Cosmological aspects of a unified dark energy and dust dark matter model dual to quadratic purely kinetic k-essence

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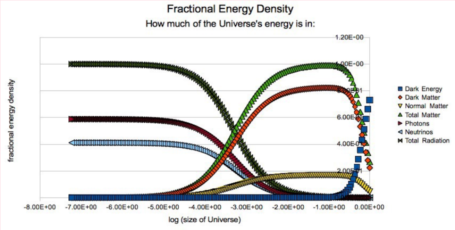
Contents

The two-mesures theory of gravity

Proposed in works by Guendelman, Nissimov and Pacheva: 2014, 2015 ([1],[2])

In this theory, the scalar Lagrangian couples symmetrically both to the standard Riemannian volume-form ($\sqrt{g_{\mu\nu}}$), as well as to another non-Riemannian volume-form ($\Phi(B)$) given in terms of an auxiliary maximal-rank antisymmetric tensor gauge field ($B_{\nu\kappa\lambda}$)

Dark Matter and Dark Energy constitute 95% of the Universe energy content. How to describe them from first principles?



(c) Image credit: E. Siegel.

$$S = S_{grav}[g_{\mu\nu}, \Gamma^{\lambda}_{\mu\nu}] + \int d^4x(\sqrt{-g} + \Phi(B))L(\phi, X)$$

$$\Phi(B) = \frac{1}{3}\epsilon^{\mu\nu\kappa\lambda}\partial_\mu B_{\nu\kappa\lambda}$$

$$L(\phi, X) = X - V(\phi), \quad X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

The model in FLRW metric

We work with the following metric:

$$ds^2 = -dt^2 + a(t)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right].$$

The Lagrangian in this case is:

$$\mathcal{L} = a(t)\left[-\dot{a(t)}^2 + \frac{1}{4}a(t)^2\left(\frac{1}{\alpha} - \frac{\dot{\phi}^2}{\alpha} + (\frac{1}{4\alpha} - 2M)\dot{\phi}^4\right)\right]$$

And the equation of motions:

$$a(t)^3\left[-\frac{1}{2\alpha}\dot{\phi} + (\frac{1}{4\alpha} - 2M)\dot{\phi}^3\right] = p_\phi \ \ (= \mathrm{const})$$

From the Friedman equations ($G_{00} = T_{00}$), one obtains for the energy density:

$$\rho = \frac{1}{8\alpha}\dot{\phi}^2 + \frac{3}{4}\frac{p_\phi}{a(t)^3}\dot{\phi} - \frac{1}{4\alpha}$$

$\dot{\phi}$ is a solution of a cubic equation: $a(t)^3\left[-\frac{1}{2\alpha}\dot{\phi}+(\frac{1}{4\alpha}-2M)\dot{\phi}^3\right]=p_\phi$
If we rewrite the equation for p_ϕ like:

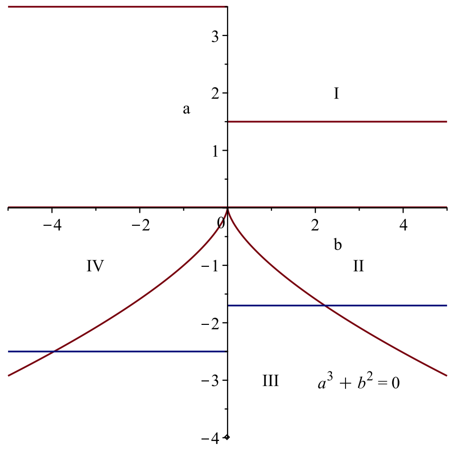
$$y^3+3\mathbf{a}y+2\mathbf{b}=0 \tag{1}$$

where $\mathbf{a}=-\frac{2}{3-24\alpha M}$ and $\mathbf{b}=-\frac{2\alpha p_\phi}{a(t)^3(1-8\alpha M)}$.
Solutions

