КТП и ЕЧ, 18.12.2020

Изходен, свободен (линеен) квантов модел

$$\hat{H}_0 = \sum_{x\in\mathcal{R}} \lambda \frac{1}{2}\left(\frac{1}{c^2}(\partial_t \hat{\varphi}_0)(x,t)^2 + (\partial_x^+ \hat{\varphi}_0)(x,t)^2 + \frac{M^2c^2}{\hbar^2}\hat{\varphi}_0(x,t)^2\right)$$

$$[\partial_t \hat{\varphi}_0(x,t), \hat{\varphi}_0(x',t)] = -i\hbar c^2 \delta_\lambda(x-x') \equiv -i\hbar c^2 \frac{1}{\lambda}\delta_{x,x'}$$

$$[\partial_t \hat{\varphi}_0(x,t), \partial_t \hat{\varphi}_0(x',t)] = 0 = [\hat{\varphi}_0(x,t), \hat{\varphi}_0(x',t)]$$

$$\hat{\tilde{\varphi}}_0(p,t) := \frac{1}{\sqrt{h}} \sum_{x\in\mathcal{R}} \lambda\, e^{ixp/\hbar}\, \hat{\varphi}_0(x,t) \quad \left(= \hat{\tilde{\varphi}}_0(-p,t)^*\right)$$

$$\hat{\varphi}_0(x,t) = \frac{1}{\sqrt{h}} \sum_{p\in\tilde{\mathcal{R}}} \tilde{\lambda}\, e^{-ixp/\hbar}\, \hat{\tilde{\varphi}}_0(p,t) \quad \left(= \hat{\varphi}_0(x,t)^*\right)$$

$$\hat{H}_0 = \sum_{p\in\tilde{\mathcal{R}}} \tilde{\lambda} \frac{1}{2}\left(\frac{1}{c^2}\left|(\partial_t \hat{\tilde{\varphi}}_0)(p,t)\right|^2 + \frac{\omega(p)^2}{c^2}\left|\hat{\tilde{\varphi}}_0(p,t)\right|^2\right)$$

$$[\partial_t \hat{\tilde{\varphi}}_0(p,t), \hat{\tilde{\varphi}}_0(p',t)] = -i\hbar c^2 \delta_{\tilde{\lambda}}(p+p') \equiv -i\hbar c^2 \frac{1}{\tilde{\lambda}}\delta_{p,-p'}$$

$$[\partial_t \hat{\tilde{\varphi}}_0(p,t), \partial_t \hat{\tilde{\varphi}}_0(p',t)] = 0 = [\hat{\tilde{\varphi}}_0(p,t), \hat{\tilde{\varphi}}_0(p',t)]$$

$$\left.\begin{aligned} \hat{a}(p) &= \sqrt{\frac{\omega(p)}{2c^2\hbar}}\, \hat{\tilde{\varphi}}_0(p,0) + i\sqrt{\frac{1}{2c^2\omega(p)\hbar}}\, \partial_t \hat{\tilde{\varphi}}_0(p,0) \\ \hat{a}(p)^* &= \sqrt{\frac{\omega(p)}{2c^2\hbar}}\, \hat{\tilde{\varphi}}_0(p,0)^* - i\sqrt{\frac{1}{2c^2\omega(p)\hbar}}\, \partial_t \hat{\tilde{\varphi}}_0(p,0)^* \end{aligned}\right\}$$

$$\hat{\tilde{\varphi}}_0(p,t) = e^{i\hat{H}_0 t}\, \hat{\tilde{\varphi}}_0(p,0)\, e^{-i\hat{H}_0 t}$$

$$= \sqrt{\frac{c^2\hbar}{2\omega(p)}}\, e^{i\omega(p)t}\, \hat{a}(p) + \sqrt{\frac{c^2\hbar}{2\omega(p)}}\, e^{-i\omega(p)t}\, \hat{a}(-p)^*$$

$$\hat{H}_0 = \sum_{p\in\tilde{\mathcal{R}}} \tilde{\lambda}\, \hbar\omega(p) \frac{1}{2}\left(\hat{a}(p)^*\hat{a}(p) + \hat{a}(p)\hat{a}(p)^*\right)$$

