К.Т.Р. и Е.Ч.

Моделът ϕ^4 - от "А" до "Я"

1. Предварителни означения

$$x = (x^\mu)_{\mu=0}^{D-1} = (x^0, \underline{x}), \quad \underline{x} = (x^j)_{j=1}^{D-1}, \quad D-1 = 3$$

$$(x_\mu) = (-x_0, \underline{x}), \quad x_\mu = \eta_{\mu\nu} x^\nu, \quad (\eta_{\mu\nu}) = diag(-,+,\dots,+)$$

$$x^\mu = \eta^{\mu\nu} x_\nu$$ ← подразбира се $\sum_{\nu=0}^{D-1}$

$$\eta_{\mu\nu}\eta^{\nu\mu'} = \delta_\mu^{\mu'} = \begin{cases} 1 & \mu = \mu' \\ 0 & \mu \neq \mu' \end{cases}$$

$$x^2 := x_\mu x^\mu = \eta_{\mu\nu} x^\mu x^\nu = -(x^0)^2 + (x^1)^2 + \dots + (x^{D-1})^2$$

Непрекъснат случай

$$x = (x^0, \underline{x}) \in \mathbb{R}^{D-1,1} = \mathbb{R} \times \mathbb{R}^{D-1}$$

$$\int d\underline{x}\, \delta(\underline{x}-\underline{y})\, f(\underline{x}) = f(\underline{y})$$

$$\int d\underline{x} \equiv \int_{\mathbb{R}^{D-1}} d^{D-1}x$$

$$\int dx \equiv \int_{\mathbb{R}^D} d^D x$$

Дискретен случай:

$$\underline{x} = (x^0, \underline{x}) \in \mathbb{R} \times \text{решетка}$$

$$\sum_{\underline{x}} d\underline{x}\; \delta(\underline{x}-\underline{y})\, f(\underline{y}) = f(\underline{x})$$

$$\sum_{\underline{x}} d\underline{x} = \sum_{\underline{x} \in \text{реш.}} \lambda^{D-1}$$

конст. на реш.

1. Изходна класическа мотивация са полевите уравнения, приведени в Хамилтонова форма (напр. посредством "3+1")

$$\partial_{x^0}\phi(x^0, \underline{x}) = \{H, \phi(x^0,\underline{x})\}$$ ← скобка на Поасон, а не антикомутатор!

$$\partial_{x^0}\pi(x^0, \underline{x}) = \{H, \pi(x^0,\underline{x})\}$$

при предположение: $:= \delta(x^1 - x'^1) \cdots \delta(x^{D-1} - x'^{D-1})$

$$\{\pi(x^0,\underline{x}), \phi(x^0,\underline{x}')\} = \delta(\underline{x}-\underline{x}') \; (= -\{\phi(x^0,\underline{x}), \pi(x^0,\underline{x}')\})$$

$$\{\phi(x^0,\underline{x}), \phi(x^0,\underline{x}')\} = 0 = \{\pi(x^0,\underline{x}), \pi(x^0,\underline{x}')\}$$

- скобки при равни времена x^0.

$$H = H\{\phi(x^0,\underline{x}), \pi(x^0,\underline{x})\}_{\underline{x}}$$ функционал по $\underline{x}$, а x^0 е външен параметър

- обикновено е полиномален алгебричен функционал и използваме после:

$$\{A, BC\} = \{A,B\}C + B\{A,C\}$$
$$\{BC, A\} = B\{C,A\} + \{C,A\}B \quad (-)$$

В частност: класически или когато $\{A,B\}$ е c-число

$$\{A^n, B\} = A^{n-1}\{A,B\} + A^{n-2}\{A,B\}A + A^{n-3}\{A,B\}A^2 + \dots$$
$$= n A^{n-1}\{A,B\}$$

тоест, В "диференцира" A^n до nA^{n-1}

(По-общо: $\{p(A), B\} = p'(A)\{A,B\}$

Изходният пункт на квантовата теория са полевите уравнения приведени в хамилтонова форма

$$\partial_{x^0}^2\phi(x^0,\underline{x}) = \partial_{\underline{x}}^2\phi(x^0,\underline{x}) - m^2\phi(x^0,\underline{x}) - \frac{\varkappa}{3!}\phi(x^0,\underline{x})^3$$

$$H_0 = \frac{1}{2}\int d\underline{x}\left(\pi(x^0,\underline{x})^2 - (\partial_{\underline{x}}\phi(x^0,\underline{x}))^2 + m^2\phi(x^0,\underline{x})^2\right)$$

$$H = H_0 + \frac{\varkappa}{4!}\int d\underline{x}\,\phi(x^0,\underline{x})^4$$

$$\Rightarrow\ \partial_{x^0}\phi(x^0,\underline{x}) = \{H,\phi(x^0,\underline{x})\} = \pi(x^0,\underline{x})$$

$$\partial_{x^0}\pi(x^0,\underline{x}) = \{H,\pi(x^0,\underline{x})\} = \partial_{\underline{x}}^2\phi(x^0,\underline{x}) - m^2\phi(x^0,\underline{x}) - \frac{\varkappa}{3!}\phi(x^0,\underline{x})^3$$

където $\partial_{\underline{x}} = (\partial_{x^i})_{i=1}^{D-1}$, $\partial_{x^\mu} := \frac{\partial}{\partial x^\mu}$

$$(\partial_{\underline{x}}F)^2 := (\partial_{x^1}F)^2 + \dots + (\partial_{x^{D-1}}F)^2$$

$$\partial_{\underline{x}}^2 F := \partial_{x^1}^2 F + \dots + \partial_{x^{D-1}}^2 F$$

Изводи: $\{H,\pi(x^0,\underline{x})\} = \frac{1}{2}\int d\underline{x}'\left\{(\partial_{\underline{x}}\phi(x^0,\underline{x}'))^2,\pi(x^0,\underline{x})\right\} + \cdots$

$$= \cancel{\frac{1}{2}}\int d\underline{x}'\,\cancel{2}(\partial_{\underline{x}}\phi(x^0,\underline{x}'))\cdot\partial_{\underline{x}}\{\phi(x^0,\underline{x}'),\pi(x^0,\underline{x})\} + \cdots$$

$$= \int d\underline{x}'(\partial_{\underline{x}}\phi(x^0,\underline{x}'))\cdot\partial_{\underline{x}}(-\delta(\underline{x}'-\underline{x})) + \cdots$$

$$= \int d\underline{x}'(+)(\partial_{\underline{x}}\cdot\partial_{\underline{x}}\phi(x^0,\underline{x}'))\,\delta(\underline{x}'-\underline{x}) + \cdots = \partial_{\underline{x}}^2\phi(x^0,\underline{x}) + \cdots$$

интегриране по части

$$\int d\underline{x}\,F(\underline{x})\,\partial_{x^i}G(\underline{x}) = \int d\underline{x}\,(-\partial_{x^i}F(\underline{x}))\,G(\underline{x})$$

обобщ. функц.