$$\begin{aligned}
y_1 &= -\frac{\mathbf{a}}{(-\mathbf{b}+\sqrt{\mathbf{a}^3+\mathbf{b}^2})^{1/3}}+(-\mathbf{b}+\sqrt{\mathbf{a}^3+\mathbf{b}^2})^{1/3} && (2)\\
y_2 &= \frac{\mathbf{a}}{(\mathbf{b}-\sqrt{\mathbf{a}^3+\mathbf{b}^2})^{1/3}}-(\mathbf{b}-\sqrt{\mathbf{a}^3+\mathbf{b}^2})^{1/3} && (3)\\
y_3 &= \frac{1-i\sqrt{3}}{2}\frac{\mathbf{a}}{(-\mathbf{b}+\sqrt{\mathbf{a}^3+\mathbf{b}^2})^{1/3}}-\frac{1+i\sqrt{3}}{2}(-\mathbf{b}+\sqrt{\mathbf{a}^3+\mathbf{b}^2})^{1/3} && (4)
\end{aligned}$$

N.B. Eq.(1) is with real coefficients. Therefore there is always one real root $\forall\mathbf{a},\mathbf{b}$. However, no real smooth solution exists in the $\mathbf{a},\mathbf{b}$ plane.

Regions of validity in the [a,b] plane



Note that $\mathbf{b} \sim a(t)^{-3}$ and for an emerging universe where $a(t) \in [0, \infty]$, $\mathbf{a} > 0$, the evolution of the universe goes along a horizontal line never crossing $\mathbf{b} = 0$ line. However, for negative $\mathbf{a}$ the line $\mathbf{a}^3 + \mathbf{b}^2 = 0$ which separate domains of reality for different roots will be always crossed and eventually phase transition will be observed.

Rewrite eq(4) in terms of $\mathbf{a}, \mathbf{b}$ and y

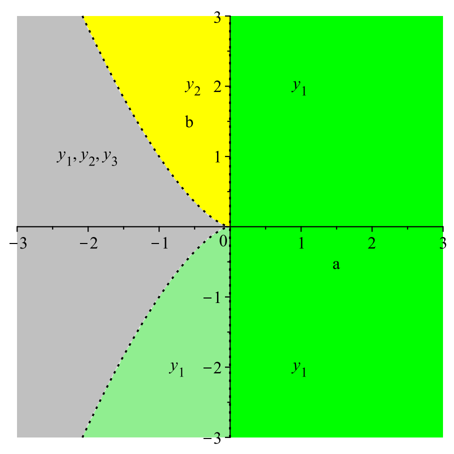
$$4\alpha\rho = \frac{1}{2}y^2 + \frac{\mathbf{b}}{\mathbf{a}}y - 1 \tag{5}$$

Changing the time scale in Freedman equation we can set $2|\alpha|/3 = 1$, so it becomes

$$\left(\frac{\dot{a}}{a}\right)^2 = \pm\rho \tag{6}$$

where plus sign is for $\alpha > 0 \bigcap \rho > 0$ and the minus sign is for $\alpha < 0 \bigcap \rho < 0$.
α eventually goes in to Hubble constant.

Regions of validity in the [a,b] plane



We can stitch different solutions real in the whole [a,b]

First we define the regions of validity of each solution:

$$reg1 = a > 0, \quad reg2 = a < 0 \wedge b < -(-a)^{3/2},$$
$$reg3 = a < 0 \wedge b < (-a)^{3/2} \wedge b > -(-a)^{3/2} \wedge b < 0,$$
$$reg4 = a1 < 0 \wedge b < (-a)^{3/2} \wedge b > 0 \wedge b > -(-a)^{3/2}, reg5 = a < 0 \wedge b > (-a)^{3/2}$$

