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Numerical modelling of compact static stars in Minimal Dilatonic Gravity

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Why modified gravity?

In order to fit the ~ 95% "dark" content of the Universe which latest cosmological data confirm, one has to either add a cosmological term (the Λ-CDM model) or to modify gravity to include new fields. Such modified gravity models have been used in cosmology for a long time,but they also have applications in modeling neutron stars.

Minimal Dilatonic Gravity

The Minimal Dilatonic Gravity (MDG) action is:

$$A_{g,\phi} = \frac{c}{2\kappa}\int d^4x\sqrt{|g|}(\Phi R - 2\Lambda U(\Phi))$$

Here $\Phi \in (0,\infty)$, $U(0) = U(\infty) = \infty$, $U(\Phi) > 0, U_{\Phi\Phi} > 0$ ($\kappa = 8\pi G_N/c^2$).
The scalar **dilaton** Φ leads to variable gravitational constant $G^{eff}(\Phi) = G_N/\Phi$, while the scalar potential $U(\Phi)$ leads to a variable cosmological factor $\Lambda^{eff}(\Phi) = \Lambda U(\Phi)$.
The field equations are:/ $\kappa = c = 1, R = 2\Lambda U_{,\Phi}(\Phi)$, $\mathbf{p} = \sqrt{\Lambda}\hbar/cm_\Phi$/:

$$\begin{aligned} &U(\Phi) = \Phi^2 + \frac{3}{16}\mathbf{p}^{-2}(\Phi - 1/\Phi)^2 \\ &\Box\Phi + 2/3(\Phi U_{,\Phi}(\Phi) - 2U(\Phi)) = 1/3T \\ &\Phi \hat{R_\alpha^\beta} = -\widehat{\nabla_\alpha\nabla^\beta}\Phi - \hat{T_\alpha^\beta} \end{aligned} \quad (1)$$

Equations for the second potential $V(\Phi)$ (the unique vacuum condition):

$$\begin{aligned} &V(\Phi) = \frac{2}{3}\int_1^\Phi (\Phi U_{,\Phi}(\Phi) - 2U(\Phi))d\Phi \\ &V_{,\Phi}(\Phi) = 2/3(\Phi U_{,\Phi}(\Phi) - 2U(\Phi)) \end{aligned}$$

To avoid conflict with existing solar system and laboratory gravitational experiments:
$m_\Phi \sim 10^{-3}eV/c^2$ ($\lambda_\Phi < 10^{-2}cm$ or $\mathbf{p} \leq 10^{-31}$)

The code

Code based on **COCAL** : Compact Object CALculator (Tsokaros et al. (2014)

- Integrator: ODEPACK routines **lsode** and **lsoda**
- Arbitrary-precision algebra: **MPFUN**
- Shooting method
- Adapted variables: $\Phi = \exp(\exp(\phi - 1))$ and $\bar{r_N} = \ln\bar{r}$,

The code needs to handle a very stiff system on three different scales: $\lambda_\Phi \in (10^{-3}cm, 10^{28}cm), r_* \approx 10^5 cm, r_\Delta \approx 10^{28}cm$.

References

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[3] Fiziev P. *arXiv:1506.08585 [gr-qc]*, 2015.

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Application to the compact starts

We apply the field equations on a static, spherically symmetric metric:
$ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2 d\Omega^2$ for a perfect fluid with $T^{\mu\nu} = diag(\epsilon, p, p, p)$

Our equations for $m(r), p(r), \Phi(r), p_\Phi(r)$ are:

$$\begin{aligned} \frac{dm}{dr} &= 4\pi r^2 \epsilon_{eff}/\Phi \\ \frac{dp}{dr} &= -\frac{p+\epsilon}{r}\frac{m + 4\pi r^3 p_{eff}/\Phi}{\Delta - 2\pi r^3 p_\Phi/\Phi} \\ \frac{d\Phi}{dr} &= -4\pi r^2 p_{eff}/\Delta \\ \frac{dp_\Phi}{dr} &= -\frac{p_\Phi}{r\Delta}\left(3r - 7m - \frac{2}{3}\Lambda r^3 + 4\pi r^3 \epsilon_{eff}/\Phi\right) - \frac{2}{r}\epsilon_\Phi \\ \frac{d\nu}{dr} &= \frac{m + 4\pi r^3 p_{eff}/\Phi}{\Delta - 2\pi r^3 p_\Phi/\Phi} \end{aligned}$$