$$[\hat{a}(p), \hat{a}(p')^*] = \delta_{\tilde{\lambda}}(p-p') = \tilde{\lambda}^{-1}\delta_{p,-p'}$$

$$[\hat{a}(p), \hat{a}(p')] = 0 = [\hat{a}(p)^*, \hat{a}(p')^*]$$

Основна помощна формула:

$$\sum_{x\in R} \lambda\, e^{\pm ixp/\hbar} = h\,\delta_{\tilde{\lambda}}(p)\,, \qquad \sum_{p\in \tilde{R}} \tilde{\lambda}\, e^{\pm ixp/\hbar} = h\,\delta_{\lambda}(x)$$

$$f(x) = \sum_{x'\in R} \lambda\ \delta_{\lambda}(x-x')\ f(x')$$

$$f(p) = \sum_{p'\in \tilde{R}} \tilde{\lambda}\ \delta_{\tilde{\lambda}}(p-p')\ f(p')$$

Връзката между регуляризационните параметри (параметрите на дискретизацията): $\tilde{\lambda} := \frac{h}{s\lambda}$

Условия на периодичност: $x + s\lambda = x$

$$p + s\tilde{\lambda} = p$$

Добавка на взаимодействие (нелинейност): $\hat{H} = \hat{H}_0 + I$

$$\hat{I} = \sum_{x\in R} \lambda \frac{1}{4!}\, g\, \hat{\varphi}(x,0)^4$$

По-нататък работим в система $\hbar = 1 = c\,,\ h = 2\pi$

Еднакви начални условия (при $t=0$)

$$\hat{\varphi}_0(x,0) = \hat{\varphi}(x,0)\,, \qquad \partial_t \hat{\varphi}_0(x,0) = \partial_t \hat{\varphi}(x,0)$$

Еволюции: $\hat{\varphi}_0(x,t) = e^{i\hat{H}_0 t}\, \hat{\varphi}_0(x,0)\, e^{-i\hat{H}_0 t}$

$$\hat{\varphi}(x,t) = e^{i\hat{H} t}\, \hat{\varphi}(x,0)\, e^{-i\hat{H} t}$$

Забележете: ако $\hat{H}_0 = H_0\{\hat{\varphi}_0(x,t)\}$ се конструира от $\hat{\varphi}_0(x,t)$, то $\hat{H}_0$ не зависи от t, понеже е интеграл на движение: $\hat{H}_0 \equiv e^{i\hat{H}_0 t}\, \hat{H}_0\, e^{-i\hat{H}_0 t}$

Ако обаче $\hat{H}_0(t) := H_0\{\hat{\varphi}(x,t)\} \equiv e^{i\hat{H}t}\, \hat{H}_0\, e^{-i\hat{H}t}$

$$\hat{I}(t) := I\{\hat{\varphi}(x,t)\} \equiv e^{i\hat{H}t}\, \hat{I}\, e^{-i\hat{H}t}$$

$$= \sum \lambda \frac{1}{4!} g\, \hat{\varphi}(x,t)^4$$

(понеже $(\hat{U}\hat{\varphi}\hat{U}^{-1})^4 = \hat{U}\hat{\varphi}^4\hat{U}^{-1}$ за $\forall \hat{U}, \hat{\varphi}$

и в частност при $\hat{U} = e^{i\hat{H}t}$, $\hat{\varphi} := \hat{\varphi}(x,0)$),

Тогава $\hat{H} = \hat{H}_0 + \hat{I} \equiv \hat{H}_0(t) + \hat{I}(t) = e^{i\hat{H}t}\underbrace{(\hat{H}_0 + \hat{I})}_{\hat{H}} e^{-i\hat{H}t}$

За пресмятане на оператора на разсейване играе роля

$$\hat{I}_0(t) := e^{i\hat{H}_0 t}\, \hat{I}\, e^{-i\hat{H}_0 t} = \sum \lambda \frac{1}{4!} g\, \hat{\varphi}_0(x,t)^4$$

$$\hat{S}^{-1} = \lim_{\substack{t_1 \to \infty \\ t_2 \to -\infty}} e^{-i\hat{H}_0 t_1} e^{i\hat{H}(t_1 - t_2)} e^{i\hat{H}_0 t_2} = T\text{-}\exp\left(\int_{-\infty}^{+\infty} d\tau\, i\hat{I}_0(\tau)\right)$$

