

$$\hat{H}_0 = \sum_{x \in \mathbb{R}} \lambda \frac{1}{2} \left( \frac{1}{c^2} (\partial_t \hat{\phi}_0)(x, t)^2 + (\partial_x^2 \hat{\phi}_0)(x, t)^2 + \frac{M^2 c^2}{\hbar^2} \hat{\phi}_0(x, t)^2 \right)$$

$$[\partial_t \hat{\phi}_0(x, t), \hat{\phi}_0(x', t)] = -i\hbar c^2 \delta_\lambda(x - x') \equiv -i\hbar c^2 \frac{1}{\lambda} \delta_{x, x'}$$

$$[\partial_t \hat{\phi}_0(x, t), \partial_t \hat{\phi}_0(x', t)] = 0 = [\hat{\phi}_0(x, t), \hat{\phi}_0(x', t)]$$

$$\begin{aligned} \hat{\tilde{\phi}}_0(p, t) &:= \frac{1}{\sqrt{\hbar}} \sum_{x \in \mathbb{R}} \lambda e^{ixp/\hbar} \hat{\phi}_0(x, t) \quad (= \hat{\tilde{\phi}}(-p, t)^*) \\ \hat{\phi}_0(x, t) &= \frac{1}{\sqrt{\hbar}} \sum_{p \in \tilde{\mathbb{R}}} \tilde{\lambda} e^{-ixp/\hbar} \hat{\tilde{\phi}}_0(p, t) \quad (= \hat{\tilde{\phi}}(x, t)^*) \end{aligned}$$

$$\hat{H}_0 = \sum_{p \in \tilde{\mathbb{R}}} \tilde{\lambda} \frac{1}{2} \left( \frac{1}{c^2} |(\partial_t \hat{\tilde{\phi}}_0)(p, t)|^2 + \frac{\omega(p)^2}{c^2} |\hat{\tilde{\phi}}_0(p, t)|^2 \right)$$

$$[\partial_t \hat{\tilde{\phi}}_0(p, t), \hat{\tilde{\phi}}_0(p', t)] = -i\hbar c^2 \delta_{\tilde{\lambda}}(p + p') \equiv -i\hbar c^2 \frac{1}{\tilde{\lambda}} \delta_{p, -p'}$$

$$[\partial_t \hat{\tilde{\phi}}_0(p, t), \partial_t \hat{\tilde{\phi}}_0(p', t)] = 0 = [\hat{\tilde{\phi}}_0(p, t), \hat{\tilde{\phi}}_0(p', t)]$$

$$\begin{aligned} \hat{a}(p) &= \left\{ \sqrt{\frac{\omega(p)}{2c^2\hbar}} \hat{\tilde{\phi}}_0(p, 0) + i \sqrt{\frac{1}{2c^2\omega(p)\hbar}} \partial_t \hat{\tilde{\phi}}_0(p, 0) \right\} \\ \hat{a}(p)^* &= \left\{ \sqrt{\frac{\omega(p)}{2c^2\hbar}} \hat{\tilde{\phi}}_0(p, 0)^* - i \sqrt{\frac{1}{2c^2\omega(p)\hbar}} \partial_t \hat{\tilde{\phi}}_0(p, 0)^* \right\} \\ \hat{\tilde{\phi}}_0(p, t) &= e^{i\hat{H}_0 t} \hat{\tilde{\phi}}_0(p, 0) e^{-i\hat{H}_0 t} \\ &= \sqrt{\frac{c^2\hbar}{2\omega(p)}} e^{i\omega(p)t} \hat{a}(p) + \sqrt{\frac{c^2\hbar}{2\omega(p)}} e^{-i\omega(p)t} \hat{a}(-p)^* \end{aligned}$$

$$\hat{H}_0 = \sum_{p \in \tilde{\mathbb{R}}} \tilde{\lambda} \hbar \omega(p) \frac{1}{2} (\hat{a}(p)^* \hat{a}(p) + \hat{a}(p) \hat{a}(p)^*)$$

$$[\hat{a}(p), \hat{a}(p')^*] = \delta_{\tilde{\lambda}}(p - p') = \tilde{\lambda}^{-1} \delta_{p, -p'}$$

$$[\hat{a}(p), \hat{a}(p')] = 0 = [\hat{a}(p)^*, \hat{a}(p')^*]$$

Основна помощна формула:

$$\sum_{x \in \mathbb{R}} \lambda e^{\pm i x p / \hbar} = h \delta_{\tilde{\lambda}}(p), \quad \sum_{p \in \tilde{\mathbb{R}}} \tilde{\lambda} e^{\pm i x p / \hbar} = h \delta_{\lambda}(x)$$

$$f(x) = \sum_{x' \in \mathbb{R}} \lambda \delta_{\lambda}(x - x') f(x')$$

$$f(p) = \sum_{p' \in \tilde{\mathbb{R}}} \tilde{\lambda} \delta_{\tilde{\lambda}}(p - p') f(p')$$

