The multi-measure cosmological model and its peculiar effective potential

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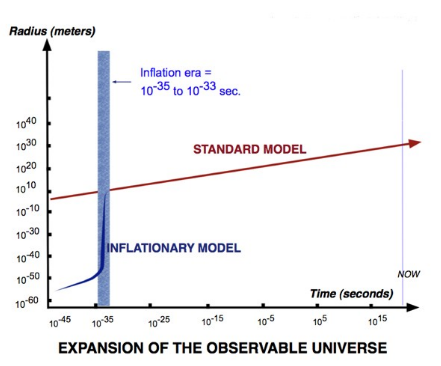
Cosmology today

– The Universe is isotropic, homogenous and flat
– The horizon problem, the flatness problem, the missing monopols problem and the large-structures formation problem
– Λ-tension, H_0-tension, σ_8 tension



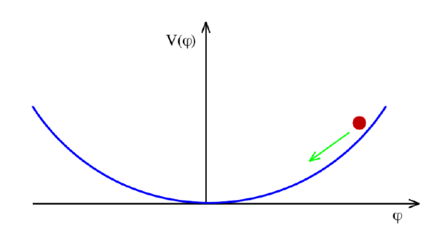
Credit:https://www.physicsoftheuniverse.com/

So one introduces inflation...

$\Lambda - CDM$ model: $H = \frac{\dot{a}}{a} = H_0\sqrt{\Omega_m a^{-3} + \Omega_{rad} a^{-4} + \Omega_\Lambda}$
Inflation:

$$H^2 = \frac{8\pi}{3m_{Pl}^2}(V(\phi) + \frac{1}{2}\dot{\phi}^2)$$
$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

Inflation occurs when $\ddot{a}(t) > 0 \leftrightarrow \dot{\phi}^2 < V(\phi)$



Credit:https://arxiv.org/pdf/astro-ph/9901124.pdf

The multimeasure model

- Model developped by Guendelman, Nissimov and Pacheva 2014, 2016
- Aimed to produce a model that describes early inflation and a smooth exit to modern times.
- The action of the model: $S = S_{darkon} + S_{inflaton}$ is :

$$S_{darkon} = \int d^4x(\sqrt{-g} + \Phi(C))L(u, Y)$$

$$S_{inflaton} = \int d^4x\Phi_1(A)(R + L^{(1)}) + \int d^4x\Phi_2(B)\left(L^{(2)} + \frac{\Phi(H)}{\sqrt{-g}}\right)$$

where $\Phi_i(X) = \frac{1}{3}\epsilon^{\mu\nu\kappa\lambda}\partial_\mu X_{\nu\kappa\lambda}$, and

$$L(u, X) = -\frac{1}{2}g^{\mu\nu}\partial_\mu u\partial_\nu u - W(u)$$

$$L^{(1)} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi),\ V(\phi) = f_1 e^{-\alpha\phi}$$

$$L^{(2)} = -\frac{b}{2}e^{-\alpha\phi}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + U(\phi),\ U(\phi) = f_2 e^{-2\alpha\phi}$$

In Einstein frame

There is a Weyl-rescaled meric $\tilde{g}$ for which the action

$$S^{(eff)} = \int d^4x \, \sqrt{-\tilde{g}}(\tilde{R} + L^{(eff)}) \tag{1}$$

with effective Lagrangian:

$$L^{(eff)} = \tilde{X} - \tilde{Y}(V + M_1 - \chi_2 b e^{-\alpha\phi}\tilde{X}) + \tilde{Y}^2(\chi_2(U + M_2) - 2M_0).$$

satisfies the Einstein equations:

$$\tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{R} = \frac{1}{2}T^{(eff)}_{\mu\nu} \tag{2}$$

Non-linear with respect to both scalar fields kinetic terms, thus of the generalized k-essence type.

The effective potential of the model is: $U_{eff}(\phi) = \frac{1}{4}\frac{(f_1 e^{-\alpha\phi} + M_1)^2}{\chi_2(f_2 e^{-2\alpha\phi} + M_2) - 2M_0}$.

The effective cosmological costant: $\Lambda_{eff} = U_+/2 = \frac{M_1^2}{8(\chi_2 M_2 - 2M_0)}$.

