Cosmological aspects of a unified dark energy and dust dark matter model

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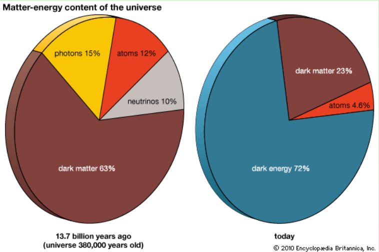
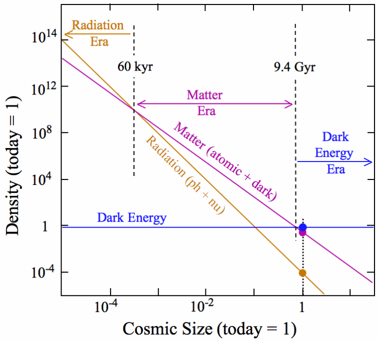
Contents

The current state of cosmology

What we know (from Planck, SNe Ia, the HST key project, the Sloan Digital Sky Survey, WMAP, COBE, large-scale galaxy formations):

- The Universe is isotropic and homogenous
- The Universe is expanding in an accelerated rate
- The Universe is flat (within error of $\sim 0.4\%$ by WMAP)
- The age of the Universe is 13.798 ± 0.037 bln. years
- The Universe contains $4.82 \pm 0.05\%$ ordinary matter, $25.8 \pm 0.4\%$ dark matter and $69 \pm 1\%$ dark energy.





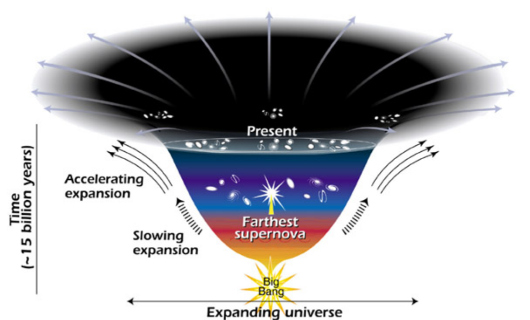
The need of inflation

The period of inflation is invoked in the theory to explain:

- the flatness problem (fine-tuning of the model)
- the horizon problem (isotropy of the CMB)
- the monopole problem
- the formation of large-scale structures of the Universe etc

Λ-CDM model + inflation are currently considered as the most successful cosmology model.

They don't explain, however, the nature of dark energy and dark matter.



The two-measures theory of gravity

Proposed in works by Guendelman, Nissimov and Pacheva: 2014, 2015 ([1],[2])
In this theory, the scalar Lagrangian couples both to the standard Riemannian volume-form ($\sqrt{-|g_{\mu\nu|}}$), as well as to another non-Riemannian volume-form ($\Phi(B)$) given in terms of an auxiliary maximal-rank antisymmetric tensor gauge field ($B_{\nu\kappa\lambda}$)

$$S = S_{grav}[g_{\mu\nu}, \Gamma^{\lambda}_{\mu\nu}] + \int d^4x(\sqrt{-g} + \Phi(B))L(\phi, X)$$

$$\Phi(B) = \frac{1}{3}\epsilon^{\mu\nu\kappa\lambda}\partial_\mu B_{\nu\kappa\lambda}$$

$$L(\phi, X) = X - V(\phi), \quad X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$$T_{\mu\nu} = -2Mg_{\mu\nu} + \left(1 + \frac{(\Phi(B)}{\sqrt{(-g)}}\right)\partial_\mu\phi\partial_\nu\phi = -2Mg_{\mu\nu} + \rho_0 u_\mu u_\nu,$$

where $u_\mu = -\frac{\partial_\mu\phi}{\sqrt{2X}}$ and $\rho_0 = \left(1 + \frac{(\Phi(B)}{\sqrt{(-g)}}\right)2X\frac{\partial L}{\partial X}$. Therefore, we can interpret $T_{\mu\nu}$ as a sum of dark energy and dust contribution with $\rho_{DE} = 2M$.

