Cosmological solutions from models with unified dark energy and dark matter and with inflaton field

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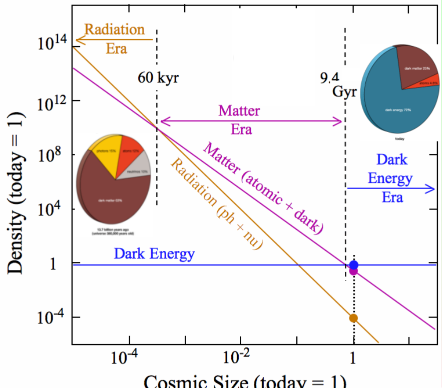
Contents

The current state of cosmology

What we know :

- The Universe is isotropic and homogeneous
- The Universe is expanding in an accelerated rate
- The Universe is flat (within error of $\sim 0.4\%$ by WMAP)
- The age of the Universe is 13.798 ± 0.037 bln. years
- The Universe contains $4.82 \pm 0.05\%$ ordinary matter, $25.8 \pm 0.4\%$ dark matter and $69 \pm 1\%$ dark energy.

Data from Planck, SNe Ia, the HST key project, the Sloan Digital Sky Survey, WMAP, COBE, large-scale galaxy



To extend Λ-CDM, we need theory with inflation and with DE/DM!

The darkon model in FLRW metric, Mod. Phys. Lett. A, **32**, 1 (2017)

If we apply the two-measures darkon model in the f(R) gravity(Guendelman, Nissimov and Pacheva ([1],[2])),

$$S_{darkon} = \int d^4x\sqrt{-g}(R(g,\Gamma) - \alpha R^2(g,\Gamma)) + \int d^4x(\sqrt{-g} + \Phi(C))L(u,X)$$

with $\Phi(C) = \frac{1}{3}\epsilon^{\mu\nu\kappa\lambda}\partial_\mu C_{\nu\kappa\lambda}$, $L(u,X) = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$

The Friedman–Lemaître–Robertson–Walker metric with $k = 0$ is:

$$ds^2 = -dt^2 + a(t)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right].$$

From the Friedman equations ($G_{00} = T_{00}$), the energy density is:

$$\rho = \frac{1}{8\alpha}\dot{u}^2 + \frac{3}{4}\frac{p_u}{a(t)^3}\dot{u} - \frac{1}{4\alpha} \qquad (1)$$

where for the constant p_u we have from the equation of motions:

$$a(t)^3\left[-\frac{1}{2\alpha}\dot{u} + (\frac{1}{4\alpha} - 2M)\dot{u}^3\right] = p_u \;\; (= \mathrm{const}) \qquad (2)$$

The miracles of a simple cubic equation

We rewrite the last cubic equation for $\dot{u}$ Eq. (2), as

$$y^3 + 3\mathbf{a}y + 2\mathbf{b} = 0 \text{ with } \mathbf{a} = -\frac{2}{3-24\alpha M} \text{ and } \mathbf{b} = -\frac{2\alpha p_u}{a(t)^3(1-8\alpha M)}.$$

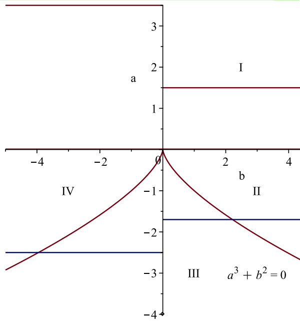
The solutions are:

$$y_1 = -\frac{\mathbf{a}}{(-\mathbf{b}+\sqrt{\mathbf{a}^3+\mathbf{b}^2})^{1/3}} + (-\mathbf{b}+\sqrt{\mathbf{a}^3+\mathbf{b}^2})^{1/3}$$

$$y_2 = \frac{\mathbf{a}}{(\mathbf{b}-\sqrt{\mathbf{a}^3+\mathbf{b}^2})^{1/3}} - (\mathbf{b}-\sqrt{\mathbf{a}^3+\mathbf{b}^2})^{1/3}$$

$$y_3 = \frac{1-i\sqrt{3}}{2}\frac{\mathbf{a}}{(-\mathbf{b}+\sqrt{\mathbf{a}^3+\mathbf{b}^2})^{1/3}} - \frac{1+i\sqrt{3}}{2}(-\mathbf{b}+\sqrt{\mathbf{a}^3+\mathbf{b}^2})^{1/3}$$

No real smooth solution exists in the $[\mathbf{a}, \mathbf{b}]$ plane.