- за разпределения това идва от определението за производна на разпределение.

$$\partial_{x^0}^2\phi(x^0,\underline{x}) = \bar\partial_{\underline{x}}^-\bar\partial_{\underline{x}}^+\phi(x^0,\underline{x}) - m^2\phi(x^0,\underline{x}) - \frac{\varkappa}{3!}\phi(x^0,\underline{x})^3$$

$$H_0 = \frac{1}{2}\sum_{\underline{x}} d\underline{x}\left(\pi(x^0,\underline{x})^2 - (\bar\partial_{\underline{x}}^+\phi(x^0,\underline{x}))^2 + m^2\phi(x^0,\underline{x})^2\right)$$

$$H = H_0 + \frac{\varkappa}{4!}\sum_{\underline{x}} d\underline{x}\,\phi(x^0,\underline{x})^4$$

$$\Rightarrow\ \partial_{x^0}\phi(x^0,\underline{x}) = \{H,\phi(x^0,\underline{x})\} = \pi(x^0,\underline{x})$$

$$\partial_{x^0}\pi(x^0,\underline{x}) = \{H,\pi(x^0,\underline{x})\} = \bar\partial_{\underline{x}}^-\bar\partial_{\underline{x}}^+\phi(x^0,\underline{x}) - m^2\phi(x^0,\underline{x}) - \frac{\varkappa}{3!}\phi(x^0,\underline{x})^3$$

където $\bar\partial_{\underline{x}}^\pm = (\bar\partial_{x^i}^\pm)_{i=1}^{D-1}$,

$$(\bar\partial_{x^i}^\pm F)(x^i) := \pm\frac{1}{\lambda}\left(F(x^i) - F(x^i \pm \lambda)\right)$$

$$(\bar\partial_{\underline{x}}F)^2 := (\bar\partial_{x^1}F)^2 + \dots + (\bar\partial_{x^{D-1}}F)^2,\quad \bar\partial_{\underline{x}}^-\bar\partial_{\underline{x}}^+F := \bar\partial_{x^1}^-\bar\partial_{x^1}^+F + \dots + \bar\partial_{x^{D-1}}^-\bar\partial_{x^{D-1}}^+F$$

Изводи: $\{H,\pi(x^0,\underline{x})\} = \frac{1}{2}\sum_{\underline{x}} d\underline{x}\left\{(\bar\partial_{\underline{x}}^+\phi(x^0,\underline{x}'))^2,\pi(x^0,\underline{x})\right\} + \cdots$

$$= \cancel{\frac{1}{2}}\sum_{\underline{x}} d\underline{x}\,\cancel{2}(\bar\partial_{\underline{x}}^+\phi(x^0,\underline{x}'))\cdot\bar\partial_{\underline{x}}^+\{\phi(x^0,\underline{x}'),\pi(x^0,\underline{x})\} + \cdots$$

$$= \sum_{\underline{x}} d\underline{x}\,(\bar\partial_{\underline{x}}^+\phi(x^0,\underline{x}'))\cdot\bar\partial_{\underline{x}}^+(-\delta(\underline{x}'-\underline{x})) + \cdots$$

$$= \sum_{\underline{x}} d\underline{x}\,(+)(\bar\partial_{\underline{x}}^-\cdot\bar\partial_{\underline{x}}^+\phi(x^0,\underline{x}'))\,\delta(\underline{x}'-\underline{x}) + \cdots = \partial_{\underline{x}}^2\phi(x^0,\underline{x}) + \cdots$$

"сумиране по части"

$$\sum_{\underline{x}} d\underline{x}\,F(\underline{x})\,\bar\partial_{x^i}^+G(\underline{x}) = \sum_{\underline{x}} d\underline{x}\,(-\bar\partial_{x^i}^-F(\underline{x}))\,G(\underline{x})$$

- идва от сумационното тъждество

$$\sum_{i\text{-цикл.}} f(i)\left(g(i) - g(i+1)\right) = \sum_{i\text{-цикл.}}\left(f(i) - f(i-1)\right)g(i)$$

V. Kac, Quantum Calculus

$$\partial^2_{x^0}\hat\phi(x^0,\underline{x}) = \partial^2_{\underline{x}}\hat\phi(x^0,\underline{x}) - m^2\hat\phi(x^0,\underline{x}) - \frac{\varkappa}{3!}:\hat\phi(x^0,\underline{x})^3:$$

$$\hat H_0 = \frac12\int d\underline{x}\left(:\hat\pi(x^0,\underline{x})^2: - :(\partial_{\underline{x}}\hat\phi(x^0,\underline{x}))^2: + m^2:\hat\phi(x^0,\underline{x})^2:\right)$$

$$\hat H = \hat H_0 + \frac{\varkappa}{4!}\int d\underline{x}:\hat\phi(x^0,\underline{x})^4:$$

$$\Rightarrow\quad \partial_{x^0}\hat\phi(x^0,\underline{x}) = i[\hat H,\hat\phi(x^0,\underline{x})] = \hat\pi(x^0,\underline{x})$$

$$\partial_{x^0}\hat\pi(x^0,\underline{x}) = i[\hat H,\hat\pi(x^0,\underline{x})] = \partial^2_{\underline{x}}\hat\phi(x^0,\underline{x}) - m^2\hat\phi(x^0,\underline{x}) - \frac{\varkappa}{3!}:\hat\phi(x^0,\underline{x})^3:$$

$$[\hat\pi(x^0,\underline{x}),\hat\phi(x^0,\underline{x}')] = -i\delta(\underline{x}-\underline{x}')\ \left(= -[\hat\phi(x^0,\underline{x}),\hat\pi(x^0,\underline{x}')]\right)$$

$$[\hat\phi(x^0,\underline{x}),\hat\phi(x^0,\underline{x}')] = 0 = [\hat\pi(x^0,\underline{x}),\hat\pi(x^0,\underline{x}')]$$

$\hat S = T\text{-}\exp\left(\frac1i\int dx\ \hat I\{\hat\phi_0(x)\}\right)$ – оператор на разсейване

където $\hat I := \hat H - \hat H_0$ и $\hat\phi_0(x)$ е решението на своб. полево ур.

$$\int dx = \int_{\mathbb{R}^D} dx^0 dx^1\cdots dx^{D-1} \equiv \int dx^0\int d\underline{x}$$

$$G_0(x-x') := \langle\Omega|T(\hat\phi_0(x)\hat\phi_0(x'))\Omega\rangle$$

$$:= \langle\Omega|\hat\phi_0(x)\hat\phi_0(x')\Omega\rangle\,\theta(x^0-x'^0) + \langle\Omega|\hat\phi_0(x')\hat\phi_0(x)\Omega\rangle\,\theta(x'^0-x^0)$$

$$(\partial^2_{x^0} - \partial^2_{\underline{x}} + m^2)\,G_0(x) = i\delta(x^0)\,\delta(\underline{x})\ \left(= i\delta(x)\right)$$

По-нататък – без шапки!

