Numerical modelling of compact static stars in Minimal Dilatonic Gravity

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Minimal Dilatonic Gravity (MDG)

We change the Einstein-Hilbert Action: $A_E = \int \frac{1}{2\kappa}(R - 2\Lambda)\sqrt{-g}\,\mathrm{d}^4x$ to

/ Fiziev, Mod. Phys. Lett. A, 15 1077 (2000) and Fiziev, PRD **87**, 044053 (2013)/

$$A_{g,\phi} = \frac{c}{2\kappa}\int d^4x\sqrt{|g|}(\Phi R - 2\Lambda U(\Phi))$$

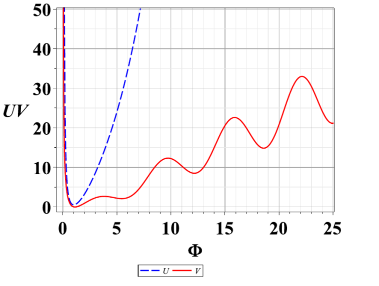
Here $\Phi \in (0,\infty)$, $U(0) = U(\infty) = \infty$, $U(\Phi) > 0$, $U_{\Phi\Phi} > 0$
$(\kappa = 8\pi G_N/c^2)$.

The scalar **dilaton** Φ leads to variable gravitational constant $G^{eff}(\Phi) = G_N/\Phi$, while the scalar potential $U(\Phi)$ leads to a variable cosmological factor $\Lambda^{eff}(\Phi) = \Lambda U(\Phi)$.

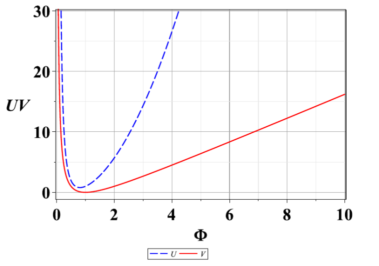
The field equations are:/ $\kappa = c = 1$, $R = 2\Lambda U_{,\Phi}(\Phi)$, $\mathbf{p} = \sqrt{\Lambda}\hbar/cm_\Phi$/:

$$\begin{aligned} &U(\Phi) = \Phi^2 + \frac{3}{16}\mathbf{p}^{-2}(\Phi - 1/\Phi)^2 \\ &\Box\Phi + 2/3(\Phi U_{,\Phi}(\Phi) - 2U(\Phi)) = 1/3T \\ &\Phi \hat{R}_\alpha^\beta = -\widehat{\nabla_\alpha\nabla^\beta}\Phi - \hat{T}_\alpha^\beta \end{aligned} \tag{1}$$

Credit: Fiziev, Physical Review D 87, 044053 (2013)



(e) Unique Einstein Vacuum and many deSitter vacuums: $U_{,\Phi\Phi} > 0$



(f) Unique Einstein Vacuum and unique deSitter vacuums: $U_{,\Phi\Phi} > 0$, $V_{,\Phi\Phi} > 0$

$$V(\Phi) = \frac{2}{3}\int_1^{\Phi} (\Phi U_{,\Phi}(\Phi) - 2U(\Phi))d\Phi$$

$$V_{,\Phi}(\Phi) = 2/3(\Phi U_{,\Phi}(\Phi) - 2U(\Phi))$$

To avoid conflict with existing solar system and laboratory gravitational experiments: $m_\Phi \sim 10^{-3} eV/c^2$ ($\lambda_\Phi < 10^{-2} cm$ or $\mathbf{p} \leq 10^{-31}$)

The equations, following Fiziev, PRD **87**, 044053 (2013)

We apply the field equations on a static, spherically symmetric metric: $ds^2 = e^{\nu(r)}dt^2 - e^{\lambda(r)}dr^2 - r^2 d\Omega^2$ for $T^{\mu\nu} = diag(\epsilon, p, p, p)$, (perfect-fluid)