We define the following piecewise functions, **real** in the whole plane [a,b]

$$y_b = \begin{cases} y_1 & \text{for } (a,b) \in \{a \geq 0\} \cup \{a < 0 \cap b < 0\} \\ y_2 & \text{for } (a,b) \in \{a < 0 \cap b > 0\} \end{cases} \qquad y_s = \begin{cases} y_1 & \text{for } b > 0 \\ y_2 & \text{for } b < 0. \end{cases}$$

Universe evolution with and without phase transition

After rescaling time by $2|\alpha|/3 = 1$ and absorbing α into Hubble constant ($\bar{\rho} = 4|\alpha|\rho$):

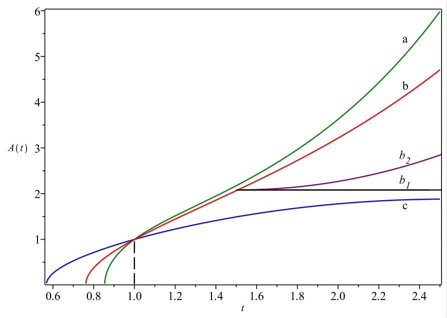
Final form of the Friedman equation:

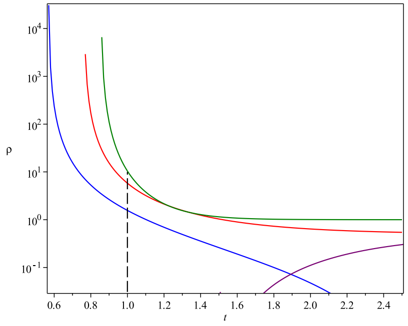
$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \bar{\rho} = \left(\frac{1}{2}y^2 + \frac{\mathbf{b}}{\mathbf{a}}y - 1\right) \quad (3)$$

$$\bar{\rho} \xrightarrow[a(t)\to\infty]{} 1 \text{ for } \mathbf{a} > 0$$

$$\bar{\rho} \xrightarrow[a(t)\to\infty]{} -\frac{3}{2}\mathbf{a} - 1 \text{ for } \mathbf{a} < 0 \quad (4)$$

We use as independent real solutions y_b (our basic solution) and y_s and integrate numerically Eq. 3 to find the evolution of the universe.





Graphics of the $a(t)$ evolution for: $\mathbf{a} = -.5, \mathbf{b} = -\frac{0.5}{a^3}$ (blue), $\mathbf{a} = -1, \mathbf{b} = -\frac{2}{a^3}$ (red), $\mathbf{a} = 1, \mathbf{b} = \frac{6}{a^3}$ (green) and $t_p = 1.5074, a_s(t_p) = 2.0825$ (purple).

Universe evolution with phase transition

- We denote $\bar{\bar{\rho}}$ the density which corresponds to solution y_s.
- For $t = 1$, $\bar{\bar{\rho}}(t_3) < 0$, but for certain moment t_p: $\bar{\bar{\rho}}(a(t_p)) = 0$ (same asymptotic for large $a(t)$).
- Therefore, for any moment after t_p we have two "states" $\bar{\rho}$ and $\bar{\bar{\rho}}$ of the Universe:

$$0 \leq \bar{\bar{\rho}} < \bar{\rho} \;\; \text{for} \;\; t \geq t_p. \tag{5}$$

  This opens the possibility the Universe to undergo **"phase transition" or "quenching" to the lower state**.
- The moment of the phase transition is crucial for the further evolution:
  – If it happens exactly at time t_p the evolution **stops** ($\bar{\bar{\rho}} = 0$).
  – If the jump occurs in any later moment, then we observe **phase transition of the first kind.**

The supernova data fit

Distance modulus d_m:

$$d_m = 5\log_{10}\left(\frac{d}{10}\right) \qquad (6)$$

where d is the distance in parsecs.

Connection between d_m and z:

$$\begin{aligned} d_m &= 5\log_{10}\left((1+z)\int_0^z dx \frac{a(x)}{\dot{a}(x)}\right) = \\ &\mathrm{const} + 5\log_{10}\left((1+z)\int_0^z dx \frac{1}{\sqrt{\rho(x)}}\right) \end{aligned} \qquad (7)$$

Where we have used $a = \frac{1}{1+z}$. Here, $\rho(x)$:

$$\rho_{CDM} \sim a(t)^{-3}, \rho_r \sim a(t)^{-4}, \rho_k \sim a(t)^{-2}, \rho_{DE} \sim \Lambda$$

From observations, we know that $\rho_k \sim 0$, $\rho_r \sim 10^{-5}$, so we use $\Omega_{CDM} + \Omega_\Lambda$ universe.