The model in FLRW metric, arXiv:1610.08368 [gr-qc]

We work with the model in the f(R) gravity, i.e.

$$S_{grav} = \int d^4x\sqrt{-g}(R(g,\Gamma) - \alpha R^2(g,\Gamma))$$

. The Friedman–Lemaître–Robertson–Walker metric with $k=0$ is:

$$ds^2 = -dt^2 + a(t)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right].$$

The Lagrangian for it is:

$$\mathcal{L} = a(t)\left[-\dot{a(t)}^2 + \frac{1}{4}a(t)^2\left(\frac{1}{\alpha} - \frac{\dot{\phi}^2}{\alpha} + (\frac{1}{4\alpha} - 2M)\dot{\phi}^4\right)\right]$$

And the equation of motions:

$$a(t)^3\left[-\frac{1}{2\alpha}\dot{\phi} + (\frac{1}{4\alpha} - 2M)\dot{\phi}^3\right] = p_\phi \;\; (= \mathrm{const})$$

From the Friedman equations ($G_{00} = T_{00}$), the energy density is:

$$\rho = \frac{1}{8\alpha}\dot{\phi}^2 + \frac{3}{4}\frac{p_\phi}{a(t)^3}\dot{\phi} - \frac{1}{4\alpha}$$

The miracles of a simple cubic equation

$\dot{\phi}$ is a solution of a cubic equation: $a(t)^3\left[-\frac{1}{2\alpha}\dot{\phi}+(\frac{1}{4\alpha}-2M)\dot{\phi}^3\right]=p_\phi$

If we rewrite the equation for p_ϕ like:

$$y^3+3\mathbf{a}y+2\mathbf{b}=0 \tag{1}$$

where $\mathbf{a}=-\frac{2}{3-24\alpha M}$ and $\mathbf{b}=-\frac{2\alpha p_\phi}{a(t)^3(1-8\alpha M)}$.

Solutions

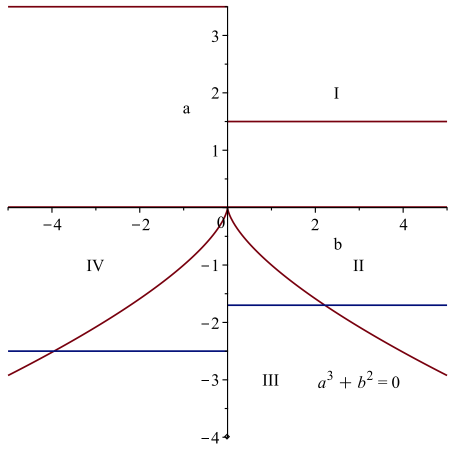
$$y_1 = -\frac{\mathbf{a}}{(-\mathbf{b}+\sqrt{\mathbf{a}^3+\mathbf{b}^2})^{1/3}}+(-\mathbf{b}+\sqrt{\mathbf{a}^3+\mathbf{b}^2})^{1/3} \tag{2}$$

$$y_2 = \frac{\mathbf{a}}{(\mathbf{b}-\sqrt{\mathbf{a}^3+\mathbf{b}^2})^{1/3}}-(\mathbf{b}-\sqrt{\mathbf{a}^3+\mathbf{b}^2})^{1/3} \tag{3}$$

$$y_3 = \frac{1-i\sqrt{3}}{2}\frac{\mathbf{a}}{(-\mathbf{b}+\sqrt{\mathbf{a}^3+\mathbf{b}^2})^{1/3}}-\frac{1+i\sqrt{3}}{2}(-\mathbf{b}+\sqrt{\mathbf{a}^3+\mathbf{b}^2})^{1/3} \tag{4}$$

N.B. Eq.(1) is with real coefficients. Therefore there is always one real root $\forall \mathbf{a},\mathbf{b}$. However, no real smooth solution exists in the $\mathbf{a},\mathbf{b}$ plane.

Regions of validity in the [a,b] plane



Note that $\mathbf{b} \sim a(t)^{-3}$ and for an emerging universe where $a(t) \in [0, \infty]$, $\mathbf{a} > 0$, the evolution of the universe goes along a horizontal line never crossing $\mathbf{b} = 0$ line. However, for negative $\mathbf{a}$ the line $\mathbf{a}^3 + \mathbf{b}^2 = 0$ which separate domains of reality for different roots will be always crossed and eventually phase transition will be observed.

