КТП и ЕЧ, 18.12.2020

Изходен, свободен (линеен) квантов модел

$$\hat H_0 = \sum_{x\in\mathbb{R}} \lambda \,\frac{1}{2}\left(\frac{1}{c^2}\left(\partial_t \hat\varphi_0\right)(x,t)^2 + \left(\partial_x^+ \hat\varphi_0\right)(x,t)^2 + \frac{M^2c^2}{\hbar^2}\,\hat\varphi_0(x,t)^2\right)$$

$$\left[\partial_t \hat\varphi_0(x,t),\, \hat\varphi_0(x',t)\right] = -i\hbar c^2 \delta_\lambda(x-x') \equiv -i\hbar c^2 \frac{1}{\lambda}\delta_{x,x'}$$

$$\left[\partial_t \hat\varphi_0(x,t),\, \partial_t\hat\varphi_0(x',t)\right] = 0 = \left[\hat\varphi_0(x,t),\, \hat\varphi_0(x',t)\right]$$

$$\hat{\tilde\varphi}_0(p,t) := \frac{1}{\sqrt{h}}\sum_{x\in\mathbb{R}} \lambda\, e^{ixp/\hbar}\,\hat\varphi_0(x,t) \quad \left(= \hat{\tilde\varphi}(-p,t)^*\right)$$

$$\hat\varphi_0(x,t) = \frac{1}{\sqrt{h}}\sum_{p\in\tilde{\mathbb{R}}} \tilde\lambda\, e^{-ixp/\hbar}\,\hat{\tilde\varphi}_0(p,t) \quad \left(= \hat\varphi(x,t)^*\right)$$

$$\hat H_0 = \sum_{p\in\tilde{\mathbb{R}}} \tilde\lambda\, \frac{1}{2}\left(\frac{1}{c^2}\left|\left(\partial_t \hat{\tilde\varphi}_0\right)(p,t)\right|^2 + \frac{\omega(p)^2}{c^2}\left|\hat{\tilde\varphi}_0(p,t)\right|^2\right)$$

$$\left[\partial_t \hat{\tilde\varphi}_0(p,t),\, \hat{\tilde\varphi}_0(p',t)\right] = -i\hbar c^2 \delta_{\tilde\lambda}(p+p') \equiv -i\hbar c^2 \frac{1}{\tilde\lambda}\delta_{p,-p'}$$

$$\left[\partial_t \hat{\tilde\varphi}_0(p,t),\, \partial_t\hat{\tilde\varphi}_0(p',t)\right] = 0 = \left[\hat{\tilde\varphi}_0(p,t),\, \hat{\tilde\varphi}_0(p',t)\right]$$

$$\left.\begin{aligned}
\hat a(p) &= \sqrt{\frac{\omega(p)}{2c^2\hbar}}\,\hat{\tilde\varphi}_0(p,0) + i\sqrt{\frac{1}{2c^2\omega(p)\hbar}}\,\partial_t\hat{\tilde\varphi}_0(p,0)\\
\hat a(p)^* &= \sqrt{\frac{\omega(p)}{2c^2\hbar}}\,\hat{\tilde\varphi}_0(p,0)^* - i\sqrt{\frac{1}{2c^2\omega(p)\hbar}}\,\partial_t\hat{\tilde\varphi}_0(p,0)^*
\end{aligned}\right\}$$

$$\begin{aligned}
\hat{\tilde\varphi}_0(p,t) &= e^{i\hat H_0 t}\,\hat{\tilde\varphi}_0(p,0)\,e^{-i\hat H_0 t}\\
&= \sqrt{\frac{c^2\hbar}{2\omega(p)}}\,e^{i\omega(p)t}\,\hat a(p) + \sqrt{\frac{c^2\hbar}{2\omega(p)}}\,e^{-i\omega(p)t}\,\hat a(-p)^*
\end{aligned}$$

$$\hat H_0 = \sum_{p\in\tilde{\mathbb{R}}} \tilde\lambda\,\hbar\omega(p)\,\frac{1}{2}\left(\hat a(p)^*\hat a(p) + \hat a(p)\hat a(p)^*\right)$$

$$\left[\hat a(p),\, \hat a(p')^*\right] = \delta_{\tilde\lambda}(p-p') = \tilde\lambda^{-1}\delta_{p,-p'}$$

$$\left[\hat a(p),\, \hat a(p')\right] = 0 = \left[\hat a(p)^*,\, \hat a(p')^*\right]$$

