КВАНТОВА ИНФОРМАТИКА, ЛЕКЦИЯ 10 / 15.12.2021

ПАСАЖИТЕ С ДОПЪЛНИТЕЛЕН МАТЕРИАЛ СА ПОСТАВЕНИ В ЗЕЛЕН ЦВЯТ

С ЛИЛАВ ЦВЯТ СА ОТРАЗИ КОРЕКЦИИ СПРЯМО ПРЕДХОДНАТА ВЕРСИЯ

ТЕ СА НА СТР.: 208-220, 244-251

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2. Система от няколко кубити: изчислителен базис
3. Система от няколко кубити: квант. трансформации
4. Дискретната трансформация на Фурие: определение
5. Дискретната трансформация на Фурие: реализация с квантова мрежа
6. Първо понятие за квантов алгоритъм

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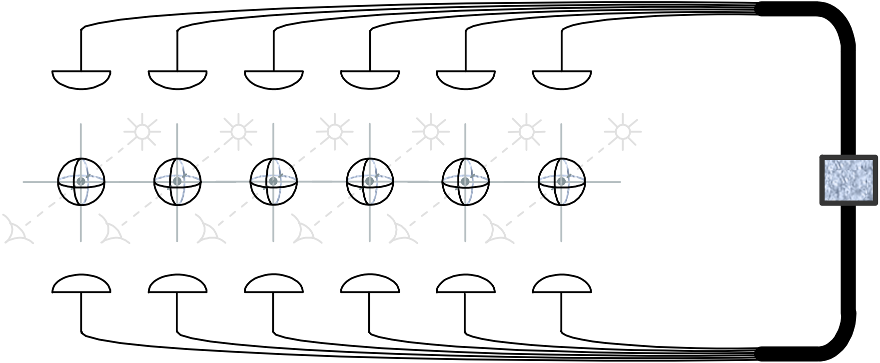

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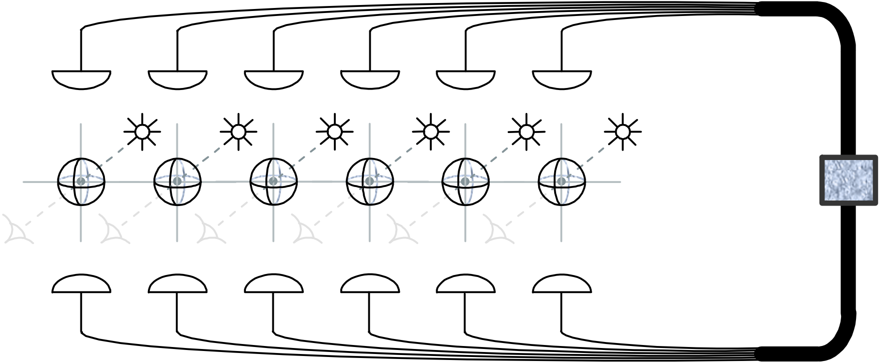

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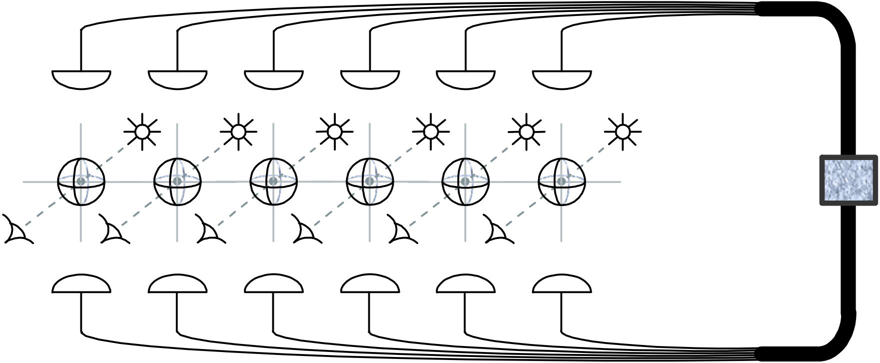

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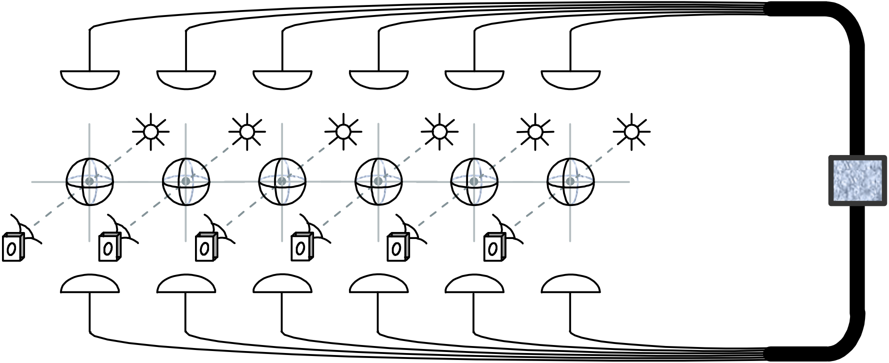

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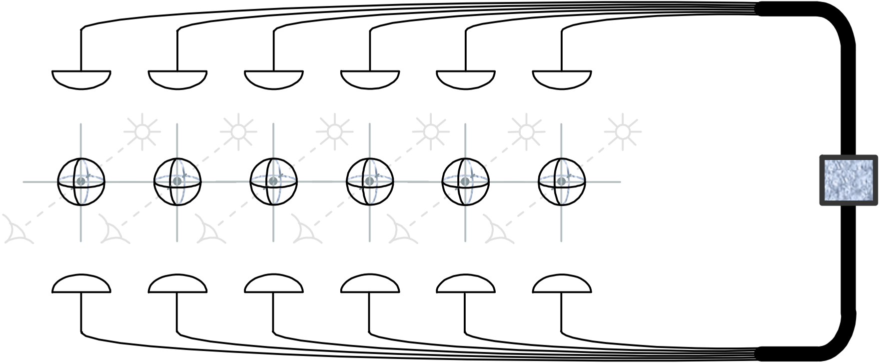

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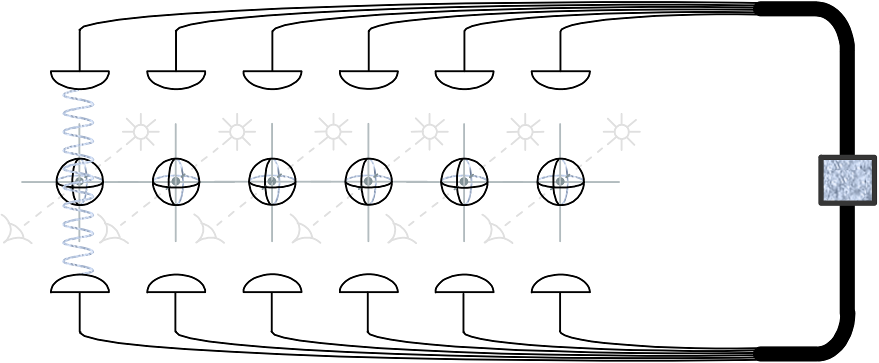

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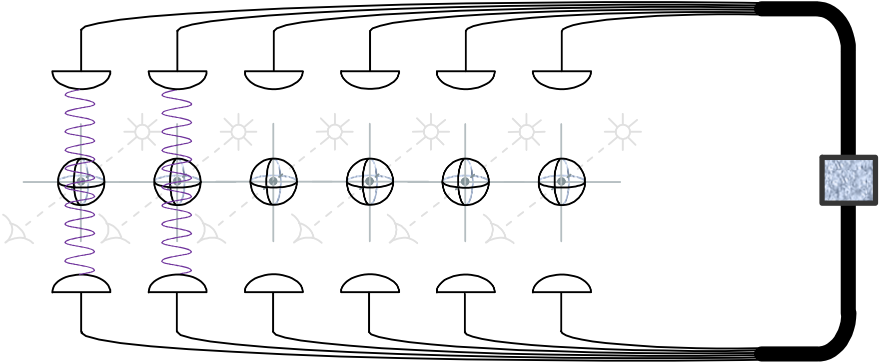

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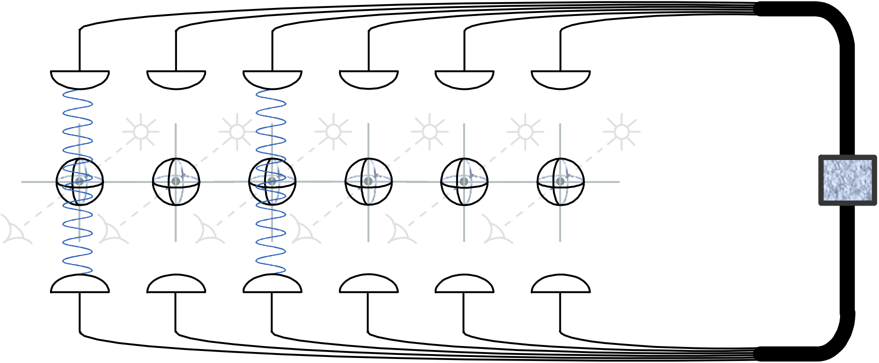

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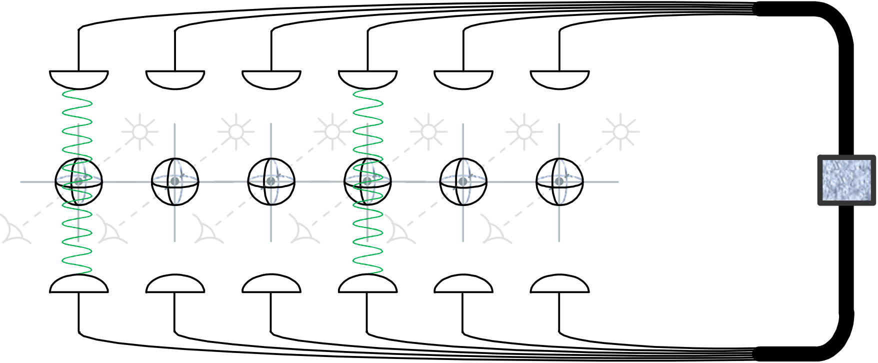

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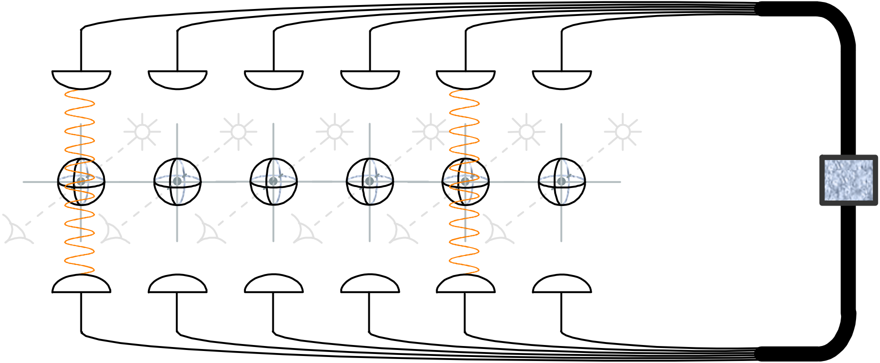

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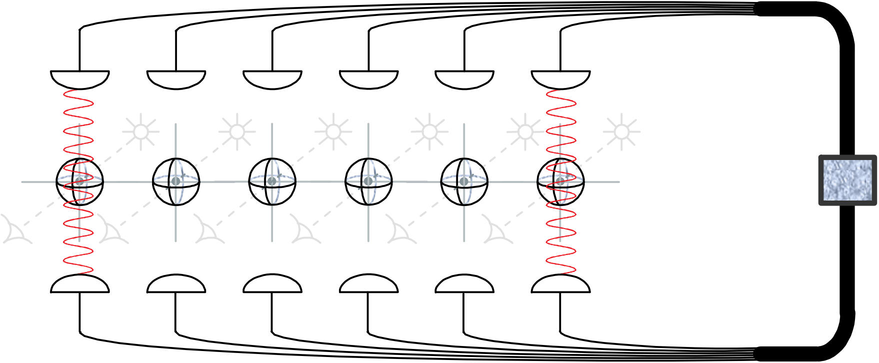

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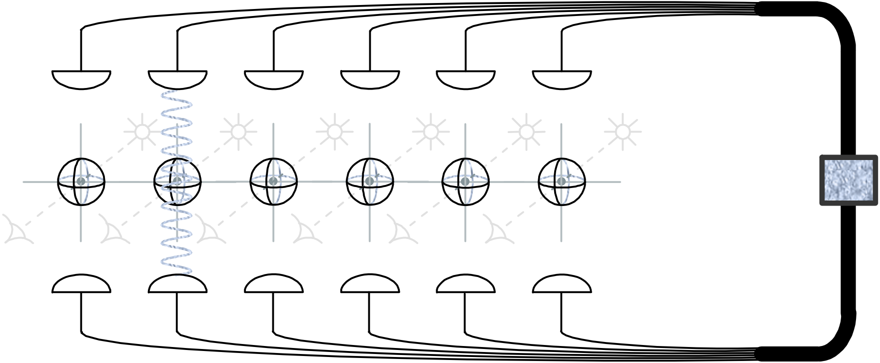

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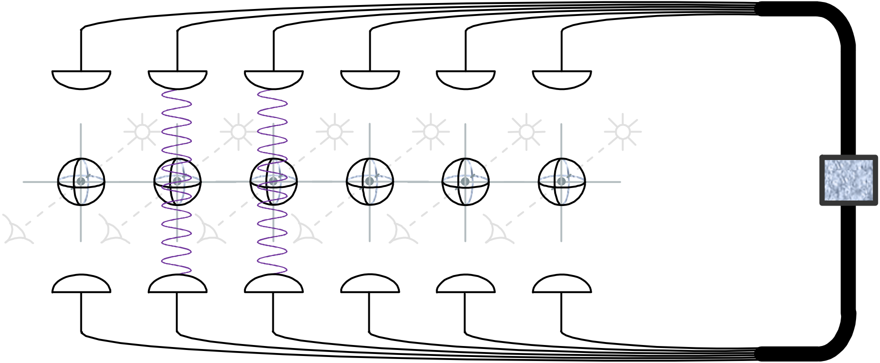

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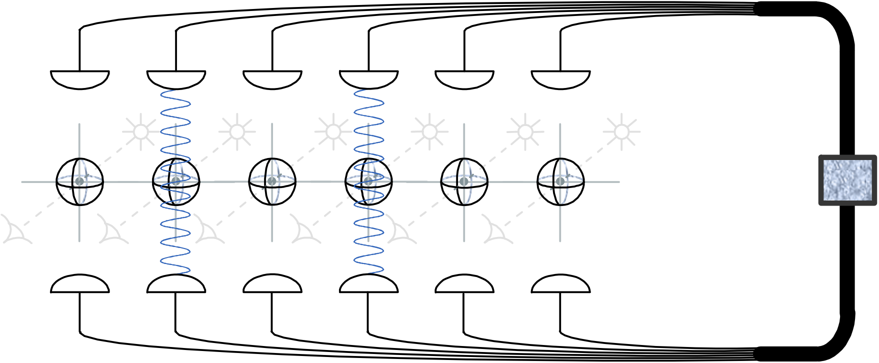

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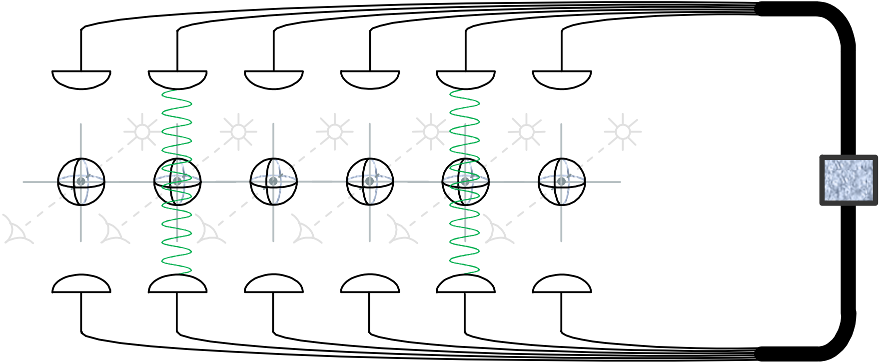

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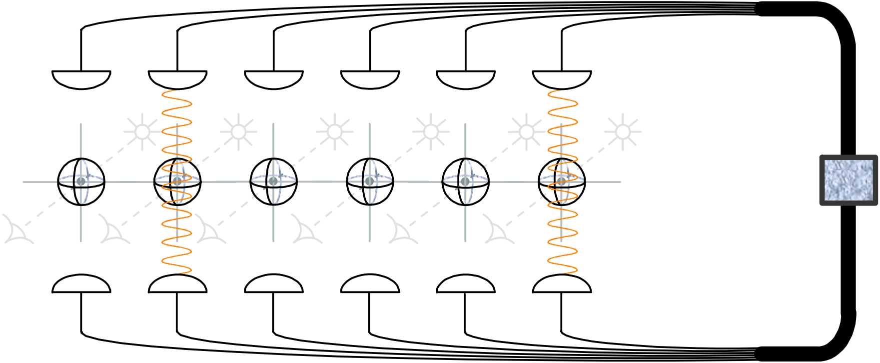

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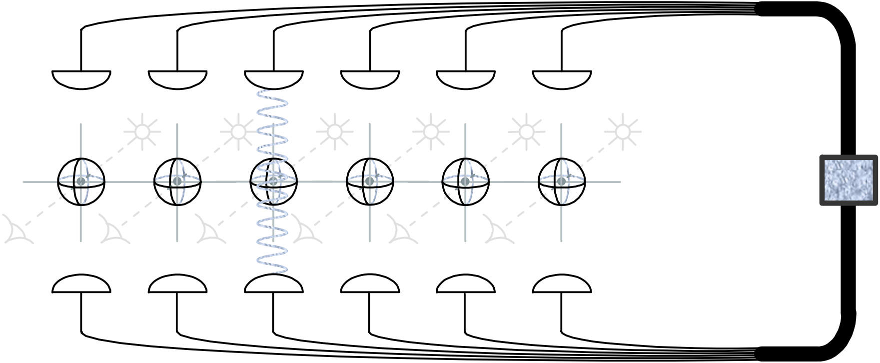

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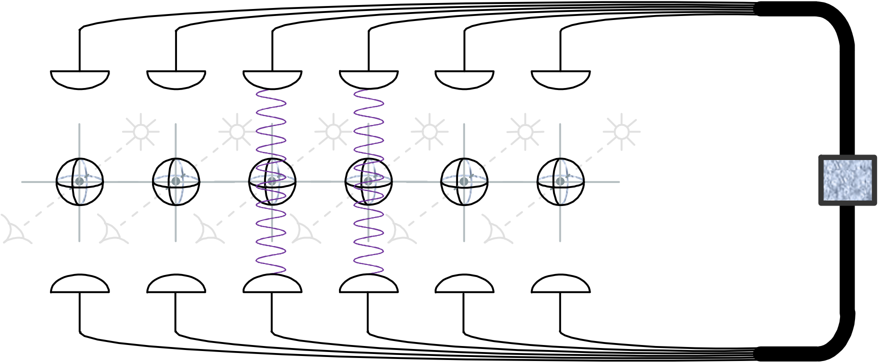

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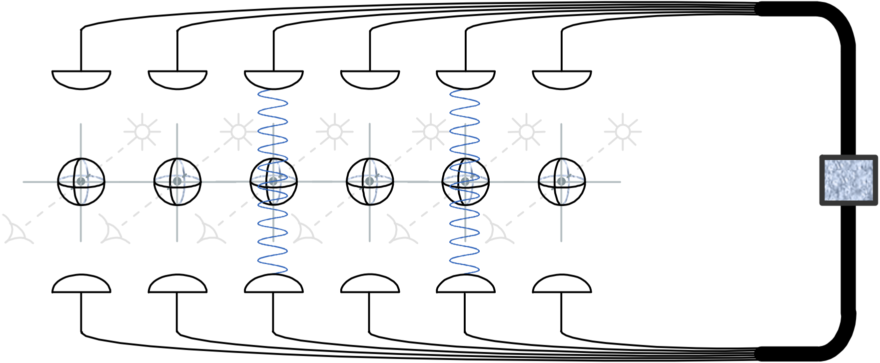