Where:

$$\begin{aligned} &\Delta = r - 2m - \frac{\Lambda r^3}{3}, \Pi = \frac{m + 4\pi r^3 p_{eff}/\Phi}{\Delta - 2\pi r^3 p_\Phi/\Phi} \\ &\epsilon_{eff} = \epsilon + \epsilon_\Phi + \epsilon_\Lambda, \\ &p_{eff} = p + p_\Phi + p_\Lambda \\ &p_\Lambda = \frac{\Lambda}{8\pi}(U(\Phi) - \frac{1}{3}\Phi) \\ &\epsilon = \epsilon(p) \\ &\epsilon_\Lambda = -p_\Lambda - \frac{\Lambda}{12\pi}\Phi, \\ &\epsilon_\Phi = p - \frac{1}{3}\epsilon + \frac{\Lambda}{8\pi}V'(\Phi) + \frac{p_\Phi}{2}\Pi \end{aligned}$$

Initial and boundary conditions:
We have 3 different domains: inside the star ($p = 0$), inside the dilasphere ($p_\Phi = 0$) and to the end of the Universe ($\Delta = 0$).
1. For the inner domain $r \in [0, r_*]$:

$$m(0) = m_c = 0, p(0) = p_c, \Phi(0) = \Phi_c, p_\Phi(0) = p_{\Phi c} = \frac{2}{3}\left(\frac{\epsilon(p)}{3} - p_c\right) - \frac{\Lambda}{12\pi}V'(\Phi_c)$$

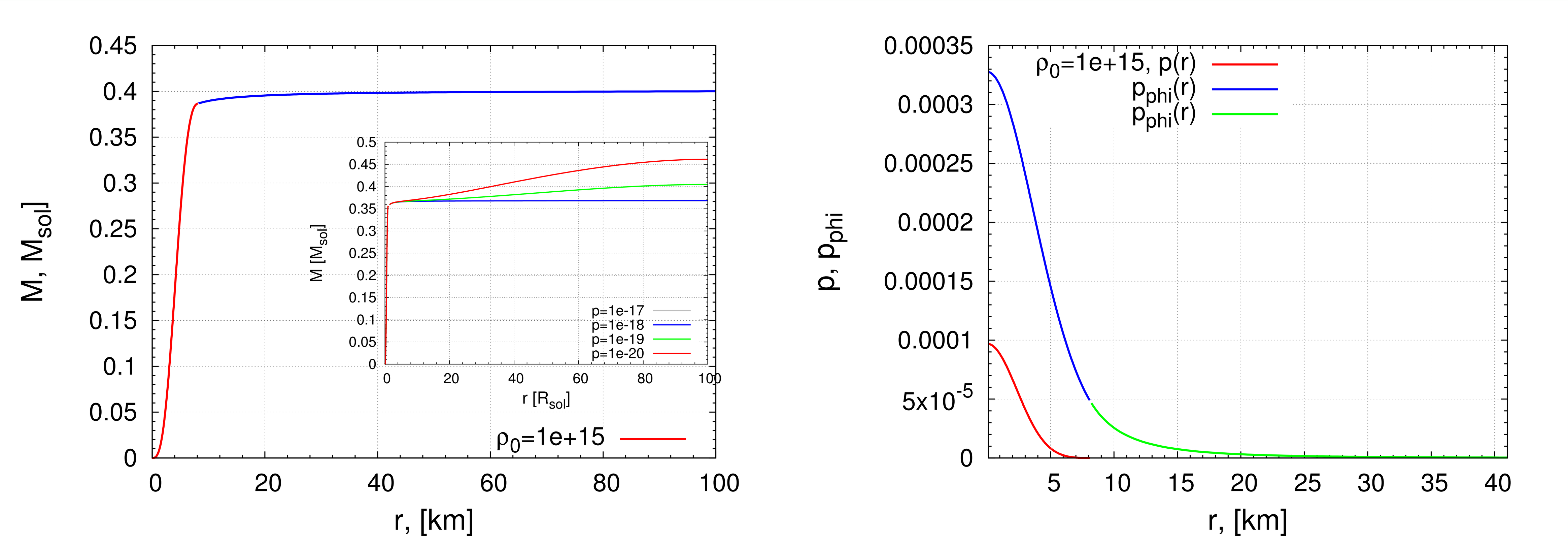
On the star's edge ($p(r^*) = 0$) we have: $m^* = m(r^*; p_c, \Phi_c), \Phi^* = \Phi(r^*, p_c, \Phi_c), p^*_\Phi = p_{\Phi c}(r^*, p_c, \Phi_c)$
2. For the outer domain $r \in (r_*, r_\Delta)$: a boundary value problem for Φ with $p = 0, \epsilon = 0, \Phi_\Delta = 1$
3. At the cosmological horizon $r = r_\Delta = 10^{22}km$, we recover GR: $\Phi = 1, U(\Phi) = 1$

