Cosmological solutions from multi-measure model

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The multi-measures model Guendelman, Nissimov and Pacheva 2014, 2016

The action of the model $S = S_{darkon} + S_{inflaton}$ is :

$$S_{darkon} = \int d^4x\sqrt{-g}(R(g,\Gamma)) + \int d^4x(\sqrt{-g} + \Phi(C))L(u,Y)$$

$$S_{inflaton} = \int d^4x \Phi_1(A)(R + L^{(1)}) + \int d^4x \Phi_2(B)\left(L^{(2)} + \frac{\Phi(H)}{\sqrt{-g}}\right)$$

where $\Phi_i(X) = \frac{1}{3}\epsilon^{\mu\nu\kappa\lambda}\partial_\mu X_{\nu\kappa\lambda}$, $L(u,X) = -\frac{1}{2}g^{\mu\nu}\partial_\mu u \partial_\nu u - W(u)$ and

$$L^{(1)} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi),\ V(\phi) = f_1 e^{-\alpha\phi}$$

$$L^{(2)} = -\frac{b}{2}e^{-\alpha\phi}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + U(\phi),\ U(\phi) = f_2 e^{-2\alpha\phi}$$

The system of equations in FLRW metric is:

$$v^3 + 3\mathbf{a}v + 2\mathbf{b} = 0 \text{ for } \mathbf{a} = \frac{-1}{3}\frac{V(\phi) + M_1 - \frac{1}{2}\chi_2 b e^{-\alpha\phi}\dot{\phi}^2}{\chi_2(U(\phi) + M_2) - 2M_0}, \mathbf{b} = \frac{-p_u}{2a(t)^3(\chi_2(U(\phi) + M_2) - 2M_0)} \quad (1)$$

$$\dot{a}(t) = \sqrt{\frac{\rho}{6}}a(t),\ \ \rho = \frac{1}{2}\dot{\phi}^2(1 + \frac{3}{4}\chi_2 b e^{-\alpha\phi}v^2) + \frac{v^2}{4}(V + M_1) + \frac{3p_u v}{4a(t)^3} \quad (2)$$

$$\frac{d}{dt}\left(a(t)^3\dot{\phi}(1 + \frac{\chi_2}{2}be^{-\alpha\phi}v^2)\right) + a(t)^3(\alpha\frac{\dot{\phi}^2}{4}\chi_2 b e^{-\alpha\phi}v^2 + \frac{1}{2}V_\phi v^2 - \chi_2 U_\phi \frac{v^4}{4}) = 0 \quad (3)$$

The parameters of this system are 12:

$$\{\alpha, b_0, M_0, M_1, M_2, f_1, f_2, p_u, \chi_2\}$$

Numerical solutions, two cases, 3 periods of evolution

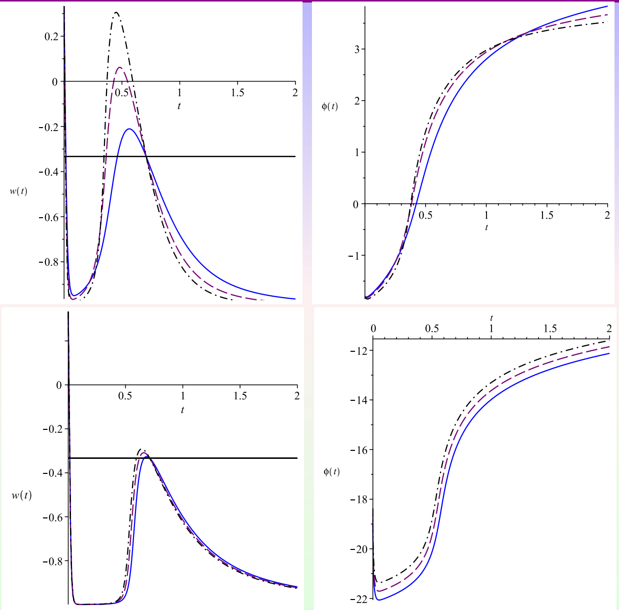
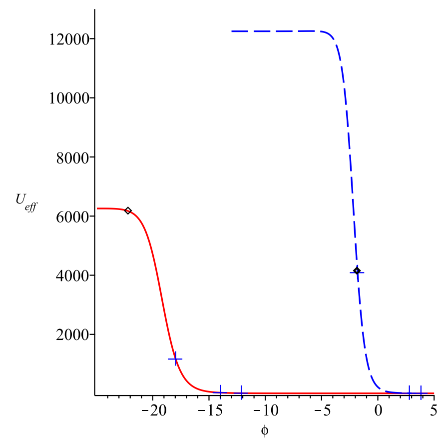


Figure: The Universe evolution for Case 1 (top): $\chi_2 \sim 1$, $M_0 \sim -0.04$, $0 < M_2 << |M_0|$. and Case 2 (bottom): $\chi_2 << 1$, $M_0 \sim -0.01$, $M_2 = 4$ $(\gg |M_0|)$.