Final form of the Friedman equation

It is convenient to rewrite the equation of state (6) like:

$$\rho = \frac{1}{4|\alpha|}\bar{\rho} = \frac{1}{4\alpha}\left(\frac{1}{2}y^2 + \frac{b}{a}y - 1\right) \tag{5}$$

Note that rescaling the time we can ensure $2|\alpha|/3 = 1$, so the Friedman equation takes the form

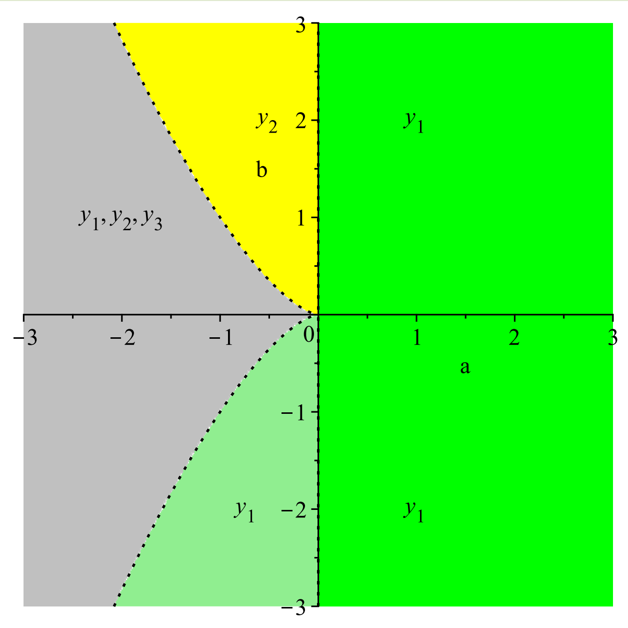
$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \bar{\rho} \tag{6}$$

α eventually goes in to Hubble constant.
The density $\bar{\rho}$ has a well defined asymptotic value for large A(t)

$$\begin{aligned} \bar{\rho} \quad & \xrightarrow[A(t)\to\infty]{} \quad 1 \ \text{ for } \ \mathbf{a} > 0 \\ \bar{\rho} \quad & \xrightarrow[A(t)\to\infty]{} \quad -\frac{3}{2}\mathbf{a} - 1 \ \text{ for } \ \mathbf{a} < 0 \end{aligned} \tag{7}$$

which can be interpreted as asymptotic value of the cosmological constant.

Regions of validity in the [a,b] plane



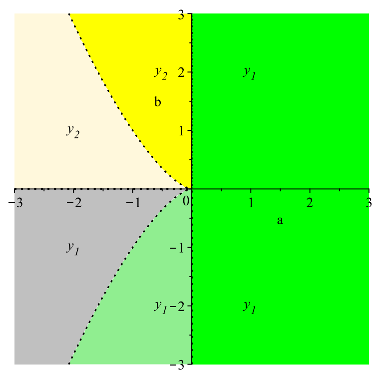
We can stitch different solutions real in the whole [a,b]

The solution y_1 is real in the domain
$\{a \geq 0\} \cup \{a < 0 \cap b < 0\} \cup \{a < 0 \cap b > 0 \cap a^3 + b^2 < 0\}$,
y_2 is real in the domain :
$\{a < 0 \cap b > 0\} \cup \{a < 0 \cap b < 0 \cap a^3 + b^2 < 0\}$
and y_3 is real in the domain:
$\{a < 0 \cap a^3 + b^2 < 0\}$.
Since y_3 gives negative densities, we won't use it to construct a solution with it. We can define the following piecewise functions, **real** in the whole plane [a,b]:

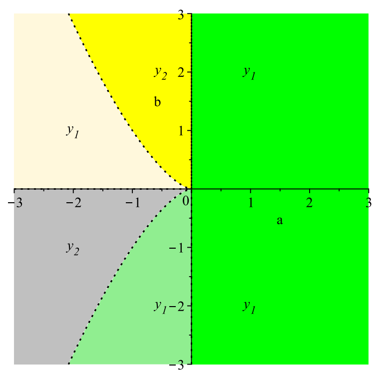
$$y_b = \begin{cases} y_1 & \text{for } (a,b) \in \{a \geq 0\} \cup \{a < 0 \cap b < 0\} \\ y_2 & \text{for } (a,b) \in \{a < 0 \cap b > 0\} \end{cases} \qquad y_s = \begin{cases} y_1 & \text{for } b > 0 \\ y_2 & \text{for } b < 0. \end{cases}$$

Then we can use as independent real solutions y_b (our basic solution), y_s and y_3.