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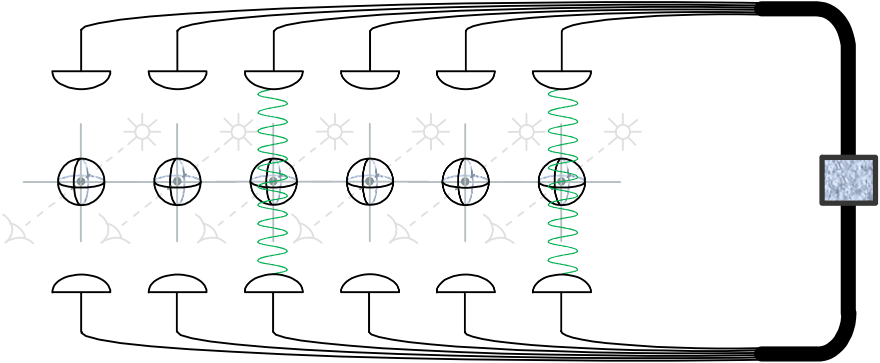

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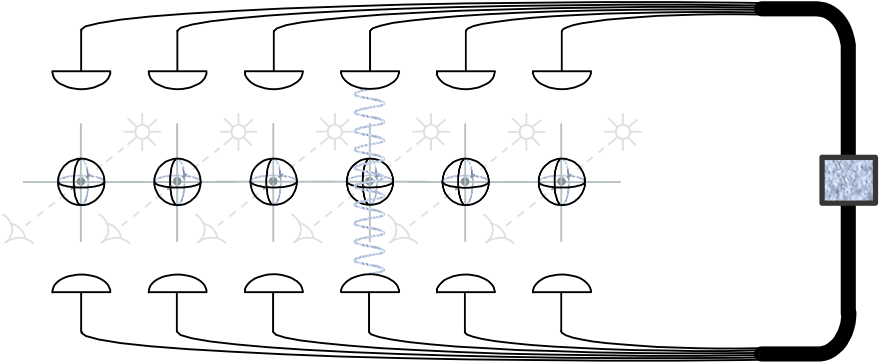

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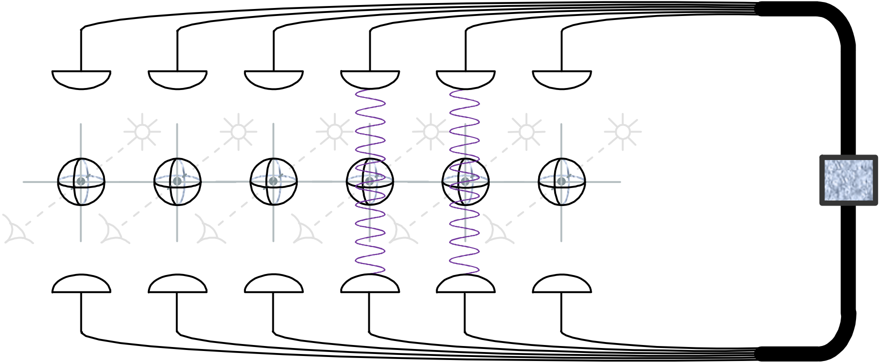

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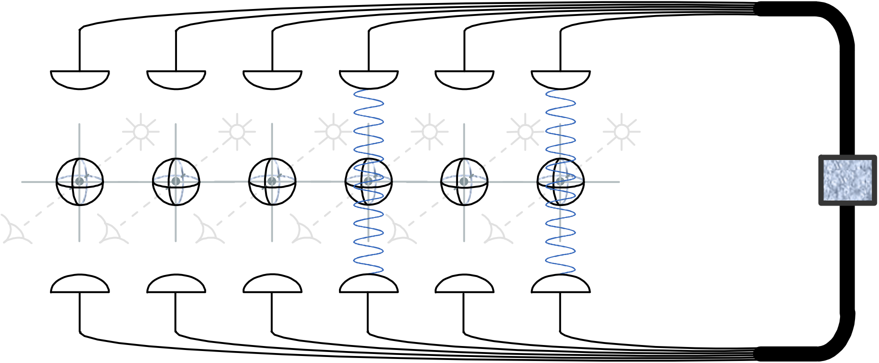

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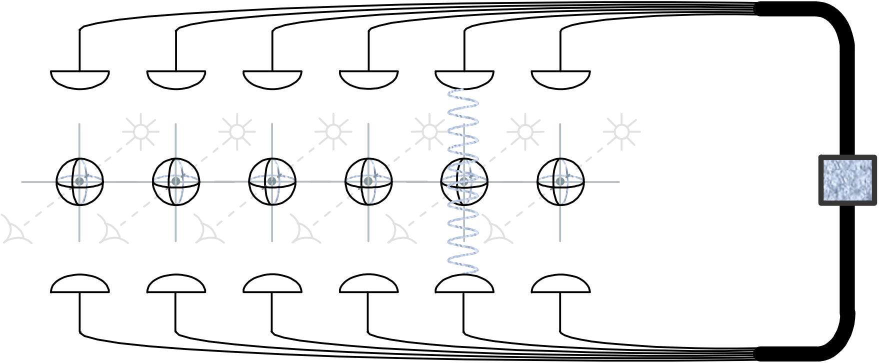

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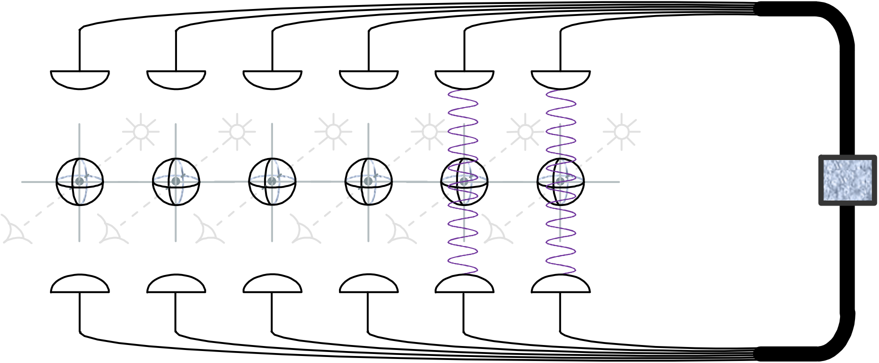

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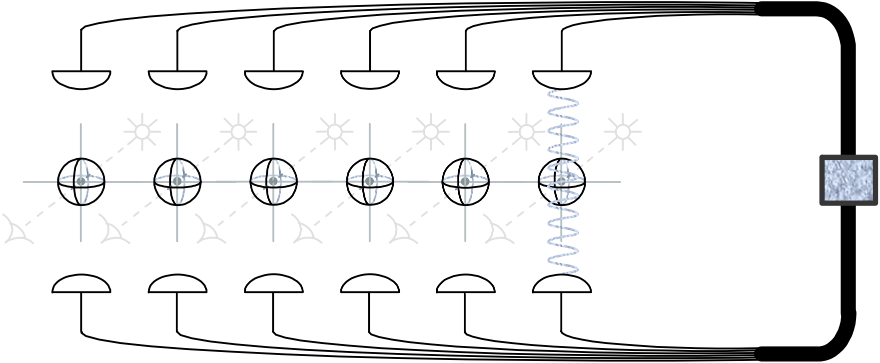

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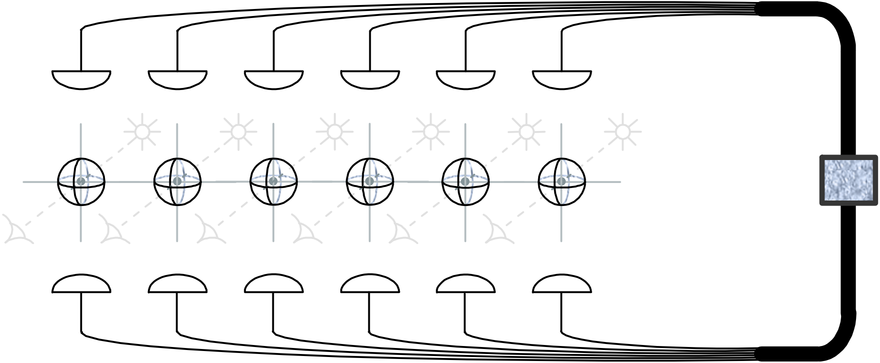

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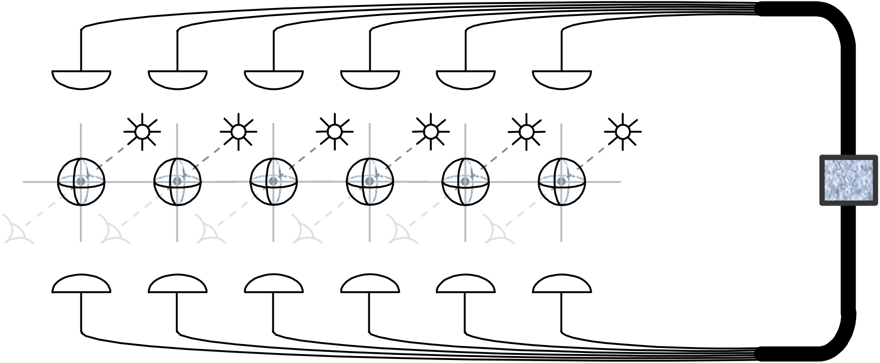

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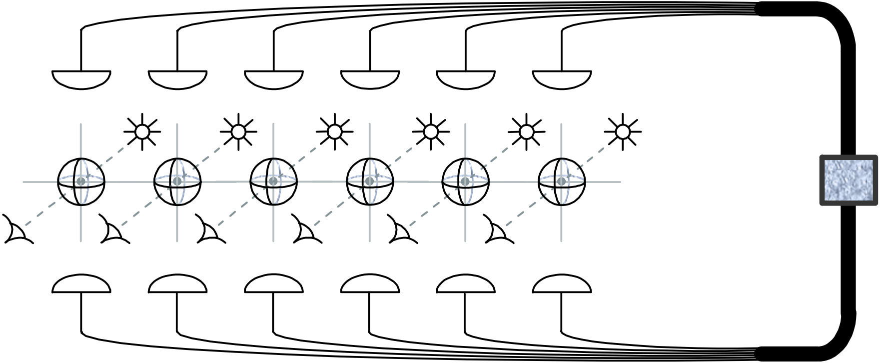

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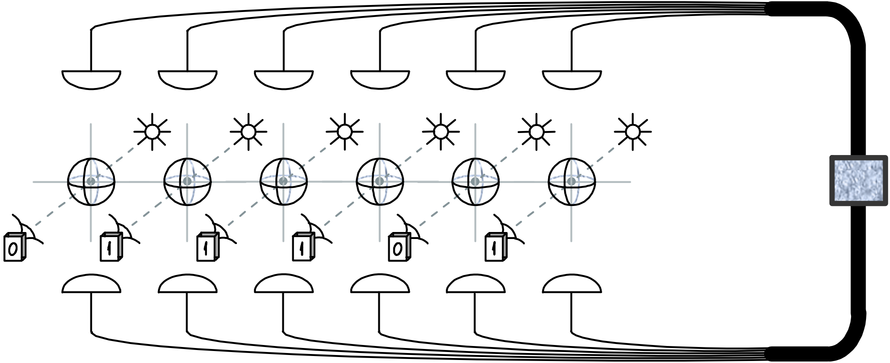

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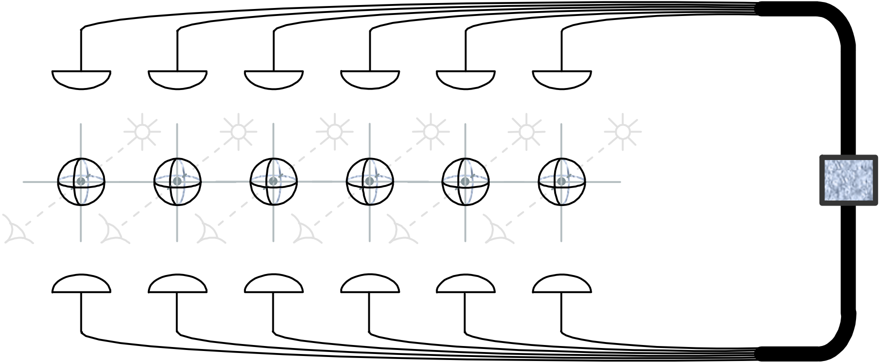

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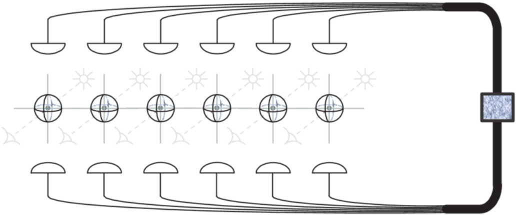

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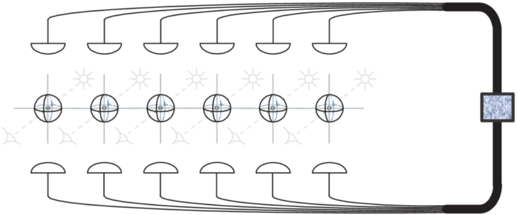

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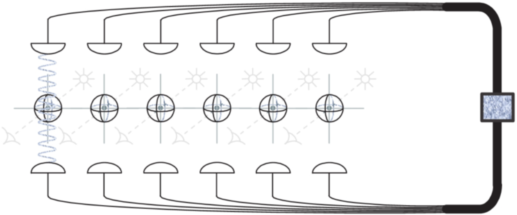

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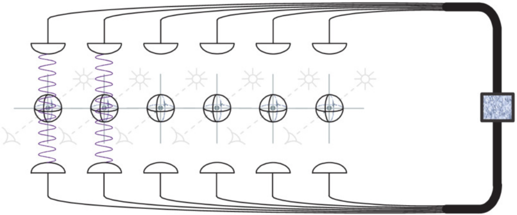

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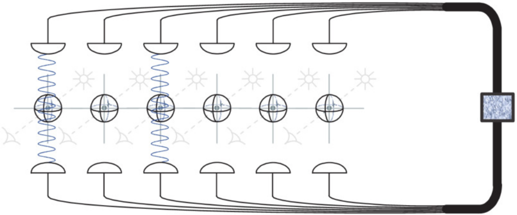

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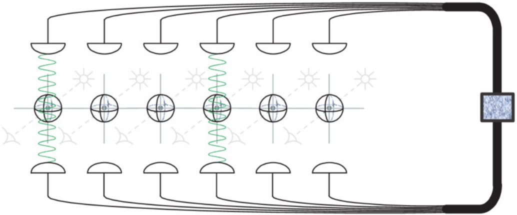

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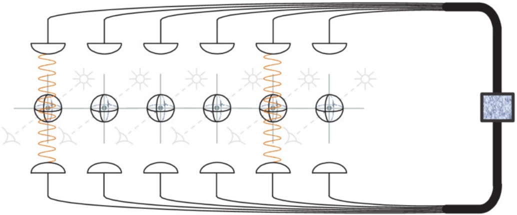

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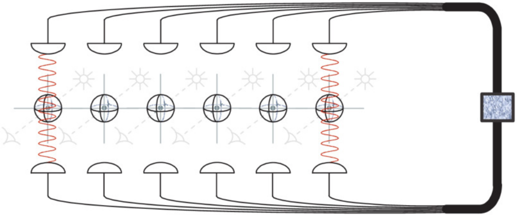

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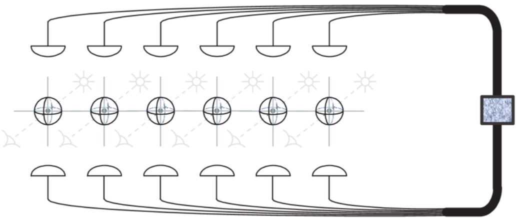

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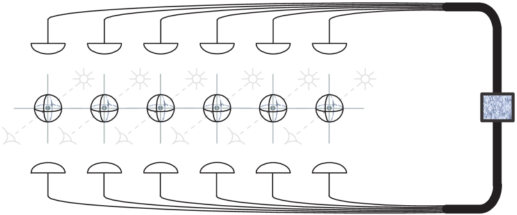

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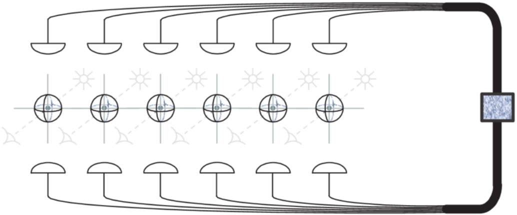

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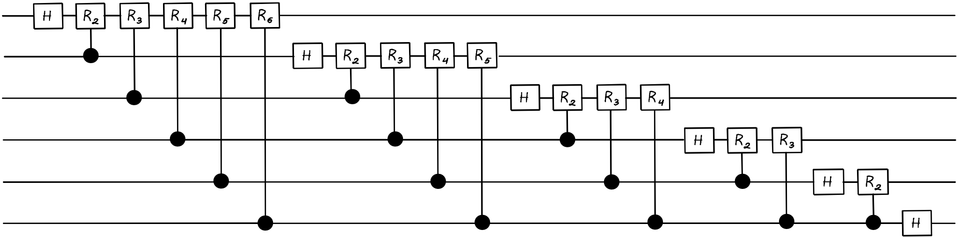

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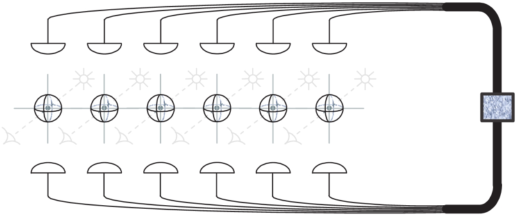

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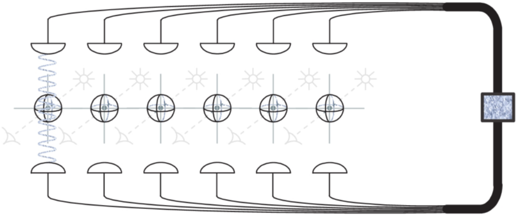

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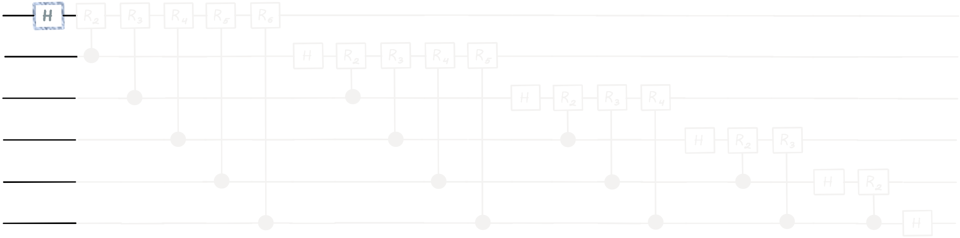

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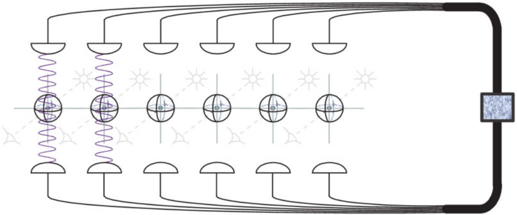

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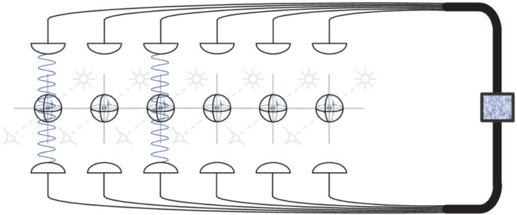

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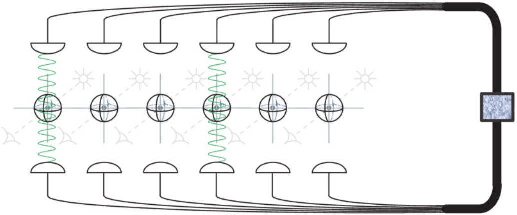

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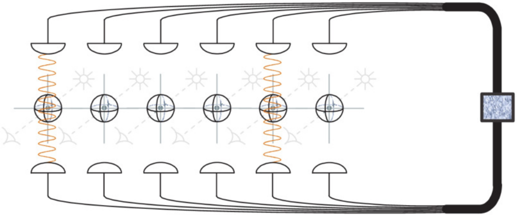

