robinson, Andrew J. Love, Hidden channels of groundwater flow in Tóthian drainage basins, Advances in Water Resources 62:71–78 · December (2013)
(4):7 независими цитати (2012, 2013а,б, 2014, 2015а,б,в):
M.A. Dariescu, C. Dariescu, “Approximative Analytic Study of Fermions in Magnetar's Crust; Ultra-relativistic Plane Waves, Heun and Mathieu Solutions and Beyond”, Astrophysics and Space Science, 2012 – Springer, 10.1007/s10509-012-1101-y. 341, I. 2, pp.429-435 (2012)
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