Основна задача: намиране на $\hat{S}$, развита в степенен ред по g (теория на пертурбациите)

$$\hat{S}^{-1} = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int_{\mathbb{R}} d\tau_1 \cdots \int_{\mathbb{R}} d\tau_n \, T\Big(\underbrace{\hat{I}_0(\tau_1)}_{\boxed{\sum_{x_1\in R} \lambda \frac{1}{4!} g \, \hat{\varphi}_0(x_1,\tau_1)^4}} \cdots \hat{I}_0(\tau_n)\Big)$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{ig}{4!}\right)^n \mathop{\not\Sigma\!\!\!\int}_{(x_1,\tau_1)} \cdots \mathop{\not\Sigma\!\!\!\int}_{(x_n,\tau_n)} T\Big(\hat{\varphi}_0(x_1,\tau_1)^4 \cdots \hat{\varphi}_0(x_n,\tau_n)^4\Big)$$

където

$$\mathop{\not\Sigma\!\!\!\int}_{(x,\tau)} := \sum_{x\in R} \lambda \int_{\mathbb{R}} d\tau$$

Преди да продължим ще направим едно предефиниране

Разлагаме по теоремата на Вик:

$$\hat{H}_0 = \sum_{x\in R} \lambda \frac{1}{2}\Big(\underbrace{\frac{1}{c^2}(\partial_t \hat{\varphi}_0)(x,0)^2}_{= \underbrace{N\left(\partial_t \hat{\varphi}_0(x,0)^2\right)}_{:\partial_t \hat{\varphi}_0(x,0)^2:} + \underbrace{\partial_t \hat{\varphi}_0(x,0)\, \partial_t \hat{\varphi}_0(x,0)}_{\text{const}}} + (\partial_x^+ \hat{\varphi}_0)(x,0)^2 + \frac{M^2c^2}{\hbar^2} \hat{\varphi}_0(x,0)^2\Big)$$

и т.н.

$$= H_0' + \text{const}'$$

$$\hat H_0' = \sum_{x\in R} \lambda \frac{1}{2}\left(\frac{1}{c^2} :(\partial_t \hat\varphi_0)(x,0)^2: + :(\partial_x^+ \hat\varphi_0)(x,0)^2: + \frac{M^2c^2}{\hbar^2} :\hat\varphi_0(x,0)^2:\right)$$

Също и с $\hat I$

$$\hat I = \sum_{x\in R} \lambda \frac{1}{4!} g\, \hat\varphi_0(x,0)^4$$

$$= \sum_{x\in R} \lambda \frac{1}{4!} g :\hat\varphi_0(x,0)^4:$$

$$+ \boxed{\sum_{x\in R} \lambda \frac{1}{4!} g\, (3.2.1)\, \underbrace{\hat\varphi_0(x,0)\,\hat\varphi_0(x,0)}_{const} :\hat\varphi_0(x,0)^2:}$$

води до корекция на M – първа пренормировка на масата

пояснение

$$\varphi_0\ \varphi_0\ \varphi_0\ \varphi_0$$

3.2.1

$$+\ const''$$ от члена с две сдвоявания

Така: $\hat H = \hat H_0'' + \hat I' + const$

$$\hat H_0'' = \sum_{x\in R} \lambda \frac{1}{2}\left(\frac{1}{c^2} :(\partial_t \hat\varphi_0)(x,0)^2: + :(\partial_x^+ \hat\varphi_0)(x,0)^2: + \frac{M'^2c^2}{\hbar^2} :\hat\varphi_0(x,0)^2:\right)$$