$$\partial^2_{x^0}\hat\phi(x^0,\underline{x}) = \partial^-_{\underline{x}}\partial^+_{\underline{x}}\hat\phi(x^0,\underline{x}) - m^2\hat\phi(x^0,\underline{x}) - \frac{\varkappa}{3!}:\hat\phi(x^0,\underline{x})^3:$$

$$\hat H_0 = \frac12\sum_{\underline{x}} d\underline{x}\left(:\hat\pi(x^0,\underline{x})^2: - :(\partial^+_{\underline{x}}\hat\phi(x^0,\underline{x}))^2: + m^2:\hat\phi(x^0,\underline{x})^2:\right)$$

$$\hat H = \hat H_0 + \frac{\varkappa}{4!}\sum_{\underline{x}} d\underline{x}:\hat\phi(x^0,\underline{x})^4:$$

$$\Rightarrow\quad \partial_{x^0}\hat\phi(x^0,\underline{x}) = i[\hat H,\hat\phi(x^0,\underline{x})] = \hat\pi(x^0,\underline{x})$$

$$\partial_{x^0}\hat\pi(x^0,\underline{x}) = i[\hat H,\hat\pi(x^0,\underline{x})] = \partial^-_{\underline{x}}\partial^+_{\underline{x}}\hat\phi(x^0,\underline{x}) - m^2\hat\phi(x^0,\underline{x}) - \frac{\varkappa}{3!}:\hat\phi(x^0,\underline{x})^3:$$

$$[\hat\pi(x^0,\underline{x}),\hat\phi(x^0,\underline{x}')] = -i\delta(\underline{x}-\underline{x}')\ \left(= -[\hat\phi(x^0,\underline{x}),\hat\pi(x^0,\underline{x}')]\right)$$

$$[\hat\phi(x^0,\underline{x}),\hat\phi(x^0,\underline{x}')] = 0 = [\hat\pi(x^0,\underline{x}),\hat\pi(x^0,\underline{x}')]$$

$\hat S = T\text{-}\exp\left(\frac1i\,⨏ dx\ \hat I\{\hat\phi_0(x)\}\right)$ – оператор на разсейване

където $\hat I := \hat H - \hat H_0$ и $\hat\phi_0(x)$ е решението на своб. полево ур.

$$⨏ dx := \int dx^0\sum_{\underline{x}} d\underline{x}$$

$$G_0(x-x') := \langle\Omega|T(\hat\phi_0(x)\hat\phi_0(x'))\Omega\rangle$$

$$:= \langle\Omega|\hat\phi_0(x)\hat\phi_0(x')\Omega\rangle\,\theta(x^0-x'^0) + \langle\Omega|\hat\phi_0(x')\hat\phi_0(x)\Omega\rangle\,\theta(x'^0-x^0)$$

$$(\partial^2_{x^0} - \partial^-_{\underline{x}}\partial^+_{\underline{x}} + m^2)\,G_0(x) = i\delta(x^0)\,\delta(\underline{x})$$

Проверка на $\left(\partial^2_{x^0} - \partial^2_{\underline{x}} + m^2\right) G_0(x) = i\,\delta(x^0)\,\delta(\underline{x})$

$$\left(\partial^2_{x^0} - \partial^2_{\underline{x}} + m^2\right) G_0(x-x')$$

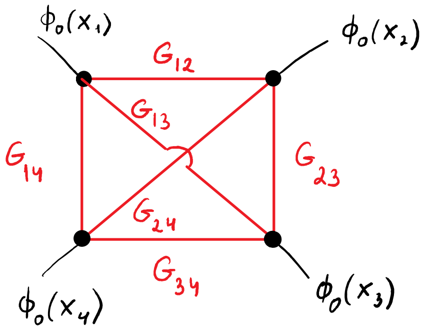
$$\begin{aligned}
&= \langle\Omega|\cancel{\partial^2_{x^0}\phi_0(x)}\,\phi_0(x')\,\Omega\rangle\,\theta(x^0-x'^0) + 2\langle\Omega|\partial_{x^0}\phi_0(x)\,\phi_0(x')\,\Omega\rangle\,\underbrace{\partial_{x^0}\theta(x^0-x'^0)}_{\delta(x^0-x'^0)} + \langle\Omega|\phi_0(x)\,\phi_0(x')\,\Omega\rangle\,\underbrace{\partial^2_{x^0}\theta(x^0-x'^0)}_{\partial_{x^0}\delta(x^0-x'^0)}\\
&- \langle\Omega|\cancel{\partial^2_{\underline{x}}\phi_0(x)}\,\phi_0(x')\,\Omega\rangle\,\theta(x^0-x'^0)\\
&+ \langle\Omega|\cancel{m^2\phi_0(x)}\,\phi_0(x')\,\Omega\rangle\,\theta(x^0-x'^0)\\
&+ \langle\Omega|\phi_0(x')\,\cancel{\partial^2_{x^0}\phi_0(x)}\,\Omega\rangle\,\theta(x'^0-x^0) + 2\langle\Omega|\phi_0(x')\,\partial_{x^0}\phi_0(x)\,\Omega\rangle\,\underbrace{\partial_{x^0}\theta(x'^0-x^0)}_{-\delta(x^0-x'^0)} + \langle\Omega|\phi_0(x')\,\phi_0(x)\,\Omega\rangle\,\underbrace{\partial^2_{x^0}\theta(x'^0-x^0)}_{-\partial_{x^0}\delta(x^0-x'^0)}\\
&- \langle\Omega|\phi_0(x')\,\cancel{\partial^2_{\underline{x}}\phi_0(x)}\,\Omega\rangle\,\theta(x'^0-x^0)\\
&+ \langle\Omega|\phi_0(x')\,\cancel{m^2\phi_0(x)}\,\Omega\rangle\,\theta(x'^0-x^0)\\
&= 2\langle\Omega|\underbrace{[\underbrace{\partial_{x^0}\phi_0(x^0,\underline{x})}_{\pi_0(x^0,\underline{x})},\ \phi_0(x^0,\underline{x}')]}_{i\,\delta(\underline{x}-\underline{x}')}\,\Omega\rangle\,\delta(x^0-x'^0) + \underbrace{\langle\Omega|[\phi_0(x),\phi_0(x')]\,\Omega\rangle\,\partial_{x^0}\delta(x^0-x'^0)}_{\begin{aligned}&\partial_{x^0}\left(\langle\Omega|[\phi_0(x),\phi_0(x')]\,\Omega\rangle\,\delta(x^0-x'^0)\right)\\ &- \langle\Omega|[\partial_{x^0}\phi_0(x),\phi_0(x')]\,\Omega\rangle\,\delta(x^0-x'^0)\\ &= 0 - \langle\Omega|[\partial_{x^0}\phi_0(x^0,\underline{x})\ \phi_0(x^0,\underline{x}')]\,\Omega\rangle\,\delta(x^0-x'^0)\end{aligned}}\\
&= i\,\delta(\underline{x}-\underline{x}')\,\delta(x^0-x'^0)
\end{aligned}$$