Climbing up the slope? arXiv1801.07133, arXiv:1806.08199, arXiv:1808.08890



Due to the strong friction term, we are unable to start from the left plateau.

But then, why it seems that the inflaton climbs up the slope?

And why the slow-roll parameters seem to not work properly?

The effective potential

The model defines as effective potential: $U_{eff}(\phi) = \frac{1}{4}\frac{(f_1 e^{-\alpha\phi}+M_1)^2}{\chi_2(f_2 e^{-2\alpha\phi}+M_2)-2M_0}$.

In "Standard" cosmology with a scalar field:

$$H^2 = 8\frac{\pi}{3m_{pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \quad (4)$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \quad (5)$$

Then the slow-roll parameters are defined as:

1) $\ddot{\phi}(t) << 3H\dot{\phi} \rightarrow \eta = -\frac{\ddot{\phi}}{H\dot{\phi}} << 1$

2) $\frac{\ddot{a}(t)}{a(t)} = \dot{H} + H^2 = H^2(1-\epsilon) \rightarrow \epsilon = -\frac{\dot{H}}{H^2} << 1$

If we write the **inflaton equation** in the form:

$$\left(v(t)^2 A(t)+1\right)\ddot{\phi}(t) - \frac{1}{2}v(t)^2\alpha A(t)\dot{\phi}(t)^2 + \left(3v(t)^2 A(t)H + 2v(t)A(t)\dot{v}(t) + 3H\right)\dot{\phi}(t) - \frac{v(t)^2 f_1\alpha}{2e^{\alpha\phi(t)}} + \frac{\chi_2 f_2 \alpha v(t)^4}{2e^{2\alpha\phi(t))}} = 0, \quad (6)$$

where $A(t) = \frac{b_0\chi_2}{2e^{\alpha\phi(t)}}$.

If one uses the slow-roll approximation (neglecting the terms $\sim\ \dot{\phi}^2,\ \dot{\phi}^3,\ \dot{\phi}^4$ and $A(t)$), Eq. (6) simplifies to:

$$\ddot{\phi} + 3H\dot{\phi} + W(\phi) = 0. \quad (7)$$

where $W(\phi) = -\frac{v(t)^2 f_1\alpha}{2e^{\alpha\phi(t)}} + \frac{\chi_2 f_2\alpha v(t)^4}{2e^{2\alpha\phi(t))}}$, i.e. $W(\phi) \neq U'_{eff}$.

Asymptotic values

In general $v(t) \sim a(t), \phi(t), \dot{\phi}(t)$ and $\sqrt[3]{\ }$.
If we use the asymptotic values of $v(t)$ for $p_u \to 0$, $b_0 \to 0$ we get:

$$v_{pb}^2 = \frac{4U_{eff}}{f_1 e^{-\alpha\phi} + M_1}$$

Then one can obtain for the effective potential term:

$$U'_{eff} \approx -\frac{v_{pb}^2}{2}\alpha e^{-\alpha\phi}\left(f_1 - v_{pb}^2 f_2 \chi_2 e^{-\alpha\phi}\right). \tag{8}$$

Then, in the limit $v \to v_{pb}$, the equations reduce to:
Case 1 $(v(t)^2 A(t) >> 1)$:

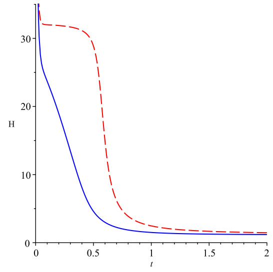
$$\ddot{\phi} + \frac{1}{2}\left(\frac{U'_{eff}}{U_{eff}} + \frac{\alpha e^{-2\alpha\phi}\chi_2 f_2 v_{pb}^4}{2U_{eff}} - \alpha\right)\dot{\phi}^2 + 3H\dot{\phi} + \frac{1}{v_{pb}^2 A(t)}U'_{eff} = 0. \tag{9}$$

Case 2 $(v(t)^2 A(t) << 1)$:

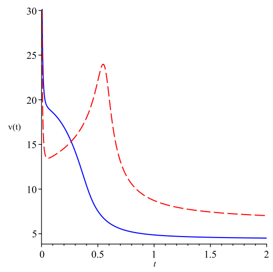
$$\ddot{\phi} + 3H\dot{\phi} + U'_{eff} = 0. \tag{10}$$

Case 2 approximates the standard equation, Case 1 – no.