Details on moving from Jordan to Einstein frame

– There are 4 dynamically generated integration constants:

$$L^{(0)} = -2M_0,$$
$$R + L^{(1)} = M_1,$$
$$L^{(2)} + \frac{\Phi(\mathcal{H})}{\sqrt{-g}} = -M_2,$$
$$\frac{\Phi(\mathcal{B})}{\sqrt{-g}} = \chi_2$$

– The transformation to Einstein frame:

$$\tilde{g}_{\mu\nu} = \chi_1 g_{\mu\nu}, \text{for } \chi_1 = \frac{\Phi(\mathcal{A})}{\sqrt{-g}}.$$
$$u \to \tilde{u} : \frac{\partial \tilde{u}}{\partial u} = (W - 2M_0)^{-\frac{1}{2}}$$
$$\tilde{Y} = -\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu \tilde{u}\partial_\nu \tilde{u}, \tilde{X} = -\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu \phi \partial_\nu \phi$$

– A Noether symmetry: $\partial_\mu \left(\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu \tilde{u} \frac{\partial \tilde{L}}{\partial \tilde{Y}} \right) = 0$

In FLRW metric ($v = \dot{u}$):

The action becomes:

$$S^{(eff)} = \int dt\ a(t)^3 \Big(-6\ \frac{\dot{a}(t)^2}{a(t)^2} + \frac{\dot{\phi}^2}{2} - \frac{v^2}{2}\left(V + M_1 - \chi_2 b e^{-\alpha\phi}\dot{\phi}^2/2\right) + \frac{v^4}{4}\left(\chi_2(U + M_2) - 2M_0\right)\Big).$$

And the equations of motion are:

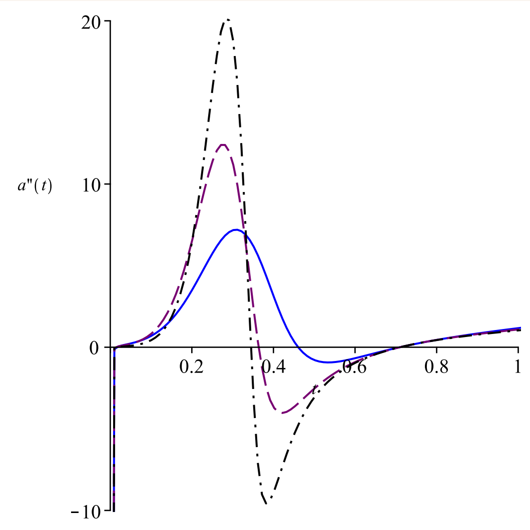
$$v^3 + 3\mathbf{a}v + 2\mathbf{b} = 0 \text{ for } \mathbf{a} = \frac{-1}{3}\frac{V(\phi) + M_1 - \frac{1}{2}\chi_2 b e^{-\alpha\phi}\dot{\phi}^2}{\chi_2(U(\phi) + M_2) - 2M_0}, \mathbf{b} = \frac{-p_u}{2a(t)^3(\chi_2(U(\phi) + M_2) - 2M_0)} \quad (3)$$

$$\dot{a}(t) = \sqrt{\frac{\rho}{6}}a(t), \quad \rho_= \frac{1}{2}\dot{\phi}^2(1 + \frac{3}{4}\chi_2 b e^{-\alpha\phi}v^2) + \frac{v^2}{4}(V + M_1) + \frac{3p_u v}{4a(t)^3} \quad (4)$$

$$\frac{d}{dt}\left(a(t)^3\dot{\phi}(1 + \frac{\chi_2}{2}b e^{-\alpha\phi}v^2)\right) + a(t)^3(\alpha\frac{\dot{\phi}^2}{4}\chi_2 b e^{-\alpha\phi}v^2 + \frac{1}{2}V_\phi v^2 - \chi_2 U_\phi\frac{v^4}{4}) = 0 \quad (5)$$

The parameters of this system are 12:
4 free parameters $\{\alpha, b_0, f_1, f_2\}$, 5 integration constants $\{M_0, M_1, M_2, \chi_2, p_u\}$ and 3 initial conditions $\{a(0), \phi(0), \dot{\phi}(0)\}$

Numerical solutions: The concept



The initial conditions and normalization are:

$$a(0) = 10^{-15}, \phi(0) = \phi_0, \dot{\phi}(0) = 0 \text{ and } a(1) = 1, \ddot{a}(0.71) = 0 \qquad (6)$$

The scale factor

Two cases: $\chi_2 \sim 1$ $(M_2 << |M_0|)$ and $\chi_2 << 1$ $(M_2 \gg |M_0|)$.