Our best fit against Supernovae data and standard fit

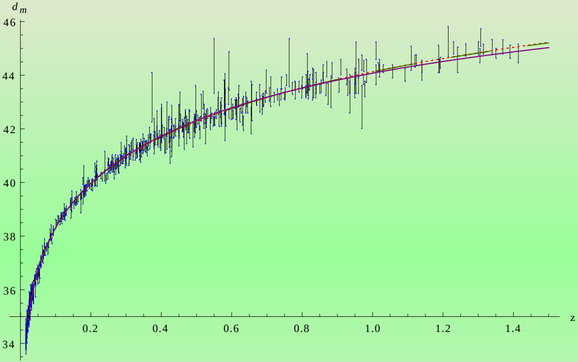


Figure: Supernovae data against Standard model fit (dotted line), $\mathbf{a} < -2/3$ fit (dashed line) and $\mathbf{a} > 1$ fit (solid line).

$\chi^2 = 562$ for $\Omega_{DE} = 0.722, \ \Omega_{CM} = 0.278, \Omega_K = 0, \Omega_R = 0$:standard fit
$\chi^2 = 571$ for $\Omega_{DE} = 0.73224, \Omega_{CM} = 0.26776, \Omega_K = 0, \Omega_R = 0$:Misho's symplectic fit
$\chi^2 \sim 562$ for $\mathbf{a} < -2/3$,
$\chi^2 \sim 578$ for $\mathbf{a} > 1$.

Supernova data [6] are available at http://www.supernova.lbl.gov/Union/figuresSCPUnion2.1_mu_vs_z.txt

The best fit of SN data using the proposed model is not unique.

We observe one parametric family of solutions which gives practically one and the same $d_m(z)$ function.

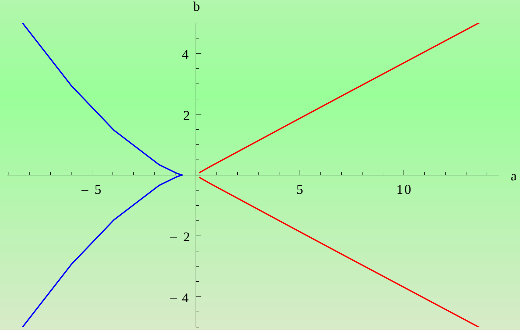
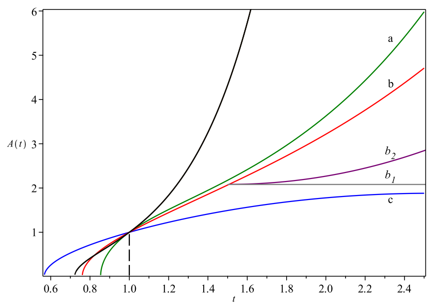
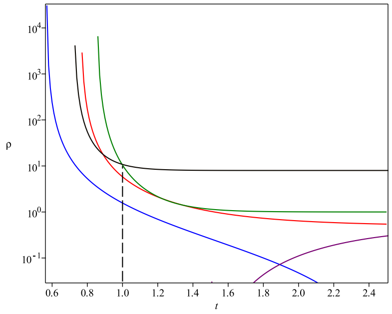


Figure: Best fit families on the parametric plane.

Using the so obtained a, b as parameters in the evolution





Here the black line corresponds to
$a = -5.98736, b = -2.93213, t_0 = 0.72226.$
We have an expanding Universe, but what about the inflation?