Връзката между регуляризационните параметри (параметрите на дискретизацията):  $\tilde{\lambda} := \frac{h}{s\lambda}$

$$\begin{aligned} \text{Условия на периодичност: } x + s\lambda &= x \\ p + s\tilde{\lambda} &= p \end{aligned}$$

Добавка на взаимодействие (нелинейност):  $\hat{H} = \hat{H}_0 + I$

$$\hat{I} = \sum_{x \in \mathbb{R}} \lambda \frac{1}{4!} \varphi \hat{\varphi}(x, 0)^4$$

По-нататък работим в система  $\hbar = 1 = c$ ,  $h = 2\pi$

Еднакви начални условия (при  $t = 0$ )

$$\hat{\varphi}_0(x, 0) = \hat{\varphi}(x, 0), \quad \partial_t \hat{\varphi}_0(x, 0) = \partial_t \hat{\varphi}(x, 0)$$

$$\text{Еволюция: } \hat{\varphi}_0(x, t) = e^{i\hat{H}_0 t} \underbrace{\hat{\varphi}_0(x, 0)}_{\hat{\varphi}(x, 0)} e^{-i\hat{H}_0 t}$$

$$\hat{\varphi}(x, t) = e^{i\hat{H} t} \underbrace{\hat{\varphi}(x, 0)}_{\hat{\varphi}(x, 0)} e^{-i\hat{H} t}$$

Забележете : ако  $\hat{H}_0 = H_0 \{ \hat{\varphi}(x,t) \}$  се конструира  
от  $\hat{\varphi}_0(x,t)$ , то  $\hat{H}_0$  не зависи от  $t$ , понеже е  
интеграл на движение :  $\hat{H}_0 \equiv e^{i\hat{H}_0 t} \hat{H}_0 e^{-i\hat{H}_0 t}$

Ако обаче  $\hat{H}_0(t) := H_0 \{ \hat{\varphi}(x,t) \} \equiv e^{i\hat{H}t} \hat{H}_0 e^{-i\hat{H}t}$   
 $\hat{I}(t) := I \{ \hat{\varphi}(x,t) \} \equiv e^{i\hat{H}t} \hat{I} e^{-i\hat{H}t}$   
 $= \sum_{x \in \mathbb{R}} \lambda \frac{1}{4!} \varphi \hat{\varphi}(x,t)^4$

(понеже  $(\hat{U} \hat{\varphi} \hat{U}^{-1})^4 = \hat{U} \hat{\varphi}^4 \hat{U}^{-1}$  за  $\forall \hat{U}, \hat{\varphi}$

и в частност при  $\hat{U} = e^{i\hat{H}t}$ ,  $\hat{\varphi} := \hat{\varphi}(x,0)$ ),

Тогава  $H = H_0 + I \equiv \hat{H}_0(t) + \hat{I}(t) = e^{i\hat{H}t} \underbrace{(\hat{H}_0 + \hat{I})}_{\hat{H}} e^{-i\hat{H}t}$

За пресмятане на оператора на разсейване играе роля

$$\hat{I}_0(t) := e^{i\hat{H}_0 t} \hat{I} e^{-i\hat{H}_0 t} = \sum_{x \in \mathbb{R}} \lambda \frac{1}{4!} \varphi \hat{\varphi}_0(x,t)^4$$

$$\hat{S}^{-1} = \lim_{\substack{t_1 \rightarrow \infty \\ t_2 \rightarrow -\infty}} e^{-i\hat{H}_0 t_1} e^{i\hat{H}(t_1 - t_2)} e^{i\hat{H}_0 t_2} = T\text{-exp} \left( \int_{-\infty}^{+\infty} d\tau i\hat{I}_0(\tau) \right)$$