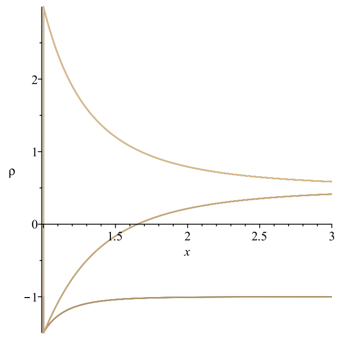
And then we use them to define a piecewise function, **real** in the whole plane [a,b]:

$$rr1(a,b) = \begin{cases} y_1, & \text{in } reg1 \\ y_1, & \text{in } reg2 \\ y_1, & \text{in } reg3 \\ y_1, & \text{in } reg4 \\ y_2, & \text{in } reg5 \end{cases} \quad rr4(a,b) = \begin{cases} y_1, & \text{in } reg1 \\ y_1, & \text{in } reg2 \\ y_1, & \text{in } reg3 \\ y_2, & \text{in } reg4 \\ y_2, & \text{in } reg5 \end{cases} \quad rr7(a,b) = \begin{cases} y_1, & \text{in } reg1 \\ y_1, & \text{in } reg2 \\ y_2, & \text{in } reg3 \\ y_3, & \text{in } reg4 \\ y_2, & \text{in } reg5 \end{cases}$$

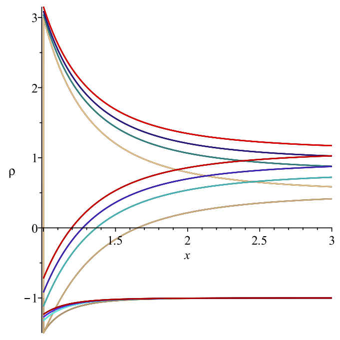
$$rr2(a,b) = \begin{cases} y_1, & \text{in } reg1 \\ y_1, & \text{in } reg2 \\ y_2, & \text{in } reg3 \\ y_2, & \text{in } reg4 \\ y_2, & \text{in } reg5 \end{cases} \quad rr5(a,b) = \begin{cases} y_1, & \text{in } reg1 \\ y_1, & \text{in } reg2 \\ y_1, & \text{in } reg3 \\ y_3, & \text{in } reg4 \\ y_2, & \text{in } reg5 \end{cases} \quad rr8(a,b) = \begin{cases} y_1, & \text{in } reg1 \\ y_1, & \text{in } reg2 \\ y_3, & \text{in } reg3 \\ y_1, & \text{in } reg4 \\ y_2, & \text{in } reg5 \end{cases}$$

$$rr3(a,b) = \begin{cases} y_1, & \text{in } reg1 \\ y_1, & \text{in } reg2 \\ y_3, & \text{in } reg3 \\ y_3, & \text{in } reg4 \\ y_2, & \text{in } reg5 \end{cases} \quad rr6(a,b) = \begin{cases} y_1, & \text{in } reg1 \\ y_1, & \text{in } reg2 \\ y_2, & \text{in } reg3 \\ y_1, & \text{in } reg4 \\ y_2, & \text{in } reg5 \end{cases} \quad rr9(a,b) = \begin{cases} y_1, & \text{in } reg1 \\ y_1, & \text{in } reg2 \\ y_3, & \text{in } reg3 \\ y_2, & \text{in } reg4 \\ y_2, & \text{in } reg5 \end{cases}$$

The behaviour of the density ρ:



(d) The 9 functions for $a = -1, b = 1/x^3$

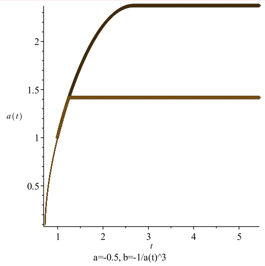


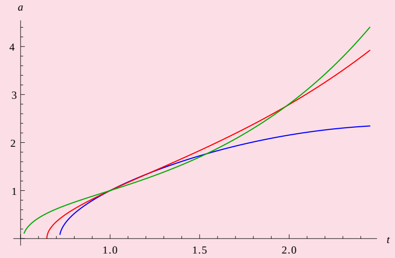
(e) The 9 functions for different $\{a, b\}$

We have entirely positive and entirely negative densities and also ones that change sign.