Our equations are:

$$\frac{dm}{dr} = 4\pi r^2 \epsilon_{eff}/\Phi$$

$$\frac{dp}{dr} = -\frac{p+\epsilon}{r}\frac{m + 4\pi r^3 p_{eff}/\Phi}{\Delta - 2\pi r^3 p_\Phi/\Phi}$$

$$\frac{d\Phi}{dr} = -4\pi r^2 p_{eff}/\Delta$$

$$\frac{dp_\Phi}{dr} = -\frac{p_\Phi}{r\Delta}\left(3r - 7m - \frac{2}{3}\Lambda r^3 + 4\pi r^3 \epsilon_{eff}/\Phi\right) - \frac{2}{r}\epsilon_\Phi$$

$$\frac{d\nu}{dr} = \frac{m + 4\pi r^3 p_{eff}/\Phi}{\Delta - 2\pi r^3 p_\Phi/\Phi}$$

Where:

$$\Delta = r - 2m - \frac{\Lambda r^3}{3}, \Pi = \frac{m + 4\pi r^3 p_{eff}/\Phi}{\Delta - 2\pi r^3 p_\Phi/\Phi}$$

$$\epsilon_{eff} = \epsilon + \epsilon_\Phi + \epsilon_\Lambda,$$

$$p_{eff} = p + p_\Phi + p_\Lambda$$

$$p_\Lambda = \frac{\Lambda}{8\pi}(U(\Phi) - \frac{1}{3}\Phi)$$

$$\epsilon = \epsilon(p)$$

$$\epsilon_\Lambda = -p_\Lambda - \frac{\Lambda}{12\pi}\Phi,$$

$$\epsilon_\Phi = p - \frac{1}{3}\epsilon + \frac{\Lambda}{8\pi}V'(\Phi) + \frac{p_\Phi}{2}\Pi$$

We have 4 unknown physical quantities : $m(r), p(r), \Phi(r), p_\Phi(r)$.

Initial and boundary conditions:

We have 3 different domains: inside the star ($p = 0$), inside the dilasphere ($p_\Phi = 0$) and to the end of the Universe ($\Delta = 0$).

1. For the inner domain $r \in [0, r_*]$:

$$m(0) = m_c = 0, p(0) = p_c, \Phi(0) = \Phi_c,$$

$$p_\Phi(0) = p_{\Phi c} = \frac{2}{3}\left(\frac{\epsilon(p)}{3} - p_c\right) - \frac{\Lambda}{12\pi} V'(\Phi_c)$$

On the star's edge ($p(r^*) = 0$) we have

$$m^* = m(r^*; p_c, \Phi_c), \Phi^* = \Phi(r^*, p_c, \Phi_c), p_\Phi^* = p_{\Phi c}(r^*, p_c, \Phi_c)$$

2. For the outer domain $r \in (r_*, r_\Delta)$: a boundary value problem for Φ:

$$p = 0, \epsilon = 0, \Phi_\Delta = 1$$

3. At $r = r_\Delta = 10^{22} km$, we recover GR: $\Phi = 1, U(\Phi) = 1$

The code:

This is a **boundary value problem**: find $\Phi(0) = \Phi_c$ such that on $r = r_\Delta$ we have $\Phi = 1$.
Three different scales: $\lambda_\Phi \in (10^{-3} cm, 10^{28} cm), r_* \approx 10^5 cm, r_\Delta \approx 10^{28} cm$.
Numerically this defines a very stiff system.
Code based on **COCAL** : Compact Object CALculator (Tsokaros et al. (2014)
Special treatment for stiff systems:

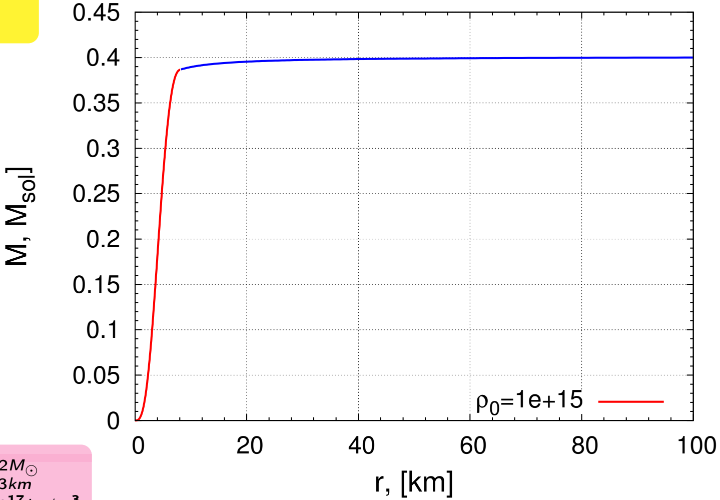
- Integrator: ODEPACK routines **lsode** and **lsoda**
- Arbitrary-precision algebra: **MPFUN**
- Shooting method
- Adapted variables: $\Phi = \exp(\exp(\phi - 1))$ and $\bar{r_N} = \ln \bar{r}$,

We need to reach $\mathbf{p} < 10^{-31}$ and $r_v = 1/\sqrt{\Lambda} = 10^{22} km$.
So far we have reached $\mathbf{p} \approx 10^{-23}$.

