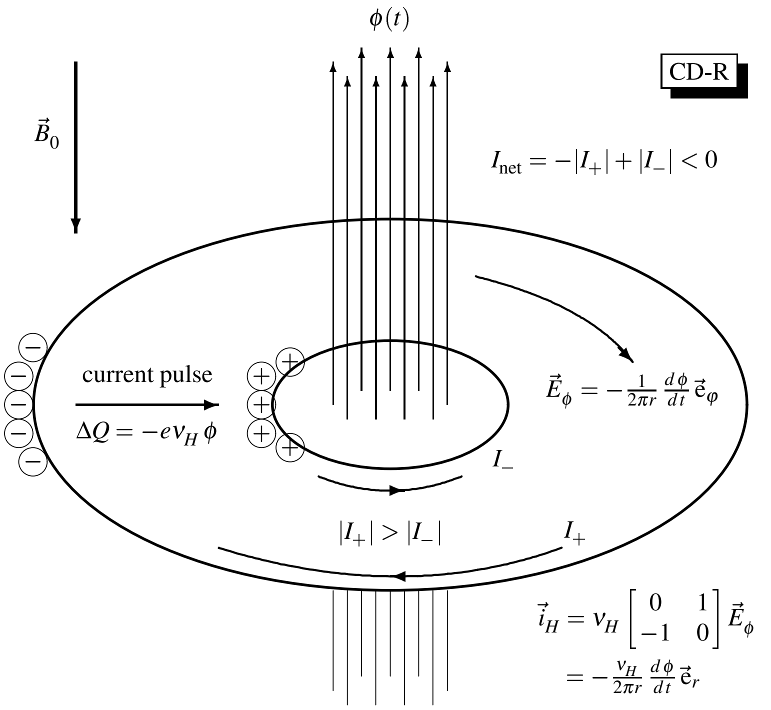
1 PERSISTENT CURRENTS: INTUITIVE PICTURE

We consider an adiabatic change of the flux threading the Corbino ring sample at zero temperature (a reversible process on each step).



1. We assume that the huge homogeneous magnetic field $\vec{B}_0 = -B_0\vec{e}_z$ (perpendicular to the plane), which points in the negative z direction[1], has driven the electron system into an incompressible FQH state on the ring. The total magnetic field is measured with respect to B_0, i.e., $B = B_{\text{tot}} - B_0$.

2. When no flux is threading the sample ,i.e., $B = 0$, the currents are concentrated on the edges and have the same magnitude but opposite directions so that the net current is zero. For a sample with negative-charge carriers (electrons) the current on the inner edge is **counter-clockwise** while that on the outer edge is **clockwise**. The intuitive way to determine these directions is given by the *skipping orbits* semi-classical picture: the electrons in the plane move along *clockwise* circles (of arbitrary radius in general) because the magnetic field points downwards. In the bulk the orbits are pinned down complete circles in such a way that there is no net current. However

[1] if you don't like this – complain to the person who defined the electron charge to be $-e$

on the inner edge the circular orbits are cut and (elastically) reflected from the edge becoming open curves resulting in a *clockwise* drift of the edge electrons producing a *counter-clockwise* current I_-. On the outer edge the drift of the cut orbits is *counter-clockwise* so that the outer edge current I_+ is *clockwise*. Quantum-Mechanically the situation is quite similar only the effect of edges is represented by a confining potential. Note that the role of the confining potential is fairly important: without it the two edges would try to completely compensate the additional AB flux (in which case both currents would be negative for small positive flux in order to produce a maximal negative total magnetic moment) so that there would be no mesoscopic effects. The confining potential makes this energetically unfavorable so that the additional flux is only partially compensated, like in an ordinary conducting ring, which opens the way for the oscillating mesoscopic currents.

3. Introducing Aharonov–Bohm type flux creates by Faraday induction an azimuthal electric field $\vec{E} = -\frac{1}{2\pi r}\frac{d\phi}{dt}\vec{e}_\varphi$.

4. The electric field $\vec{E}$ cannot create an azimuthal bulk current since $\sigma_{xx} = 0$ but induces a radial current pulse

$$\vec{i} = \nu_H \left[\begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array}\right] \vec{E}_\phi = -\nu_H \frac{d\phi}{dt}\frac{1}{2\pi r}\vec{e}_r$$

   due to the non-zero $\sigma_{xy} = \nu_H e^2/h$.