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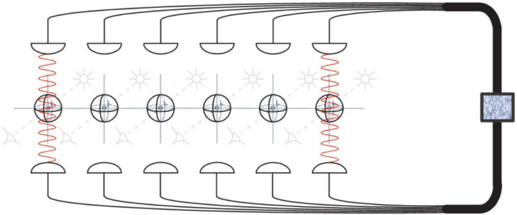

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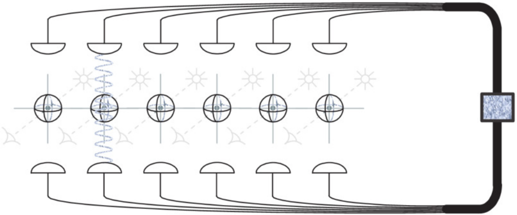

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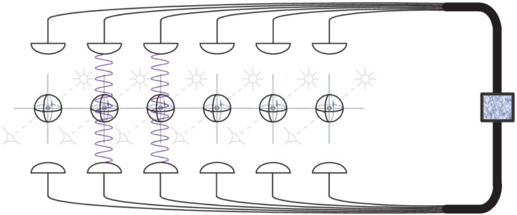

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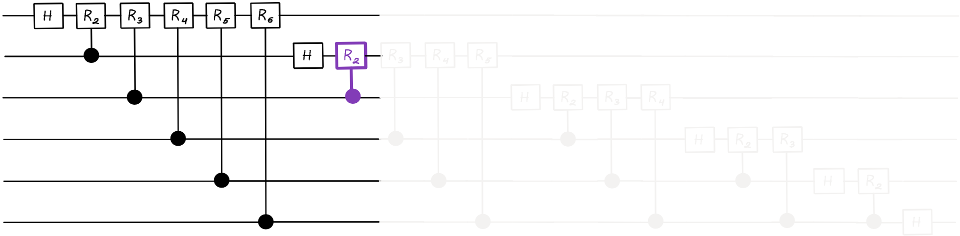

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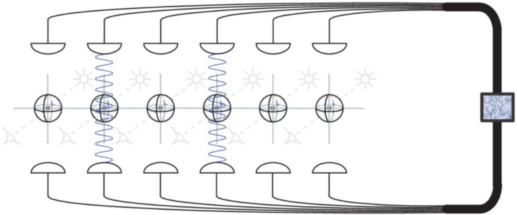

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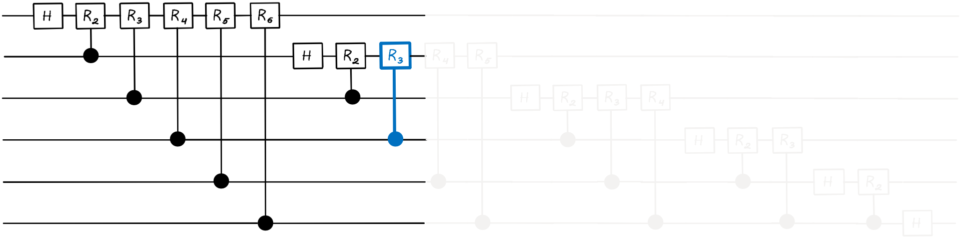

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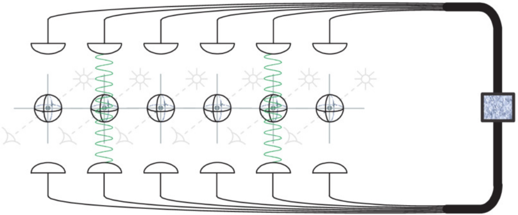

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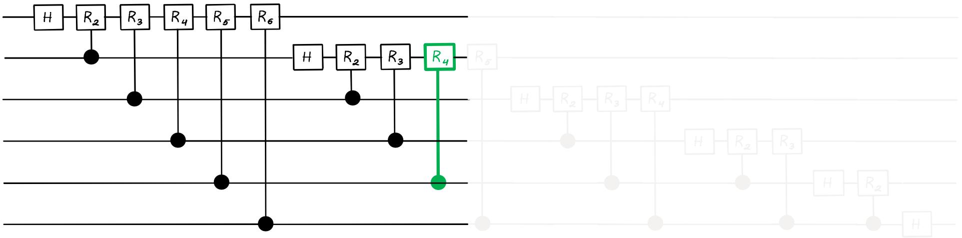

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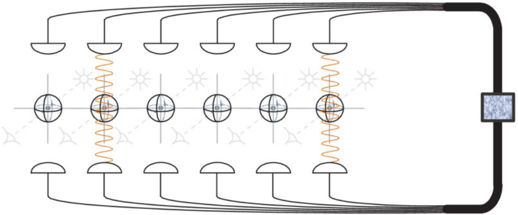

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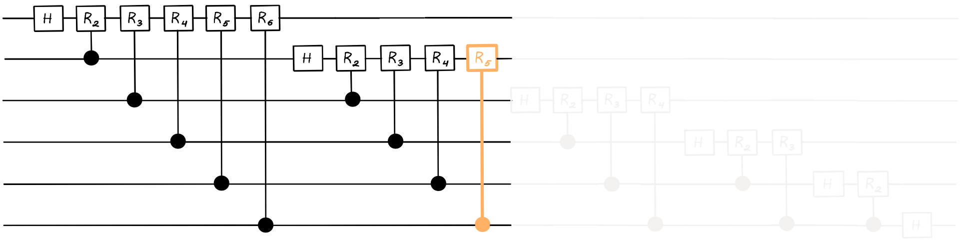

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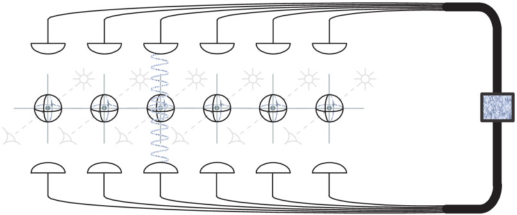

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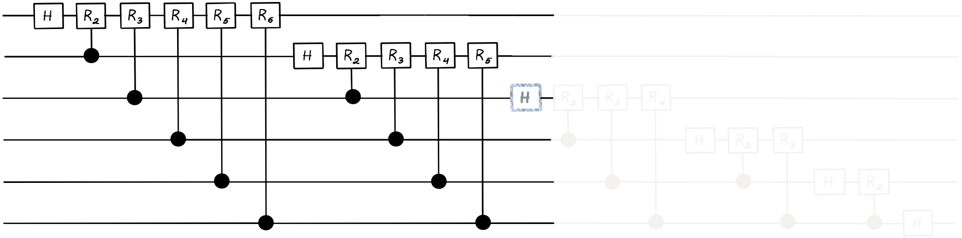

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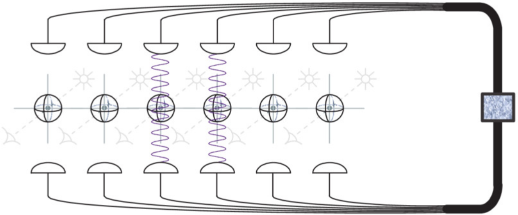

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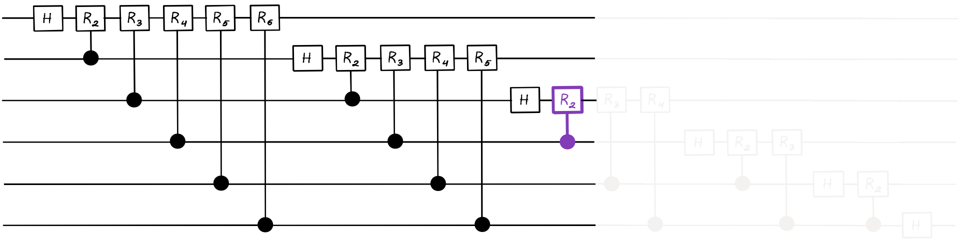

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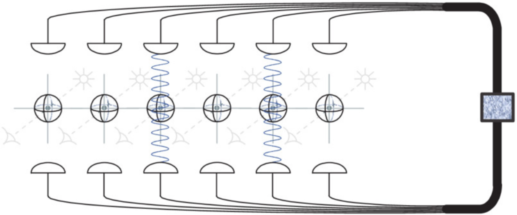

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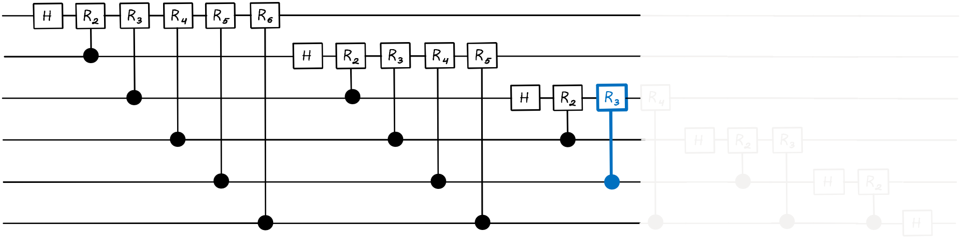

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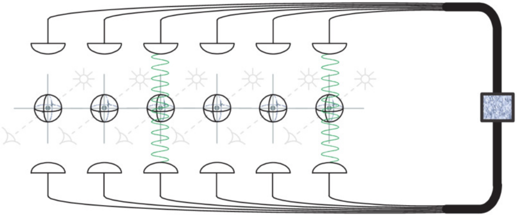

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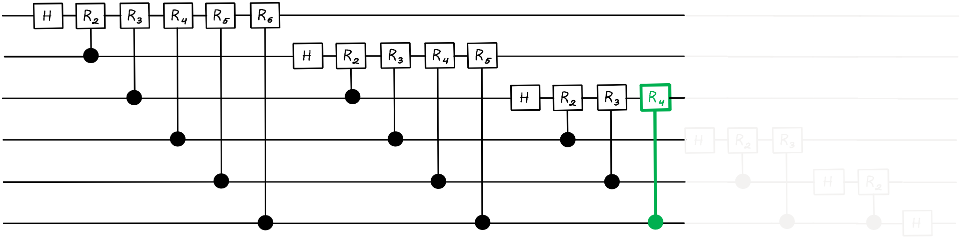

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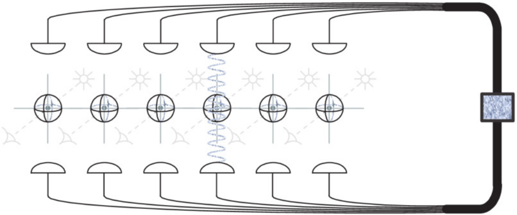

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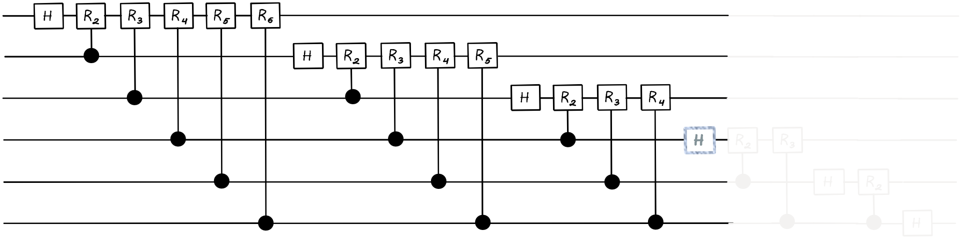

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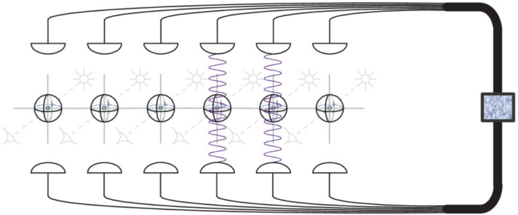

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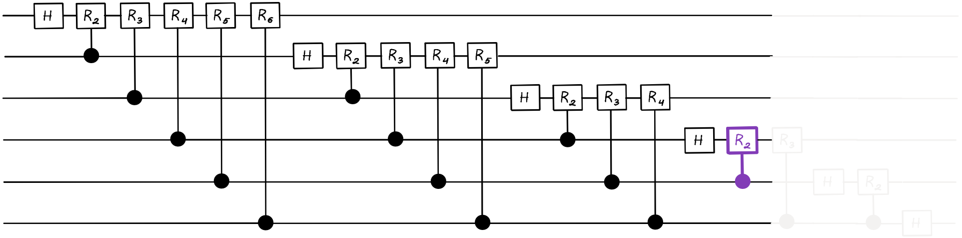

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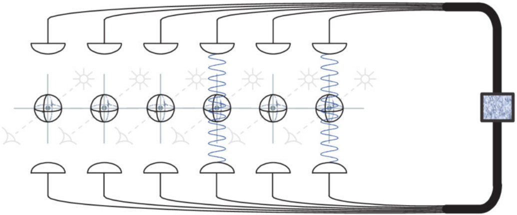

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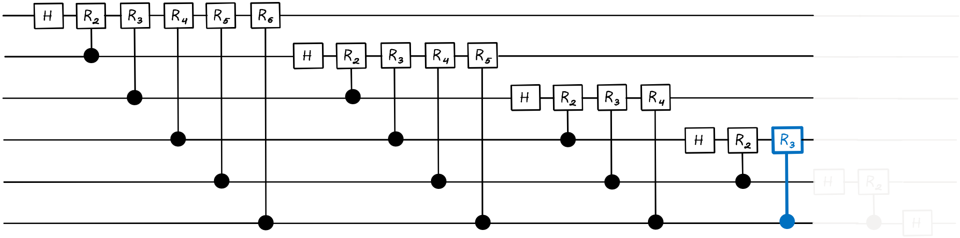

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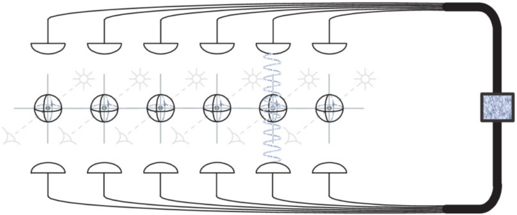

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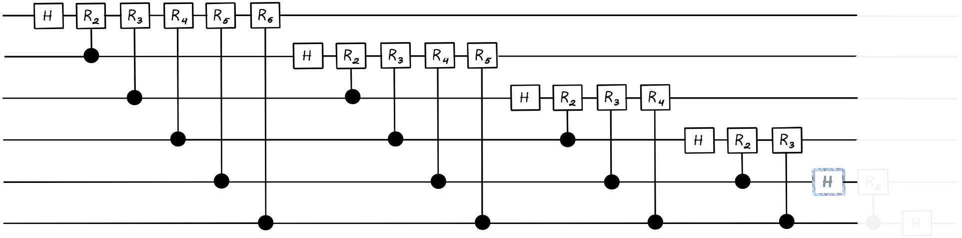

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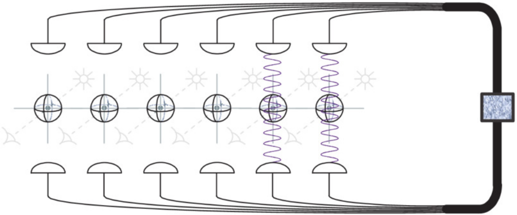

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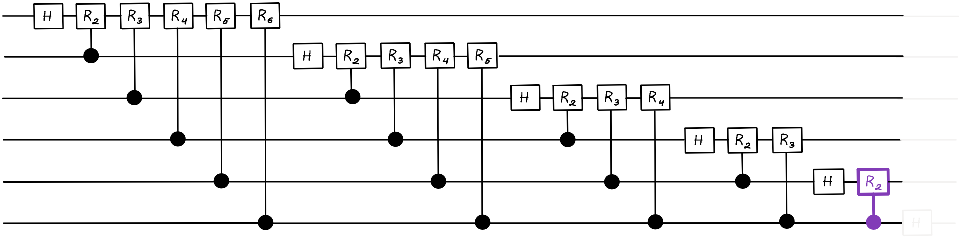

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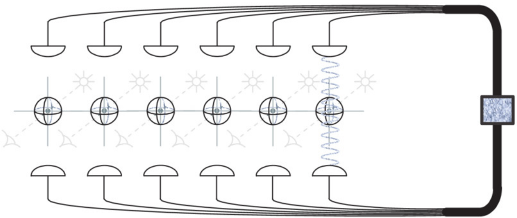

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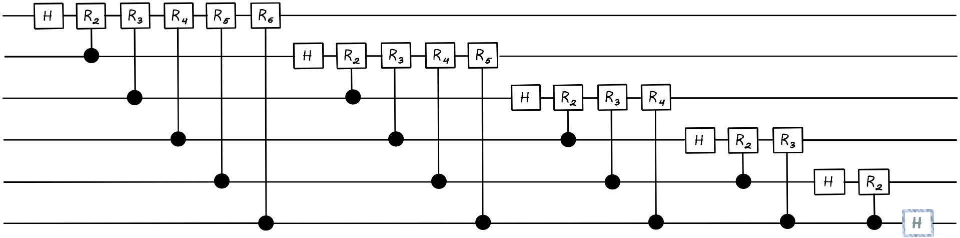

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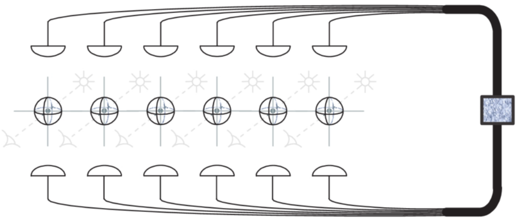

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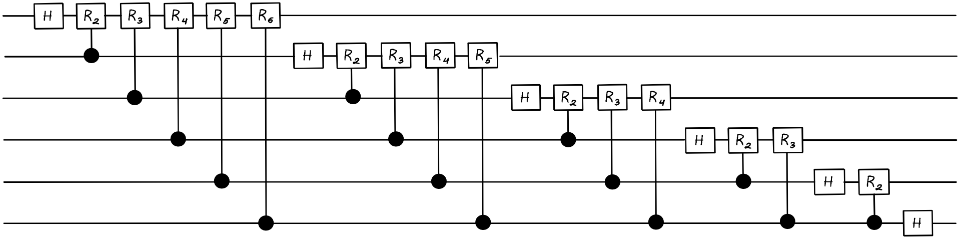

1. Квантови изчисления: начална илюстрация
2. Система от няколко кубити: изчислителен базис
3. Система от няколко кубити: квант. трансформации
4. Дискретната трансформация на Фурие: определение
5. Дискретната трансформация на Фурие: реализация с квантова мрежа
6. Първо понятие за квантов алгоритъм

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- За стандартния о.н.б. $e_0, \dots, e_{N-1} \in \mathbb{C}^N$

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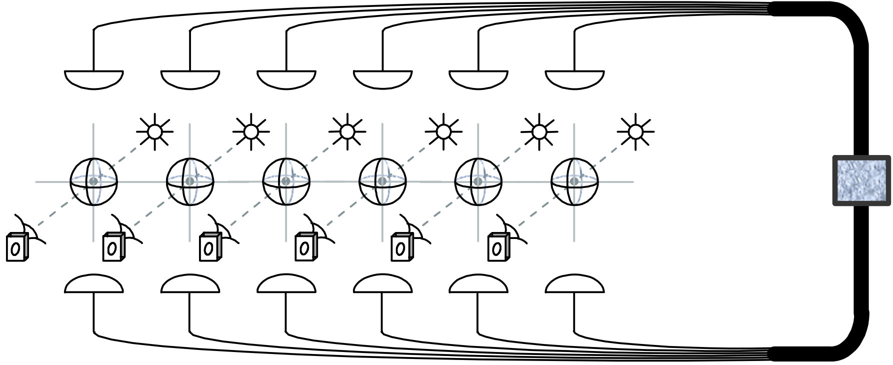
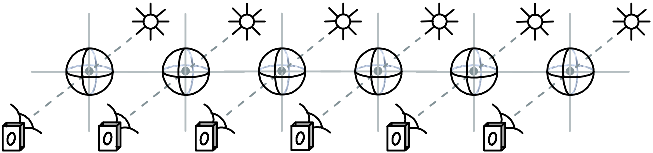
Сега обаче $|0\rangle \equiv e_0 \neq 0$ - нулевия вектор в $\mathbb{C}^N$!!!

