Cosmological solutions from two-measures model with inflaton field

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The darkon model in FLRW metric, Mod. Phys. Lett. A, **32**, 1 (2017)

If we apply the two-measures darkon model in the f(R) gravity(Guendelman, Nissimov and Pacheva ([1],[2])),

$$S_{darkon} = \int d^4x\sqrt{-g}(R(g,\Gamma) - \alpha R^2(g,\Gamma)) + \int d^4x(\sqrt{-g} + \Phi(C))L(u,X)$$

with $\Phi(C) = \frac{1}{3}\epsilon^{\mu\nu\kappa\lambda}\partial_\mu C_{\nu\kappa\lambda}$, $L(u,X) = -\frac{1}{2}g^{\mu\nu}\partial_\mu u\partial_\nu u - V(u)$

The Friedman–Lemaître–Robertson–Walker metric with $k = 0$ is:

$$ds^2 = -dt^2 + a(t)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right].$$

From the Friedman equations ($G_{00} = T_{00}$), the energy density is:

$$\rho = \frac{1}{8\alpha}\dot{u}^2 + \frac{3}{4}\frac{p_u}{a(t)^3}\dot{u} - \frac{1}{4\alpha} \quad (1)$$

where for the constant p_u we have from the equations of motion:

$$a(t)^3\left[-\frac{1}{2\alpha}\dot{u} + (\frac{1}{4\alpha} - 2M)\dot{u}^3\right] = p_u \;\; (= \mathrm{const}) \quad (2)$$

The miracles of a simple cubic equation

We rewrite the last cubic equation for $\dot{u}$ Eq. (2), as

$y^3 + 3\mathbf{a}y + 2\mathbf{b} = 0$ with $\mathbf{a} = -\frac{2}{3-24\alpha M}$ and $\mathbf{b} = -\frac{2\alpha p_u}{a(t)^3(1-8\alpha M)}$.

The solutions are ($y_3 = \frac{y_2 - i\sqrt{3}y_1}{2}$):

$$y_1 = -\frac{\mathbf{a}}{(-\mathbf{b}+\sqrt{\mathbf{a}^3+\mathbf{b}^2})^{1/3}} + (-\mathbf{b}+\sqrt{\mathbf{a}^3+\mathbf{b}^2})^{1/3}$$

$$y_2 = \frac{\mathbf{a}}{(\mathbf{b}-\sqrt{\mathbf{a}^3+\mathbf{b}^2})^{1/3}} - (\mathbf{b}-\sqrt{\mathbf{a}^3+\mathbf{b}^2})^{1/3}$$

We define the following piecewise functions, **real** in the whole plane $[\mathbf{a}, \mathbf{b}]$:

$$y_b = \begin{cases} y_1 & \text{for } (a,b) \in \{a \geq 0\} \cup \{a < 0 \cap b < 0\} \\ y_2 & \text{for } (a,b) \in \{a < 0 \cap b > 0\} \end{cases} \qquad y_s = \begin{cases} y_1 & \text{for } b > 0 \\ y_2 & \text{for } b < 0. \end{cases}$$

After rescaling time by $2|\alpha|/3 = 1$ and absorbing α into Hubble constant ($\bar{\rho} = 4|\alpha|\rho$):

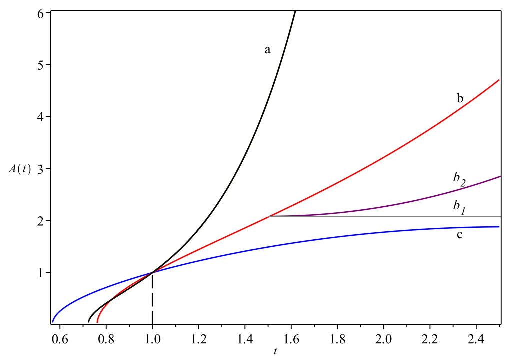
Final form of the Friedman equation: With asymptotic:

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \bar{\rho} = \left(\frac{1}{2}y^2 + \frac{\mathbf{b}}{\mathbf{a}}y - 1\right) \qquad (3)$$

$$\bar{\rho} \xrightarrow[a(t)\to\infty]{} \begin{cases} 1 & \text{for } \mathbf{a} > 0 \\ -\frac{3}{2}\mathbf{a} - 1 & \text{for } \mathbf{a} < 0 \end{cases}$$

Universe evolution with and without phase transition

Using y_b (corresponding to $\bar{\rho}$) and y_s (to $\bar{\bar{\rho}}$) we integrate numerically Eq.3:

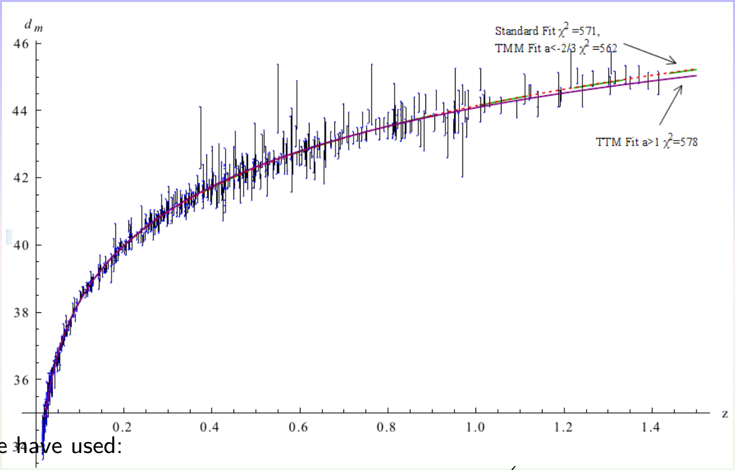


"Phase transition" or "quenching" to the lower state:
If at $t = 0$, $\bar{\rho} > 0$ and $\bar{\bar{\rho}} < 0$, but for certain moment t_p: $\bar{\bar{\rho}}(a(t_p)) = 0$, then for $t > t_p$ we have two possible "states" $\bar{\rho}$ and $\bar{\bar{\rho}}$ of the Universe:

$$0 \leq \bar{\bar{\rho}} < \bar{\rho} \ \text{for} \ \ t \geq t_p. \quad (4)$$

We can observe **phase transition of the first kind.**

Our best fit against Supernovae data and standard fit



where we have used:

$$d_m = 5\log_{10}\left((1+z)\int_0^z dx\,\frac{a(x)}{\dot a(x)}\right) = \mathrm{const} + 5\log_{10}\left((1+z)\int_0^z dx\,\frac{1}{\sqrt{\rho(x)}}\right) \qquad (5)$$

to obtain one parametric family of solutions:

$$b_\pm = \sum_0^4 \pm c_i a^i + O(a^5),\; c_i = [0.337906, 0.376679, -0.0251697, 0.00148545, 0.11272710^{-3}]$$

Including inflation into the model

Following Guendelman, Nissimov and Pacheva [4, 5] (where in S_{darkon} $\alpha = 0$))

$$S = S_{darkon} + \int d^4x \Phi_1(A)(R + L^{(1)}) + \int d^4x \Phi_2(B) \left(L^{(2)} + \frac{\Phi(H)}{\sqrt{-g}} \right)$$

where we have

$$L^{(1)} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi), \; V(\phi) = f_1 e^{-\alpha\phi}$$

$$L^{(2)} = -\frac{b}{2} e^{-\alpha\phi} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + U(\phi), \; U(\phi) = f_2 e^{-2\alpha\phi}$$

From the equations of motion we have:

$$p = -2M_0 = const, \frac{\Phi_2(B)}{\sqrt{-g}} = \chi_2 = const$$

$$R + L^{(1)} = -M_1 = const, L^{(2)} + \frac{\Phi(H)}{\sqrt{-g}} = -M_2 = const$$

$$U_{eff}(\phi) = \frac{(V_1(\phi) + M_1)^2}{4\chi_2(U(\phi) + M_2)} \text{with } U_- = \frac{f_1^2}{4\chi_2 f_2}, U_+ = \frac{M_1^2}{4\chi_2 M_2}$$