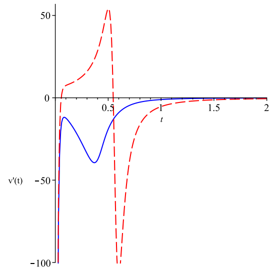
Evolution of some physical parameters in the two cases



H(t)

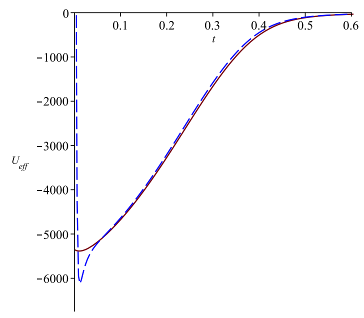


v(t)

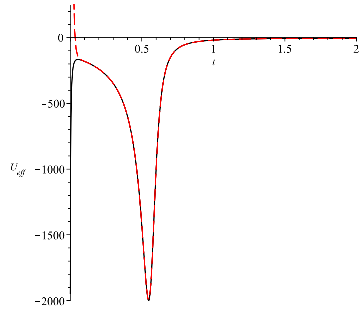


$\dot{v}(t)$

Climbing up the slope? Comparison between U'_{eff} and T_5



(a)



(b)

Figure: Comparison of the term T_5 (dash) with the derivative of the effective potential U'_{eff} (solid) in the two cases. (Note, case 1 is zoomed in)

The effective potential fits the numerical term very well, except for the initial period. But this is when the climbing up the slope happens!

Numerical integration

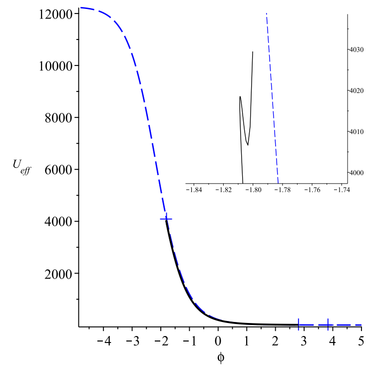
In summary we observe that:

1. Independent of the value of $\phi(0)$, there is a period for which ϕ increases in absolute value – i.e. it climbs up the slope of the effective potential
2. This effect becomes more pronounced the more we increase in absolute value $\phi(0)$
3. The effect does not depend on $\dot{\phi}(0)$. That is to say that this effect is not connected with the inflaton gaining kinetic energy so that it can climb the slope.
4. The time during which this happens puts it in the interval when the effective potential is not a good approximation of the potential term.

The effective potential might not be a good approximation!

- We reconstruct the effective potential U_{eff}^{num} from T_5 trough the means of numerical integration. I.e. we assume that $U_{eff}^{num} \sim \int T_5(\phi)d\phi$.
- We use numerical integration of the data points $[\phi(t_i), T_5(t_i)]$ with a modified Simpson's rule adapted to work in Maple with 1000 datapoints.
- The constant of integration has been determined by comparing the numerical potential and the effective one at late times.

And the results...



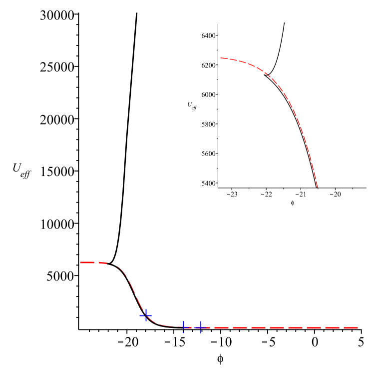


Figure: a) The effect in Case 2 is much more pronounced. The inflaton does not climb up the slope, it goes down a slope of a complicated shape.

Conclusions:

1. We have numerically confirmed that the multi-measure model can be applied to the inflaton+darkon case.
2. We have shown that it is not possible to start from the left plateau and to obtain physically realistic solutions
3. There is friction term which stops the evolution of the inflaton
4. The theory can produce "realistic" Universe if evolution starts from the slope.
5. The effective potential is a good approximation to the actual potential term only after certain moment.

Summary of the viable (left) and nonviable (right) Scalar-Tensor theories after GW170817 [Ezquiaga and Zumalacárregui (2017)]

	$c_g = c$	$c_g \neq c$
Horndeski	General Relativity quintessence/k-essence [47] Brans-Dicke/$f(R)$ [48, 49] Kinetic Gravity Braiding [51]	quartic/quintic Galileons [13, 14] Fab Four [15] de Sitter Horndeski [50] $G_{\mu\nu}\phi^\mu\phi^\nu$ [5], $f(\phi)\cdot$Gauss-Bonnet [53]
beyond H.	Derivative Conformal (19) [17] Disformal Tuning (21) quadratic DHOST with $A_1 = 0$	quartic/quintic GLPV [18] quadratic DHOST [20] with $A_1 \neq 0$ cubic DHOST [23]
	Viable after GW170817	Non-viable after GW170817

$$\mathcal{L} = G_2 - G_3\nabla^2\phi + G_4 R + G_{4,X}[\nabla\nabla\phi]^2 + G_5 G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5,X}}{6}[\nabla\nabla\phi]^3$$

arXiv: 1710.05901, arXiv: 1712.06556

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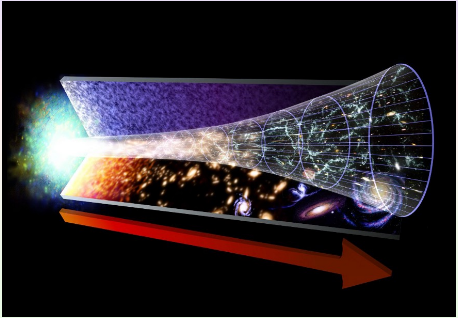

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