Система от няколко кубити: изчислителен базис

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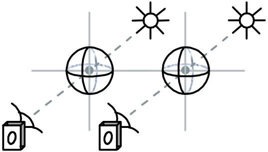
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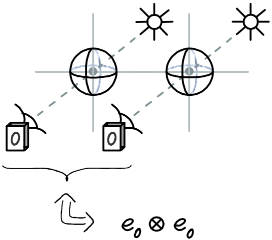
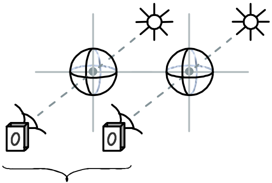


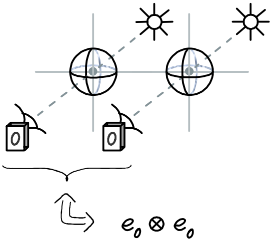
$e_0 \otimes e_0$

$$e_0 \otimes e_0 \equiv |e_0\rangle \otimes |e_0\rangle$$



$$\Longrightarrow \quad e_0 \otimes e_0$$

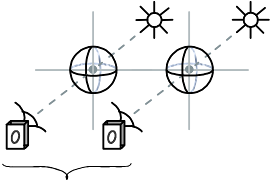
$$e_0 \otimes e_0 \equiv |e_0\rangle \otimes |e_0\rangle \equiv |0\rangle \otimes |0\rangle$$



$$e_0 \otimes e_0$$

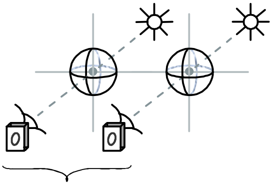
$$e_0 \otimes e_0 \equiv |e_0\rangle \otimes |e_0\rangle \equiv |0\rangle \otimes |0\rangle \boxed{\equiv |0\rangle|0\rangle \equiv |0,0\rangle}$$

нови означения



$e_0 \otimes e_0$

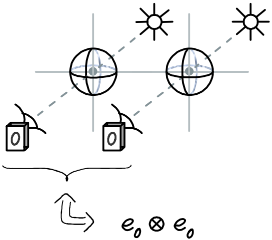
$$e_0 \otimes e_0 \equiv |e_0\rangle \otimes |e_0\rangle \equiv |0\rangle \otimes |0\rangle \equiv |0\rangle|0\rangle \equiv |0,0\rangle$$



$e_0 \otimes e_0$

$$\underbrace{e_0 \otimes e_0}_{\rotatebox{90}{$\equiv$}} \equiv |e_0\rangle \otimes |e_0\rangle \equiv |0\rangle \otimes |0\rangle \equiv |0\rangle|0\rangle \equiv |0,0\rangle$$

$$\overbrace{\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}}$$



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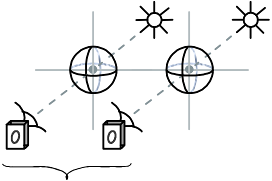
$$\underbrace{e_0 \otimes e_0}_{\equiv} \equiv |e_0\rangle \otimes |e_0\rangle \equiv |0\rangle \otimes |0\rangle \equiv |0\rangle|0\rangle \equiv |0,0\rangle$$

$$\overbrace{\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

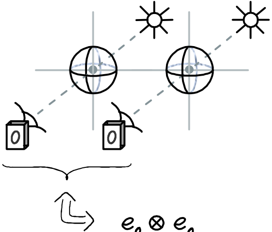


$\Leftrightarrow \quad e_0 \otimes e_0$

$$\underbrace{e_0 \otimes e_0} \equiv |e_0\rangle \otimes |e_0\rangle \equiv |0\rangle \otimes |0\rangle \equiv |0\rangle|0\rangle \equiv |0,0\rangle$$

$$\equiv$$

$$\overbrace{\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \equiv e_0$$



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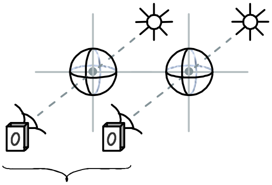
$$\equiv$$

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \equiv e_0 \in \{e_0, e_1, e_2, e_3\}$$ – стандартния о.н.б. в $\mathbb{C}^4$



$$e_0 \otimes e_0$$

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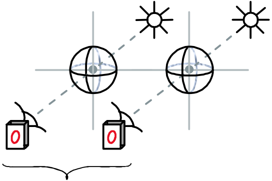
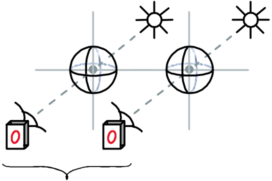


$e_0 \otimes e_0$

$$\underbrace{e_0 \otimes e_0}_{} \equiv |e_0\rangle \otimes |e_0\rangle \equiv |0\rangle \otimes |0\rangle \equiv |0\rangle|0\rangle \equiv |0,0\rangle \equiv |\underbrace{[00]_2}_{}\rangle \equiv |0\rangle$$

числото "0" в двоична система

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \equiv e_0 \in \{e_0, e_1, e_2, e_3\}$$ – стандартния о.н.б. в $\mathbb{C}^4$



$\Longleftrightarrow \quad e_0 \otimes e_0$

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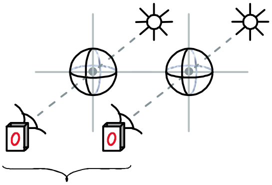

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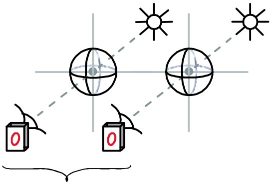
$$\underbrace{e_0 \otimes e_0}_{} \equiv |e_0\rangle \otimes |e_0\rangle \equiv |0\rangle \otimes |0\rangle \equiv |0\rangle|0\rangle \equiv |0,0\rangle \equiv |\underbrace{[00]_2}_{\text{числото "0" в двоична система}}\rangle \equiv |0\rangle \leftarrow \begin{cases}\text{има} \\ \text{различен смисъл !!!}\end{cases}$$

$$\equiv$$

$$\underbrace{\begin{pmatrix}1\\0\end{pmatrix}}_{e_0 \in \mathbb{C}^2} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \equiv e_0 \in \{e_0, e_1, e_2, e_3\} \text{ - стандартния о.н.б. в } \mathbb{C}^4$$

имат различен смисъл !!!



$\Updownarrow$ $e_0 \otimes e_0$

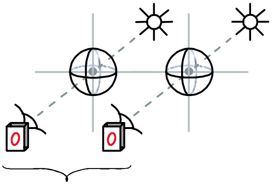
$$\underbrace{e_0^{(2)} \otimes e_0^{(2)}} \equiv |e_0^{(2)}\rangle \otimes |e_0^{(2)}\rangle \equiv |0\rangle \otimes |0\rangle \equiv |0\rangle|0\rangle \equiv |0,0\rangle \equiv |\underbrace{[00]_2}_{\text{числото "0" в двоична система}}\rangle \equiv |0\rangle \leftarrow \left\{\begin{array}{l}\text{има} \\ \text{различен смисъл !!!}\end{array}\right.$$

$$|||$$

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \equiv e_0^{(4)} \in \{e_0^{(4)}, e_1^{(4)}, e_2^{(4)}, e_3^{(4)}\} \text{ - стандартния о.н.б. в } \mathbb{C}^4$$

$e_0^{(2)} \in \mathbb{C}^2$

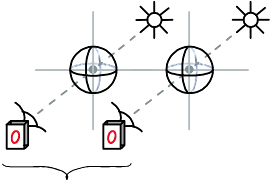
имат
различен смисъл !!!



$$e_0^{(2)} \otimes e_0^{(2)}$$

$$\underbrace{e_0 \otimes e_0}_{\equiv} \equiv |e_0\rangle \otimes |e_0\rangle \equiv |0\rangle \otimes |0\rangle \equiv |0\rangle|0\rangle \equiv |0,0\rangle \equiv |\underbrace{[00]_2}_{\text{числото "0" в двоична система}}\rangle \equiv |0\rangle$$

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \equiv e_0 \in \{e_0, e_1, e_2, e_3\} \text{ - стандартния о.н.б. в } \mathbb{C}^4$$



$e_0 \otimes e_0$

$$\underbrace{e_0 \otimes e_0}_{\equiv} \equiv |e_0\rangle \otimes |e_0\rangle \equiv |0\rangle \otimes |0\rangle \equiv |0\rangle|0\rangle \equiv |0,0\rangle \equiv |\underbrace{[00]_2}_{\text{числото "0" в двоична система}}\rangle \equiv |0\rangle$$

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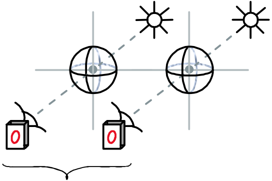


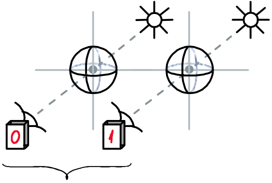
$$e_0 \otimes e_0$$

$$\underbrace{e_0 \otimes e_1}_{} \equiv |e_0\rangle \otimes |e_1\rangle \equiv |0\rangle \otimes |1\rangle \equiv |0\rangle|1\rangle \equiv |0,1\rangle \equiv |\underbrace{[01]}_{}{}_2\rangle \equiv |1\rangle$$

числото "1" в двоична система

$$\equiv$$

$$\overbrace{\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix}} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \equiv e_1 \in \{e_0, e_1, e_2, e_3\}$$ – стандартния о.н.б. в $\mathbb{C}^4$



$$e_0 \otimes e_1$$

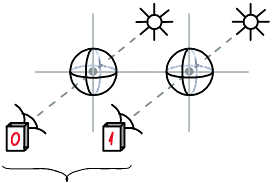
$$\underbrace{e_0 \otimes e_1}_{\displaystyle \equiv \; \overbrace{\begin{pmatrix}1\\0\end{pmatrix}}^{} \otimes \begin{pmatrix}0\\1\end{pmatrix}} \equiv |e_0\rangle \otimes |e_1\rangle \equiv |0\rangle \otimes |1\rangle \equiv |0\rangle|1\rangle \equiv |0,1\rangle \equiv |\underbrace{[01]_2}_{\text{числото „1" в двоична система}}\rangle \equiv |1\rangle$$

← има различен смисъл !!!

$$\underbrace{\begin{pmatrix}1\\0\end{pmatrix}}_{e_0} \otimes \underbrace{\begin{pmatrix}0\\1\end{pmatrix}}_{e_1} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \equiv e_1 \in \{e_0, e_1, e_2, e_3\}$$ – стандартния о.н.б. в $\mathbb{C}^4$

$e_0, e_1 \in \mathbb{C}^2$

имат различен смисъл !!!



$\Updownarrow \quad e_0 \otimes e_1$

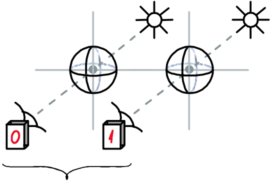
$$e_0^{(2)}\otimes e_1^{(2)} \equiv |e_0^{(2)}\rangle\otimes|e_1^{(2)}\rangle \equiv |0\rangle\otimes|1\rangle \equiv |0\rangle|1\rangle \equiv |0,1\rangle \equiv |\underbrace{[01]_2}_{\text{числото „1" в двоична система}}\rangle \equiv |1\rangle$$

← { има различен смисъл !!!

$$\underbrace{e_0^{(2)}\otimes e_1^{(2)}}_{\equiv} \equiv \underbrace{\begin{pmatrix}1\\0\end{pmatrix}}_{e_0^{(2)}}\otimes\underbrace{\begin{pmatrix}0\\1\end{pmatrix}}_{e_1^{(2)}} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \equiv e_1^{(4)} \in \{e_0^{(4)}, e_1^{(4)}, e_2^{(4)}, e_3^{(4)}\}$$ – стандартния о.н.б. в $\mathbb{C}^4$

$e_0^{(2)},\ e_1^{(2)} \in \mathbb{C}^2$

имат
различен смисъл !!!

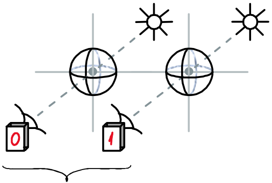


$\Longleftrightarrow\ e_0^{(2)}\otimes e_1^{(2)}$

$$\underbrace{e_0 \otimes e_1}_{} \equiv |e_0\rangle \otimes |e_1\rangle \equiv |0\rangle \otimes |1\rangle \equiv |0\rangle|1\rangle \equiv |0,1\rangle \equiv |\underbrace{[01]_2}_{\text{числото "1" в двоична система}}\rangle \equiv |1\rangle$$

$$\text{|||}$$

$$\overbrace{\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix}} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \equiv e_1 \in \{e_0, e_1, e_2, e_3\} \text{ – стандартния о.н.б. в } \mathbb{C}^4$$

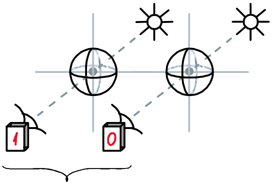


$$\Updownarrow \quad e_0 \otimes e_1$$

$$\underbrace{e_1 \otimes e_0}_{} \equiv |e_1\rangle \otimes |e_0\rangle \equiv |1\rangle \otimes |0\rangle \equiv |1\rangle|0\rangle \equiv |1,0\rangle \equiv |\underbrace{[10]_2}_{\text{числото „2" в двоична система}}\rangle \equiv |2\rangle$$

$$\equiv$$

$$\overbrace{\begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}}^{} = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \equiv e_2 \in \{e_0, e_1, e_2, e_3\}$$ – стандартния о.н.б. в $\mathbb{C}^4$

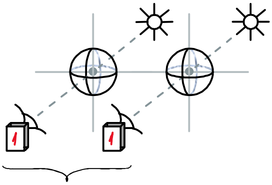


$$\Longleftrightarrow \quad e_1 \otimes e_0$$

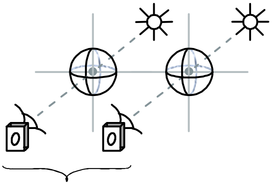
$$\underbrace{e_1 \otimes e_1}_{} \equiv |e_1\rangle \otimes |e_1\rangle \equiv |1\rangle \otimes |1\rangle \equiv |1\rangle|1\rangle \equiv |1,1\rangle \equiv |\underbrace{[11]_2}_{\text{числото „3" в двоична система}}\rangle \equiv |3\rangle$$

$$\equiv$$

$$\begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \equiv e_3 \in \{e_0, e_1, e_2, e_3\} \text{ – стандартния о.н.б. в } \mathbb{C}^4$$

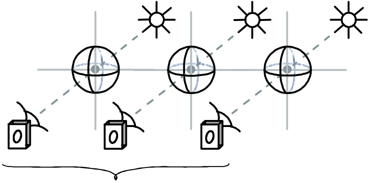


$$e_1 \otimes e_1$$



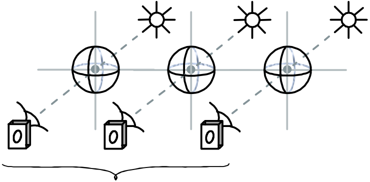
$e_0 \otimes e_0$

Система от няколко кубити: изчислителен базис



$e_0 \otimes e_0 \otimes e_0$

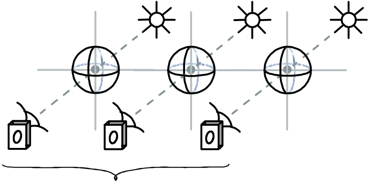
$$\underbrace{e_0 \otimes e_0 \otimes e_0}_{\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}}$$



$$e_0 \otimes e_0 \otimes e_0$$

$$\underbrace{e_0 \otimes (e_0 \otimes e_0)}$$

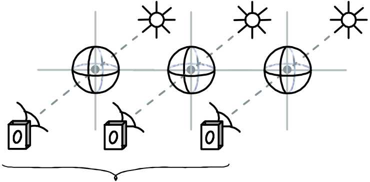
$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \left(\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}\right)$$



$$e_0 \otimes e_0 \otimes e_0$$

$$\underbrace{e_0 \otimes (e_0 \otimes e_0)}$$

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \left(\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}\right) = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

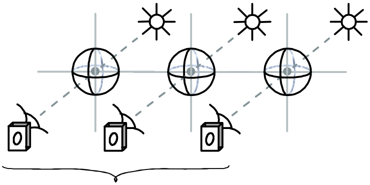


$$e_0 \otimes e_0 \otimes e_0$$

$$\left(\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right)\otimes\begin{pmatrix}1\\0\end{pmatrix}$$

$$(e_0 \otimes (e_0) \otimes e_0)$$

$$\begin{pmatrix}1\\0\end{pmatrix}\otimes\left(\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right) = \begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

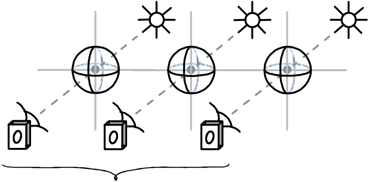


$$e_0 \otimes e_0 \otimes e_0$$

$$\left(\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}\right) \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}$$

$$(e_0 \otimes (e_0) \otimes e_0)$$

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \left(\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}\right) = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$



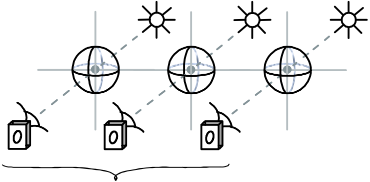
$$e_0 \otimes e_0 \otimes e_0$$

$$\left(\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right)\otimes\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}$$

$$\overbrace{\underbrace{(e_0 \otimes (e_0) \otimes e_0)}}$$

$$\begin{pmatrix}1\\0\end{pmatrix}\otimes\left(\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right) = \begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

$$= \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix}$$



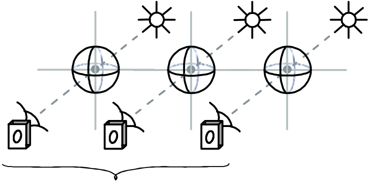
$$e_0 \otimes e_0 \otimes e_0$$

Система от няколко кубити: изчислителен базис

$$\left(\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right)\otimes\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}$$

$$(e_0 \otimes (e_0) \otimes e_0) = \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix}$$

$$\begin{pmatrix}1\\0\end{pmatrix}\otimes\left(\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right) = \begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$



$$e_0 \otimes e_0 \otimes e_0$$

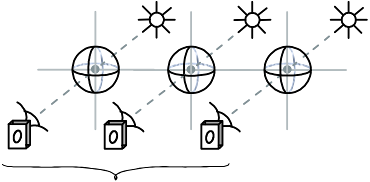
Система от няколко кубити: изчислителен базис

$$\left(\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right)\otimes\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}$$

$$(e_0 \otimes (e_0) \otimes e_0) = \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix}$$

$$\begin{pmatrix}1\\0\end{pmatrix}\otimes\left(\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right) = \begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

Тензорното произведение е асоциативно
(т.е., скобите могат да се местят)



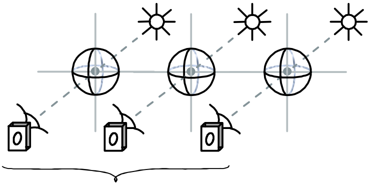
$$e_0 \otimes e_0 \otimes e_0$$

Система от няколко кубити: изчислителен базис

$$\left(\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right)\otimes\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}1\\0\\0\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}$$

$$(e_0\otimes(e_0)\otimes e_0) = \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix}$$

$$\begin{pmatrix}1\\0\end{pmatrix}\otimes\left(\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right)=\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

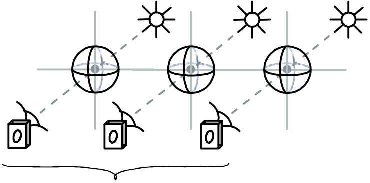


$$e_0\otimes e_0\otimes e_0$$

$$\left(\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right)\otimes\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}$$

$$(e_0 \otimes (e_0) \otimes e_0) = \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} = e_0$$

$$\begin{pmatrix}1\\0\end{pmatrix}\otimes\left(\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right) = \begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$



$$e_0 \otimes e_0 \otimes e_0$$

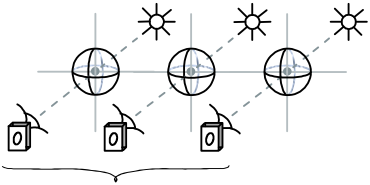
Система от няколко кубити: изчислителен базис

$$\left(\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right)\otimes\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}$$

$$(e_0 \otimes (e_0) \otimes e_0) \;=\!=\; \longrightarrow \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} = e_0 \in \{e_0, e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$$