Numerical integration - Universe evolution without phase transition

Using the so-defined solutions for y, we can numerically integrate the Friedman equations to find the evolution $a(t)$.





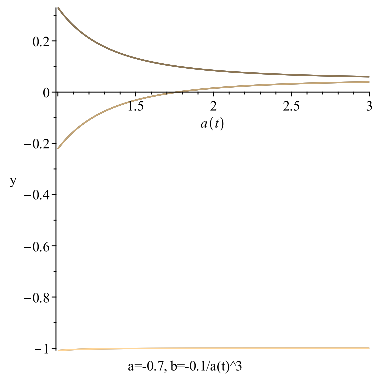
Graphics of the $a(t)$ evolution for $\mathbf{a} = -.5, \mathbf{b} = -\frac{1}{a^3}$ (blue), $\mathbf{a} = -1, \mathbf{b} = -\frac{1}{a^3}$ (red), $\mathbf{a} = 1, \mathbf{b} = \frac{1}{a^3}$ (green)

Here we have used y_1 with IC $a(1) = 1$ and the GEAR method for the numerical integrations.

Note that $t = 1$ ($a(1) = 1$) is the present moment while the moment t_0 at which $a(t_0) = 0$ represents the Big Bang.

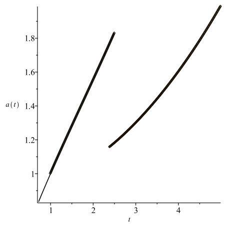
Stitching the solutions so that $\rho > 0$

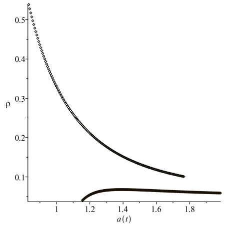
To find $a(t) : \rho(a(t)) = 0$, we use 2d Muller algorithm. For example:
Entirely positive solutions: 1, 4, 5
Entirely negative solutions: 3, 8, 9
Changing sign: [1, [1, 0.883-1.529i]], [2, [1, 1.765]],
[6, [1, 1.765I]], [7, [1, 1.765]], [8, [1, 0.883+1.529i]]

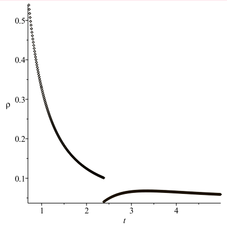


Universe evolution with phase transition 1

1) We find that $\rho_2 > 0$ at $t0 = 2.391$ with ICS for both DE: $a_1(1) = a_2(1) = 1$



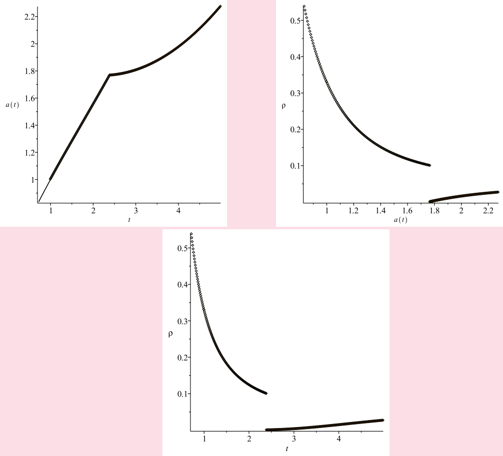




Here there is a jump in $\rho(t)$ and in $a(t)$.

Universe evolution with phase transition 2

2) Here still $t0 = 2.391$ but the ICS are: $a_1(1) = 1, a_2(t0) = 1.765$



Here there is no jump in $a(t)$, but there is a phase transition in ρ at the moment $t0$.