The numerical results

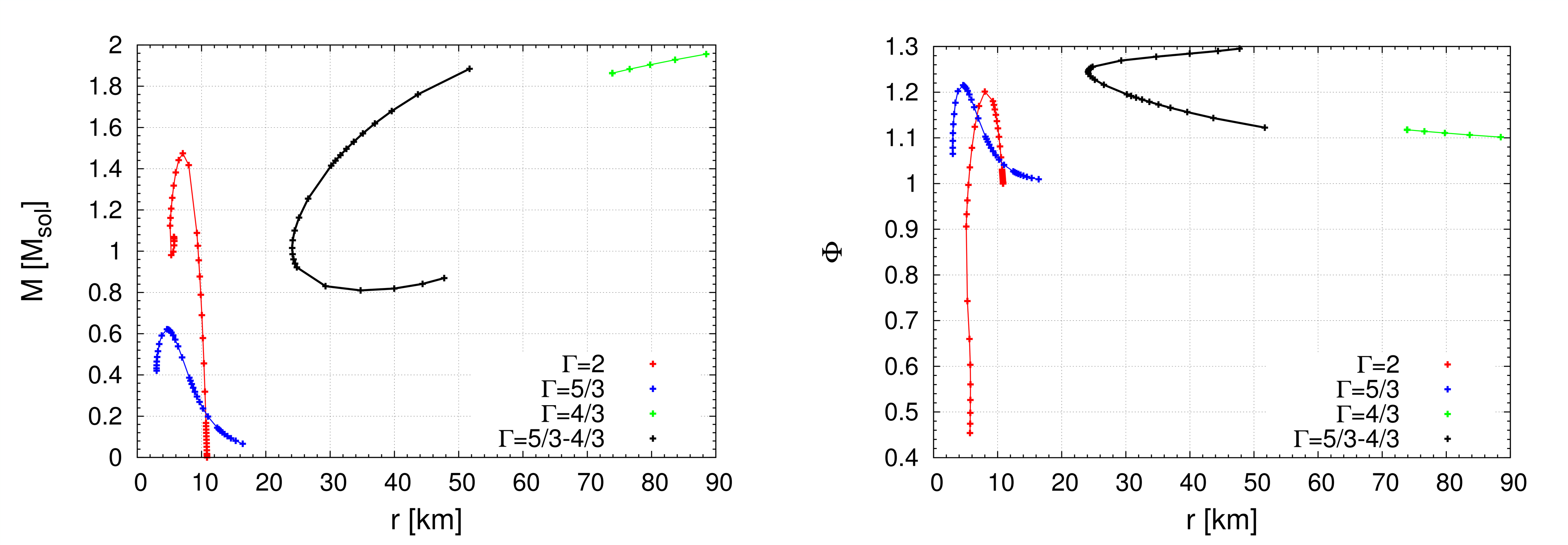
In this model, we the following form of the withholding potentials (p_d – the dilaton mass parameter),:

$$V(\Phi) = \frac{1}{2}p_d^{-2}(\Phi + 1/\Phi - 2), U(\Phi) = \Phi^2 + \frac{3}{16}p_d^{-2}(\Phi - 1/\Phi)^2$$

Characteristic for this model is the existence of the dilasphere – the space between the radius of the star and the cosmological horizon.
We start with the simplest EOS, a polytropes with $\gamma = 5/3$, $\rho_0 = 10^{15}, p_d = 10^{-18}$ ($p = K\epsilon^\gamma$).
On the plots, one can see that the mass of the star continues to increases outside of the star until it reaches a constant value at infinity. Also, while at the radius of the star the pressure $p = 0$, the dilatonic pressure $p_\Phi \neq 0$.



The study of the effect of different equations of states on the mass-radius relation and also, of the non-trivial way the final radius of the star depends on Φ_{ini} can be seen below ($p_d = 10^{-19}$):



The increase in the final mass of the star is 10-30%. Additionally, there is a non-trivial dependence of the final mass of the star on the mass of the dilaton.
Continuing work on the model focuses on more realistic EOS (by P. Fiziev and K. Marinov) and also the numerically challenging calculations for $p_d < 10^{-22}$ (by P. Fiziev and D. Staicova).