По-нататък: от хронологичната теорема на Вик

$$S = T\text{-}\exp\left(\frac{1}{i}\frac{\varkappa}{4!}\int dx \,:\phi_0(x)^4:\right) = \sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{1}{i}\frac{\varkappa}{4!}\right)^n \int dx_1\cdots dx_n \; T\left(:\phi_0(x_1)^4::\phi_0(x_2)^4:\cdots:\phi_0(x_n)^4:\right)$$

$$= \sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{1}{i}\frac{\varkappa}{4!}\right)^n \int dx_1\cdots dx_n \sum_{\substack{\forall\ \text{хронологични}\\ \text{сдвоявания}\\ \text{м/у различни групи}}} :\phi_0(x_1)^4::\phi_0(x_2)^4:\cdots:\phi_0(x_k)^4:\cdots:\phi_0(x_n)^4:$$

$$= \sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{1}{i}\frac{\varkappa}{4!}\right)^n \int dx_1\cdots dx_n \sum_{\substack{\forall\ \text{хронологични}\\ \text{сдвоявания}\\ \text{м/у различни групи}\\ \text{до диагр. }\Gamma}} G_\Gamma(x_1,\dots,x_n)\; :M_\Gamma(\phi_0(x_1),\dots,\phi_0(x_n)):$$



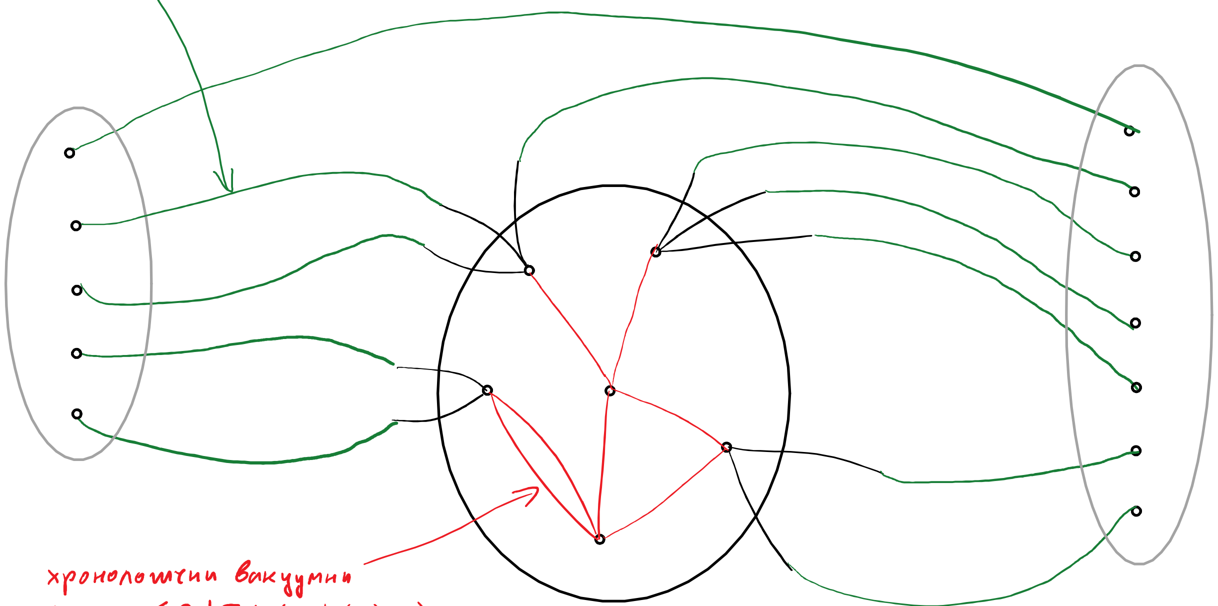
$$G_{jk} := G_0(x_j - x_k)$$

$$\overbrace{\phi_0(x)\,\phi_0}(x') := \langle\Omega|T\phi_0(x)\,\phi_0(x')\Omega\rangle \equiv G_0(x-x')$$

това са вече обикновени вакуумни средни $\langle\Omega|\phi_0(x)\,\phi_0(x')\Omega\rangle$

$$\langle :\phi_0(y_1)\cdots\phi_0(y_k):\Omega \,|\, S :\phi_0(z_1)\cdots\phi_0(z_\ell):\Omega\rangle$$

$$= \langle\Omega| :\phi_0(y_1)\cdots\phi_0(y_n): S :\phi_0(z_1)\cdots\phi_0(z_n):\Omega\rangle$$



хронологични вакуумни средни $\langle\Omega|T\,\phi_0(x)\,\phi_0(x')\Omega\rangle$

2. Импулсно представяне / momentum space picture

а) Транслационна ковариантност

$$\hat U_0(1,a)\,\hat\phi_0(x)\,\hat U_0(1,a)^{-1} = \hat\phi_0(x+a)\,, \quad \hat U_0(1,a) = e^{ia^\mu \hat P^{(0)}_\mu}\,, \quad \hat P^{(0)}_0 \equiv \hat H_0$$

т.е., $$e^{ia^\mu P^{(0)}_\mu}\,\phi_0(x)\,e^{-ia^\mu P^{(0)}_\mu} = \phi_0(x+a)$$

$$e^{itP^{(0)}_\mu}\,\phi_0(x)\,e^{-itP^{(0)}_\mu} = \phi_0(x+te_\mu) \qquad \Big\lfloor\, \partial/\partial t\,\Big|_{t=0}$$

$$\Rightarrow \quad iP^{(0)}_\mu\phi_0(x) - i\phi_0(x)P^{(0)}_\mu = \partial_{x^\mu}\phi_0(x)$$

т.е., $$[P^{(0)}_\mu, \phi_0(x)] = -i\partial_{x^\mu}\phi_0(x)$$

б) Фурие-трансформация

$$\phi_0(p) := (2\pi)^{-\frac{D}{2}}\int dx\, e^{-ipx}\phi_0(x)\,, \quad \phi_0(x) = (2\pi)^{-\frac{D}{2}}\int dp\, e^{ixp}\phi_0(p)$$

където $xp \equiv px := x^\mu p_\mu$

$$\boxed{\begin{aligned} &\int dx\, e^{\pm ipx} = (2\pi)^D\delta(p)\,, \quad \delta(-x) = \delta(x) \\ &\int dp\, e^{\pm ixp} = (2\pi)^D\delta(x)\,, \quad D = \dim x = \dim p \end{aligned}}$$

- пример на прилагане:

$$\phi_0(x) = (2\pi)^{-\frac{D}{2}}\int dp\, e^{ixp}\phi_0(p) = (2\pi)^{-\frac{D}{2}}\int dp\,(2\pi)^{-\frac{D}{2}}\int dx'\, e^{-ipx'}\phi_0(x')$$

$$= (2\pi)^{-D}\int dp\int dx\; e^{ip(x-x')}\phi_0(x')$$

$$= (2\pi)^{-D}\int dx\,\underbrace{\left(dp\; e^{ip(x-x')}\right)}_{(2\pi)^D\delta(x-x')}\phi_0(x') = \int dx'\,\delta(x-x')\,\phi_0(x')$$