Including inflation into the model

Following Guendelman, Nissimov and Pacheva [4, 5] (where in S_{darkon} $\alpha = 0$))

$$S = S_{darkon} + \int d^4x \Phi_1(A)(R + L^{(1)}) + \int d^4x \Phi_2(B) \left(L^{(2)} + \frac{\Phi(H)}{\sqrt{-g}} \right)$$

where we have

$$L^{(1)} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi), \; V(\phi) = f_1 e^{-\alpha\phi} \tag{8}$$

$$L^{(2)} = -\frac{b}{2} e^{-\alpha\phi} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + U(\phi), \; U(\phi) = f_2 e^{-2\alpha\phi} \tag{9}$$

(10)

From the equations of motion we have:

$$p = -2M_0 = const, \frac{\Phi_2(B)}{\sqrt{-g}} = \chi_2 = const$$

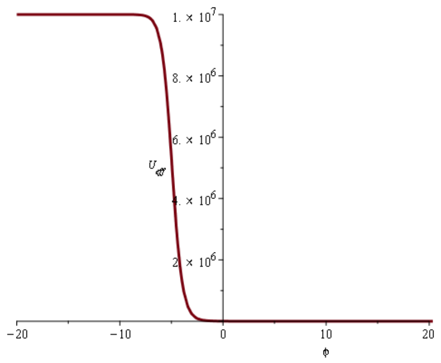
$$R + L^{(1)} = -M_1 = const, L^{(2)} + \frac{\Phi(H)}{\sqrt{-g}} = -M_2 = const$$

$$U_{eff}(\phi) = \frac{(V_1(\phi) + M_1)^2}{4\chi_2(U(\phi) + M_2)} \text{with } U_- = \frac{f_1^2}{4\chi_2 f_2}, U_+ = \frac{M_1^2}{4\chi_2 M_2}$$

From the requirement that the vacuum energy density of the early Universe U_- should be much bigger than that of the late Universe U_+ follows that:

$$\frac{f_1^2}{f_2} >> \frac{M_1^2}{M_2}$$

The effective potential



In order to give the mode physical meaning, one expects:

$$|M_1| \sim M_{EW}^4, M_2 \sim M_{Pl}^4, f_1 \sim f_2 \sim 10^{-8} M_{Pl}^4$$

The equations in FLRW

The system of equations that need to be solved in order to obtain the evolution of the Universe is the following:

$$v^3 + 3av + 2b = 0 \text{ for } a = \frac{-1}{3}\frac{V(\phi) + M_1 - \frac{1}{2}\chi_2 b e^{-\alpha\phi}\dot{\phi}^2}{\chi_2(U(\phi) + M_2) - 2M_0}, \; b = \frac{-p_u}{2a(t)^3(\chi_2(U(\phi) + M_2) - 2M_0)} \quad (11)$$

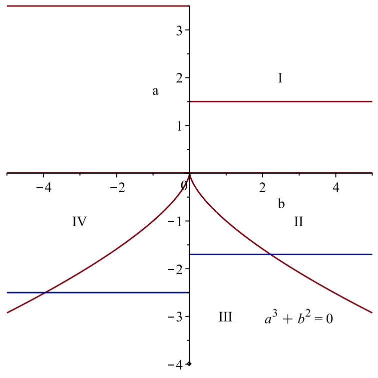
$$\dot{a}(t) = \sqrt{\frac{\rho}{6}}a(t), \; \rho = \frac{1}{2}\dot{\phi}^2(1 + \frac{3}{4}\chi_2 b e^{-\alpha\phi}v^2) + \frac{v^2}{4}(V + M_1) + \frac{3p_u v}{4a(t)^3} \quad (12)$$

$$\ddot{a}(t) = -\frac{1}{12}(\rho + 3p)a(t), \; p = \frac{1}{2}\dot{\phi}^2(1 + \frac{1}{4}\chi_2 b e^{-\alpha\phi}v^2) - \frac{v^2}{4}(V + M_1) + \frac{p_u v}{4a(t)^3} \quad (13)$$

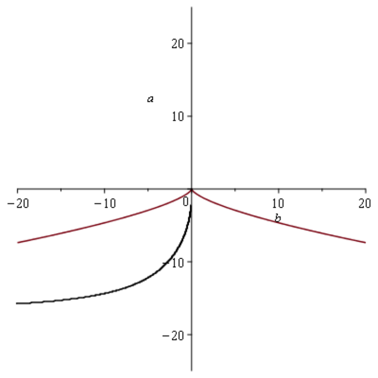
$$\frac{d}{dt}\left(a(t)^3\dot{\phi}(1 + \frac{\chi_2}{2}b e^{-\alpha\phi}v^2)\right) + a(t)^3(\alpha\frac{\dot{\phi}^2}{4}\chi_2 b e^{-\alpha\phi}v^2 + \frac{1}{2}V_\phi v^2 - \chi_2 U_\phi \frac{v^4}{4}) = 0 \quad (14)$$

Note that here Eq. (13) is optional and it offers an independent way to evaluate $\ddot{a}(t)$.
The differential system above is of first order with respect to $a(t)$ and of second order with respect to $\phi(t)$.
To evaluate it we use the implemented in Maple Fehlberg fourth-fifth order Runge-Kutta method with degree four interpolant.