– стандартния о.н.б. в $\mathbb{C}^8$

$$\begin{pmatrix}1\\0\end{pmatrix}\otimes\left(\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right) = \begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$



$e_0 \otimes e_0 \otimes e_0$

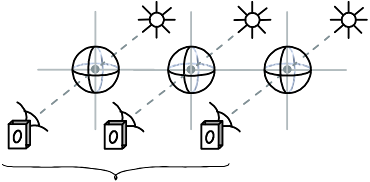
Система от няколко кубити: изчислителен базис

$$\left(\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right)\otimes\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}$$

$$\overbrace{\underbrace{\left(e_0^{(2)}\otimes\left(e_0^{(2)}\right)\otimes e_0^{(2)}\right)}}=\!= \longrightarrow \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} = e_0^{(8)} \in \{e_0^{(8)}, e_1^{(8)}, e_2^{(8)}, e_3^{(8)}, e_4^{(8)}, e_5^{(8)}, e_6^{(8)}, e_7^{(8)}\}$$

– стандартния о.н.б. в $\mathbb{C}^8$

$$\begin{pmatrix}1\\0\end{pmatrix}\otimes\left(\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right) = \begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

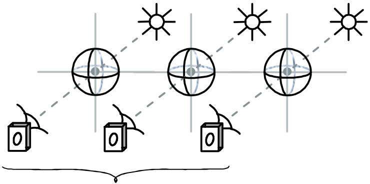


$$e_0^{(2)}\otimes e_0^{(2)}\otimes e_0^{(2)}$$

$$\underbrace{e_0^{(2)} \otimes \left(e_0^{(2)} \otimes e_0^{(2)}\right)}$$

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \left(\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}\right)$$

$$= \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} = e_0^{(8)} \in \{e_0^{(8)}, e_1^{(8)}, e_2^{(8)}, e_3^{(8)}, e_4^{(8)}, e_5^{(8)}, e_6^{(8)}, e_7^{(8)}\}$$ – стандартния о.н.б. в $\mathbb{C}^8$

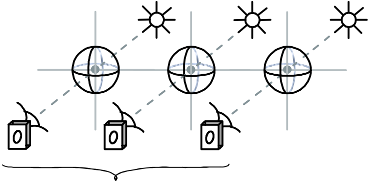


$$e_0^{(2)} \otimes e_0^{(2)} \otimes e_0^{(2)}$$

$$\underbrace{e_0^{(2)} \otimes \left(e_0^{(2)} \otimes e_0^{(2)} \right)} \equiv |e_0\rangle \otimes |e_0\rangle \otimes |e_0\rangle$$

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \left(\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} \right)$$

$$= \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} = e_0^{(8)} \in \{ e_0^{(8)}, e_1^{(8)}, e_2^{(8)}, e_3^{(8)}, e_4^{(8)}, e_5^{(8)}, e_6^{(8)}, e_7^{(8)} \}$$ – стандартния о.н.б. в $\mathbb{C}^8$

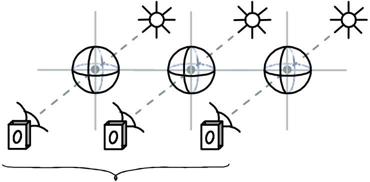


$$e_0^{(2)} \otimes e_0^{(2)} \otimes e_0^{(2)}$$

$$\underbrace{e_0^{(2)} \otimes \left(e_0^{(2)} \otimes e_0^{(2)}\right)}_{} \equiv |e_0\rangle \otimes |e_0\rangle \otimes |e_0\rangle \equiv |0\rangle \otimes |0\rangle \otimes |0\rangle$$

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \left(\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}\right)$$

$$= \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} = e_0^{(8)} \in \{e_0^{(8)}, e_1^{(8)}, e_2^{(8)}, e_3^{(8)}, e_4^{(8)}, e_5^{(8)}, e_6^{(8)}, e_7^{(8)}\}$$ – стандартния о.н.б. в $\mathbb{C}^8$



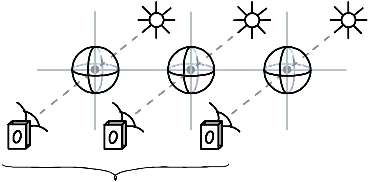
$$e_0^{(2)} \otimes e_0^{(2)} \otimes e_0^{(2)}$$

$$\underbrace{e_0^{(2)} \otimes \left(e_0^{(2)} \otimes e_0^{(2)}\right)}_{} \equiv |e_0\rangle \otimes |e_0\rangle \otimes |e_0\rangle \equiv |0\rangle \otimes |0\rangle \otimes |0\rangle$$

$$\equiv |0\rangle|0\rangle|0\rangle$$

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \left(\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}\right)$$

$$= \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} = e_0^{(8)} \in \{e_0^{(8)}, e_1^{(8)}, e_2^{(8)}, e_3^{(8)}, e_4^{(8)}, e_5^{(8)}, e_6^{(8)}, e_7^{(8)}\}$$ – стандартния о.н.б. в $\mathbb{C}^8$



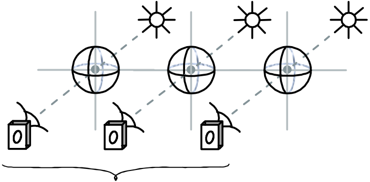
$$e_0^{(2)} \otimes e_0^{(2)} \otimes e_0^{(2)}$$

$$\underbrace{e_0^{(2)} \otimes \left(e_0^{(2)} \otimes e_0^{(2)}\right)}_{} \equiv |e_0\rangle \otimes |e_0\rangle \otimes |e_0\rangle \equiv |0\rangle \otimes |0\rangle \otimes |0\rangle$$

$$\equiv |0\rangle|0\rangle|0\rangle \equiv |0,0,0\rangle$$

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \left(\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}\right)$$

$$= \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} = e_0^{(8)} \in \{e_0^{(8)}, e_1^{(8)}, e_2^{(8)}, e_3^{(8)}, e_4^{(8)}, e_5^{(8)}, e_6^{(8)}, e_7^{(8)}\}$$ – стандартния о.н.б. в $\mathbb{C}^8$



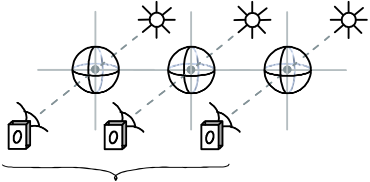
$$e_0^{(2)} \otimes e_0^{(2)} \otimes e_0^{(2)}$$

$$\underbrace{e_0^{(2)} \otimes \left(e_0^{(2)} \otimes e_0^{(2)}\right)} \equiv |e_0\rangle \otimes |e_0\rangle \otimes |e_0\rangle \equiv |0\rangle \otimes |0\rangle \otimes |0\rangle$$

$$\equiv |0\rangle|0\rangle|0\rangle \equiv |0,0,0\rangle \equiv |\underbrace{[000]_2}_{\text{числото „0" в двоична система}}\rangle$$

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \left(\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}\right)$$

$$= \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} = e_0^{(8)} \in \left\{e_0^{(8)}, e_1^{(8)}, e_2^{(8)}, e_3^{(8)}, e_4^{(8)}, e_5^{(8)}, e_6^{(8)}, e_7^{(8)}\right\}$$ – стандартния о.н.б. в $\mathbb{C}^8$



$$e_0^{(2)} \otimes e_0^{(2)} \otimes e_0^{(2)}$$

Система от няколко кубити: изчислителен базис

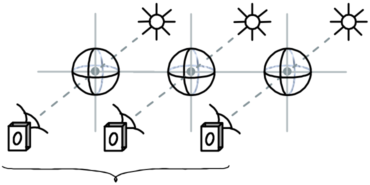
$$\underbrace{e_0^{(2)} \otimes \left(e_0^{(2)} \otimes e_0^{(2)}\right)} \equiv |e_0\rangle \otimes |e_0\rangle \otimes |e_0\rangle \equiv |0\rangle \otimes |0\rangle \otimes |0\rangle$$

$$\equiv |0\rangle|0\rangle|0\rangle \equiv |0,0,0\rangle \equiv |\underbrace{[000]_2}\rangle \equiv |0\rangle$$

числото "0" в двоична система

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \left(\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}\right)$$

$$= \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} = e_0^{(8)} \in \{e_0^{(8)}, e_1^{(8)}, e_2^{(8)}, e_3^{(8)}, e_4^{(8)}, e_5^{(8)}, e_6^{(8)}, e_7^{(8)}\}$$ – стандартния о.н.б. в $\mathbb{C}^8$



$$e_0^{(2)} \otimes e_0^{(2)} \otimes e_0^{(2)}$$

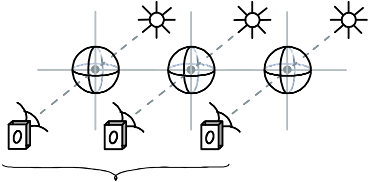
$$\underbrace{e_0 \otimes (e_0 \otimes e_0)} \equiv |e_0\rangle \otimes |e_0\rangle \otimes |e_0\rangle \equiv |0\rangle \otimes |0\rangle \otimes |0\rangle$$

$$\equiv |0\rangle|0\rangle|0\rangle \equiv |0,0,0\rangle \equiv |\underbrace{[000]_2}\rangle \equiv |0\rangle$$

числото "0" в двоична система

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \left(\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}\right)$$

$$= \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} = e_0 \in \{e_0, e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$$ – стандартния о.н.б. в $\mathbb{C}^8$



$$e_0 \otimes e_0 \otimes e_0$$

Система от няколко кубити: изчислителен базис

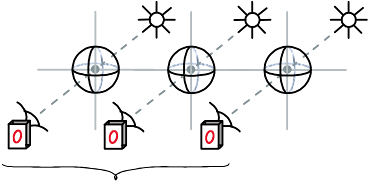
$$\underbrace{e_0 \otimes (e_0 \otimes e_0)} \equiv |e_0\rangle \otimes |e_0\rangle \otimes |e_0\rangle \equiv |0\rangle \otimes |0\rangle \otimes |0\rangle$$

$$\equiv |0\rangle|0\rangle|0\rangle \equiv |0,0,0\rangle \equiv |\underbrace{[000]_2}\rangle \equiv |0\rangle$$

числото "0" в двоична система

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \left(\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} \right)$$

$$= \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} = e_0 \in \{e_0, e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$$ – стандартния о.н.б. в $\mathbb{C}^8$



$$e_0 \otimes e_0 \otimes e_0$$

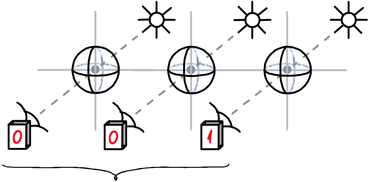
$$\underbrace{e_0 \otimes (e_0 \otimes e_1)} \equiv |e_0\rangle \otimes |e_0\rangle \otimes |e_1\rangle \equiv |0\rangle \otimes |0\rangle \otimes |1\rangle$$

$$\equiv |0\rangle|0\rangle|1\rangle \equiv |0,0,1\rangle \equiv |\underbrace{[001]_2}\rangle \equiv |1\rangle$$

числото "1" в двоична система

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \left(\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix}\right)$$

$$= \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\\0\\0\end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\\0\\0\\0\\0\end{pmatrix} = e_1 \in \{e_0, e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$$ – стандартния о.н.б. в $\mathbb{C}^8$



$$e_0 \otimes e_0 \otimes e_1$$

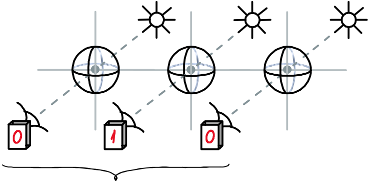
$$\underbrace{e_0 \otimes (e_1 \otimes e_0)}_{} \equiv |e_0\rangle \otimes |e_1\rangle \otimes |e_0\rangle \equiv |0\rangle \otimes |1\rangle \otimes |0\rangle$$

$$\equiv |0\rangle|1\rangle|0\rangle \equiv |0,1,0\rangle \equiv |\underbrace{[010]_2}\rangle \equiv |2\rangle$$

числото "2" в двоична система

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \left(\begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}\right)$$

$$= \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\1\\0\\0\\0\\0\\0\end{pmatrix} = e_2 \in \{e_0, e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$$ – стандартния о.н.б. в $\mathbb{C}^8$



$$e_0 \otimes e_1 \otimes e_0$$

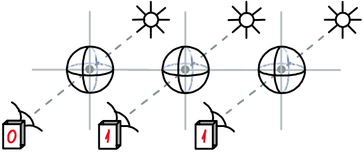
$$\underbrace{e_0 \otimes (e_1 \otimes e_1)} \equiv |e_0\rangle \otimes |e_1\rangle \otimes |e_1\rangle \equiv |0\rangle \otimes |1\rangle \otimes |1\rangle$$

$$\equiv |0\rangle|1\rangle|1\rangle \equiv |0,1,1\rangle \equiv |[\underbrace{011}]_2\rangle \equiv |3\rangle$$

числото "3" в двоична система

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \left(\begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} \right)$$

$$= \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\\1\\0\\0\\0\\0\end{pmatrix} = e_3 \in \{e_0, e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$$ – стандартния о.н.б. в $\mathbb{C}^8$



$$e_0 \otimes e_1 \otimes e_1$$

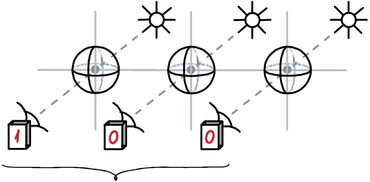
$$\underbrace{e_1 \otimes (e_0 \otimes e_0)} \equiv |e_1\rangle \otimes |e_0\rangle \otimes |e_0\rangle \equiv |1\rangle \otimes |0\rangle \otimes |0\rangle$$

$$\equiv |1\rangle|0\rangle|0\rangle \equiv |1,0,0\rangle \equiv |\underbrace{[100]_2}\rangle \equiv |4\rangle$$

числото "4" в двоична система

$$\begin{pmatrix}0\\1\end{pmatrix} \otimes \left(\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}\right)$$

$$= \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\\1\\0\\0\\0\end{pmatrix} = e_4 \in \{e_0, e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$$ – стандартния о.н.б. в $\mathbb{C}^8$



$$e_1 \otimes e_0 \otimes e_0$$

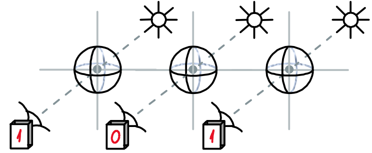
$$\underbrace{e_1 \otimes (e_0 \otimes e_1)}_{} \equiv |e_1\rangle \otimes |e_0\rangle \otimes |e_1\rangle \equiv |1\rangle \otimes |0\rangle \otimes |1\rangle$$

$$\equiv |1\rangle|0\rangle|1\rangle \equiv |1,0,1\rangle \equiv |\underbrace{[101]_2}\rangle \equiv |5\rangle$$

числото "5" в двоична система

$$\begin{pmatrix}0\\1\end{pmatrix} \otimes \left(\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix}\right)$$

$$= \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}0\\1\\0\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\\0\\1\\0\\0\end{pmatrix} = e_5 \in \{e_0, e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$$ – стандартния о.н.б. в $\mathbb{C}^8$



$$e_1 \otimes e_0 \otimes e_1$$

Система от няколко кубити: изчислителен базис

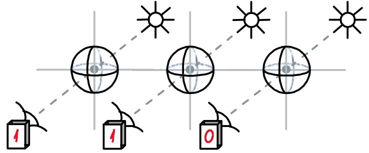
$$\underbrace{e_1 \otimes (e_1 \otimes e_0)} \equiv |e_1\rangle \otimes |e_1\rangle \otimes |e_0\rangle \equiv |1\rangle \otimes |1\rangle \otimes |0\rangle$$

$$\equiv |1\rangle|1\rangle|0\rangle \equiv |1,1,0\rangle \equiv |\underbrace{[110]_2}\rangle \equiv |6\rangle$$

числото "6" в двоична система

$$\begin{pmatrix}0\\1\end{pmatrix} \otimes \left(\begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix}\right)$$

$$= \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}0\\0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\\0\\0\\1\\0\end{pmatrix} = e_6 \in \{e_0, e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$$ – стандартния о.н.б. в $\mathbb{C}^8$



$$\underbrace{\qquad\qquad}_{e_1 \otimes e_1 \otimes e_0}$$

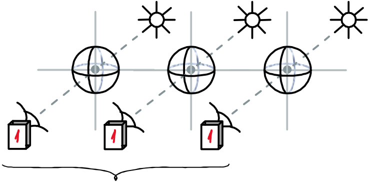
$$\underbrace{e_1 \otimes (e_1 \otimes e_1)} \equiv |e_1\rangle \otimes |e_1\rangle \otimes |e_1\rangle \equiv |1\rangle \otimes |1\rangle \otimes |1\rangle$$

$$\equiv |1\rangle|1\rangle|1\rangle \equiv |1,1,1\rangle \equiv |\underbrace{[111]_2}\rangle \equiv |7\rangle$$

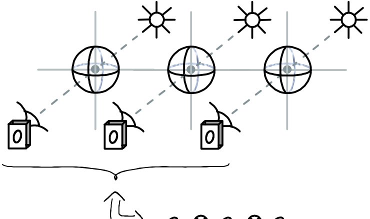
числото "7" в двоична система

$$\begin{pmatrix}0\\1\end{pmatrix} \otimes \left(\begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix}\right)$$

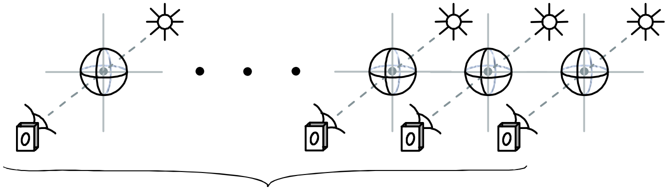
$$= \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}0\\0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\\0\\0\\0\\1\end{pmatrix} = e_7 \in \{e_0, e_1, e_2, e_3, e_4, e_5, e_6, e_7\} \text{ – стандартния о.н.б. в } \mathbb{C}^8$$



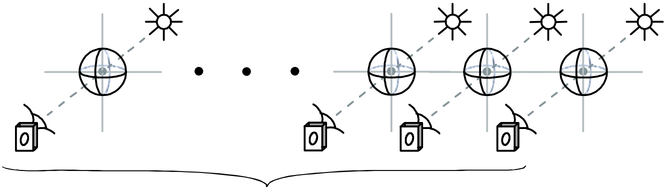
$$e_1 \otimes e_1 \otimes e_1$$



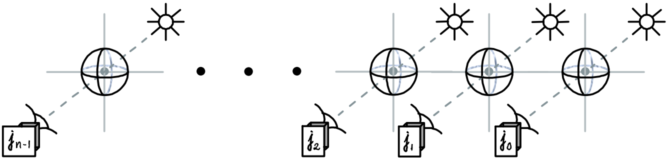
$$e_0 \otimes e_0 \otimes e_0$$



$$e_0 \otimes \cdots e_0 \otimes e_0 \otimes e_0$$

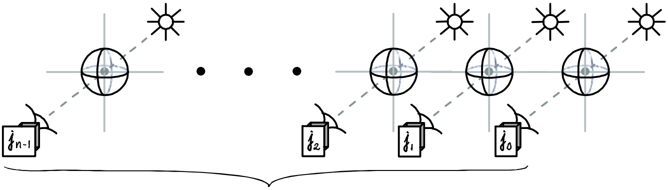


$$e_0 \otimes \; \cdots \; e_0 \otimes e_0 \otimes e_0 = |0\rangle \in \mathbb{C}^N$$



$$e_{j_{n-1}} \otimes \cdots e_{j_2} \otimes e_{j_1} \otimes e_{j_0} = |\ell\rangle \in \mathbb{C}^N$$

Система от няколко кубити: изчислителен базис

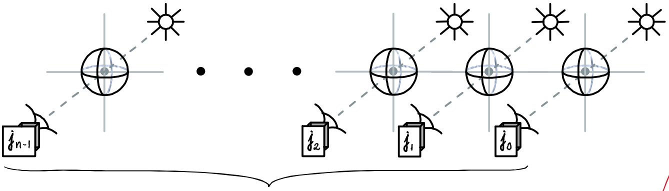


$$e_{j_{n-1}} \otimes \cdots e_{j_2} \otimes e_{j_1} \otimes e_{j_0} = |\ell\rangle \in \mathbb{C}^N$$