$$\hat I' := \sum_{x\in R} \lambda \frac{1}{4!} g :\hat\varphi_0(x,0)^4:$$

Предефинираме $\hat H_0 \mapsto \hat H_0''$

$\hat I \mapsto \hat I'$

$$\Rightarrow \quad \hat{S}^{-1} =$$

$$\sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{i g}{4!}\right)^n \oint_{(x_1,\tau_1)} \cdots \oint_{(x_n,\tau_n)} T\left(:\hat{\varphi}_0(x_1,\tau_1)^4: \cdots :\hat{\varphi}_0(x_n,\tau_n)^4: \right)$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{i g}{4!}\right)^n \oint_{(x_1,\tau_1)} \cdots \oint_{(x_n,\tau_n)} \sum_{\substack{\text{по} \\ \text{всевъзможните} \\ \text{сдвоявания от} \\ \text{различни групи}}} :\hat{\varphi}_0(x_1,\tau_1)^4: \cdots :\hat{\varphi}_0(x_n,\tau_n)^4:$$

Хронологична теорема на Вик

където сега: $\hat{\varphi}_0(x_1,t_1)\,\hat{\varphi}_0(x_2,t_2)$

$$:= \langle \Omega | T\left(\hat{\varphi}_0(x_1,t_1)\,\hat{\varphi}_0(x_2,t_2)\right) \Omega \rangle$$

- хронологично виково сдвояване

Нарича се още пропагатор.

Пресмятане на пропагатора $G_0^{(2)} :=$

$$\langle \Omega | T(\hat{\varphi}_0(x_1,t_1)\hat{\varphi}_0(x_2,t_2))\Omega\rangle$$

$$= \theta(t_1-t_2)\langle\Omega|\hat{\varphi}_0(x_1,t_1)\hat{\varphi}_0(x_2,t_2)\Omega\rangle$$

$$+ \theta(t_2-t_1)\langle\Omega|\hat{\varphi}_0(x_2,t_2)\hat{\varphi}_0(x_1,t_1)\Omega\rangle$$

Пресмятане на т. нар. дву-точкова (two-point) функция

$$\langle\Omega|\hat{\varphi}_0(x_1,t_1)\hat{\varphi}_0(x_2,t_2)\Omega\rangle = ? \qquad (\hbar = 1 = c\,,\ h = 2\pi)$$

$$= \langle\Omega|\sum_{p_1\in\tilde{R}}\tilde{\lambda}\sum_{p_2\in\tilde{R}}\tilde{\lambda}\, e^{-i(x_1p_1+x_2p_2)}\,\hat{\tilde{\varphi}}_0(p_1,t_1)\hat{\tilde{\varphi}}_0(p_2,t_2)\Omega\rangle\frac{1}{h}$$

$$= \sum_{p_1\in\tilde{R}}\tilde{\lambda}\sum_{p_2\in\tilde{R}}\tilde{\lambda}\, e^{-i(x_1p_1+x_2p_2)}\langle\Omega|\hat{\tilde{\varphi}}_0(p_1,t_1)\hat{\tilde{\varphi}}_0(p_2,t_2)\Omega\rangle\frac{1}{h}$$

$$= \sum_{p_1\in\tilde{R}}\tilde{\lambda}\sum_{p_2\in\tilde{R}}\tilde{\lambda}\, e^{-i(x_1p_1+x_2p_2)}\frac{1}{h}$$

$$\cdot \langle\Omega|\left(\sqrt{\frac{1}{2\omega(p_1)}}\,e^{i\omega(p_1)t_1}\hat{a}(p_1) + \sqrt{\frac{1}{2\omega(p_1)}}\,e^{-i\omega(p_1)t_1}\hat{a}(-p_1)^*\right)$$

$$\left(\sqrt{\frac{1}{2\omega(p_2)}}\,e^{i\omega(p_2)t_2}\hat{a}(p_2) + \sqrt{\frac{1}{2\omega(p_2)}}\,e^{-i\omega(p_2)t_2}\hat{a}(-p_2)^*\right)\Omega\rangle$$

$$= \sum_{p\in\tilde{R}}\tilde{\lambda}\,\frac{1}{2\omega(p)}\,e^{-i(x_1-x_2)p\,+\,i(t_1-t_2)\,\omega(p)}$$

$$=: W_0^{(2)}(x_1-x_2, t_1-t_2)$$ – свободна 2-точкова функция на Уайтман (Wightman)

$$\Rightarrow\ G_0^{(2)} = G_0^{(2)}(x_1-x_2, t_1-t_2)$$
$$= \theta(t_1-t_2)\,W_0^{(2)}(x_1-x_2, t_1-t_2)$$
$$+ \theta(t_2-t_1)\,W_0^{(2)}(x_2-x_1, t_2-t_1)$$

– свободна функция на Грийн
Green function

Уравнения за свободните корелационни функции (общо наименование на Уайтмановите и Грийновите функц.).