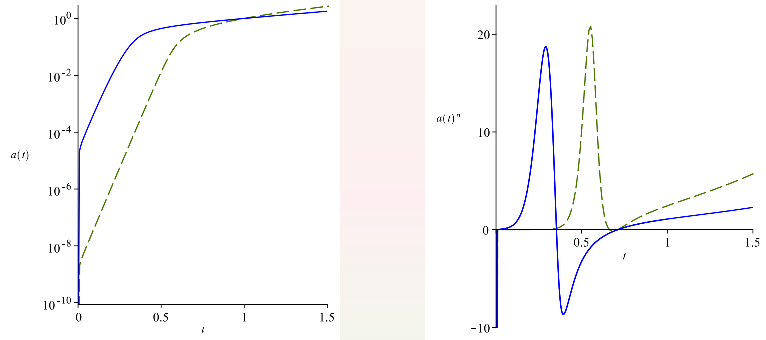
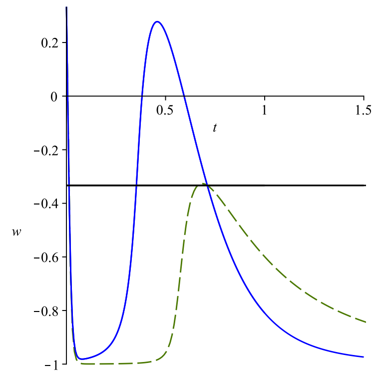
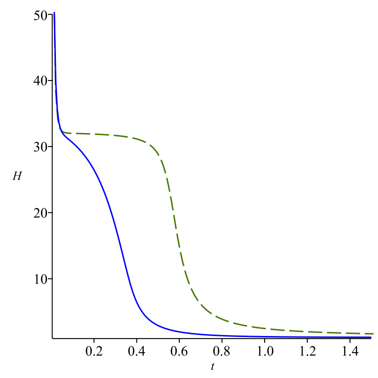


Figure: The Universe evolution: Case 1 (solid lines): $\chi_2 = 1,\ M_0 = -0.04,\ M_1 = 1.53,\ M_2 = 10^{-3},\ \alpha = 1.4,\ b_0 = 0.016,\ p_u = 11.5 \times 10^{-12},\ f_1 = 5.86,\ f_2 = 10^{-3},\ \phi(0) = -1.8$. Case 2 (dashed lines): $\chi_2 = 4 \times 10^{-5},\ M_0 = -0.01,\ M_1 = 0.763,\ M_2 = 4,\ \alpha = 0.64,\ b_0 = 1.52 \times 10^{-7},\ p_u = 6.5 \times 10^{-24},\ f_1 = 10^{-4},\ f_2 = 10^{-8},\ \phi(0) = -18$.

Number of e-folds of early inflation – about 18. $(N = \ln(a_{SD}/a_{EI}))$

The equation of state and the Hubble parameter





3 stages of the universe: early inflation, matter-radiation dominatio and late-time expansion

Limits for the equation of state (EOS) $w = p/\rho$:

$$w \xrightarrow[a(t)=0]{} 1/3 \text{ and } w \xrightarrow[a(t)\to\infty]{} -1 \tag{7}$$

The slow-roll parameters

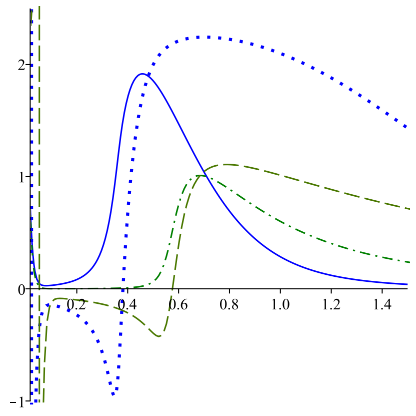
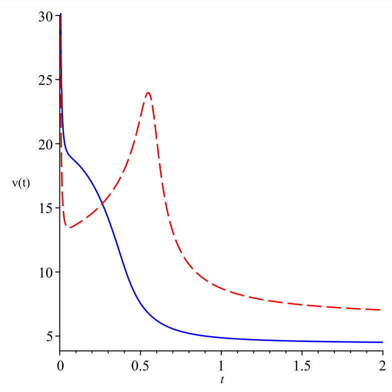