Основна помощна формула:

$$\sum_{x\in R}\lambda\, e^{\pm ixp/\hbar} = h\,\delta_{\tilde\lambda}(p)\,,\qquad \sum_{p\in\tilde R}\tilde\lambda\, e^{\pm ixp/\hbar} = h\,\delta_\lambda(x)$$

$$f(x) = \sum_{x'\in R}\lambda\;\delta_\lambda(x-x')\;f(x')$$

$$f(p) = \sum_{p'\in\tilde R}\tilde\lambda\;\delta_{\tilde\lambda}(p-p')\;f(p')$$

Връзката между регуляризационните параметри (параметрите на дискретизацията): $\tilde\lambda := \frac{h}{s\lambda}$

Условия на периодичност: $x + s\lambda = x$

$p + s\tilde\lambda = p$

Добавка на взаимодействие (нелинейност): $\hat H = \hat H_0 + I$

$$\hat I = \sum_{x\in R}\lambda\,\frac{1}{4!}\,g\,\hat\varphi(x,0)^4$$

По-нататък работим в система $\hbar = 1 = c\,,\ h = 2\pi$

Еднакви начални условия (при $t=0$)

$$\hat\varphi_0(x,0) = \hat\varphi(x,0)\,,\qquad \partial_t\hat\varphi_0(x,0) = \partial_t\hat\varphi(x,0)$$

Еволюции: $\hat\varphi_0(x,t) = e^{i\hat H_0 t}\,\hat\varphi_0(x,0)\,e^{-i\hat H_0 t}$

$$\hat\varphi(x,t) = e^{i\hat H t}\,\hat\varphi(x,0)\,e^{-i\hat H t}$$

Забележете: ако $\hat{H}_0 = H_0\{\hat{\varphi}(x,t)\}$ се конструира от $\hat{\varphi}_0(x,t)$, то $\hat{H}_0$ не зависи от t, понеже е интеграл на движение: $\hat{H}_0 \equiv e^{i\hat{H}_0 t}\,\hat{H}_0\,e^{-i\hat{H}_0 t}$

Ако обаче
$$\hat{H}_0(t) := H_0\{\hat{\varphi}(x,t)\} \equiv e^{i\hat{H}t}\,\hat{H}_0\,e^{-i\hat{H}t}$$
$$\hat{I}(t) := I\{\hat{\varphi}(x,t)\} \equiv e^{i\hat{H}t}\,\hat{I}\,e^{-i\hat{H}t}$$
$$= \sum_{x\in R} \lambda \frac{1}{4!}\, g\, \hat{\varphi}(x,t)^4$$

(понеже $\left(\hat{U}\hat{\varphi}\hat{U}^{-1}\right)^4 = \hat{U}\,\hat{\varphi}^4\,\hat{U}^{-1}$ за $\forall\, \hat{U}, \hat{\varphi}$

и в частност при $\hat{U} = e^{i\hat{H}t}$, $\hat{\varphi} := \hat{\varphi}(x,0)$),

Тогава $H = H_0 + I \equiv \hat{H}_0(t) + \hat{I}(t) = e^{i\hat{H}t}\underbrace{\left(\hat{H}_0 + \hat{I}\right)}_{\hat{H}}e^{-i\hat{H}t}$

За пресмятане на оператора на разсейване играе роля
$$\hat{I}_0(t) := e^{i\hat{H}_0 t}\,\hat{I}\,e^{-i\hat{H}_0 t} = \sum_{x\in R} \lambda \frac{1}{4!}\, g\, \hat{\varphi}_0(x,t)^4$$

$$\hat{S}^{-1} = \lim_{\substack{t_1\to\infty \\ t_2\to -\infty}} e^{-i\hat{H}_0 t_1}\, e^{i\hat{H}(t_1-t_2)}\, e^{i\hat{H}_0 t_2} = T\text{-}\exp\left(\int_{-\infty}^{+\infty} d\tau\; i\hat{I}_0(\tau)\right)$$

Основна задача: намиране на $\hat{S}$, развита в степенен ред по g (теория на пертурбациите)

$$\hat{S}^{-1} = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int_{\mathbb{R}} d\tau_1 \cdots \int_{\mathbb{R}} d\tau_n \, T\Big(\underbrace{\hat{I}_0(\tau_1)}_{\boxed{\sum_{x_1\in R} \lambda \frac{1}{4!} g \hat{\varphi}_0(x_1,\tau_1)^4}} \cdots \hat{I}_0(\tau_n) \Big)$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{ig}{4!}\right)^n \not{\int}_{(x_1,\tau_1)} \cdots \not{\int}_{(x_n,\tau_n)} T\left(\hat{\varphi}_0(x_1,\tau_1)^4 \cdots \hat{\varphi}_0(x_n,\tau_n)^4 \right)$$