From the requirement that the vacuum energy density of the early Universe U_- should be much bigger than that of the late Universe U_+ follows that:

$$\frac{f_1^2}{f_2} >> \frac{M_1^2}{M_2}$$

The equations in FLRW

The system of equations that need to be solved in order to obtain the evolution of the Universe is the following:

$$v^3 + 3av + 2b = 0 \text{ for } a = \frac{-1}{3} \frac{V(\phi) + M_1 - \frac{1}{2}\chi_2 b e^{-\alpha\phi}\dot{\phi}^2}{\chi_2(U(\phi) + M_2) - 2M_0}, \ b = \frac{-p_u}{2a(t)^3(\chi_2(U(\phi) + M_2) - 2M_0)} \tag{6}$$

$$\dot{a}(t) = \sqrt{\frac{\rho}{6}} a(t), \ \rho = \frac{1}{2}\dot{\phi}^2(1 + \frac{3}{4}\chi_2 b e^{-\alpha\phi} v^2) + \frac{v^2}{4}(V + M_1) + \frac{3p_u v}{4a(t)^3} \tag{7}$$

$$\ddot{a}(t) = -\frac{1}{12}(\rho + 3p)a(t), \ p = \frac{1}{2}\dot{\phi}^2(1 + \frac{1}{4}\chi_2 b e^{-\alpha\phi} v^2) - \frac{v^2}{4}(V + M_1) + \frac{p_u v}{4a(t)^3} \tag{8}$$

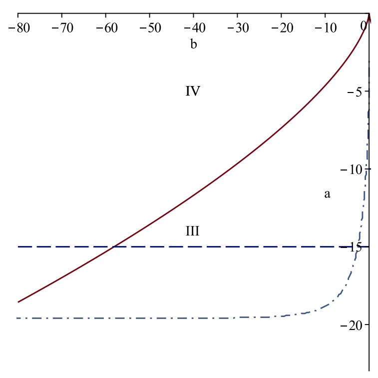
$$\frac{d}{dt}\left(a(t)^3\dot{\phi}(1 + \frac{\chi_2}{2} b e^{-\alpha\phi} v^2)\right) + a(t)^3(\alpha\frac{\dot{\phi}^2}{4}\chi_2 b e^{-\alpha\phi} v^2 + \frac{1}{2}V_\phi v^2 - \chi_2 U_\phi \frac{v^4}{4}) = 0 \tag{9}$$

Note that here Eq. (8) is optional and it offers an independent way to evaluate $\ddot{a}(t)$.
The differential system above is of first order with respect to $a(t)$ and of second order with respect to $\phi(t)$.
To evaluate it we use the implemented in Maple Fehlberg fourth-fifth order Runge-Kutta method with degree four interpolant.

Initial and boundary conditions:

I. $a(0) = 10^{-12}, \phi(0) = \phi_0, \dot{\phi}(0) = 0$, II. $a(1) = 1$, III. $a''(t) = 0$ in 3 points + correct sign

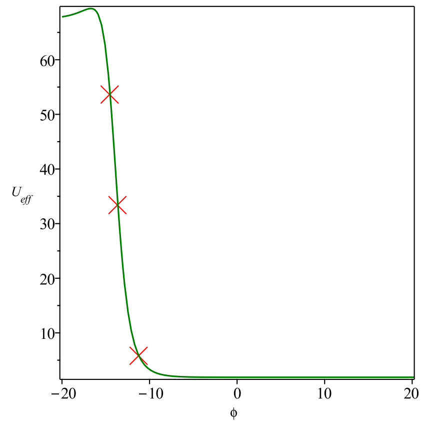
The evolution of the Universe in the $[a, b]$ plane



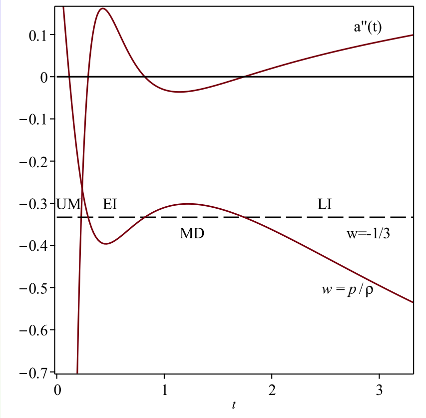
Evolution of the parameters $[\mathbf{a}(t), \mathbf{b}(t)]$ for the darkon (dash) and the inflaton (dot-dash). With solid line is the line where y_i change validity