Основна задача : намиране на  $\hat{S}$ , развита в степенен ред по  $g$  (теория на пертурбациите)

$$\hat{S}^{-1} = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int_{\mathbb{R}} d\tau_1 \cdots \int_{\mathbb{R}} d\tau_n T(\underbrace{\hat{I}_0(\tau_1) \cdots \hat{I}_0(\tau_n)}_{\parallel})$$

$$\sum_{x \in \mathbb{R}} \lambda \frac{1}{4!} g \hat{\phi}_0(x, \tau)^4$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{i g}{4!} \right)^n \oint_{(x_1, \tau_1)} \cdots \oint_{(x_n, \tau_n)} T(\hat{\phi}_0(x_1, \tau_1)^4 \cdots \hat{\phi}_0(x_n, \tau_n)^4)$$

където  $\oint_{(x, \tau)} := \sum_{x \in \mathbb{R}} \lambda \int_{\mathbb{R}} d\tau$

Преди да продължим ще направим едно предефиниране

Разлагаме по теоремата на Вик :

$$\hat{H}_0 = \sum_{x \in \mathbb{R}} \lambda \frac{1}{2} \left( \frac{1}{c^2} (\partial_t \hat{\phi}_0)(x, 0)^2 + (\partial_x^+ \hat{\phi}_0)(x, 0)^2 + \frac{M^2 c^2}{\hbar^2} \hat{\phi}_0(x, 0)^2 \right)$$

$$= \underbrace{N(\partial_t \hat{\phi}_0(x, 0)^2)}_{: \partial_t \hat{\phi}_0(x, 0)^2 :} + \underbrace{\partial_t \hat{\phi}_0(x, 0) \partial_t \hat{\phi}_0(x, 0)}_{const} \text{ и т.н.}$$

$$= H'_0 + const'$$

$$\hat{H}_0' = \sum_{x \in \mathbb{R}} \lambda \frac{1}{2} \left( \frac{1}{c^2} :(\partial_t \hat{\phi}_0)(x, 0)^2: + :(\partial_x^+ \hat{\phi}_0)(x, 0)^2: + \frac{M^2 c^2}{\hbar^2} : \hat{\phi}_0(x, 0)^2 : \right)$$

Гоуо и с  $\hat{I}$

води до корекция на  
M - първа пренормировка  
на масата

$$\begin{aligned} \hat{I} &= \sum_{x \in \mathbb{R}} \lambda \frac{1}{4!} g : \hat{\phi}_0(x, 0)^4 : \\ &= \sum_{x \in \mathbb{R}} \lambda \frac{1}{4!} g : \hat{\phi}_0(x, 0)^4 : \end{aligned}$$

$$+ \sum_{x \in \mathbb{R}} \lambda \frac{1}{4!} g (3.2.1) \underbrace{\hat{\phi}_0(x, 0) \hat{\phi}_0(x, 0)}_{const} : \hat{\phi}_0(x, 0)^2 :$$

+ const'' ← от члена с две сдвоявания

$$\text{Така : } \hat{H} = \hat{H}_0'' + \hat{I}' + \text{const}$$

$$\hat{H}_0'' = \sum_{x \in \mathbb{R}} \lambda \frac{1}{2} \left( \frac{1}{c^2} :(\partial_t \hat{\phi}_0)(x, 0)^2: + :(\partial_x^+ \hat{\phi}_0)(x, 0)^2: + \frac{M'^2 c^2}{\hbar^2} : \hat{\phi}_0(x, 0)^2 : \right)$$

$$\hat{I}' := \sum_{x \in \mathbb{R}} \lambda \frac{1}{4!} g : \hat{\phi}_0(x, 0)^4 :$$

Предефинираме

$$\hat{H}_0 \mapsto \hat{H}_0''$$

$$\hat{I} \mapsto \hat{I}'$$

$$\Rightarrow \hat{S}^{-1} = \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{i\mu}{4!} \right)^n \int_{(x_1, \tau_1)} \cdots \int_{(x_n, \tau_n)} T \left( : \hat{\varphi}_0(x_1, \tau_1)^4 : \cdots : \hat{\varphi}_0(x_n, \tau_n)^4 : \right)$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{i\mu}{4!} \right)^n \int_{(x_1, \tau_1)} \cdots \int_{(x_n, \tau_n)} \sum_{\text{по всевъзможните съвоявания от различни групи}} : \hat{\varphi}_0(x_1, \tau_1)^4 : \cdots : \hat{\varphi}_0(x_n, \tau_n)^4 :$$

↑  
Хронологична теорема на Вика

където сега:

$$\hat{\varphi}_0(x_1, t_1) \hat{\varphi}_0(x_2, t_2) := \langle \Omega | T(\hat{\varphi}_0(x_1, t_1) \hat{\varphi}_0(x_2, t_2)) | \Omega \rangle$$

- хронологично виково съвояване

Нарича се още пропагатор.