в) Транслационна ковариантност и импулсно представяне

$$\boxed{\begin{aligned} &[P^{(0)}_\mu, \phi_0(p)] = p_\mu\phi_0(p) \\ &P^{(0)}_\mu\big(\phi_0(p_1)\cdots\phi_0(p_n)\Omega\big) \\ &= (p_{1,\mu}+\cdots+p_{n,\mu})\,\phi_0(p_1)\cdots\phi_0(p_n)\Omega \end{aligned}}$$

Извод:

- $$[P^{(0)}_\mu, \phi_0(p)] = [P^{(0)}_\mu, (2\pi)^{-\frac{D}{2}}\int dx\, e^{-ipx}\phi_0(x)]$$
$$= (2\pi)^{-\frac{D}{2}}\int dx\; e^{-ipx}\,[P^{(0)}_\mu, \phi_0(x)]$$
$$= (2\pi)^{-\frac{D}{2}}\int dx\; e^{-ipx}\,(-i\partial_{x^\mu})\phi_0(x)$$
$$= (2\pi)^{-\frac{D}{2}}\int dx\,\left(+i\partial_{x^\mu}e^{-ip\cdot x}\right)\phi_0(x)$$
$$= p_\mu(2\pi)^{-\frac{D}{2}}\int dx\, e^{-ip\cdot x}\phi_0(x) = p_\mu\phi_0(p)$$

- $$P^{(0)}_\mu A\Omega = P^{(0)}_\mu A\Omega - AP^{(0)}_\mu\Omega = [P^{(0)}_\mu, A]\Omega$$

$$\Rightarrow \quad P^{(0)}_\mu\big(\phi_0(p_1)\cdots\phi_0(p_n)\Omega\big) = [P^{(0)}_\mu, \phi_0(p_1)\cdots\phi_0(p_n)]\Omega$$
$$= [P^{(0)}_\mu, \phi_0(p_1)]\phi_0(p_2)\cdots\phi_0(p_n)\Omega$$
$$+ \phi_0(p_1)[P^{(0)}_\mu, \phi_0(p_2)]\cdots\phi_0(p_n)\Omega + \cdots$$
$$+ \phi_0(p_1)\phi_0(p_2)\cdots[P^{(0)}_\mu, \phi_0(p_n)]\Omega$$
$$= p_{1,\mu}\phi_0(p_1)\phi_0(p_2)\cdots\phi_0(p_n)\Omega + p_{2,\mu}\phi_0(p_1)\phi_0(p_2)\cdots\phi_0(p_n)\Omega + \cdots$$

Извод:

$:\phi_0(p_1)\cdots\phi_0(p_n):\Omega$ е асимптотично състояние на n свободни частици с импулси $p_1,\dots,p_n$
$\Rightarrow$

$$\langle :\phi_0(p_1)\cdots\phi_0(p_n):\Omega \,|\, S :\phi_0(p'_1)\cdots\phi_0(p'_m):\Omega\rangle$$
$$= \langle\Omega| :\phi_0(p_1)\cdots\phi_0(p_n): S :\phi_0(p'_1)\cdots\phi_0(p'_m):\Omega\rangle$$

е амплитуда на разсейване на n частици с входни импулси: $p_1,\dots,p_n$ и изходни $p'_1,\dots,p'_m$.

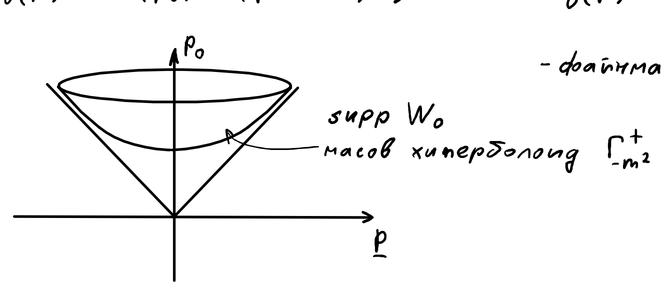
$$W_0(p) := (2\pi)^{-\frac{D}{2}}\int dx\, e^{-ipx}\, W_0(x)\,,\quad W_0(x-x') := \langle\Omega|\,\phi_0(x)\,\phi_0(x')\,\Omega\rangle$$

$$G_0(p) := (2\pi)^{-\frac{D}{2}}\int dx\, e^{-ipx}\, G_0(p)\,,\quad G_0(x-x') := \langle\Omega|\,T\phi_0(x)\,\phi_0(x')\,\Omega\rangle$$

$$G_0(x) = W(x)\,\theta(x^0) + W(-x)\,\theta(-x^0)$$

$$W_0(p) = \theta(p_0)\,\delta(p^2+m^2)\,,\qquad G_0(p) = \frac{1}{p^2+m^2-i0}$$

- файнманов пропагатор

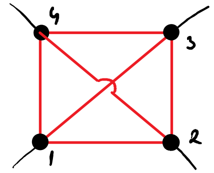


следва от теоремата на Вигнер:

$\mathrm{Span}\{\phi_0(p)\Omega \,|\, p\in\Gamma^+_{-m^2}\}$ - неприводимо за гр. на П.

Диаграми и амплитуди на Файнман в импулсно предст.

Нека $\Gamma =$



и
$$S_\Gamma = \int dx_1dx_2dx_3dx_4\, G_\Gamma(x_1,x_2,x_3,x_4) \cdot :M_\Gamma(\phi_0(x_1)\,\phi_0(x_2)\,\phi_0(x_3)\,\phi_0(x_4)):$$

$$= \int dx_1dx_2dx_3dx_4\, G_\Gamma(x_1,x_2,x_3,x_4)\; :\phi_0(x_1)\,\phi_0(x_2)\,\phi_0(x_3)\,\phi_0(x_4):$$

$$G_\Gamma = G_0(x_1-x_2)\,G_0(x_2-x_3)\,G_0(x_3-x_4)\,G_0(x_4-x_1) \cdot G_0(x_1-x_3)\,G_0(x_2-x_4)$$

Един принос в $\langle\Omega| :\phi_0(y_1)\,\phi_0(y_2): S_\Gamma :\phi_0(y_3)\,\phi_0(y_4):\Omega\rangle$