за $j_0, \dots, j_{n-1} = 0, 1$ и $N = 2^n$

Система от няколко кубити: изчислителен базис

$$\ell = [j_{n-1}\cdots j_1 j_0]_2 := j_{n-1}\,2^{n-1} + \cdots + j_1\,2^1 + j_0\,2^0$$



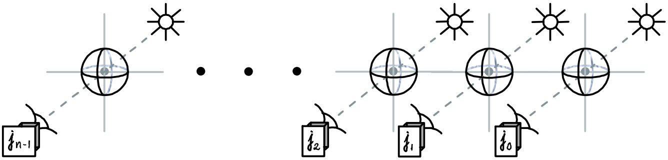
където

$$e_{j_{n-1}} \otimes \cdots \otimes e_{j_2} \otimes e_{j_1} \otimes e_{j_0} = |\ell\rangle \in \mathbb{C}^N$$

за $j_0, \dots, j_{n-1} = 0, 1$ и $N = 2^n$

двоично разлагане

$$\ell = [j_{n-1} \cdots j_1 j_0]_2 := j_{n-1}\, 2^{n-1} + \cdots + j_1\, 2^1 + j_0\, 2^0$$



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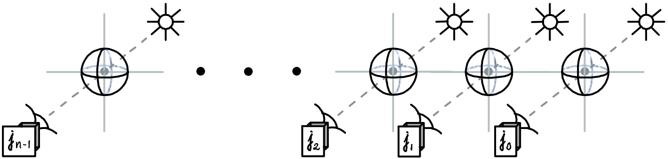
където



$$e_{j_{n-1}} \otimes \cdots e_{j_2} \otimes e_{j_1} \otimes e_{j_0} = |\ell\rangle \in \mathbb{C}^N$$

за $j_0,\dots,j_{n-1} = 0,1$ и $N = 2^n$

Изводи :

Изводи: хилбертовото пространство на състоянията на система от n квантови бита (кубити) е $\mathbb{C}^N$ (N-мерно) за $N = 2^n$

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Стандартния базис $e_0 \equiv |0\rangle$ и $e_1 \equiv |1\rangle$ в $\mathbb{C}^2$ пораждат стандартния базис в $\mathbb{C}^N$: $|[j_{n-1} \cdots j_0]_2\rangle = e_{j_{n-1}} \otimes \cdots \otimes e_{j_0}$

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1. Квантови изчисления: начална илюстрация
2. Система от няколко кубити: изчислителен базис
3. Система от няколко кубити: квант. трансформации
4. Дискретната трансформация на Фурие: определение
5. Дискретната трансформация на Фурие: реализация с квантова мрежа
6. Първо понятие за квантов алгоритъм

1. Комбиниране на квантови трансформации

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Нека: $U: \mathbb{C}^N \to \mathbb{C}^N$, $V: \mathbb{C}^M \to \mathbb{C}^M$ - унитарни оператори на квантови трансформации

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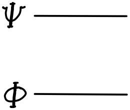

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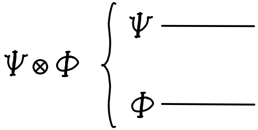

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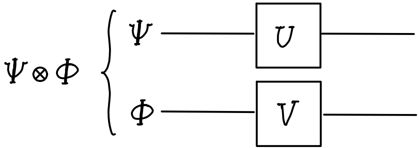

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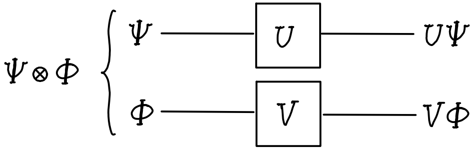

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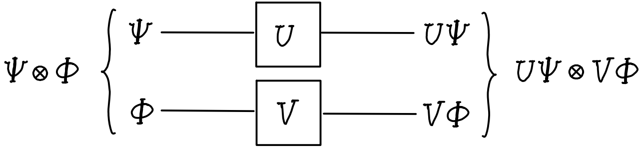

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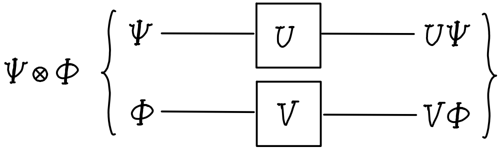

$$U\Psi \otimes V\Phi = (U \otimes V)(\Psi \otimes \Phi)$$

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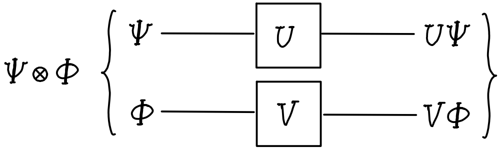

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– така се трябва да се трансформират несплетените състояния – независимо.

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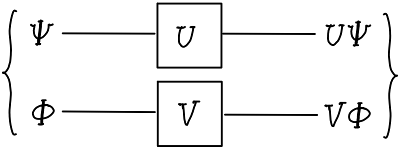
$\Psi \otimes \Phi$



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Но векторите на несплетените състояния пораждат линейно всички вектори на състояния от $\mathbb{C}^N \otimes \mathbb{C}^M = \mathbb{C}^{NM}$.

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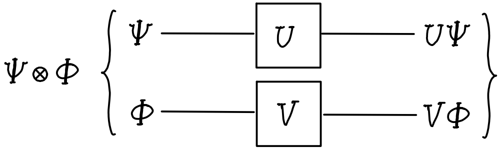

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$U \otimes V$ - унитарна $NM \times NM$ - матрица

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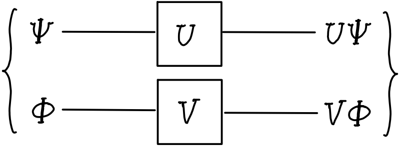

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$$\left((U \otimes V)^* (U \otimes V) = U^* U \otimes V^* V = \hat{1}. \right)$$

2. Примери:

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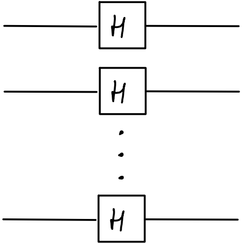

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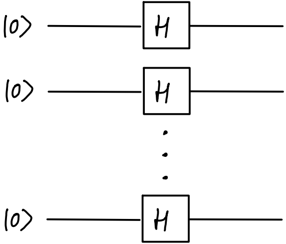

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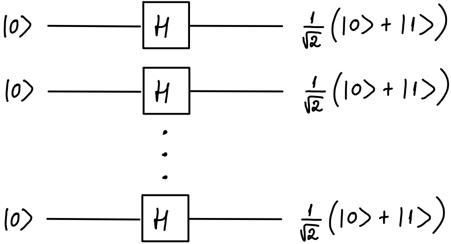

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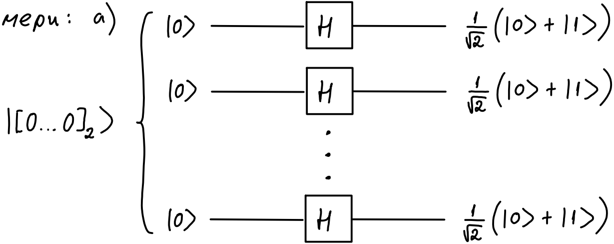
$|[0\ldots0]_2\rangle$
$|0\rangle$
H
$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
$|0\rangle$
H
$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
$|0\rangle$
H
$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

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$$|[0\ldots0]_2\rangle \left\{ \begin{array}{lcl} |0\rangle & \text{---}\boxed{H}\text{---} & \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle) \\ |0\rangle & \text{---}\boxed{H}\text{---} & \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle) \\ & \vdots & \\ |0\rangle & \text{---}\boxed{H}\text{---} & \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle) \end{array} \right\}$$

$$2^{-\frac{n}{2}}(|0\rangle+|1\rangle)(|0\rangle+|1\rangle)\cdots(|0\rangle+|1\rangle)$$

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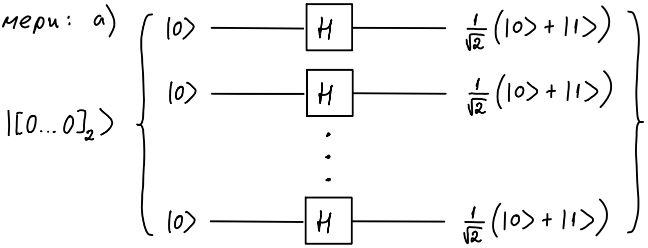

$$2^{-\frac{n}{2}}(|0\rangle+|1\rangle)\otimes(|0\rangle+|1\rangle)\otimes\cdots\otimes(|0\rangle+|1\rangle)$$

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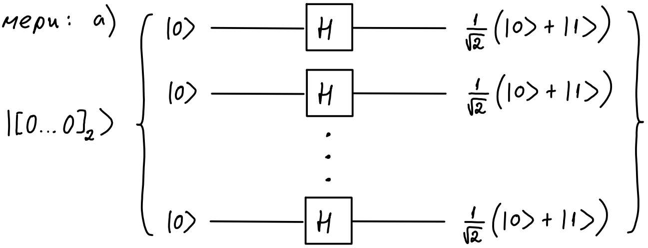

$$2^{-\frac{n}{2}}\left(|0\rangle+|1\rangle\right)\otimes\left(|0\rangle+|1\rangle\right)\otimes\cdots\otimes\left(|0\rangle+|1\rangle\right) =$$

$\boxed{\text{тензорното произведение е дистрибутивно (полилинейно)}}$

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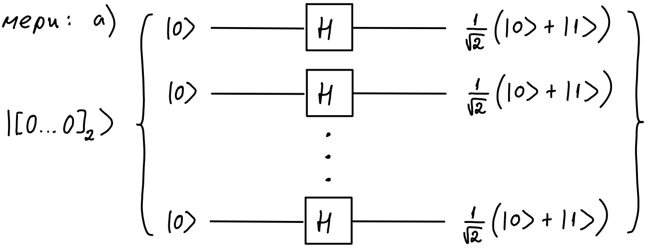

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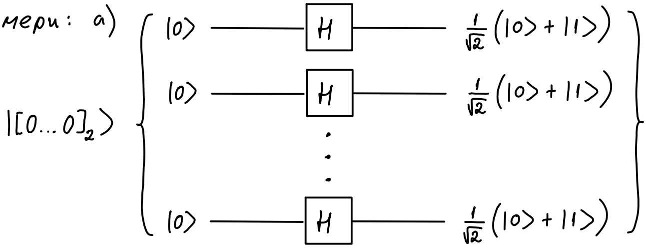

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$$= 2^{-\frac{n}{2}}\left(\underbrace{|0\rangle|0\rangle\cdots|0\rangle}_{|[00\ldots0]_2\rangle} + \underbrace{|0\rangle|0\rangle\cdots|1\rangle}_{|[00\ldots1]_2\rangle} + \cdots + \underbrace{|1\rangle|1\rangle\cdots|1\rangle}_{|[11\ldots1]_2\rangle}\right)$$

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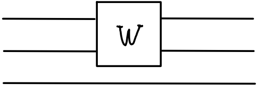

където $W : \mathbb{C}^N \otimes \mathbb{C}^M \to \mathbb{C}^N \otimes \mathbb{C}^M$

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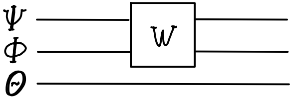

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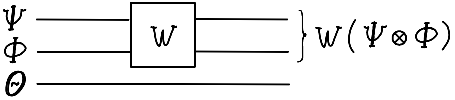

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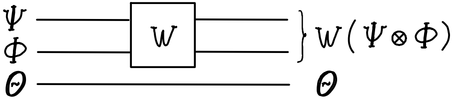

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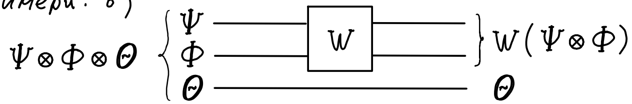

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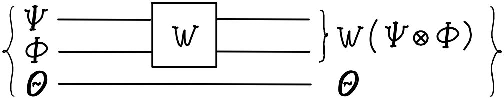

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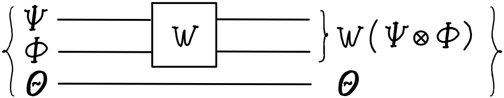

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където $W : \mathbb{C}^N\otimes\mathbb{C}^M \to \mathbb{C}^N\otimes\mathbb{C}^M$ и получената трансформация е

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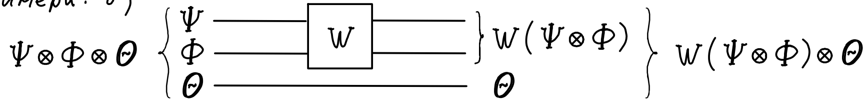

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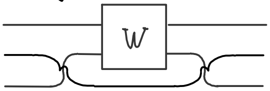
Възможно е и с допълнително пренареждане



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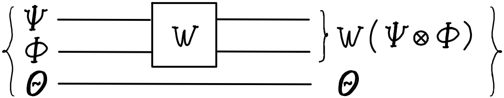

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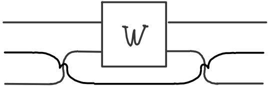

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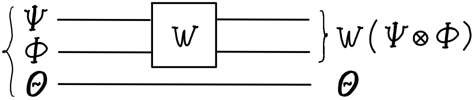

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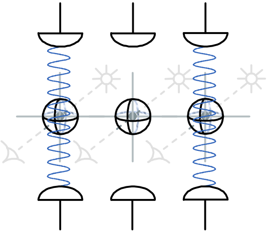

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– за да се коригира понякога избират σ^{-1} вместо σ.

4. Условни операции

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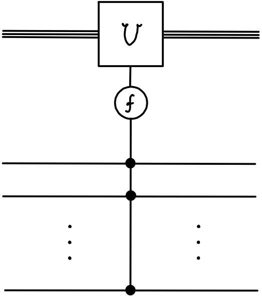

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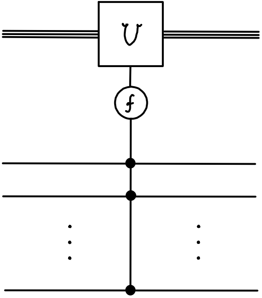

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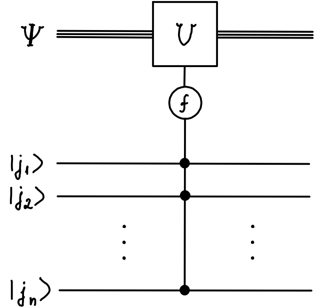

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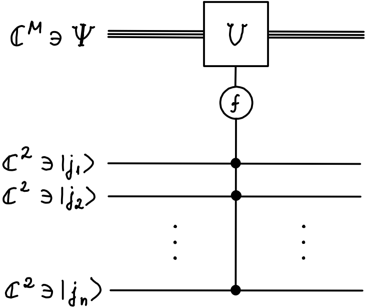

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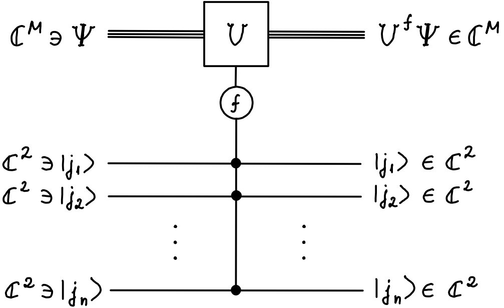

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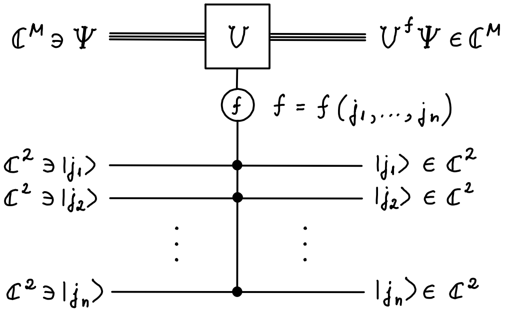
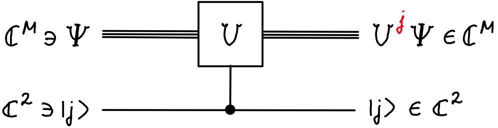


определя унитарна $\tilde{U} : \mathbb{C}^M \otimes \mathbb{C}^N \to \mathbb{C}^M \otimes \mathbb{C}^N$, $N = 2^n$

4. Условни операции ("IF $f=1$ THEN U ELSE $\hat{1}$") / controlled gate

В частност, за

$U: \mathbb{C}^M \to \mathbb{C}^M$ - унитарна



определя унитарна $\tilde{U}: \mathbb{C}^M \otimes \mathbb{C}^2 \to \mathbb{C}^M \otimes \mathbb{C}^2$

1. Квантови изчисления: начална илюстрация
2. Система от няколко кубити: изчислителен базис
3. Система от няколко кубити: квант. трансформации
4. Дискретната трансформация на Фурие: определение
5. Дискретната трансформация на Фурие: реализация с квантова мрежа
6. Първо понятие за квантов алгоритъм

Нека $N = 2, 3, \ldots,\ \xi := \xi_N := e^{\frac{2\pi i}{N}}$

Нека $N = 2, 3, \dots,$ $\xi := \xi_N := e^{\frac{2\pi i}{N}}$ - примитивен, N-ти корен на 1

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$$F := \left(f_{jk}\right)_{j,k=0}^{N-1} = \frac{1}{\sqrt{N}} \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \xi & \xi^2 & \cdots & \xi^{N-1} \\ 1 & \xi^2 & \xi^4 & \cdots & \xi^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \xi^{N-1} & \xi^{(N-1)2} & \cdots & \xi^{(N-1)^2} \end{pmatrix}$$

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Матрица на дискретното преобразование на Фурие

Нека $N = 2, 3, \dots,\quad \xi := \xi_N := e^{\frac{2\pi i}{N}}$ - примитивен, N-ти корен на 1: $\xi^N = 1$

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Матрица на дискретного преобразование на Фурие

Нека $N = 2, 3, \ldots$, $\xi := \xi_N := e^{\frac{2\pi i}{N}}$ - примитивен, N-ти корен на 1: $\xi^N = 1$

$$F := \left(f_{jk}\right)_{j,k=0}^{N-1} = \frac{1}{\sqrt{N}} \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \xi & \xi^2 & \cdots & \xi^{N-1} \\ 1 & \xi^2 & \xi^4 & \cdots & \xi^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \xi^{N-1} & \xi^{(N-1)2} & \cdots & \xi^{(N-1)^2} \end{pmatrix}, \qquad f_{jk} = N^{-\frac{1}{2}} \xi^{jk} \equiv N^{-\frac{1}{2}} e^{\frac{2\pi i}{N} jk}$$

матрица на дискретното преобразование на Фурие

Нека $N = 2, 3, \ldots$, $\xi := \xi_N := e^{\frac{2\pi i}{N}}$ - примитивен, N-ти корен на 1: $\xi^N = 1$

$$F := \left(f_{jk}\right)_{j,k=0}^{N-1} = \frac{1}{\sqrt{N}} \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \xi & \xi^2 & \cdots & \xi^{N-1} \\ 1 & \xi^2 & \xi^4 & \cdots & \xi^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \xi^{N-1} & \xi^{(N-1)2} & \cdots & \xi^{(N-1)^2} \end{pmatrix}, \quad f_{jk} = N^{-\frac{1}{2}} \xi^{jk} \equiv N^{-\frac{1}{2}} e^{\frac{2\pi i}{N} jk}$$

$$= N^{-\frac{1}{2}} \left(\cos \frac{2\pi jk}{N} + i \sin \frac{2\pi jk}{N} \right)$$

матрица на дискретното преобразование на Фурие

Нека $N = 2, 3, \dots$, $\xi := \xi_N := e^{\frac{2\pi i}{N}}$ - примитивен, N-ти корен на 1: $\xi^N = 1$

$$F := (f_{jk})_{j,k=0}^{N-1} = \frac{1}{\sqrt{N}} \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \xi & \xi^2 & \dots & \xi^{N-1} \\ 1 & \xi^2 & \xi^4 & \dots & \xi^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \xi^{N-1} & \xi^{(N-1)2} & \dots & \xi^{(N-1)^2} \end{pmatrix}, \quad f_{jk} = N^{-\frac{1}{2}} \xi^{jk} \equiv N^{-\frac{1}{2}} e^{\frac{2\pi i}{N} jk}$$