От $\frac{1}{c^2}\partial_t^2 \hat{\varphi}_0(x,t) - \partial_x^2 \hat{\varphi}_0(x,t) + \frac{M^2c^2}{\hbar^2}\hat{\varphi}_0(x,t) = 0$

$$\Rightarrow \left(\frac{1}{c^2}\partial_t^2 - \partial_x^2 + \frac{M^2c^2}{\hbar^2}\right) W_0^{(2)}(x,t) = 0$$

$$\left(\frac{1}{c^2}\partial_t^2 - \partial_x^2 + \frac{M^2c^2}{\hbar^2}\right) G_0^{(2)}(x,t) = \delta_\lambda(x)\,\delta(t)$$

С помощта на $G_0^{(2)}$ полевото уравнение на взаимод. поле

$$\frac{1}{c^2}\partial_t^2 \hat{\varphi}(x,t) - \partial_x^2 \hat{\varphi}(x,t) + \frac{M^2c^2}{\hbar^2}\hat{\varphi}(x,t) = -\frac{g}{3!} :\hat{\varphi}(x,t)^3:$$

се записва като интегрално уравнение

– уравнение на Дайсън - Швингер / Dyson - Schwinger

$$\hat{\varphi}(x,t) = \hat{\varphi}_0(x,t) - \frac{g}{3!}\oint_{(x,\tau)} G_0^{(2)}(x-x', t-t') :\hat{\varphi}(x',t')^3:$$

Представяне на развитието на $\hat{S}$-матрицата с диаграми на Файнман

$$\hat{S} - 1 =$$

$$\sum_{n=0}^{\infty} \frac{1}{n!}\left(\frac{i g}{4!}\right)^n \sum\!\!\!\!\!\!\!\int_{(x_1,\tau_1)} \cdots \sum\!\!\!\!\!\!\!\int_{(x_n,\tau_n)} \sum_{\substack{\text{по}\\ \text{всевъзможните}\\ \text{сдвоявания от}\\ \text{различни групи}}} :\hat{\varphi}_0(x_1,\tau_1)^4: \cdots :\hat{\varphi}_0(x_n,\tau_n)^4:$$

$$= \sum_{n=0}^{\infty} \sum_{\Gamma_n} \frac{1}{|\mathrm{Symm}\,\Gamma_n|} \sum\!\!\!\!\!\!\!\int_{(x_1,\tau_1)} \cdots \sum\!\!\!\!\!\!\!\int_{(x_n,\tau_n)} G_{\Gamma_n}(x_1,\tau_1;\dots;x_n,\tau_n)$$

$$\cdot :M_{\Gamma_n}(x_1,\tau_1;\dots;x_n,\tau_n):$$

Пример: $\int\limits_{\mathbb{R}^4} dx_k \leftrightarrow \sum\!\!\!\!\!\!\int\limits_{(x_k,\tau_k)}$, $x_k = (x_k, \tau_k)$

$$\hat{S}^{-1} = \hat{1} + \frac{(ig)}{24} \int\limits_{\mathbb{R}^4} d^4x_1 \, \underbrace{G(x_1)}_{1} : \hat{\varphi}(x_1)^4 :$$

$$+ \frac{(ig)^2}{1152} \int\limits_{\mathbb{R}^{4.2}} d^4x_1 \, d^4x_2 \, \underbrace{G(x_1, x_2)}_{1} : \hat{\varphi}(x_1)^4 \hat{\varphi}(x_2)^4 :$$

$$+ \frac{(ig)^2}{72} \int\limits_{\mathbb{R}^{4.2}} d^4x_1 \, d^4x_2 \, \underbrace{G(x_1, x_2)}_{G_0(x_1 - x_2)} : \hat{\varphi}(x_1)^3 \hat{\varphi}(x_2)^3 :$$

$$+ \frac{(ig)^2}{16} \int\limits_{\mathbb{R}^{4.2}} d^4x_1 \, d^4x_2 \, \underbrace{G(x_1, x_2)}_{G_0(x_1 - x_2)^2} : \hat{\varphi}(x_1)^2 \hat{\varphi}(x_2)^2 :$$