Пресмятане на пропагатора  $G_0^{(2)} :=$

$$\begin{aligned} & \langle \Omega | T(\hat{\varphi}_0(x_1, t_1) \hat{\varphi}_0(x_2, t_2)) | \Omega \rangle \\ &= \theta(t_1 - t_2) \langle \Omega | \hat{\varphi}_0(x_1, t_1) \hat{\varphi}_0(x_2, t_2) | \Omega \rangle \\ &+ \theta(t_2 - t_1) \langle \Omega | \hat{\varphi}_0(x_2, t_2) \hat{\varphi}_0(x_1, t_1) | \Omega \rangle \end{aligned}$$

Пресмятане на с. нар. дву-точкова (two-point) функция

$$\langle \Omega | \hat{\varphi}_0(x_1, t_1) \hat{\varphi}_0(x_2, t_2) | \Omega \rangle = ? \quad (\hbar = 1 = c, \hbar = 2\pi)$$

$$= \langle \Omega | \sum_{p_1 \in \tilde{R}} \tilde{\lambda} \sum_{p_2 \in \tilde{R}} \tilde{\lambda} e^{-i(x_1 p_1 + x_2 p_2)} \hat{\varphi}_0(p_1, t_1) \hat{\varphi}_0(p_2, t_2) | \Omega \rangle$$

$$= \sum_{p_1 \in \tilde{R}} \tilde{\lambda} \sum_{p_2 \in \tilde{R}} \tilde{\lambda} e^{-i(x_1 p_1 + x_2 p_2)} \langle \Omega | \hat{\varphi}_0(p_1, t_1) \hat{\varphi}_0(p_2, t_2) | \Omega \rangle$$

$$= \sum_{p_1 \in \tilde{R}} \tilde{\lambda} \sum_{p_2 \in \tilde{R}} \tilde{\lambda} e^{-i(x_1 p_1 + x_2 p_2)}$$

$$\cdot \langle \Omega | \left( \sqrt{\frac{1}{2\omega(p_1)}} e^{i\omega(p_1)t_1} \hat{a}(p_1) + \sqrt{\frac{1}{2\omega(p_1)}} e^{-i\omega(p_1)t_1} \hat{a}(-p_1)^* \right) \left( \sqrt{\frac{1}{2\omega(p_2)}} e^{i\omega(p_2)t_2} \hat{a}(p_2) + \sqrt{\frac{1}{2\omega(p_2)}} e^{-i\omega(p_2)t_2} \hat{a}(-p_2)^* \right) | \Omega \rangle$$

$$= \sum_{p \in \tilde{R}} \tilde{\lambda} \frac{1}{2\omega(p)} e^{-i(x_1 - x_2)p + i(t_1 - t_2)\omega(p)}$$

$$=: W_0^{(2)}(x_1 - x_2, t_1 - t_2) - \text{свободна 2-точкова функция на Уайтман (Wightman)}$$

$$\begin{aligned} \Rightarrow G_0^{(2)} &= G_0^{(2)}(x_1 - x_2, t_1 - t_2) \\ &= \theta(t_1 - t_2) W_0^{(2)}(x_1 - x_2, t_1 - t_2) \\ &+ \theta(t_2 - t_1) W_0^{(2)}(x_2 - x_1, t_2 - t_1) \end{aligned}$$

- свободна функция на Грийн Green function

Уравнения за свободните корелационни функции  
(общо наименование на Уайтмановите и Грийновите функц.).