Choice of solutions y_b and y_s

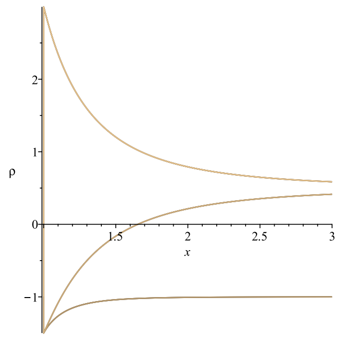


(f) y_b



(g) y_s

The behavior of the density ρ:

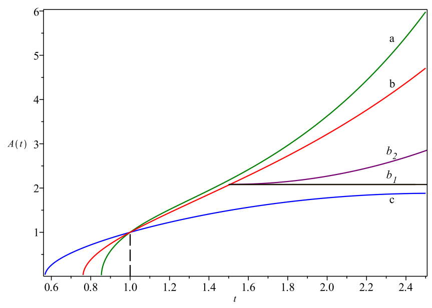


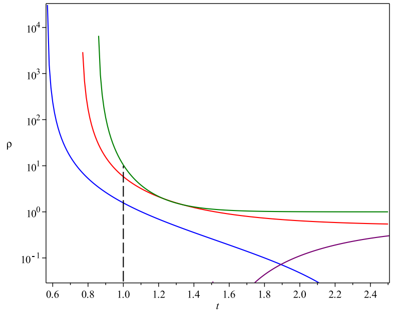
(h) $a = -1, b = 1/x^3$

We have entirely positive and entirely negative densities and also ones that change sign.

Universe evolution with and without phase transition

Using the so-defined solutions for y, we can numerically integrate the Friedman equations to find the evolution $a(t)$.





Graphics of the $a(t)$ evolution for $\mathbf{a} = -.5, \mathbf{b} = -\frac{0.5}{a^3}$ (blue), $\mathbf{a} = -1, \mathbf{b} = -\frac{2}{a^3}$ (red), $\mathbf{a} = 1, \mathbf{b} = \frac{6}{a^3}$ (green) and $t_p = 1.5074, a_s(t_p) = 2.0825$ (purple).

The initial boundary conditions are: $a_b(1) = 1$ for the universes without phase transition and $a_b(1) = 1, a_s(t_p) = a_b(t_p)$, for those with phase transition.

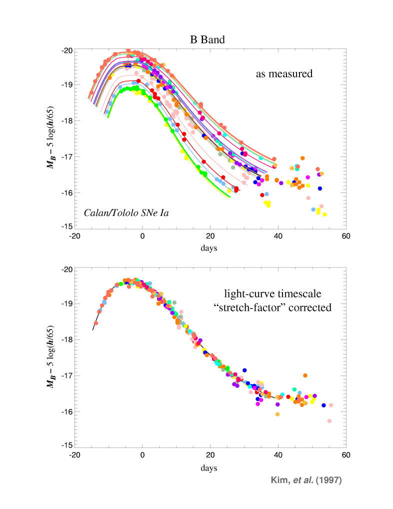
Note that $t = 1$ ($a(1) = 1$) is the present moment while the moment t_0 at which $a(t_0) = 0$ represents the Big Bang.