The evolution of the Universe in the $[a, b]$ plane



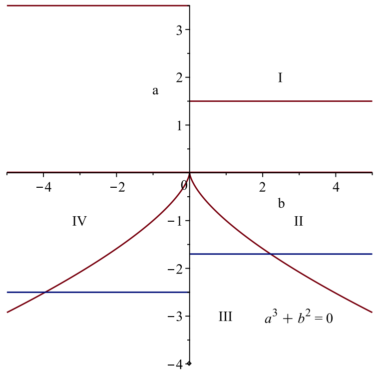
(d) Darkon case



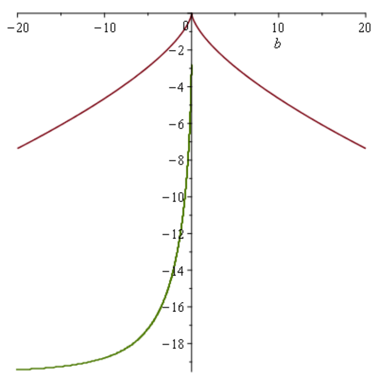
(e) Inflaton case

Here the evolution of the Universe starts from $b \to -\infty$ and finishes at $b \to 0$
We have chosen the parameters in such a way that $b = \frac{-p_u}{2a(t)^3(\chi_2(U+M_2)-2M_0)} < 0$.

Or if we zoom in



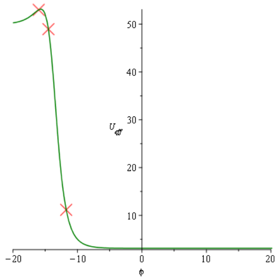
(f) Darkon case



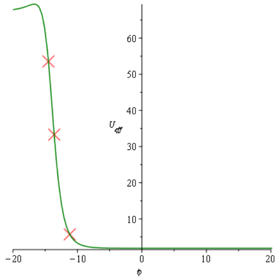
(g) Inflaton case

We see that the horizontal lines of the simpler darkon case transform into the curved lines of the inflaton case.

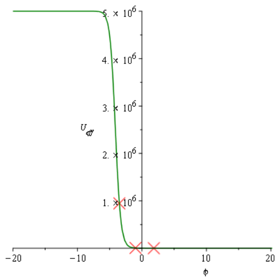
Numerical solutions - 1) The potentials



(h) Case 1



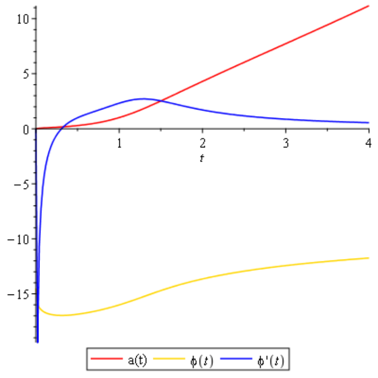
(i) Case 2



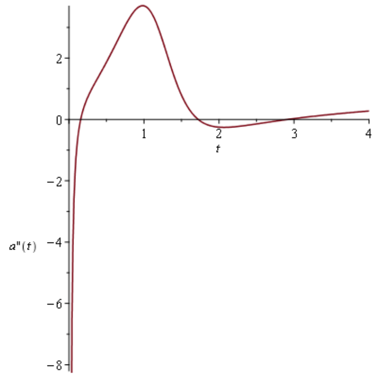
(j) Case 3

Case 1:
$M_0 = -0.01$, $M_1 = 0.1$, $M_2 = 4$, $\alpha = .72$, $b_0 = 1 \times 10^{-5}$, $p_u = 9 \times 10^{-3}$, $\chi_2 = 2 \times 10^{-4}$, $f_1 = 2 \times 10^{-5}$, $f_2 = 1 \times 10^{-8}$
Case 2:
$M_0 = -0.01$, $M_1 = 0.1$, $M_2 = 4$, $\alpha = .7$, $b_0 = 1 \times 10^{-5}$, $p_u = .15$, $\chi_2 = 3.3 \times 10^{-4}$, $f_1 = 3 \times 10^{-5}$, $f_2 = 1 \times 10^{-8}$
Case 3:
$M_0 = -.1$, $M_1 = .667$, $M_2 = 0.001$, $\alpha = 1$, $b_0 = 0.05$, $p_u = 0.19 \times 10^{-1}$, $\chi_2 = .125$, $f_1 = .5$, $f_2 = 5 \times 10^{-8}$