The results for some simple equations of state

Polytropes with $\gamma = 5/3$, $\rho 0 = 1e + 15$, $\mathbf{p} = 1e - 18$:

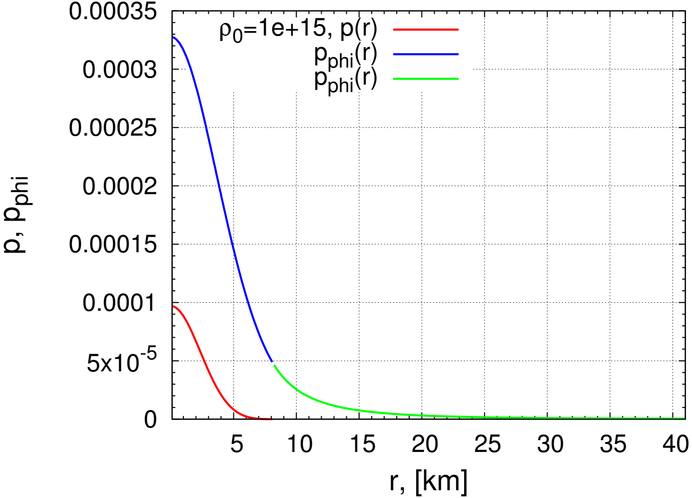
Polytropes:
$p_{NR} = K_{NR}\epsilon^{\frac{5}{3}}$,
$p_{R} = K_{R}\epsilon^{\frac{4}{3}}$,



$M_{NS} = 1.4 - 2 M_{\odot}$
$R_{NS} = 12 - 13 km$
$\rho_{NS} = 10^{9} – 10^{17} kg/m^{3}$

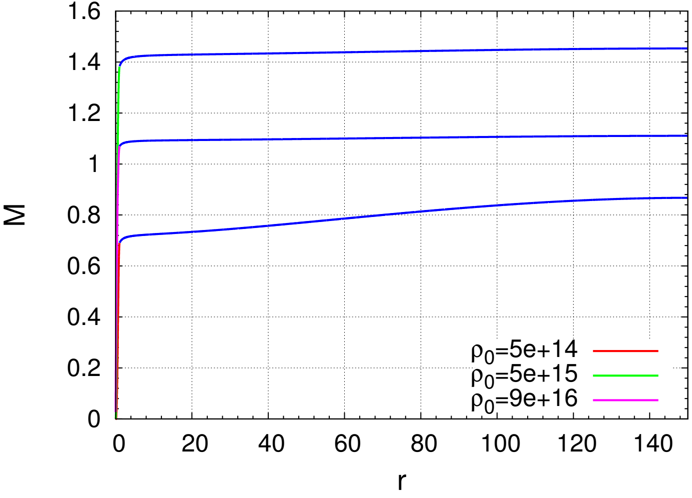
The results for some simple equations of state

Polytropes with $\gamma = 5/3,\ \rho 0 = 1e + 15, p = 1e - 18$:

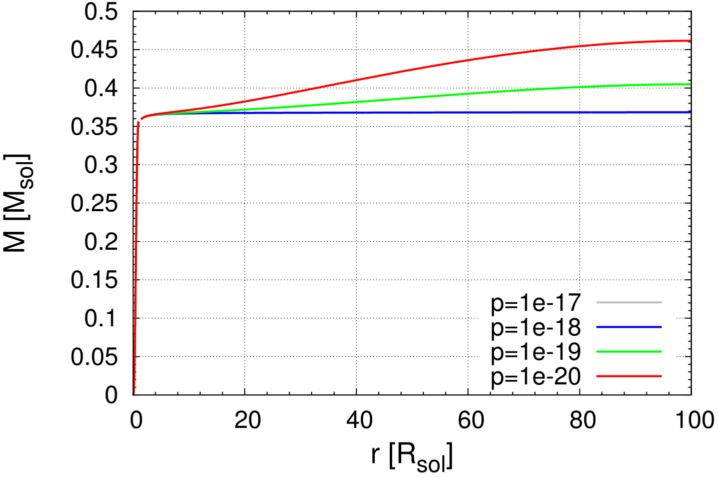


The results for some simple equations of state

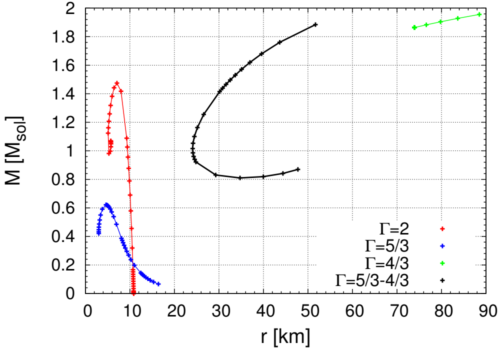
Polytropes with $\gamma = 2$, $\rho 0 = 1e + 15$:

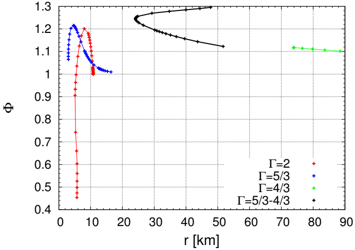


The dependence of the dilaton mass

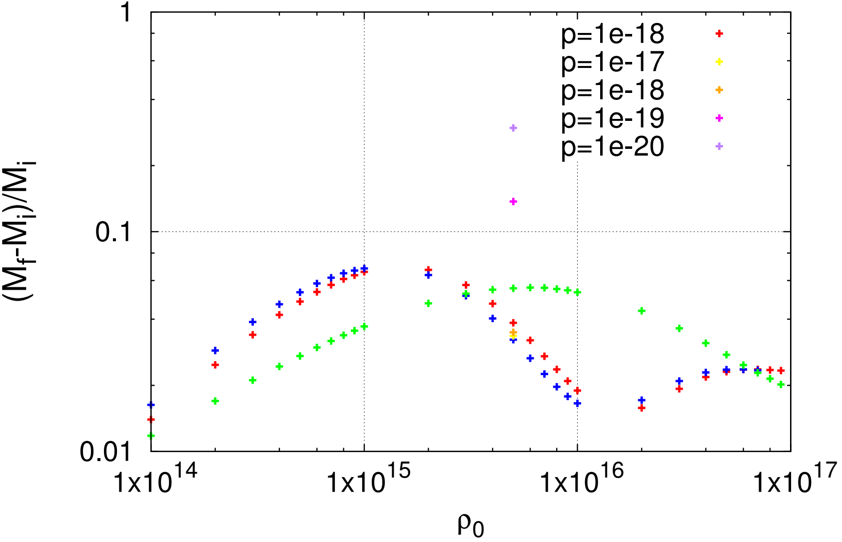


The complete picture for NS for different EOS, dilaton mass $p = 10^{-19}$





The mass of the dilasphere with respect to the mass of the star



It is work in progress!

Thank you for the attention!

Earlier works on MDG

Some of the works where details on the MDG model have been worked out.

A theory in development

P.Fiziev, The mass of dark scalar and phase space analysis of realistic models of static spherically symmetric objects, Talk at the MG14, 12-19 July 2015, Roma, Italy

P.Fiziev, A realistic model of a neutron star in minimal dilatonic gravity, arXiv:1506.08585

P. Fiziev, K. Marinov, Compact statis stars with polytropic equation of state in minimal dilatonic gravity, Bulgarian Astronomical Journal, Vol. 23, p.3, (2015)

P. Fiziev, Dark Energy and Dark Matter in Stars Physic , "Frontiers of Fundamental Physics 14", Marseille, France, July, 15-18, 2014, PoS(FFP14)080

P.P. Fiziev, Physical Review D 87, 044053 (2013)

P. Fiziev, Georgieva D., Phys. Rev. D 67 064016 (2003).

Plamen P. Fiziev, arXiv:gr-qc/0202074

Fiziev P. P., Yazadjiev S., Boyadjiev T., Todorov M., Phys. Rev. D 61 124018 (2000).