Universe evolution with phase transition

- We denote $\bar{\bar{\rho}}$ the density which corresponds to solution y_s.
- For $t = 1$, $\bar{\bar{\rho}}(t_3) < 0$, but for certain moment t_p: $\bar{\bar{\rho}}(A(t_p)) = 0$ (same asymptotic for large $A(t)$).
- Therefore, for any moment after t_p we have two "states" $\bar{\rho}$ and $\bar{\bar{\rho}}$ of the Universe:
$$0 \le \bar{\bar{\rho}} < \bar{\rho} \ \text{ for } \ t \ge t_p. \tag{8}$$
This opens the possibility the Universe to undergo **"phase transition" or "quenching" to the lower state**.
- The moment of the phase transition is crucial for the further evolution:
  – If it happens exactly at time t_p the evolution **stops** ($\bar{\bar{\rho}} = 0$).
  – If the jump occurs in any later moment, then we observe **phase transition of the first kind.**

Type Ia Supernovae as “standard candles

- Supernovae occurring in a binary system, with one partner – white dwarf
- No H in their spectra
- Smooth decaying light curve, powered by radioactive decay of ^{56}Ni to ^{56}Co to ^{56}Fe
- Maximal mass of the white dwarf before explosion – $\sim 1.4 M_{sol}$
- Width of the lightcurve peak correlates with the peak luminosity
- After correction, light curves coincide – standard candles



Or at least standartizable candles

Some problems with SN type IA

- First confirmation of the white dwarf origin: SN 2011fe Nature 512, 406408 (2014)
- The Chandrasekhar limit - fusing carbon mystery
- Super-Chandras 50 (SN 2003fg SN 2007bi, SN 2009dc $M > 2M_{sol}$)
- Dim Supernova 30 Type 1ax (SN 2012az) arXiv:1212.2209 [astro-ph.SR], ApJ 767, 1, 2013
- Less than 1% of companions red giants, most – white dwarfs
- New study of 740 Type Ia supernovae show only 3 sigma evidences for expansion (not 5 sigma) J. T. Nielsen et al." Marginal evidence for cosmic acceleration from Type Ia supernovae" Scientific Reports (2016)

The supernova data fit

Distance modulus d_m:

$$d_m = 5\log_{10}\left(\frac{d}{10}\right) \tag{9}$$

where d is the distance in parsecs.

Connection between d_m and z:

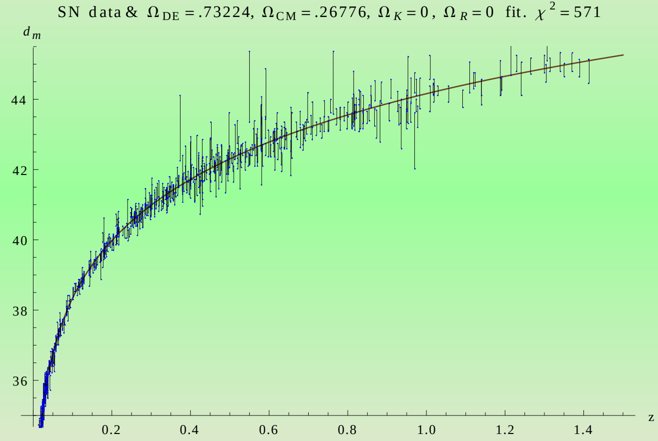
$$\begin{aligned} d_m &= 5\log_{10}\left((1+z)\int_0^z dx \frac{a(x)}{\dot{a}(x)}\right) = \\ &\mathrm{const} + 5\log_{10}\left((1+z)\int_0^z dx \frac{1}{\sqrt{\rho(x)}}\right) \end{aligned} \tag{10}$$

Where we have used $a = \frac{1}{1+z}$. Here, $\rho(x)$:

$$\rho_{CDM} \sim a(t)^{-3}, \rho_r \sim a(t)^{-4}, \rho_k \sim a(t)^{-2}, \rho_{DE} \sim \Lambda$$

From observations, we know that $\rho_k \sim 0$, $\rho_r \sim 10^{-5}$, so we use $\Omega_{CDM} + \Omega_\Lambda$ universe.

The standard fit against Supernovae data: $\Omega_{DE} = 0.722, \ \ \Omega_M = 0.278, \ \ \chi^2 = 562$.