Supernova data fit

Distance modulus d_m:

$$d_m = 5\log_{10}\left(\frac{d}{10}\right) \tag{7}$$

where d is the distance in parsecs. Connection between a and red shift z:

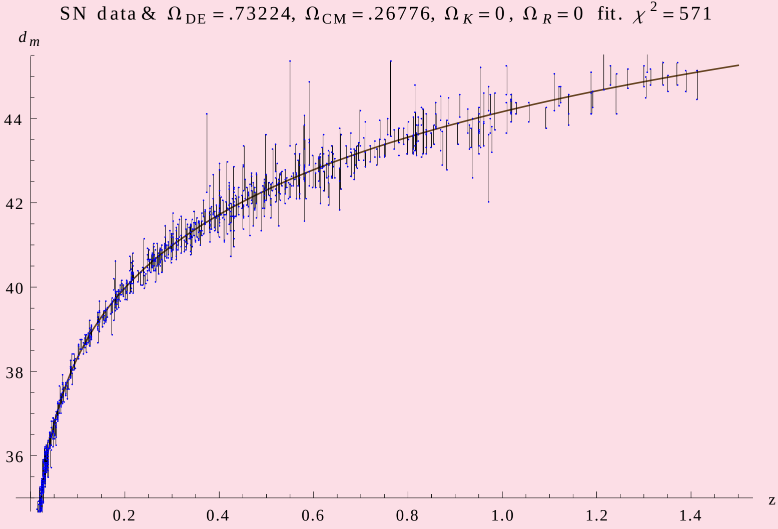
$$a = \frac{1}{1+z} \tag{8}$$

Connection between d_m and z:

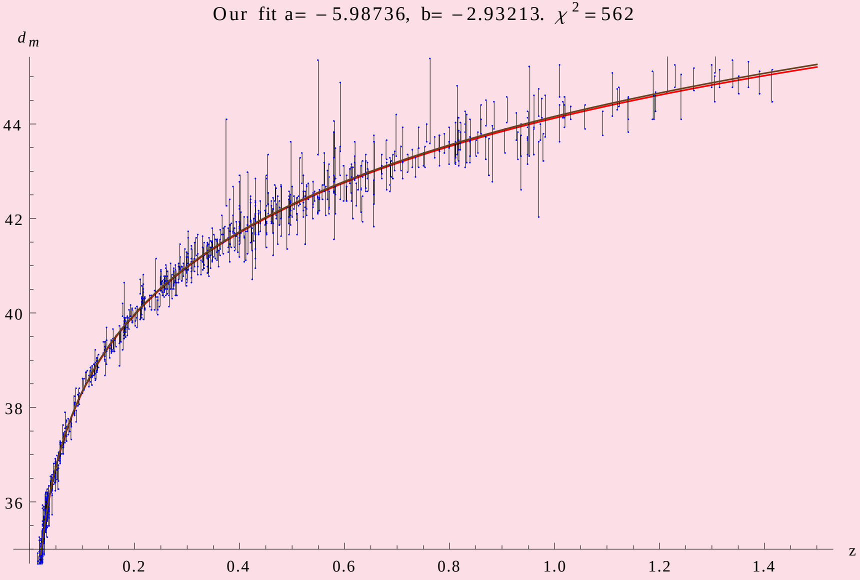
$$\begin{aligned} d_m &= 5\log_{10}\left((1+z)\int_0^z dx \frac{a(x)}{\dot{a}(x)}\right) = \\ & \mathrm{const} + 5\log_{10}\left((1+z)\int_0^z dx \frac{1}{\sqrt{\rho(x)}}\right) \end{aligned} \tag{9}$$

Supernova data [4] are available at
http://www.supernova.lbl.gov/Union/figuresSCPUnion2.1_mu_vs_z.txt

The standard fit with *ad hoc* mixture of dark energy, matter, curvature and radiation gives:

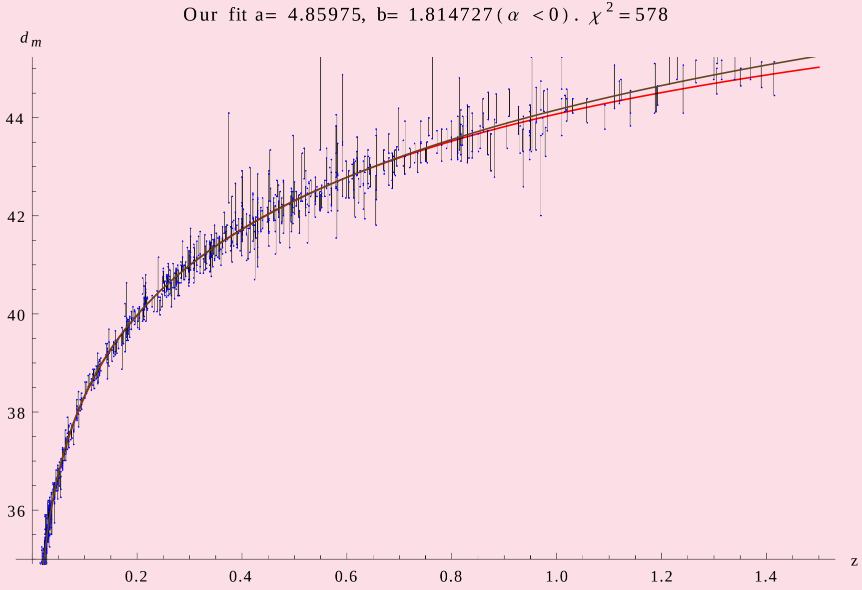


Our fit (red line) for $\alpha > 0$ compare to standard one (brown one)



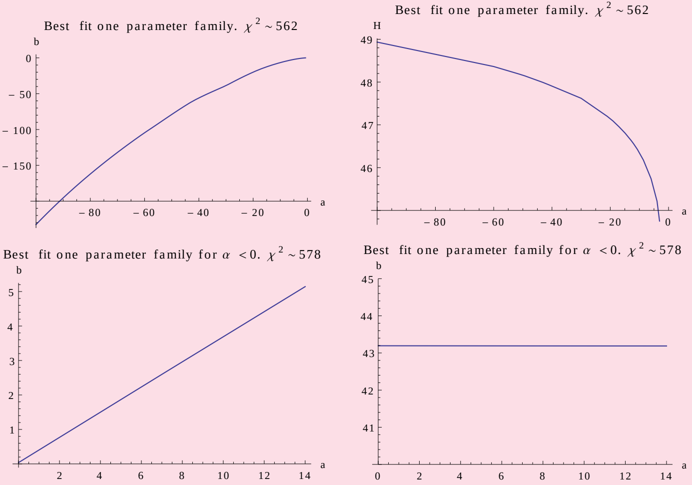
N.B. $\mathbf{b} = -2.93213(1+z)^3$.

Our fit (red line) for $\alpha < 0$ compare to standard one (brown one)



N.B. $\mathbf{b} = 1.814727(1+z)^3$.

The best fit of SN data using the proposed model is not unique. We observe one parametric family of solutions which gives practically one and the same $d_m(z)$ function. The family is represented below as functions of **a**. H is the constant in eq.(9).



$\alpha > 0$

$\alpha < 0$

Conclusions

- We integrated numerically the Friedman equations in the K-essence theory
- The dependence of the evolution of the Universe on the parameters [a,b] was examined
- It was shown that we can obtain both a Universe with and without phase transition
- A new data fit of the SNe data was presented
- The values of [a,b] deduced from the data fit have been used to calculate possible evolutions of the Univese

Note, since the phase transition, when existing, occurs after our epoch, while the SN fit is from before our epoch, the scenarios for evolution of our Universe are still open.

Thank you for you attention!

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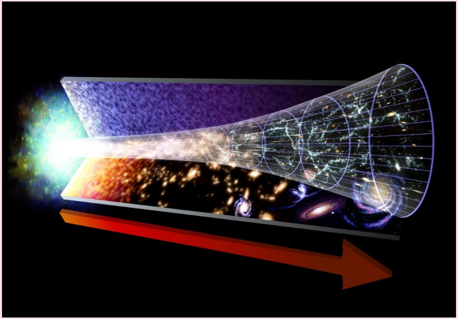

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