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→ матрица на дискретното преобразование на Фурие

$F^* F \overset{?}{=} I$ - унитарност

Нека $N = 2, 3, \ldots$, $\xi := \xi_N := e^{\frac{2\pi i}{N}}$ - примитивен, N-ти корен на 1: $\xi^N = 1$

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матрица на дискретното преобразование на Фурие

$$F^* F \stackrel{?}{=} I \text{ - унитарност} \quad \left(F^* = \left(\bar{f}_{kj}\right)_{j,k}, \ \bar{f}_{kj} = \bar{\xi}^{kj} = \xi^{-jk} \right)$$

Нека $N = 2, 3, \ldots$, $\xi := \xi_N := e^{\frac{2\pi i}{N}}$ - примитивен, N-ти корен на 1 : $\xi^N = 1$

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матрица на дискретното преобразование на Фурие

$$\underbrace{F^* F}_{(c_{jk})_{j,k}} \stackrel{?}{=} I \text{ - унитарност} \quad \left(F^* = (\overline{f}_{kj})_{j,k}, \quad \overline{f}_{kj} = \overline{\xi}^{kj} = \xi^{-jk} \right)$$

Нека $N = 2, 3, \dots$, $\xi := \xi_N := e^{\frac{2\pi i}{N}}$ - примитивен, N-ти корен на 1: $\xi^N = 1$

$$F := (f_{jk})_{j,k=0}^{N-1} = \frac{1}{\sqrt{N}} \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \xi & \xi^2 & \dots & \xi^{N-1} \\ 1 & \xi^2 & \xi^4 & \dots & \xi^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \xi^{N-1} & \xi^{(N-1)2} & \dots & \xi^{(N-1)^2} \end{pmatrix}, \quad f_{jk} = N^{-\frac{1}{2}} \xi^{jk} \equiv N^{-\frac{1}{2}} e^{\frac{2\pi i}{N} jk}$$

$$= N^{-\frac{1}{2}} \left(\cos \frac{2\pi jk}{N} + i \sin \frac{2\pi jk}{N} \right)$$

$\to$ матрица на дискретного преобразование на Фурие

$\underbrace{F^* F}_{} \overset{?}{=} I$ - унитарност $\left(F^* = (\bar{f}_{kj})_{j,k}, \ \bar{f}_{kj} = \bar{\xi}^{kj} = \xi^{-jk} \right)$

$(c_{jk})_{j,k}$,

$$c_{jk} = \sum_{l=0}^{N-1} \bar{f}_{lj} f_{lk}$$

Нека $N = 2, 3, \dots$, $\xi := \xi_N := e^{\frac{2\pi i}{N}}$ - примитивен, N-ти корен на 1: $\xi^N = 1$

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Матрица на дискретното преобразование на Фурие

$F^* F \overset{?}{=} I$ - унитарност $\quad \left(F^* = \left(\bar{f}_{kj}\right)_{j,k}, \ \bar{f}_{kj} = \bar{\xi}^{kj} = \xi^{-jk} \right)$

$\left(c_{jk}\right)_{j,k}$,

$$c_{jk} = \sum_{l=0}^{N-1} \bar{f}_{lj} f_{lk} = \sum_{l=0}^{N-1} \frac{1}{N} \xi^{(k-j)l}$$

Нека $N=2,3,\dots$, $\xi := \xi_N := e^{\frac{2\pi i}{N}}$ - примитивен, N-ти корен на 1: $\xi^N = 1$

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$$= N^{-\frac{1}{2}} \left(\cos \frac{2\pi jk}{N} + i \sin \frac{2\pi jk}{N} \right)$$

F → матрица на дискретното преобразование на Фурие

$\underbrace{F^* F}_{} \overset{?}{=} I$ - унитарност $\left(F^* = \left(\bar{f}_{kj} \right)_{j,k}, \; \bar{f}_{kj} = \bar{\xi}^{kj} = \xi^{-jk} \right)$

$\left(c_{jk} \right)_{j,k}$,

$$c_{jk} = \sum_{\ell=0}^{N-1} \bar{f}_{\ell j} f_{\ell k} = \sum_{\ell=0}^{N-1} \frac{1}{N} \xi^{(k-j)\ell} = \Bigg\{$$

Нека $N = 2, 3, \dots,$ $\xi := \xi_N := e^{\frac{2\pi i}{N}}$ - примитивен, N-ти корен на 1: $\xi^N = 1$

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→ матрица на дискретното преобразование на Фурие

$\underbrace{F^* F}_{} \overset{?}{=} I$ - унитарност $\left(F^* = (\bar{f}_{kj})_{j,k}, \ \bar{f}_{kj} = \bar{\xi}^{kj} = \xi^{-jk} \right)$

$(c_{jk})_{j,k}$,

$$c_{jk} = \sum_{\ell=0}^{N-1} \bar{f}_{\ell j} f_{\ell k} = \sum_{\ell=0}^{N-1} \frac{1}{N} \xi^{(k-j)\ell} = \begin{cases} \frac{1}{N} N = 1, & j = k \\ & \end{cases}$$

Нека $N = 2, 3, \dots$, $\xi := \xi_N := e^{\frac{2\pi i}{N}}$ - примитивен, N-ти корен на 1: $\xi^N = 1$

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матрица на дискретното преобразование на Фурие

$$\underbrace{F^* F}_{} \overset{?}{=} I \text{ - унитарност} \quad \left(F^* = (\bar{f}_{kj})_{j,k}, \ \bar{f}_{kj} = \bar{\xi}^{kj} = \xi^{-jk} \right)$$

$(c_{jk})_{j,k}$,

$$c_{jk} = \sum_{\ell=0}^{N-1} \bar{f}_{\ell j} f_{\ell k} = \sum_{\ell=0}^{N-1} \frac{1}{N} \xi^{(k-j)\ell} = \begin{cases} \frac{1}{N} N = 1, & j = k \\ \dfrac{\xi^{(k-j)N} - 1}{\xi^{k-j} - 1} = 0, & j \neq k \end{cases}$$

Нека $N = 2, 3, \ldots$, $\xi := \xi_N := e^{\frac{2\pi i}{N}}$ - примитивен, N-ти корен на 1 : $\xi^N = 1$

Действие:

Нека $N = 2, 3, \ldots,$ $\xi := \xi_N := e^{\frac{2\pi i}{N}}$ - примитивен, N-ти корен на 1: $\xi^N = 1$

Действие:

$$F|j\rangle = \frac{1}{\sqrt{N}}\left(|0\rangle + e^{\frac{2\pi i}{N} j\cdot 1}|1\rangle + e^{\frac{2\pi i}{N} j\cdot 2}|2\rangle + e^{\frac{2\pi i}{N} j\cdot 3}|3\rangle + \cdots\right),$$

Нека $N = 2, 3, \ldots,\quad \xi := \xi_N := e^{\frac{2\pi i}{N}}$ - примитивен, N-ти корен на 1: $\xi^N = 1$

Действие:

$$F|j\rangle = \frac{1}{\sqrt{N}}\left(|0\rangle + e^{\frac{2\pi i}{N} j\cdot 1}|1\rangle + e^{\frac{2\pi i}{N} j\cdot 2}|2\rangle + e^{\frac{2\pi i}{N} j\cdot 3}|3\rangle + \cdots\right),$$

В частност, $F|0\rangle = \frac{1}{\sqrt{N}}\left(|0\rangle + |1\rangle + |2\rangle + \cdots + |N-1\rangle\right).$

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https://en.wikipedia.org/wiki/Quantum_Fourier_transform

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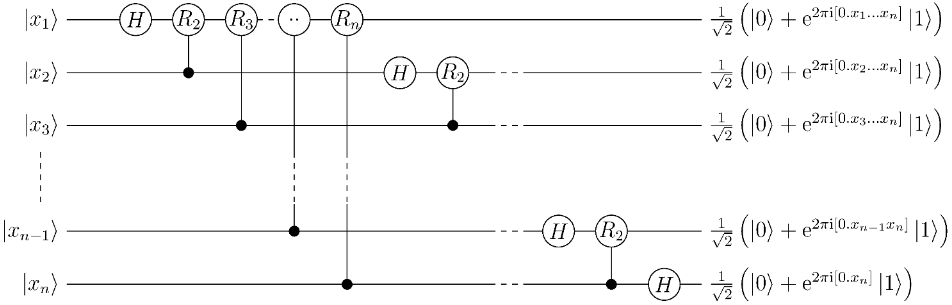

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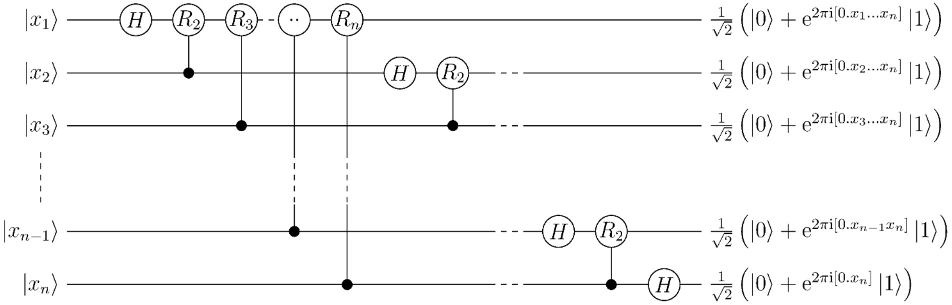

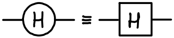

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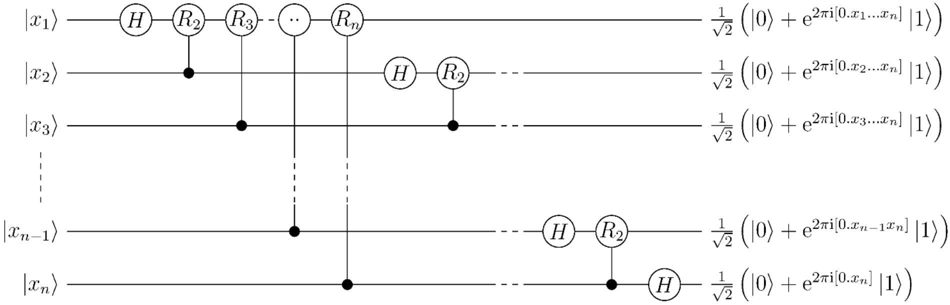
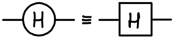
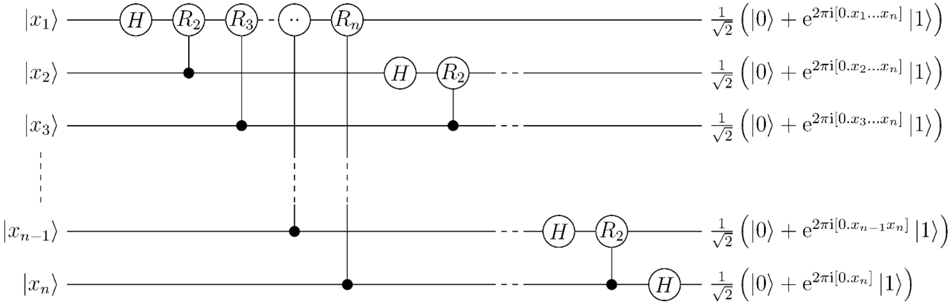
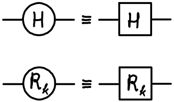




– гейт на Адамар $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

https://en.wikipedia.org/wiki/Quantum_Fourier_transform





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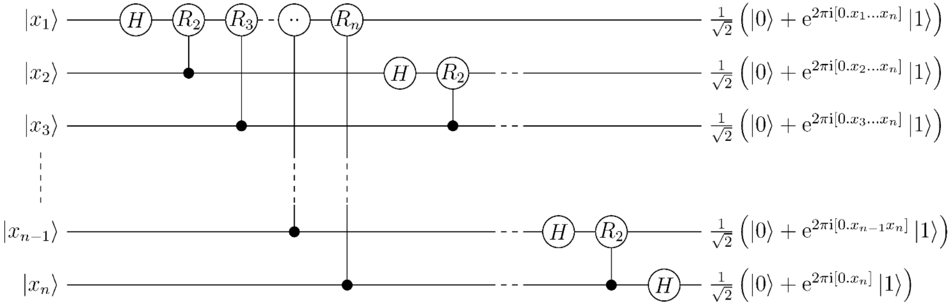
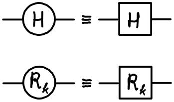
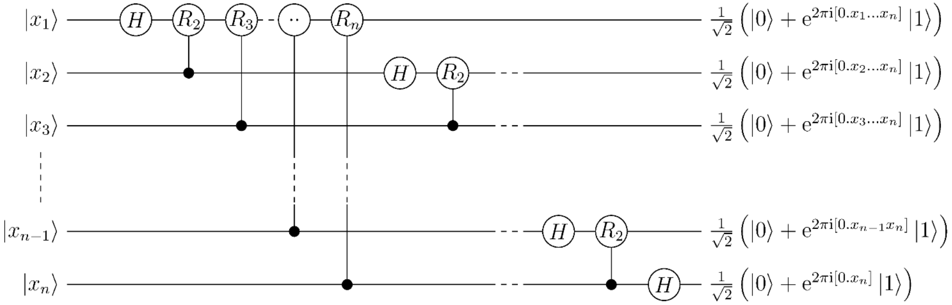




– гейт на Адамар $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$$R_k = \begin{pmatrix} 1 & 0 \\ 0 & e^{2\pi i/2^k} \end{pmatrix}$$

https://en.wikipedia.org/wiki/Quantum_Fourier_transform



$\hookrightarrow |[x_1 \ldots x_n]_2\rangle \equiv |j\rangle$

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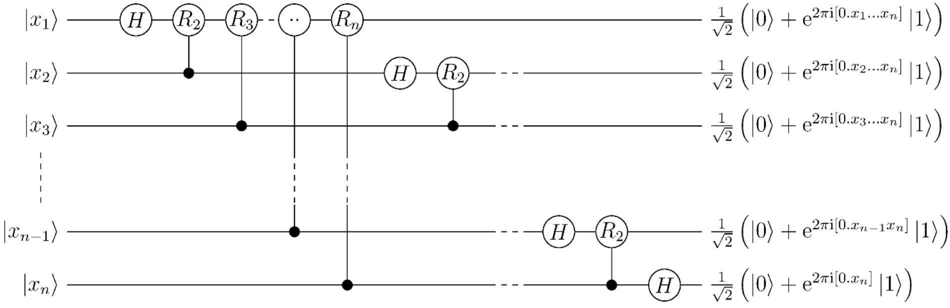

$\hookrightarrow |[x_1 \dots x_n]_2\rangle \equiv |j\rangle$

за $x_1, \dots, x_n \in \{0,1\}$

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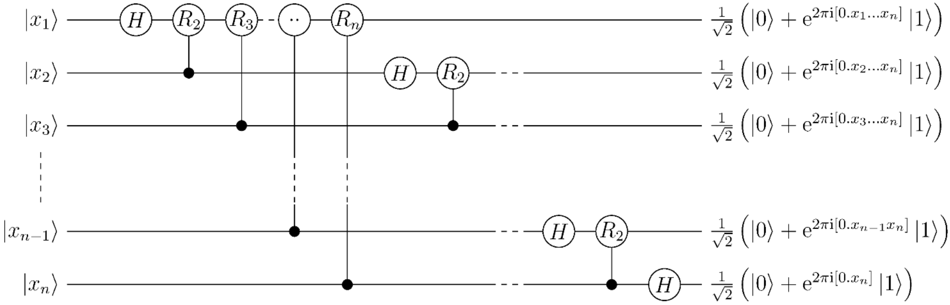

$|[x_1 \dots x_n]_2\rangle \equiv |j\rangle$

за $x_1, \dots, x_n \in \{0, 1\}$, $j \in \{0, \dots, N-1\}$

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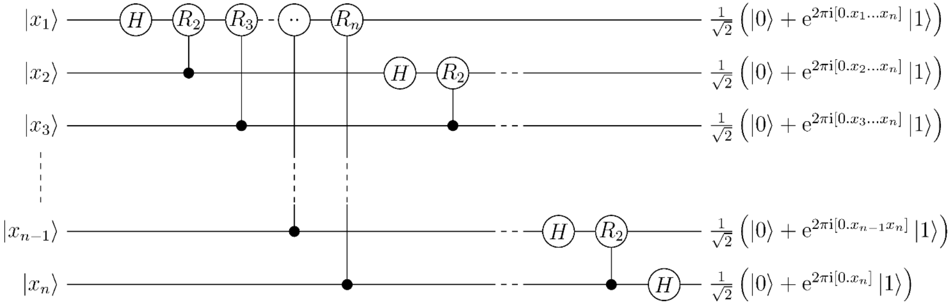

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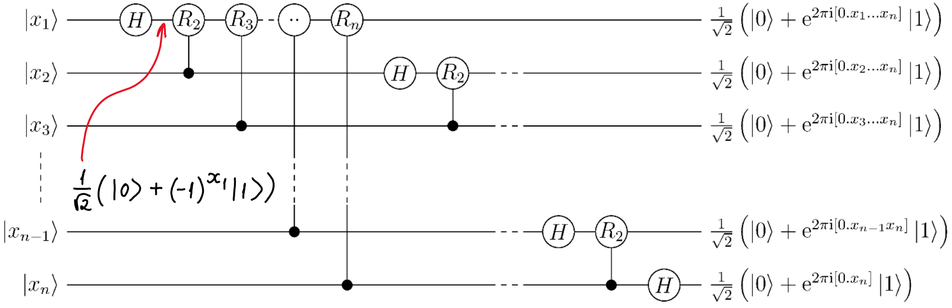

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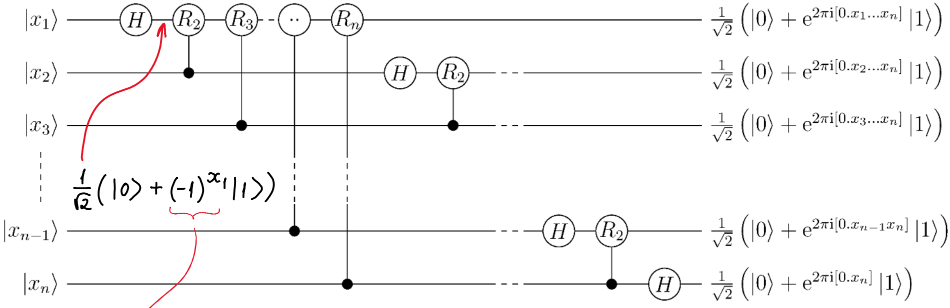
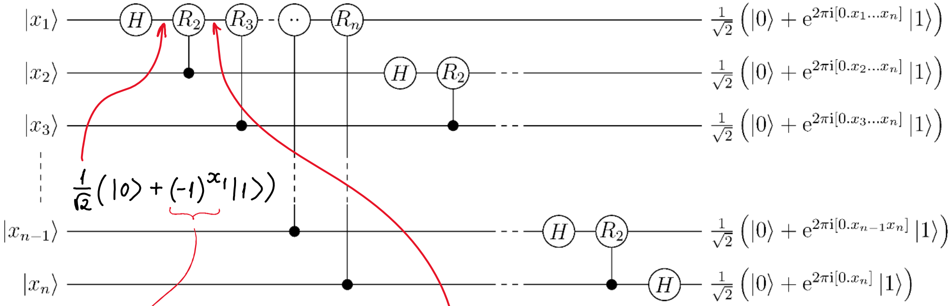


$$\frac{1}{\sqrt{2}}\left(|0\rangle + \underbrace{(-1)}^{x_1}|1\rangle\right)$$

$$e^{\pi i x_1} = e^{2\pi i \frac{x_1}{2}} = e^{2\pi i [0.x_1]}$$

https://en.wikipedia.org/wiki/Quantum_Fourier_transform

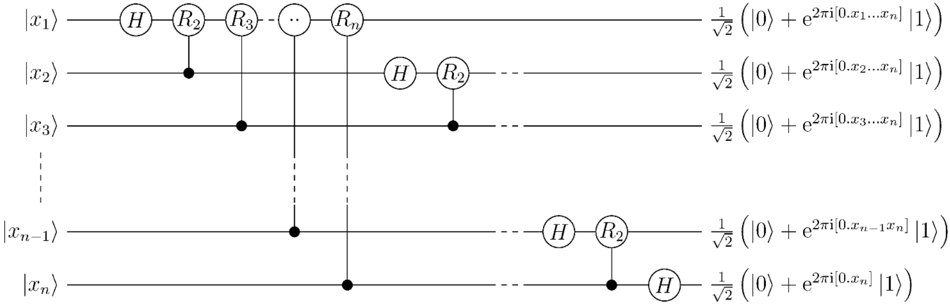


$$\frac{1}{\sqrt{2}}\left(|0\rangle + (-1)^{x_1}|1\rangle\right)$$

$$e^{\pi i x_1} = e^{2\pi i \frac{x_1}{2}} = e^{2\pi i [0.x_1]}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & e^{2\pi i [0.0x_2]} \end{pmatrix}$$

https://en.wikipedia.org/wiki/Quantum_Fourier_transform



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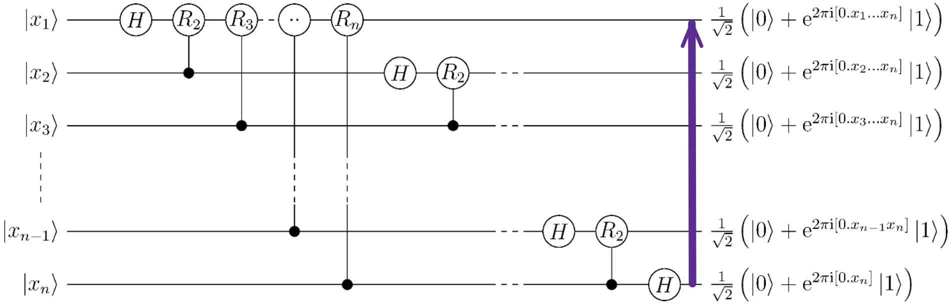