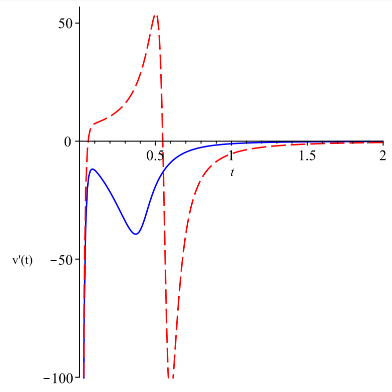
Figure: The slow roll parameter ϵ (Case 1: solid line, Case 2: dashes) and η (Case 1: dots Case 2: dash-dotted line)

$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta = -\frac{\ddot{\phi}}{H\dot{\phi}}.$$

The evolution of $v(t)$



v(t)



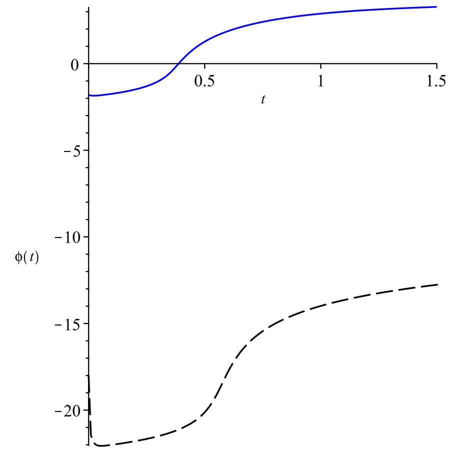
$\dot{v}(t)$

Figure: With solid lines (Case 1) and dashed (Case 2), we display the darkon field $v(t)$ and its first derivative $\dot{v}(t)$.

The asymptotic of v for $a(t) = exp(Ht)$ is:

$$v = \begin{cases} \sqrt{M_1/(\chi_2 M_2 - 2M_0)} \text{ for } \phi \to \infty \\ \sqrt{f_1/(\chi_2 M_2 - 2M_0)} \text{ for } \phi \to -\infty \\ \sqrt{(f_1 + M_1)/(\chi_2 M_2 - 2M_0)} \text{ for } \phi = 0 \text{ or } \alpha = 0 \end{cases} \quad (8)$$

The evolution of the inflaton field



Due to the strong friction term, we cannot start our evolution from the upper platea and get a “realistic” evolution.

For $a(t) = e^{Ht}$, then

$\ddot{\phi}(t) + 3\dot{\phi}(t)H = 0$ with a general solution $\phi(t) = C_1 + C_2 e^{-3Ht} \rightarrow const$ for $H > 0$

Very sensitive to the choice of parameters

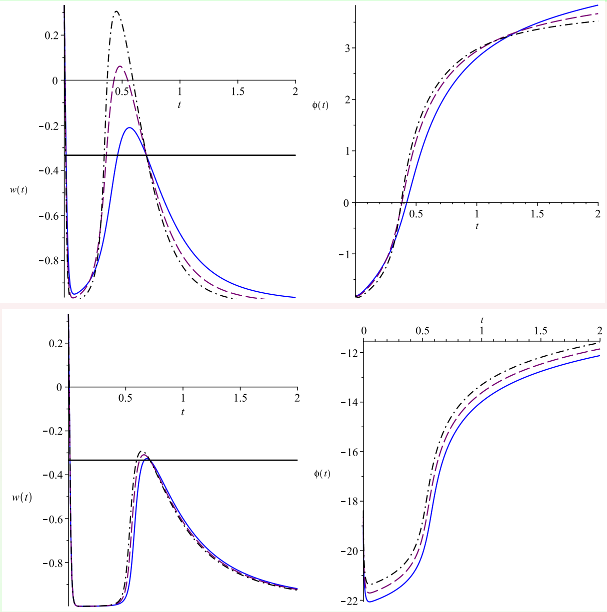
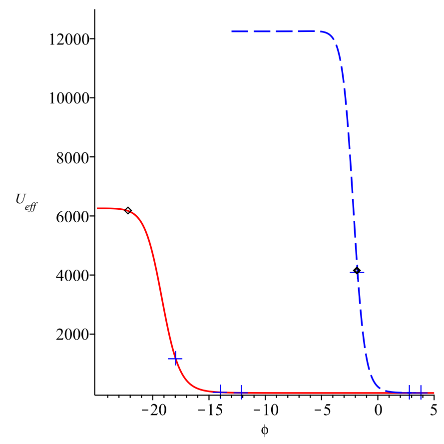