е $A(y_1,y_2,y_3,y_4)$

$$:= \int dx_1dx_2dx_3dx_4\, W_0(y_1-x_1)\,W_0(y_2-x_2)\,W_0(x_3-y_3)\,W_0(x_4-y_4)$$
$$\cdot\, G_0(x_1-x_2)\,G_0(x_2-x_3)\,G_0(x_3-x_4)\,G_0(x_4-x_1)$$
$$\cdot\, G_0(x_1-x_3)\,G_0(x_2-x_4)$$

$$A(q_1,q_2,q_3,q_4) := (2\pi)^{-4\frac{D}{2}}\int dy_1dy_2dy_3dy_4\, e^{-iq_1y_1-iq_2y_2-iq_3y_3-iq_4y_4} \cdot A(y_1,y_2,y_3,y_4)$$

$$A(q_1,q_2,q_3,q_4) := (2\pi)^{-4\frac{D}{2}} \int dy_1\, dy_2\, dy_3\, dy_4 \; e^{-iq_1y_1 - iq_2y_2 - iq_3y_3 - iq_4y_4} \int dx_1\, dx_2\, dx_3\, dx_4 \; W_0(y_1-x_1)\, W_0(y_2-x_2)\, W_0(x_3-y_3)\, W_0(x_4-y_4)$$

$$\bullet\; G_0(x_1-x_2)\, G_0(x_2-x_3)\, G_0(x_3-x_4)\, G_0(x_4-x_1)\, G_0(x_1-x_3)\, G_0(x_2-x_4)$$

$$= (2\pi)^{-4\frac{D}{2}} \int dy_1\, dy_2\, dy_3\, dy_4 \int dx_1\, dx_2\, dx_3\, dx_4 \; e^{-iq_1y_1 - iq_2y_2 - iq_3y_3 - iq_4y_4} \, (2\pi)^{-\frac{D}{2}} \int dq'_1\, e^{i(y_1-x_1)q'_1} W_0(q'_1)\, (2\pi)^{-\frac{D}{2}} \int dq'_2\, e^{i(y_2-x_2)q_2} W_0(q'_2)$$

$$\bullet\; (2\pi)^{-\frac{D}{2}} \int dq'_3\, e^{i(x_3-y_3)q'_3} W_0(q'_3)\, (2\pi)^{-\frac{D}{2}} \int dq'_4\, e^{i(x_4-y_4)q'_4} W_0(q'_4)\, (2\pi)^{-\frac{D}{2}} \int dp_1\, e^{i(x_1-x_2)p_1} G_0(p_1)\, (2\pi)^{-\frac{D}{2}} \int dp_2\, e^{i(x_2-x_3)p_2} G_0(p_2)$$

$$\bullet\; (2\pi)^{-\frac{D}{2}} \int dp_3\, e^{i(x_3-x_4)p_3} G_0(p_3)\, (2\pi)^{-\frac{D}{2}} \int dp_4\, e^{i(x_4-x_1)p_4} G_0(p_4)\, (2\pi)^{-\frac{D}{2}} \int dp_5\, e^{i(x_1-x_3)p_5} G_0(p_5)\, (2\pi)^{-\frac{D}{2}} \int dp_6\, e^{i(x_2-x_4)p_6} G_0(p_6)$$

$$= (2\pi)^{-\frac{D}{2}(4+10)} \int dp_1\, dp_2\, dp_3\, dp_4\, dp_5\, dp_6\; dq'_1\, dq'_2\, dq'_3\, dq'_4\; W_0(q'_1)\, W_0(q'_2)\, W_0(q'_3)\, W_0(q'_4)$$

$$\bullet\; G_0(p_1)\, G_0(p_2)\, G_0(p_3)\, G_0(p_4)\, G_0(p_5)\, G_0(p_6)$$

$$\bullet\; \int dy_1\, e^{iy_1(-q_1+q'_1)} \int dy_2\, e^{iy_2(-q_2+q'_2)} \int dy_3\, e^{iy_3(-q_3-q'_3)} \int dy_4\, e^{iy_4(-q_4-q'_4)}$$

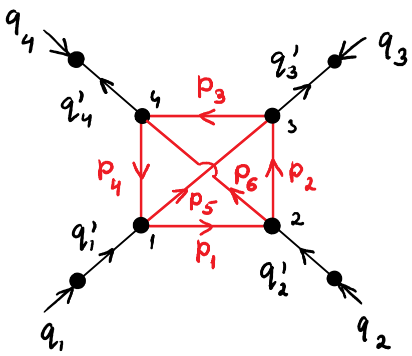
$$\bullet\; \int dx_1\, e^{ix_1(-q'_1+p_1-p_4+p_5)} \int dx_2\, e^{ix_2(-q'_2-p_1+p_2+p_6)} \int dx_3\, e^{ix_3(q'_3-p_2+p_3-p_5)} \int dx_4\, e^{ix_4(q'_4-p_3+p_4-p_6)}$$

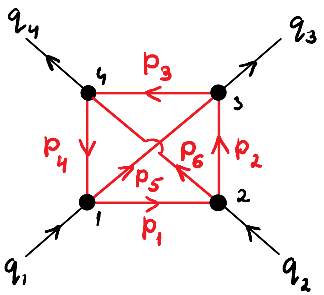
$$= (2\pi)^{-\frac{D}{2}(4+10)+D8} \int dp_1\, dp_2\, dp_3\, dp_4\, dp_5\, dp_6\; dq'_1\, dq'_2\, dq'_3\, dq'_4\; W_0(q'_1)\, W_0(q'_2)\, W_0(q'_3)\, W_0(q'_4)$$

$$\bullet\; G_0(p_1)\, G_0(p_2)\, G_0(p_3)\, G_0(p_4)\, G_0(p_5)\, G_0(p_6)$$

$$\bullet\; \delta(-q_1+q'_1)\, \delta(-q_2+q'_2)\, \delta(-q_3-q'_3)\, \delta(-q_4-q'_4)$$

$$\bullet\; \delta(-q'_1-p_1+p_2+p_6)\, \delta(-q'_2-p_1+p_2+p_6)\, \delta(q'_3-p_2+p_3-p_6)\, \delta(q'_4-p_3+p_4-p_6)$$





$$A(q_1,q_2,q_3,q_4) = 2\pi\, W_0(q_1)\, W_0(q_2)\, W_0(-q_3)\, W_0(-q_4)$$
$$\cdot \int dp_1\, dp_2\, dp_3\, dp_4\, dp_5\, dp_6\;\; G_0(p_1)\, G_0(p_2)\, G_0(p_3)\, G_0(p_5)\, G_0(p_6)$$
$$\cdot\; \delta(-q_1+p_1+p_4+p_5)\; \delta(-q_2-p_1+p_2+p_6)\; \delta(q_3-p_2+p_3-p_5)\; \delta(q_4-p_3+p_4-p_6)$$

$$\left|\begin{aligned} -q_1+p_1+p_4+p_5 &= 0\\ -q_2-p_1+p_2+p_6 &= 0\\ q_3-p_2+p_3-p_5 &= 0\\ q_4-p_3+p_4-p_6 &= 0 \end{aligned}\right.$$

6 интеграц. променливи, 4-уравнения
Оказва се, че: 3 могат да се изразят

Теорема. Броят на независимите интеграционни променливи е = на броя на независимите примки (loops) на диаграмата