където

$$\not{\int}_{(x,\tau)} := \sum_{x\in R} \lambda \int_{\mathbb{R}} d\tau$$

Преди да продължим ще направим едно предефиниране

Разлагаме по теоремата на Вик:

$$\hat{H}_0 = \sum_{x\in R} \lambda \frac{1}{2} \Big(\frac{1}{c^2} \underbrace{(\partial_t \hat{\varphi}_0)(x,0)^2}_{= \underbrace{N\left(\partial_t \hat{\varphi}_0(x,0)^2\right)}_{:\partial_t \hat{\varphi}_0(x,0)^2:} + \underbrace{\partial_t \hat{\varphi}_0(x,0)\, \partial_t \hat{\varphi}_0(x,0)}_{\text{const}}} + (\partial_x^+ \hat{\varphi}_0)(x,0)^2 + \frac{M^2 c^2}{\hbar^2} \hat{\varphi}_0(x,0)^2 \Big)$$

и т.н.

$$= H_0' + \text{const}'$$

$$\hat{H}_0' = \sum_{x\in R} \lambda \frac{1}{2}\left(\frac{1}{c^2} :(\partial_t \hat{\varphi}_0)(x,0)^2: + :(\partial_x^+ \hat{\varphi}_0)(x,0)^2: + \frac{M^2c^2}{\hbar^2} :\hat{\varphi}_0(x,0)^2:\right)$$

Също и с $\hat{I}$

$$\hat{I} = \sum_{x\in R} \lambda \frac{1}{4!} g\, \hat{\varphi}_0(x,0)^4$$

$$= \sum_{x\in R} \lambda \frac{1}{4!} g :\hat{\varphi}_0(x,0)^4:$$

$$+ \boxed{\sum_{x\in R} \lambda \frac{1}{4!} g \,(3.2.1)\, \underbrace{\hat{\varphi}_0(x,0)\,\hat{\varphi}_0(x,0)}_{\text{const}} :\hat{\varphi}_0(x,0)^2:}$$

Води до корекция на M – първа пренормировка на масата

$$+ \text{const}''$$

от члена с две сдвоявания

Така : $\hat{H} = \hat{H}_0'' + \hat{I}' + \text{const}$

$$\hat{H}_0'' = \sum_{x\in R} \lambda \frac{1}{2}\left(\frac{1}{c^2} :(\partial_t \hat{\varphi}_0)(x,0)^2: + :(\partial_x^+ \hat{\varphi}_0)(x,0)^2: + \frac{M'^2c^2}{\hbar^2} :\hat{\varphi}_0(x,0)^2:\right)$$

$$\hat{I}' := \sum_{x\in R} \lambda \frac{1}{4!} g :\hat{\varphi}_0(x,0)^4:$$

Предефинираме

$$\hat{H}_0 \mapsto \hat{H}_0''$$

$$\hat{I} \mapsto \hat{I}'$$

$$\Rightarrow \hat{S}^{-1} =$$

$$\sum_{n=0}^{\infty} \frac{1}{n!}\left(\frac{i\lambda}{4!}\right)^n \oint_{(x_1,\tau_1)} \cdots \oint_{(x_n,\tau_n)} T\left(:\hat{\varphi}_0(x_1,\tau_1)^4: \cdots :\hat{\varphi}_0(x_n,\tau_n)^4:\right)$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!}\left(\frac{i\lambda}{4!}\right)^n \oint_{(x_1,\tau_1)} \cdots \oint_{(x_n,\tau_n)} \sum_{\substack{\text{по} \\ \text{всевъзможните} \\ \text{сдвоявания от} \\ \text{различни групи}}} :\hat{\varphi}_0(x_1,\tau_1)^4: \cdots :\hat{\varphi}_0(x_n,\tau_n)^4:$$

Хронологична теорема на Вик

където сега: $\hat{\varphi}_0(x_1,t_1)\,\hat{\varphi}_0(x_2,t_2)$
$:= \langle \Omega | T\left(\hat{\varphi}_0(x_1,t_1)\,\hat{\varphi}_0(x_2,t_2)\right) \Omega \rangle$

- хронологично виково сдвояване

Нарича се още пропагатор.