

# The multi-measure cosmological model and its peculiar effective potential

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## The multimeasure model by Guendelman, Nissimov and Pacheva

- In Jordan frame, the multimeasure action of the model:  $S = S_{darkon} + S_{inflaton}$  is :

$$S_{darkon} = \int d^4x \left[ \sqrt{-g} + \Phi(C) \right] L(u, Y)$$

- Riemannian measure
- Non-Riemannian measure
- Lagrangian of the darkon scalar field  $u$

$$S_{inflaton} = \int d^4x \Phi_1(A) (R + L^{(1)}) + \int d^4x \Phi_2(B) \left( L^{(2)} + \frac{\Phi(H)}{\sqrt{-g}} \right)$$

- Lagrangians of the inflaton scalar field  $\phi$

$$\text{where } X(\phi) = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi,$$

$$Y(u) = -\frac{1}{2} g^{\mu\nu} \partial_\mu u \partial_\nu u$$

$$\Phi_i(Z) = \frac{1}{3} \epsilon^{\mu\nu\kappa\lambda} \partial_\mu Z_\nu \partial_\kappa Z_\lambda$$

$$L(u, X) = Y(u) - W(u)$$

$$L^{(1)} = X(\phi) - V(\phi), \quad V(\phi) = f_1 e^{-\alpha\phi}$$

$$L^{(2)} = b e^{-\alpha\phi} X(\phi) + U(\phi), \quad U(\phi) = f_2 e^{-2\alpha\phi}$$

- In Einstein frame (for which  $\tilde{R}_{\mu\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{R} = \frac{1}{2} T_{\mu\nu}^{(eff)}$ ):

$$S^{(eff)} = \int d^4x \sqrt{-\tilde{g}} (\tilde{R} + L^{(eff)}) \quad (1)$$

with effective Lagrangian of the k-essence type:

$$L^{(eff)} = \tilde{X} - \tilde{Y} (V + M_1 - \chi_2 b e^{-\alpha\phi} \tilde{X}) + \tilde{Y}^2 (\chi_2 (U + M_2) - 2M_0).$$

The effective potential of the model is:  $U_{eff}(\phi) = \frac{1}{4} \frac{(f_1 e^{-\alpha\phi} + M_1)^2}{\chi_2 (f_2 e^{-2\alpha\phi} + M_2) - 2M_0}$ .

The effective cosmological constant:  $\Lambda_{eff} = U_+ / 2 = \frac{M_1^2}{8(\chi_2 M_2 - 2M_0)}$ .

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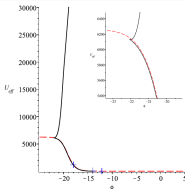
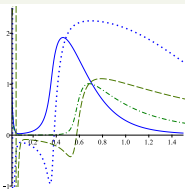
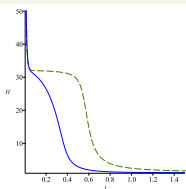
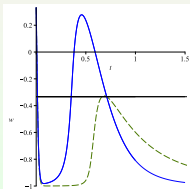
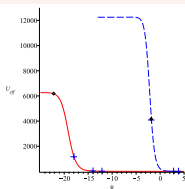
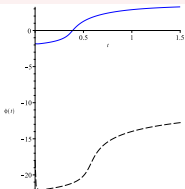
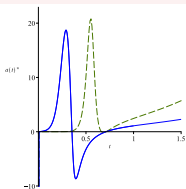
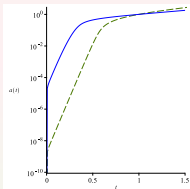
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# In FLRW metric the equations of motions are ( $v = \dot{u}$ ):

$$v^3 + 3av + 2b = 0 \text{ for } \mathbf{a} = \frac{-1}{3} \frac{V(\phi) + M_1 - \frac{1}{2}\chi_2 b e^{-\alpha\phi} \dot{\phi}^2}{\chi_2(U(\phi) + M_2) - 2M_0}, \mathbf{b} = \frac{-p_u}{2a(t)^3(\chi_2(U(\phi) + M_2) - 2M_0)} \quad (2)$$

$$\dot{a}(t) = \sqrt{\frac{\rho}{6}} a(t), \quad \rho = \frac{1}{2} \dot{\phi}^2 (1 + \frac{3}{4} \chi_2 b e^{-\alpha\phi} v^2) + \frac{v^2}{4} (V + M_1) + \frac{3p_u v}{4a(t)^3} \quad (3)$$

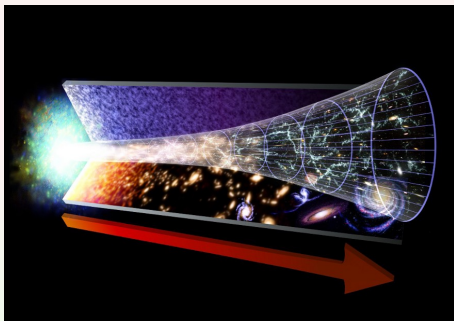
$$\frac{d}{dt} \left( a(t)^3 \dot{\phi} (1 + \frac{\chi_2}{2} b e^{-\alpha\phi} v^2) \right) + a(t)^3 \left( \alpha \frac{\dot{\phi}^2}{4} \chi_2 b e^{-\alpha\phi} v^2 + \frac{1}{2} V_\phi v^2 - \chi_2 U_\phi \frac{v^4}{4} \right) = 0 \quad (4)$$



- 3 Universe epochs
- Strong friction term
- Can't start from the plateau
- Climbing up the slope
- E-folds': N=18

# Thank you for you attention!

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For more details: [arXiv:1610.08368](https://arxiv.org/abs/1610.08368), [1801.07133](https://arxiv.org/abs/1801.07133), [1806.08199](https://arxiv.org/abs/1806.08199), [1808.08890](https://arxiv.org/abs/1808.08890)