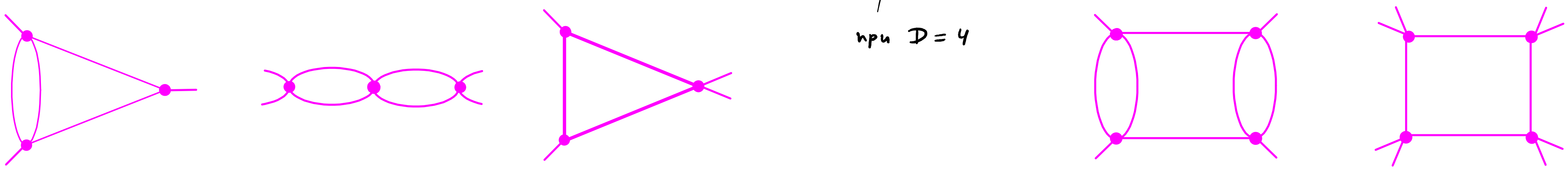
В теорията на пертурбациите се доказва, че # примки = реда по $\hbar$ при разлагане
$\Rightarrow$ Квазикласическо прибл.
= 1-примково прибл.

$$\Rightarrow \quad \begin{cases} p_6 = p_4 - p_3 - q_4\\ p_5 = p_2 - p_3 - q_3\\ p_4 = q_1 - p_1 - p_5 = q_1 + q_3 - p_1 - p_2 + p_3 \end{cases}$$

$$A(q_1,q_2,q_3,q_4) = 2\pi\, W_0(q_1)\, W_0(q_2)\, W_0(-q_3)\, W_0(-q_4)\;\delta(-q_1-q_2+q_3+q_4)$$
$$\cdot \int dp_1\, dp_2\, dp_3\;\; G_0(p_1)\, G_0(p_2)\, G_0(p_3)\, G_0(q_1+q_3-p_1-p_2+p_3)\, G_0(p_2-p_3-q_3)\, G_0(p_4-p_3-q_4)$$

Ръст при $|p| \to \infty$ $\quad \int^{\infty} |p|^{3D-1}\, d|p|\; |p|^{-2\cdot 6} \sim \int^{\infty} |p|^{3D-13}\, d|p| = \int^{\infty} |p|^{-1}\, d|p| = \log\infty$ – логаритмична разходимост

при $D = 4$



Общо-моделни ("над-моделни") въпроси от КТП

1. Структура на спектъра

Ако $\Psi_1(x),\dots,\Psi_N(x)$ са квантовите полета, които пораждат един КТП-модел, т.е.

$$\mathcal{H} = \overline{\mathrm{Span}}\,\{\Psi_{k_1}[f_1]\cdots\Psi_{k_n}[f_n]\,\Omega\}$$

тогава $P_\mu\left(\Psi_{k_1}(p_1)\cdots\Psi_{k_n}(p_n)\,\Omega\right) = p_\mu\,\Psi_{k_1}(p_1)\cdots\Psi_{k_n}(p_n)\,\Omega$

$p = p_1+\dots+p_n \in \overline{V}^+$ - собств. векторна стойност за $(P_\mu)_\mu$

съдържа инфо за спектъра частици (свързани състояния) bound states

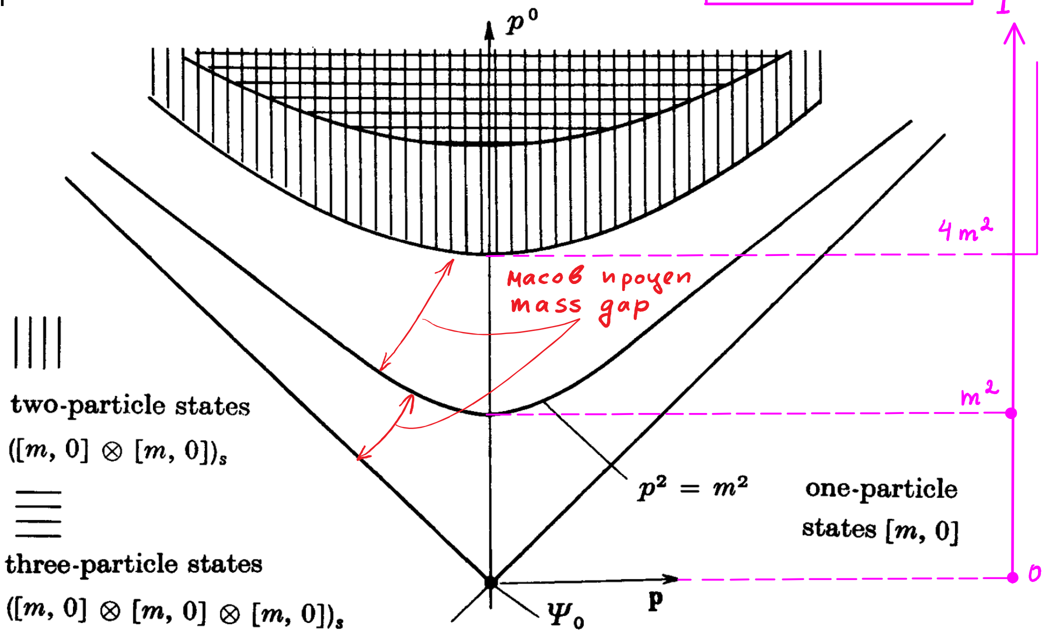


FIGURE I–3. The spectrum of a theory of neutral scalar mesons of mass m without bound states.

http://theo.inrne.bas.bg/~mitov/QFTandEP2020/Streater_R.F.,_Wightman_A.S.,_PCT,_Spin_And_Statistics,_And_All_That_[2000].pdf, стр. 26

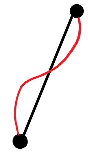
Защото, ако имаме "анти-неравенство на триъгълника"

$$\left|(p_1+\dots+p_n)^2\right|^{1/2} \geq \left|p_1^2\right|^{1/2}+\dots+\left|p_n^2\right|^{1/2}$$

- ако $p_1,\dots,p_n \in V^+$.

факт от псевдо-евкл. геометрия

(Следствие. Времеподобните геодезични са с най-голяма дължина



⇒ "парадокса на близнаците".)