The evolution of the Universe starts from $b \to -\infty$ and finishes at $b \to 0$
We have chosen the parameters in such a way that:
$b = \frac{-p_u}{2a(t)^3(\chi_2(U+M_2)-2M_0)} < 0.$

Numerical solutions, Staicova & Stoilov, arXiv:1801.07133



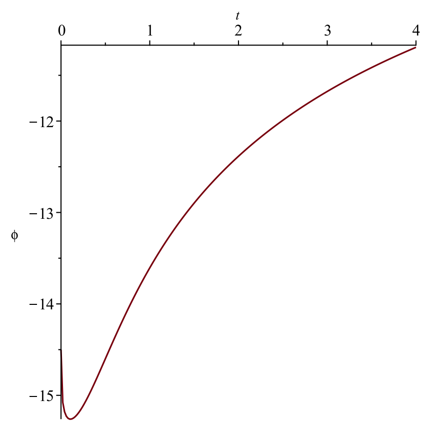
(d)



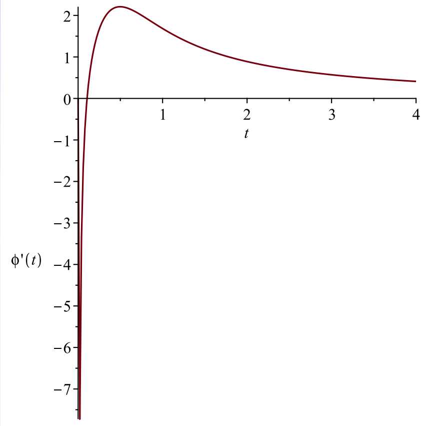
(e)

Figure: a) The effective potential with crosses at: $t = 0, 1, 4$. b) Plot of $\ddot{a}(t)$ and the EOS $w = p/\rho$, where UM is ultra-relativistic matter domination , EI – the early inflation, MD – the matter domination (MD) and LI – the late inflation.

Numerical solutions - the inflaton



(a) $\phi(t)$



(b) $\dot{\phi}(t)$

Note, the inflation experience friction, due to which inflation stops before reaching the U_+ part of the potential

Where are we?

The main requirement of the model:

$$f_1^2/f_2 >> M_1^2/M_2$$

is satisfied for all our numerical results.

Parameter	Theory	Numerics
M_1	$\sim M_{EW}^4 = 4.10^{-60}$	$1/250 = 4 \times 10^{-3}$
M_2	$\sim M_{Pl}^4 = 4$	4
f_1	$\sim 10^{-8}$	1×10^{-5}
f_2	$\sim 10^{-8}$	10^{-8}
M_0	$\Lambda^{Pl} \sim 10^{-122}$	$\Lambda = 1 \times 10^{-4}$
α	$10^{-20} - 0.2$	0.54

Time scale considerations:
Matter domination is considered to start at $a_{MD}(t) \sim 3 \times 10^{-4}$, the accelerated expansion – at $a_{AE}(t) \gtrsim 0.6$. Our current best result $a_{MD} = 0.2, a_{AE} = 1.2$

Conclusion:

The two-measures model with darkon and inflaton scalar fields is able to reproduce the required epochs of the evolution of the Universe, but further study of the parameter space is required.

Thank you for you attention!

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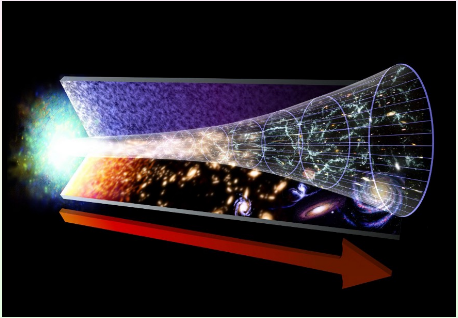

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