Numerical solutions - the evolution



(k) Case 1

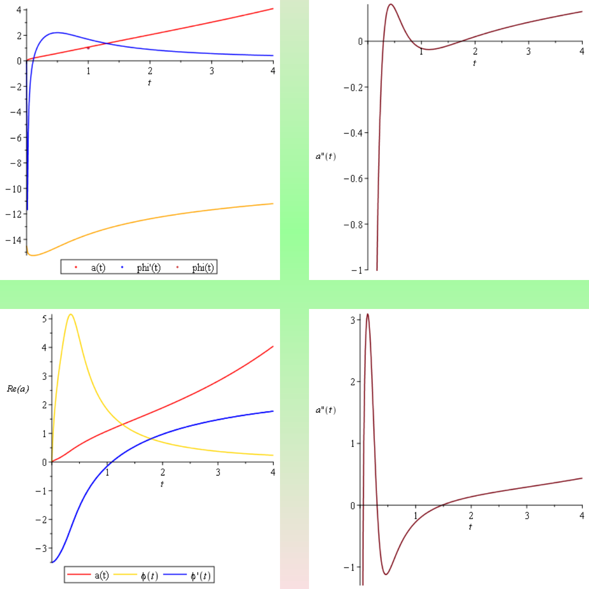


(l) Case 1

Initial and boundary conditions:

1) $a(0) = 10^{-12}, \phi(0) = \phi_0, \dot{\phi}(0) = 0$
2) $a(1) = 1$
3) $a''(t) = 0$ in 3 points + correct sign

Case 2 (up) and Case 3 (bottom)



Where are we?

The main requirement of the model:

$$f_1^2/f_2 >> M_1^2/M_2$$

is satisfied for all our numerical results.

Parameter	Theory	Numerics	Comment
M_1	$\sim M_{EW}^4 = 4.10^{-60}$	$1/15 = 6.67 \times 10^{-2}$	gauge $M_{Pl} = \sqrt{2}$
M_2	$\sim M_{Pl}^4 = 4$	4	
f_1	$\sim 10^{-8}$	2×10^{-5}	$f_1^2/f_2 = 4 \times 10^{-8}$, $f_1 \sim f_2$
f_2	$\sim 10^{-8}$	10^{-8}	
M_0	$\Lambda^{Pl} \sim 10^{-122}$	$\Lambda = 1.156 \times 10^{-5}$	$2\Lambda = \frac{M_1^2}{4(\chi_2 M_2 - 2M_0)}$
α	$10^{-20} - 0.2$	0.64	$\alpha > 0.2$ – scalar-tensor ratio $\rightarrow 0$

Time scale considerations:
Matter domination is considered to start at $a_{MD}(t) \sim 3 \times 10^{-4}$, the accelerated expansion – at $a_{AE}(t) \gtrsim 0.6$.
Current best result $a_{MD} = 0.2$, $a_{AE} = 1.2$

Conclusions

- We integrated numerically the Friedman equations in the K-essence theory in the darkon and the inflaton case
- The dependence of the evolution of the Universe on the parameters [a,b] was examined in both cases
- It was shown that in the darkon model we can obtain both a Universe with and without phase transition
- A new data fit of the SNe data was presented
- In the case of inflaton model, the parameter space of the model was studied
- Solutions with two inflationary epochs and 1 matter dominated were found
- It was shown that the inflation experience friction, due to which inflation stops before reaching the U_+ part of the potential

Thank you for you attention!

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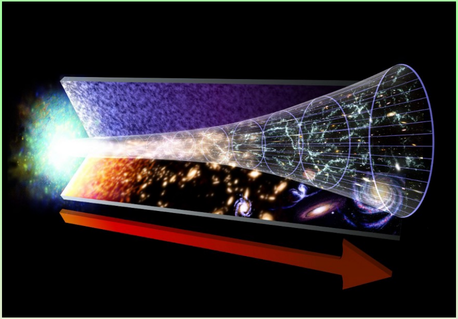

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