$\Rightarrow$ ако $p_1,\dots,p_n \in \Gamma^+_{-m^2}$ (т.е. $p_j^2 = -m^2$)

то $\left|(p_1+\dots+p_n)^2\right|^{1/2} \geq n m$

Всъщност $\left|(p_1+\dots+p_n)^2\right|^{1/2} - n m =:$ вътрешна кинетична енергия

При наличие на mass-gap е доказано, че винаги $\exists$ асимптотични свободни състояния и оператор на разсейв. - теория на Хааг-Рюел

https://www.theorie.physik.uni-goettingen.de/forschung/qft/research/theses/bach/Spratte.pdf

В присъствие на фотони обаче - масовия процеп изчезва. Изчезват и собствените стойности на P^2 - "инфра-частици"

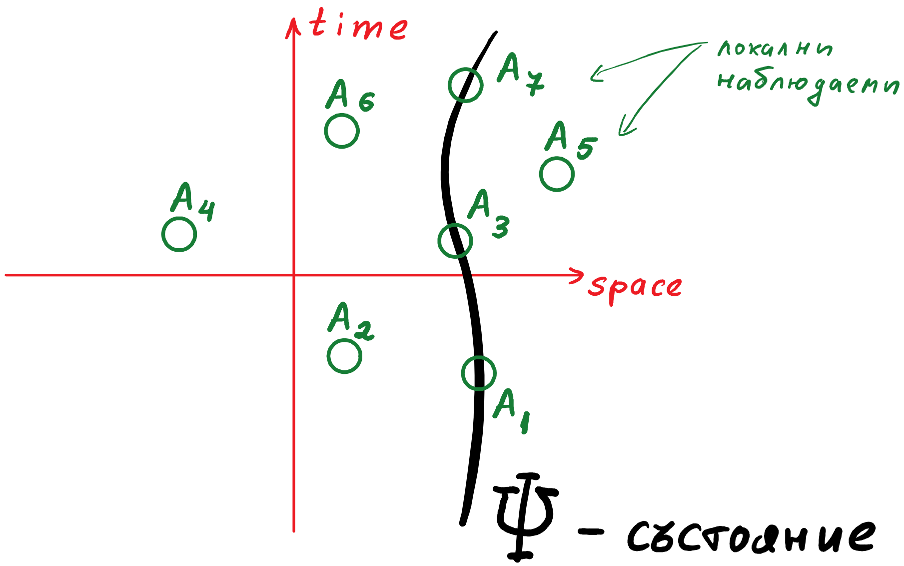
https://en.wikipedia.org/wiki/Infraparticle

2. Локалност и сплитане (entanglement) в релативистка КТП

Запомнете: локални са наблюденията (т.е., наблюдаемите, полетата), а състоянията са винаги глобални

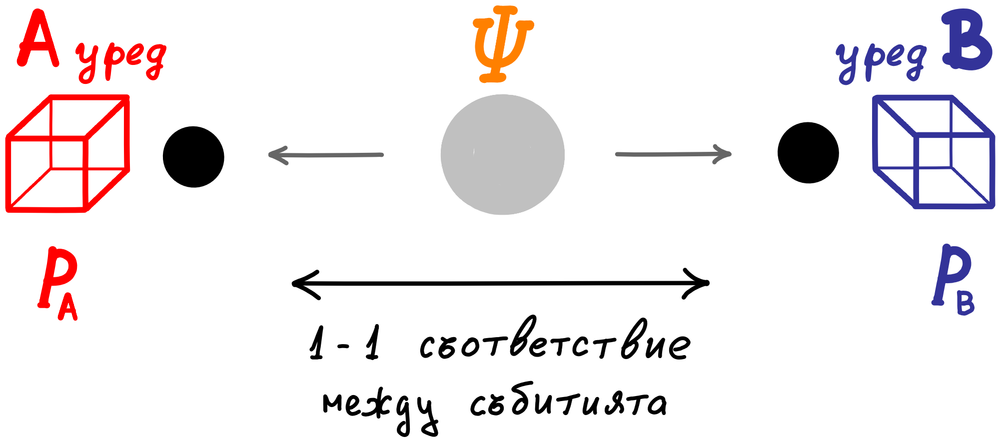
В КТП се приема картината на Хайзенберг, което значи, че състоянието е като мировата линия

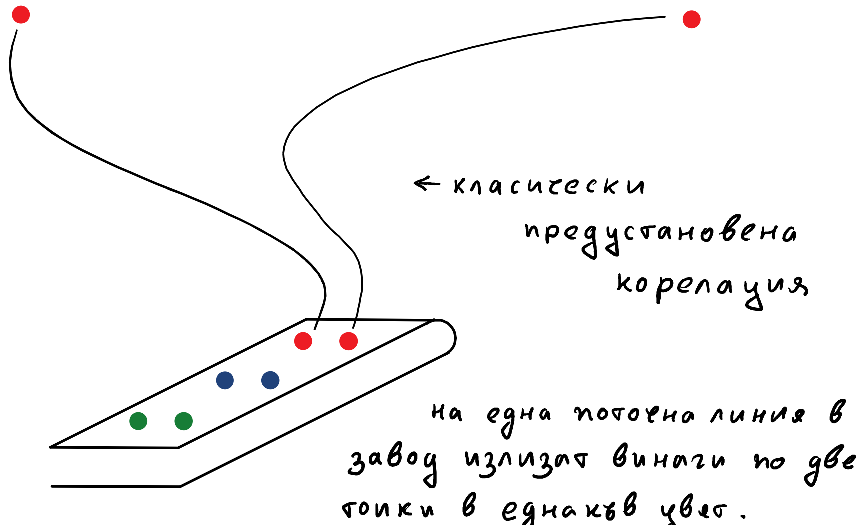
- глобална характеристика



Парадоксът АПР в КТП:

АПР/Бел постановка





Като излезе една двойка топки те се пакетират и се изпращат, като подаръци в различни галактики с известие, че са с еднакъв цвят. ⇒ в момента в който отворят единия подарък и видят цвета, те "моментално" разбират и другия цвят.

Теорема на Рее-Шлидер / Reeh-Schlieder

Нека $\mathcal{O} \subseteq \mathbb{R}^{D-1,1}$ - ограничена област и нека:

$$\mathcal{H}(\mathcal{O}) := \overline{\mathrm{Span}_{\mathbb{C}}} \{ \psi_{k_1}[f_1] \cdots \psi_{k_n}[f_n] \Omega \mid \mathrm{supp}\, f_j \subseteq \mathcal{O} \}$$

Тогава $\mathcal{H}(\mathcal{O}) = \mathcal{H}$.

Тоест, всяко състояние може да се възпроизведе с произволна отнапред зададена точност, посредством действието на полета локализирани в $\mathcal{O}$ върху вакуума Ω.

<u>Следствие.</u> Нека A е локална наблюдаема

$$A\Omega = 0 \implies A = 0$$

В частност, единствените локални събития, на които вакуумът Ω, може да бъде собствен вектор са тривиалните събития

$$0 \text{ и } \hat{1}.$$

3. В релативистката КТП няма оператори на координатите.

Ние не говорим за измерване на координатите (x, y, z) на частица, а говорим за измерване на ефект (т.е., наблюдаема) в точката с координати (x, y, z) (и разбира се, в някакъв момет от време t). See Newton-Wigner coordinate

4. Релативистките уравнения, като

$$\left(-\partial_{x^0}^2 + \partial_{\underline{x}}^2 - m^2\right)\phi(x) = 0 \quad \text{- на Клайн-Гордън}$$

$$\left(i\gamma^\mu \partial_{x^\mu} + m\right)\Psi(x) = 0 \quad \text{- на Дирак}$$

са уравнения за полета, а не за частици!!!

Всъщност, $\{\phi(x)\Omega\}$ и $\{\Psi(x)\Omega\}$ пораждат унитарни представяния на групата на Поанкаре, които съответстват на частица със спин 0 и 1/2 съответно.