5. The current pulse $\vec{i}$, corresponding to the Hall current which is purely bulk in this geometry, transfers electric charge $\Delta Q = -e\nu_H\Delta\phi$ between the two edges (where ϕ is dimensionless) which destroys the perfect balance between them. We are interested in the case $\Delta\phi = 1$ which creates one or more quasiholes and quasiparticles.

6. In the case when the charge carriers are electrons this current pulse means that electrons from the inner edge are transferred to the outer one.

7. As a result of this change $\pm e\Delta n$ of carrier densities on both edges, the current on the outer edge **increases** while the inner-edge current *decreases* and the net current is in the direction of the bigger one, i.e., **the net current is clockwise** which is in the direction of the electric field. (This is under the reasonable assumption that the Fermi velocities do not change with the AB flux.) Note that the mesoscopic net current fluctuates with period 1 and can therefore be also opposite to the electric field as flux changes. The above argument also implies that **the mesoscopic persistent currents (i.e.,**

the changes in the edge currents due to the additional AB flux) on both edges have the same direction (clockwise) and both produce negative magnetic moments.

8. On the other hand, the azimuthal electric field $\vec{E}$ should apply an additional force to the electrons on the edge: even if this changes the Fermi velocities on the edges and/or create azimuthal bulk net current in the direction of the electric field the relative change would be as small as the ratio of the flux quantum and the flux of the background magnetic field, i.e., of the order of the number of flux quanta. That's why we consider the Fermi velocities on the edges as independent of the additional flux.

9. Note that the mesoscopic edge currents for $T = 0$ can be obtained from the transferred charge between the edges:

$$\Delta I = e v_F \Delta n = v_F \frac{\Delta Q}{L} = \pm \nu_H \frac{e v_F}{L} \phi$$

where we have divided ΔQ by the circumference L in order to get the one-dimensional charge density on the edge ($eL\Delta n = \Delta Q$). Thus we get

$$I_{\text{out}} = -\nu_H \frac{e v_F^{(\text{out})}}{2\pi R_{\text{out}}} \phi, \quad I_{\text{in}} = \nu_H \frac{e v_F^{(\text{in})}}{2\pi R_{\text{in}}} \phi$$

The mesoscopic edge currents are just small fluctuation to be added on top of the much bigger non-mesoscopic edge currents. Their directions are determined by the Lenz rule for the Faraday induction as follows: the mesoscopic contributions on both edges are equal in magnitude (for symmetry reasons) but opposite in directions (due to the confining potential which acts like in the skipping orbits semi-classical picture by reflecting the electron orbits) so that the mesoscopic net current could be zero. Nevertheless, the corresponding magnetization

$$M = \frac{1}{\text{Area}} \mu, \quad \vec{\mu} = \frac{1}{2} \iiint \vec{r} \times \vec{j}(\vec{r})\, d^3 r = \mu \vec{e}_z, \quad \mu = \pi \int_0^\infty r^2 j(r) d r,$$

where $\vec{j}(\vec{r}) = -j(r)\vec{e}_\varphi$ is the current density, could be non-zero because the current on the outer edge (having bigger radius) has stronger influence on it than the inner edge one. From the Lenz rule we know that the magnetization of the responding net current should counter-act the changes of the flux that has created it. Therefore, the dominating mesoscopic edge current (the one on the outer edge) should be definitely negative for small positive flux, i.e., the outer edge mesoscopic current should always be diamagnetic. According to the discussion in point 7. above, the mesoscopic current on the inner edge should be also diamagnetic.

10. Why B_0 is in the negative z direction: if adding positive flux is supposed to increase the current on the outer edge (it is a matter of convention – one may choose the opposite) this means that the mesoscopic contribution has the same direction like the non-mesoscopic one. However the direction of the mesoscopic part of the current is determined by the Lenz rule which says that the current must be negative (i.e., clockwise). This means that the non-mesoscopic current on the outer edge should also be clockwise and fixes the direction of the background magnetic field to be like in the Figure.