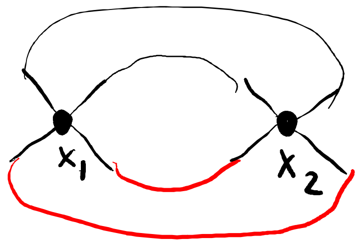
$$+ \frac{(ig)^2}{12} \int\limits_{\mathbb{R}^{4.2}} d^4x_1 \, d^4x_2 \, \underbrace{G(x_1, x_2)}_{G_0(x_1 - x_2)^3} : \hat{\varphi}(x_1) \hat{\varphi}(x_2) :$$

$$+ \frac{(ig)^3}{32} \int\limits_{\mathbb{R}^{4.3}} d^4x_1 \, d^4x_2 \, d^4x_3 \, \underbrace{G(x_1, x_2, x_3)}_{G_0(x_1 - x_2)^2 G_0(x_1 - x_3) G_0(x_2 - x_3)} : \hat{\varphi}(x_1) \hat{\varphi}(x_2) \hat{\varphi}(x_3)^2 :$$

$+ \dots$ (още членове от трети и по-високи порядъци)

$$x_k = (x_k, \tau_k)$$

$$\frac{2\binom{4}{2}^2}{(4!)^2}$$

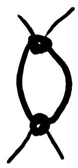


$\leftarrow$ диаграма на Файнман

$$:\hat\varphi_0(x_1,\tau_1)^4: \quad :\hat\varphi_0(x_2,\tau_2)^4:$$

$$:\cancel{\hat\varphi_0'}\,\hat\varphi_0'\,\cancel{\hat\varphi_0'}\,\hat\varphi_0': \cdot :\cancel{\hat\varphi_0''}\,\hat\varphi_0''\,\hat\varphi_0''\,\cancel{\hat\varphi_0''}:$$

$$= \underbrace{G_0(x_1-x_2)\,G_0(x_1-x_2)}_{G_\Gamma(x_1,x_2)}\;\overbrace{\underbrace{:\hat\varphi_0(x_1)^2\,\hat\varphi_0(x_2)^2:}_{:M_\Gamma(x_1,x_2):}}^{\text{общо норм. ир-е}}$$



Фермиони – Koszul

Кошул

Edward Nelso

Quantization is a mistery but the second quantization is a functor

"Функтор в категорията на лин. пр."

$$= \begin{cases} \text{обектно съответствие} : V \longmapsto \Gamma(V) \\ \text{съответствие на морфизми} \quad \begin{array}{c} V_1 \\ F \downarrow \\ V_2 \end{array} \longmapsto \begin{array}{c} \Gamma(V_1) \\ \downarrow \Gamma(F) \\ \Gamma(V_2) \end{array} \end{cases}$$

$$\Gamma(F \circ G) = \Gamma(F) \circ \Gamma(G)$$

$$\Gamma(id_V) = id_{\Gamma(V)}$$

$$\mathcal{H}_1 \longmapsto \Gamma(\mathcal{H}_1) \equiv Fock(\mathcal{H}_1)$$

$$= \bigoplus_{n=0}^{\infty} S_n\left(\mathcal{H}_1^{\otimes n}\right)$$

$$\begin{array}{c} \mathcal{H}_1 \\ U \Big\downarrow \text{ кв. трансф.} \\ \mathcal{H}_1 \end{array} \quad \longmapsto \quad \begin{array}{c} \Gamma(\mathcal{H}_1) \\ \Gamma(U) \Big\downarrow \\ \Gamma(\mathcal{H}_1) \end{array}$$