$$\text{от } \frac{1}{c^2} \partial_t^2 \hat{\varphi}_0(x, t) - \partial_x^2 \hat{\varphi}_0(x, t) + \frac{M^2 c^2}{\hbar^2} \hat{\varphi}_0(x, t) = 0$$

$$\Rightarrow \left( \frac{1}{c^2} \partial_t^2 - \partial_x^2 + \frac{M^2 c^2}{\hbar^2} \right) W_0^{(2)}(x, t) = 0$$

$$\left( \frac{1}{c^2} \partial_t^2 - \partial_x^2 + \frac{M^2 c^2}{\hbar^2} \right) G_0^{(2)}(x, t) = \delta_\lambda(x) \delta(t)$$

(помощта на  $G_0^{(2)}$  полевого уравнение на взаимод. поле

$$\frac{1}{c^2} \partial_t^2 \hat{\varphi}(x, t) - \partial_x^2 \hat{\varphi}(x, t) + \frac{M^2 c^2}{\hbar^2} \hat{\varphi}(x, t) = -\frac{\hbar}{i} : \hat{\varphi}(x, t)^3 :$$

се записва като интегрално уравнение

- уравнение на Дайсен - Швингер / Dyson - Schwinger

$$\hat{\varphi}(x, t) = \hat{\varphi}_0(x, t) - \frac{\hbar}{3!} \oint_{(x, \tau)} G_0^{(2)}(x-x', t-t') : \hat{\varphi}(x', t')^3 :$$

развитието на  
Представяне на  $\hat{S}$ -матрицата с диаграми на Файнман

$$\hat{S}^{-1} =$$

$$\sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{i\mu}{4!} \right)^n \int_{(x_1, \tau_1)} \cdots \int_{(x_n, \tau_n)} \sum_{\text{по всевъзможните състояния от различни групи}} \text{diagram} \cdot \hat{\varphi}_0(x_1, \tau_1)^4 \cdots \hat{\varphi}_0(x_n, \tau_n)^4 :$$

$$= \sum_{n=0}^{\infty} \sum_{\Gamma_n} \frac{1}{|\text{Symm } \Gamma_n|} \int_{(x_1, \tau_1)} \cdots \int_{(x_n, \tau_n)} G_{\Gamma_n}(x_1, \tau_1; \dots; x_n, \tau_n) \cdot : M_{\Gamma_n}(x_1, \tau_1; \dots; x_n, \tau_n) :$$

<http://theo.inrne.bas.bg/~mitov/renormalization2020/notes/contents.html>

Пример:  $\int_{\mathbb{R}^4} dx_k \leftrightarrow \sum_{(x_k, \tau_k)} \text{ , } x_k = (x_k, \tau_k)$

$$\hat{S}^{-1} = \hat{1} + \frac{(i)}{24} \int_{\mathbb{R}^4} d^4 x_1 \underbrace{G(x_1)}_1 : \hat{\varphi}(x_1)^4 :$$

$$+ \frac{(i)^2}{1152} \int_{\mathbb{R}^{4,2}} d^4 x_1 d^4 x_2 \underbrace{G(x_1, x_2)}_1 : \hat{\varphi}(x_1)^4 \hat{\varphi}(x_2)^4 :$$

$$+ \frac{(i)^2}{72} \int_{\mathbb{R}^{4,2}} d^4 x_1 d^4 x_2 \underbrace{G(x_1, x_2)}_{G_0(x_1 - x_2)} : \hat{\varphi}(x_1)^3 \hat{\varphi}(x_2)^3 :$$

$$+ \frac{(ig)^2}{16} \int_{\mathbb{R}^{4,2}} d^4 x_1 d^4 x_2 \underbrace{G(x_1, x_2)}_{G_0(x_1 - x_2)^2} : \hat{\varphi}(x_1)^2 \hat{\varphi}(x_2)^2 :$$

$$+ \frac{(ig)^2}{12} \int_{\mathbb{R}^{4,2}} d^4 x_1 d^4 x_2 \underbrace{G(x_1, x_2)}_{G_0(x_1 - x_3)^3} : \hat{\varphi}(x_1) \hat{\varphi}(x_2) :$$

$$+ \frac{(ig)^3}{32} \int_{\mathbb{R}^{4,3}} d^4 x_1 d^4 x_2 d^4 x_3 \underbrace{G(x_1, x_2, x_3)}_{G_0(x_1 - x_2)^2 G_0(x_1 - x_3) G_0(x_2 - x_3)} : \hat{\varphi}(x_1) \hat{\varphi}(x_2) \hat{\varphi}(x_3)^2 :$$

+ ... (еще слагаемые от трети и по-высоки порядку)