Supernova data [4] are available at
http://www.supernova.lbl.gov/Union/figuresSCPUnion2.1_mu_vs_z.txt

Our best fit against Supernovae data and standard fit

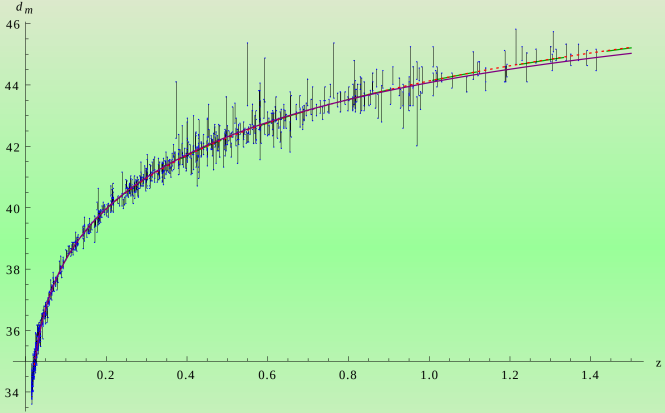


Figure: Supernovae data against Standard model fit (dotted line), $\mathbf{a} < -2/3$ fit (dashed line) and $\mathbf{a} > 1$ fit (solid line).

$\chi^2 \sim 562$ for $\mathbf{a} < -2/3$,
$\chi^2 \sim 578$ for $\mathbf{a} > 1$.

The best fit of SN data using the proposed model is not unique.

We observe one parametric family of solutions which gives practically one and the same $d_m(z)$ function.

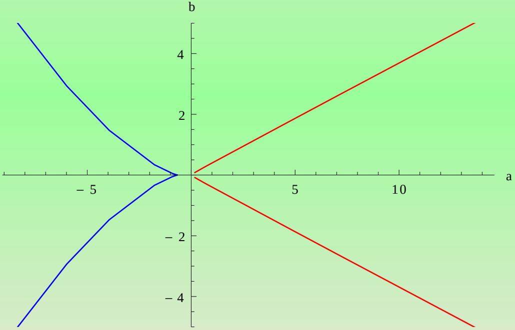
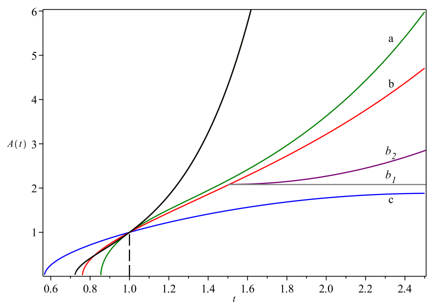
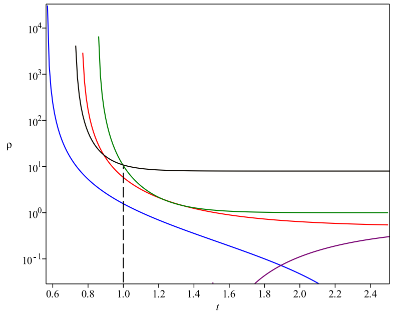


Figure: Best fit families on the parametric plane.

Using the so obtained a, b as parameters in the evolution





Here the black line corresponds to
$a = -5.98736$, $b = -2.93213$, $t_0 = 0.72226$.

Conclusions

- We integrated numerically the Friedman equations in the K-essence theory
- The dependence of the evolution of the Universe on the parameters [a,b] was examined
- It was shown that we can obtain both a Universe with and without phase transition
- A new data fit of the SNe data was presented
- The one-parametric familly of solutions corresponding to the observational data has been obtained
- The values of [a,b] deduced from the data fit have been used to calculate possible evolution of the Universe

Note, since the phase transition, when existing, occurs after our epoch, while the SN fit is from before our epoch, the scenarios for evolution of our Universe are still open.

Thank you for you attention!

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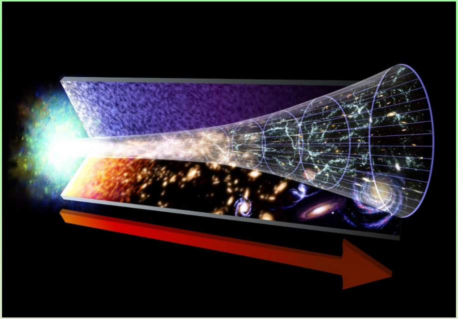

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