$$\Gamma(U)\underbrace{\left(\Psi_1 \otimes \cdots \otimes \Psi_n\right)}_{\in\ \mathcal{H}_1^{\otimes n}} = \underbrace{\left(U\Psi_1\right) \otimes \cdots \otimes \left(U\Psi_n\right)}_{=\ U^{\otimes n}\left(\Psi_1 \otimes \cdots \otimes \Psi_n\right)}$$

$$U^{\otimes n} \circ S_n = S_n \circ U^{\otimes n}$$

$$\Rightarrow \quad U^{\otimes n}\left(S_n\left(\mathcal{H}_1^{\otimes n}\right)\right) \subseteq S_n\left(\mathcal{H}_1^{\otimes n}\right)$$

$$\Gamma(U)\Big|_{S_n\left(\mathcal{H}_1^{\otimes n}\right)} = U^{\otimes n}\Big|_{S_n\left(\mathcal{H}_1^{\otimes n}\right)}$$

Нека

$$U = U_t = e^{it\hat{h}}$$

$\hat{h}$ - едночастичен хамилтониан

$$\Gamma(U_t)\Big|_{S_n(\mathcal{H}_1^{\otimes n})} = e^{iH_n t}$$

$$H_n(\Psi_1 \otimes \cdots \otimes \Psi_n) = \frac{1}{i}\frac{d}{dt} U_t^{\otimes n}(\cdots)\Big|_{t=0}$$

$$= \frac{1}{i}\frac{d}{dt}\left(U_t\Psi_1 \otimes \cdots \otimes U_t \Psi_n\right)_{t=0}$$

$$= \hat{h}\Psi_1 \otimes \Psi_2 \otimes \cdots \otimes \Psi_n$$

$$+ \Psi_1 \otimes \hat{h}\Psi_2 \otimes \cdots \otimes \Psi_n$$

$$\vdots$$

$$+ \Psi_1 \otimes \cdots \otimes \Psi_{n-1} \otimes \hat{h}\Psi_n$$

$$= \Big(\underbrace{\hat h \otimes \hat 1 \otimes \cdots \otimes 1}_{\hat h_{(1)}} + \underbrace{1 \otimes \hat h \otimes \cdots \otimes 1}_{\hat h_{(2)}}$$

$$+ \cdots \Big)\big(\Psi_1 \otimes \cdots \otimes \Psi_n\big)$$

$$H_n = \hat h_{(1)} + \cdots + \hat h_{(n)}$$

$$d\Gamma(h)\big|_{S_n(\mathcal{H}_1^{\otimes n})} = \hat h_{(1)} + \cdots + \hat h_{(n)}$$

теор. $\parallel$

$$\sum_{j,k} h_{j,k}\, \hat a_j^* \hat a_k \,\Big|\, \Psi_1 \otimes \cdots \otimes \Psi_n$$

$$\sum_{\substack{j_1 j_2 \\ k_1 k_2}} V_{j_1 j_2 k_1 k_2}\, \underbrace{\hat a^*_{j_1} \hat a^*_{j_2} \hat a_{k_1} \hat a_{k_2}} = \sum_{a<b} \hat V_{(a,b)}$$

Запазва
броя на частиците

$$\begin{array}{ccc} \vdots & & \vdots \\ \mathcal{H}_3 & \longrightarrow & \mathcal{H}_3 \\ \oplus & & \oplus \\ \mathcal{H}_2 & \xrightarrow[h_1+h_2+V_{12}]{} & \mathcal{H}_2 \\ \oplus & & \oplus \\ \mathcal{H}_1 & \xrightarrow[h]{} & \mathcal{H}_1 \\ \oplus & & \oplus \\ \mathbb{C}\Omega & \xrightarrow[1]{} & \mathbb{C}\Omega \end{array}$$

no particle production

$$\hat{\varphi}^4 = (\alpha \hat{a} + \bar{\alpha} a^*)^4$$

$$= \beta \hat{a} \hat{a} \hat{a} \hat{a}$$

$$+ \cdots + \gamma \hat{a}^* \hat{a}^* \hat{a} \hat{a} + \cdots$$