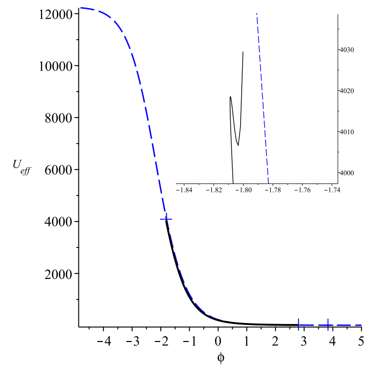
Figure: The Universe evolution for Case 1 (top): $\chi_2 \sim 1, \quad M_0 \sim -0.04, \quad 0 < M_2 << |M_0|$. and Case 2 (bottom): $\chi_2 << 1, \quad M_0 \sim -0.01, \quad M_2 = 4 \;\; (\gg |M_0|)$. $(w = p/\rho)$

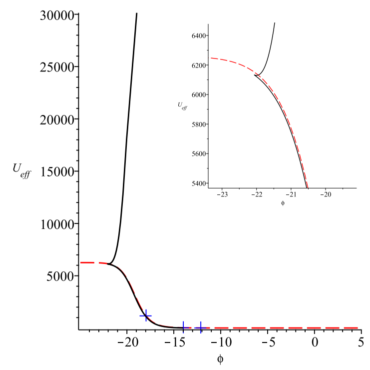
Climbing up the slope? arXiv1801.07133, arXiv:1806.08199, arXiv:1808.08890



Due to the strong friction term, we are unable to start from the left plateau.
But then, why it seems that the inflaton climbs up the slope?

If we integrate the inflaton equation





The **inflaton equation** (with $(A(t) = \frac{b_0 \chi_2}{2e^{\alpha\phi(t)}})$

$$\left(v(t)^2 A(t)+1\right)\ddot{\phi}(t) - \frac{1}{2} v(t)^2 \alpha A(t) \dot{\phi}(t)^2 + \left(3v(t)^2 A(t) H + 2v(t)A(t)\dot{v}(t) + 3H\right)\dot{\phi}(t)$$

$$- \frac{v(t)^2 f_1 \alpha}{2e^{\alpha\phi(t)}} + \frac{\chi_2 f_2 \alpha v(t)^4}{2e^{2\alpha\phi(t))}} = 0, \qquad (9)$$

The effective potential describes extremely well the numerical potential except for a small moment in beginning of the integration.

Conclusions and questions:

1. We have a model that qualitatively produces a "realistic" Universe if evolution starts from the slope of the effective potential.
2. There is friction term which stops the evolution of the inflaton
3. The effective potential is a good approximation to the actual potential term only after certain moment.
4. Question: We get only ~ 18 e-folds of inflation. Is there a way to do better?

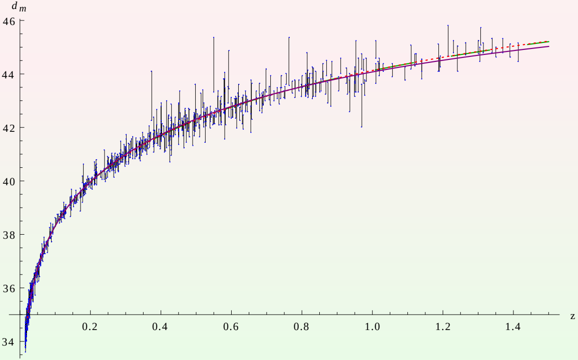


Figure: Supernovae data against Standard model fit (dotted line), $\mathbf{a} < -2/3$ fit (dashed line) and $\mathbf{a} > 1$ fit (solid line).

Thank you for you attention!

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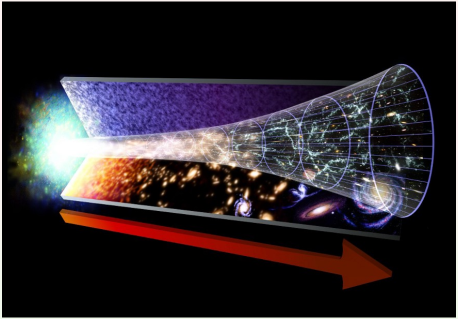

For more details: arXiv:1610.08368, 1801.07133, 1806.08199, 1808.08890