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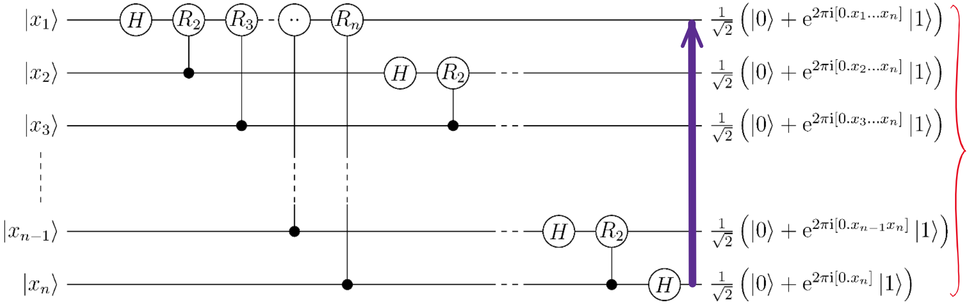

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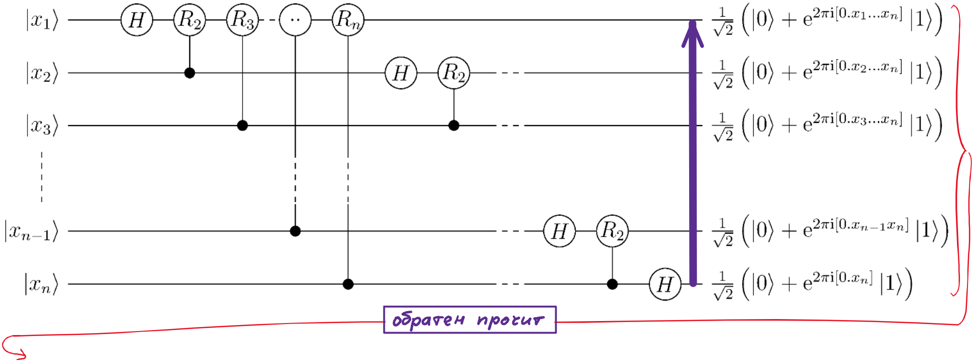

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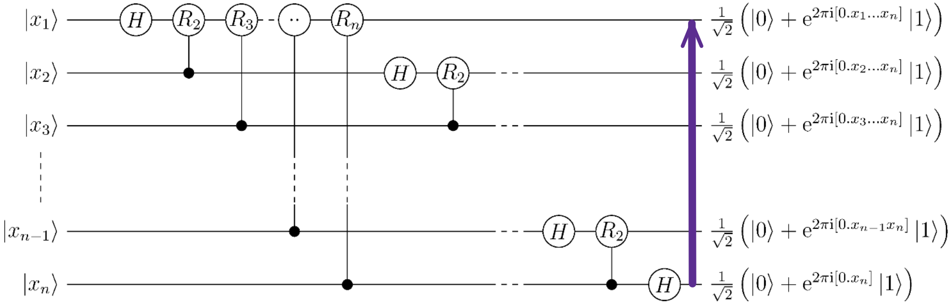
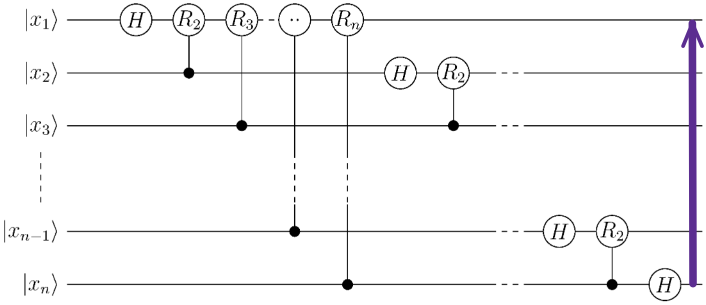


$\boxed{\text{обратен прочит}}$

$2^{-n/2}\Big(\qquad\qquad\qquad\qquad\qquad\qquad \Big)$

https://en.wikipedia.org/wiki/Quantum_Fourier_transform



$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i[0.x_1 \ldots x_n]}\,|1\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i[0.x_2 \ldots x_n]}\,|1\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i[0.x_3 \ldots x_n]}\,|1\rangle\right)$$

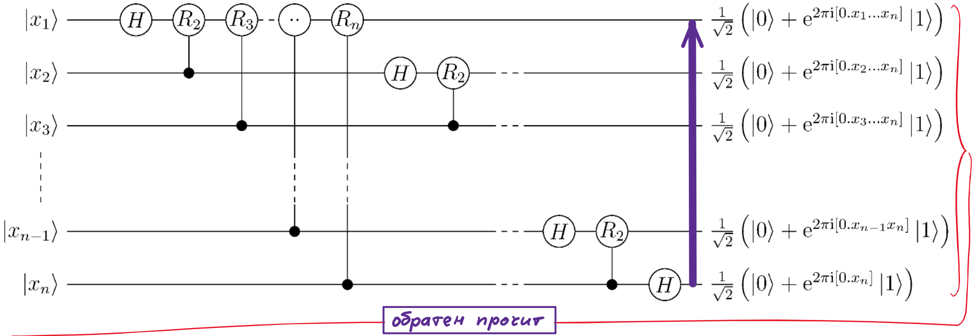
$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i[0.x_{n-1} x_n]}\,|1\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i[0.x_n]}\,|1\rangle\right)$$

обратен прочит

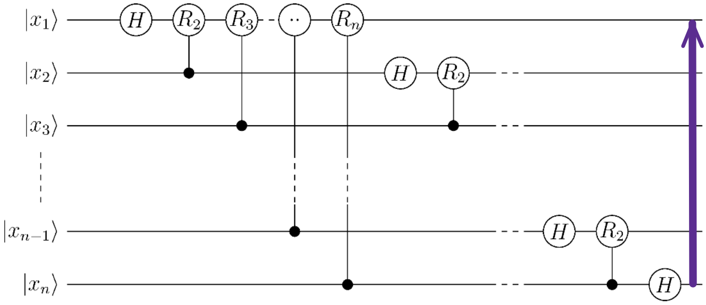
$$2^{-n/2}\left(|0\rangle \cdots |0\rangle + \qquad\qquad\qquad\qquad \right)$$

https://en.wikipedia.org/wiki/Quantum_Fourier_transform



$$2^{-n/2}\left(|0\rangle\cdots|0\rangle + e^{2\pi i[0.x_1\cdots x_n]}|0\rangle\cdots|0\rangle|1\rangle + \qquad\qquad\right)$$

https://en.wikipedia.org/wiki/Quantum_Fourier_transform



$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i[0.x_1\ldots x_n]}\,|1\rangle\right)$$

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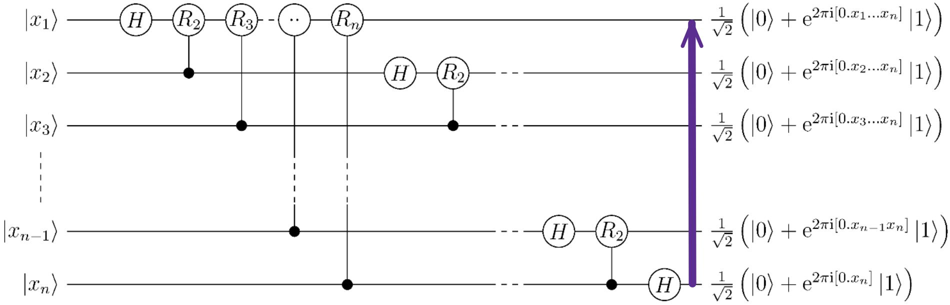
$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i[0.x_n]}\,|1\rangle\right)$$

обратен прочит

$$2^{-n/2}\left(|0\rangle\cdots|0\rangle + e^{2\pi i[0.x_1\cdots x_n]}|0\rangle\cdots|0\rangle|1\rangle + e^{2\pi i[0.x_2\cdots x_n]}|0\rangle\cdots|1\rangle|0\rangle + \cdots\right)$$

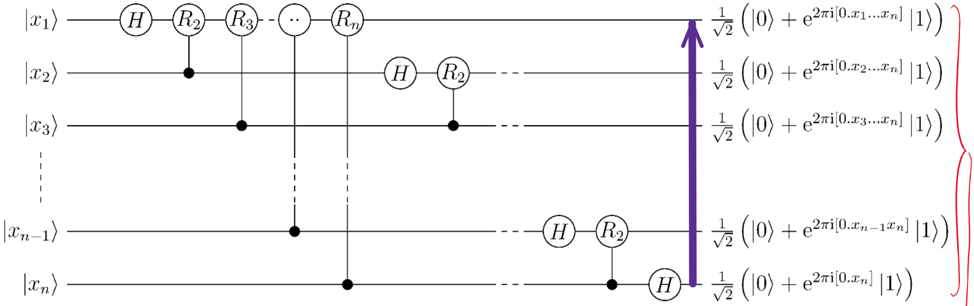
https://en.wikipedia.org/wiki/Quantum_Fourier_transform



$\boxed{\text{обратен прочит}}$

$$\underbrace{2^{-n/2}}_{N^{-\frac{1}{2}}}\left(|0\rangle\cdots|0\rangle + e^{2\pi i[0.x_1\cdots x_n]}|0\rangle\cdots|0\rangle|1\rangle + e^{2\pi i[0.x_2\cdots x_n]}|0\rangle\cdots|1\rangle|0\rangle + \cdots\right)$$

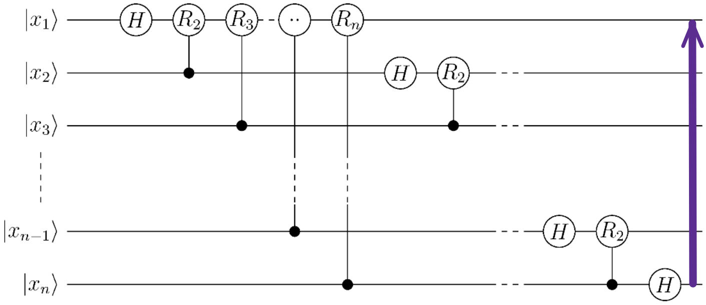
https://en.wikipedia.org/wiki/Quantum_Fourier_transform



обратен прочит

$$\underbrace{2^{-n/2}}_{N^{-\frac{1}{2}}}\Big(\underbrace{|0\rangle\cdots|0\rangle}_{\substack{|[0\ldots0]_2\rangle\\ =|0\rangle}} + e^{2\pi i[0.x_1\cdots x_n]}|0\rangle\cdots|0\rangle|1\rangle + e^{2\pi i[0.x_2\cdots x_n]}|0\rangle\cdots|1\rangle|0\rangle + \cdots\Big)$$

https://en.wikipedia.org/wiki/Quantum_Fourier_transform



$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i[0.x_1\ldots x_n]}\,|1\rangle\right)$$
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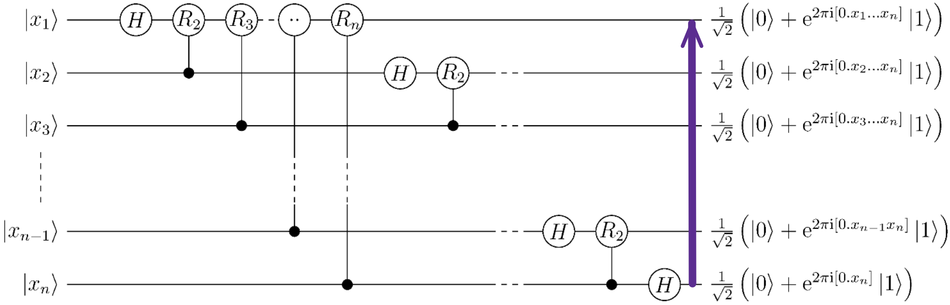
$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i[0.x_3\ldots x_n]}\,|1\rangle\right)$$
$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i[0.x_{n-1}x_n]}\,|1\rangle\right)$$
$$\frac{1}{\sqrt{2}}\left(|0\rangle + e^{2\pi i[0.x_n]}\,|1\rangle\right)$$

обратен прочит

$$\underbrace{2^{-n/2}}_{N^{-\frac{1}{2}}}\Big(\underbrace{|0\rangle\cdots|0\rangle}_{\substack{|[0\ldots0]_2\rangle\\ =|0\rangle}} + e^{2\pi i[0.x_1\cdots x_n]}\underbrace{|0\rangle\cdots|0\rangle|1\rangle}_{\substack{|[0\ldots01]_2\rangle\\ =|1\rangle}} + e^{2\pi i[0.x_2\cdots x_n]}|0\rangle\cdots|1\rangle|0\rangle + \cdots\Big)$$

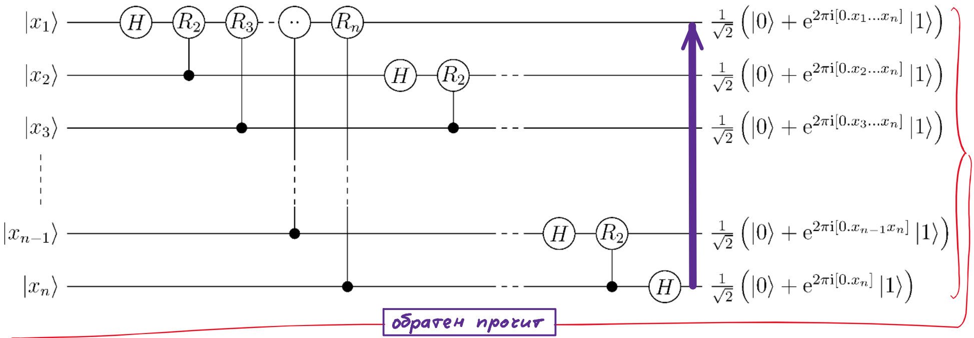
https://en.wikipedia.org/wiki/Quantum_Fourier_transform



$\boxed{\text{обратен прочит}}$

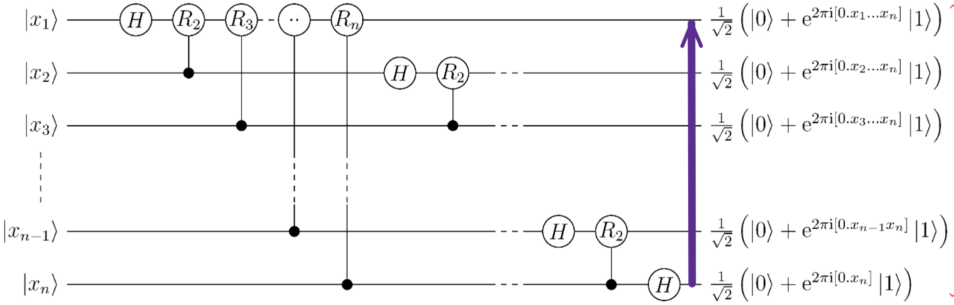
$$\underbrace{2^{-n/2}}_{N^{-\frac{1}{2}}} \Big(\underbrace{|0\rangle\cdots|0\rangle}_{\substack{|[0\ldots0]_2\rangle \\ =|0\rangle}} + e^{2\pi i[0.x_1\cdots x_n]} \underbrace{|0\rangle\cdots|0\rangle|1\rangle}_{\substack{|[0\ldots01]_2\rangle \\ =|1\rangle}} + e^{2\pi i[0.x_2\cdots x_n]} \underbrace{|0\rangle\cdots|1\rangle|0\rangle}_{\substack{|[0\ldots10]_2\rangle \\ =|2\rangle}} + \cdots \Big)$$

https://en.wikipedia.org/wiki/Quantum_Fourier_transform



$$\underbrace{2^{-n/2}}_{N^{-\frac{1}{2}}}\Big(\underbrace{|0\rangle\cdots|0\rangle}_{\substack{|[0\ldots0]_2\rangle\\ =|0\rangle}} + \underbrace{e^{2\pi i[0.x_1\cdots x_n]}}_{e^{\frac{2\pi i}{2^n}[x_1\cdots x_n]\cdot 1}}\underbrace{|0\rangle\cdots|0\rangle|1\rangle}_{\substack{|[0\ldots01]_2\rangle\\ =|1\rangle}} + e^{2\pi i[0.x_2\cdots x_n]}\underbrace{|0\rangle\cdots|1\rangle|0\rangle}_{\substack{|[0\ldots10]_2\rangle\\ =|2\rangle}} + \cdots\Big)$$

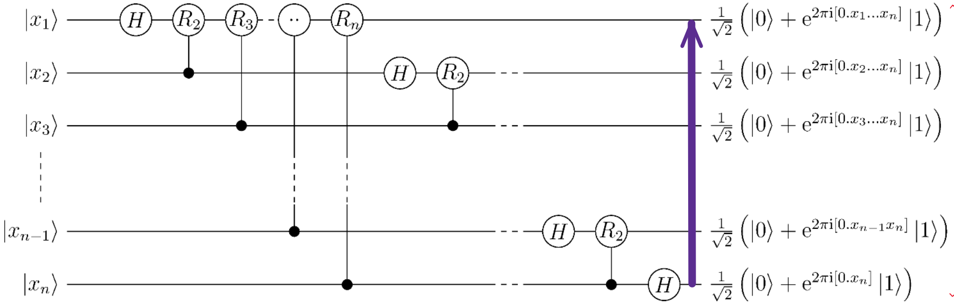
https://en.wikipedia.org/wiki/Quantum_Fourier_transform



обратен прочит

$$\underbrace{2^{-n/2}}_{N^{-\frac{1}{2}}} \Big(\underbrace{|0\rangle\cdots|0\rangle}_{|[0\ldots0]_2\rangle = |0\rangle} + \underbrace{e^{2\pi i[0.x_1\cdots x_n]}}_{e^{\frac{2\pi i}{2^n}[x_1\cdots x_n]\cdot 1}} \underbrace{|0\rangle\cdots|0\rangle|1\rangle}_{|[0\ldots01]_2\rangle = |1\rangle} + \underbrace{e^{2\pi i[0.x_2\cdots x_n]}}_{e^{\frac{2\pi i}{2^n}[x_1\cdots x_n]\cdot 2}} \underbrace{|0\rangle\cdots|1\rangle|0\rangle}_{|[0\ldots10]_2\rangle = |2\rangle} + \cdots \Big)$$

https://en.wikipedia.org/wiki/Quantum_Fourier_transform



обратен прочит

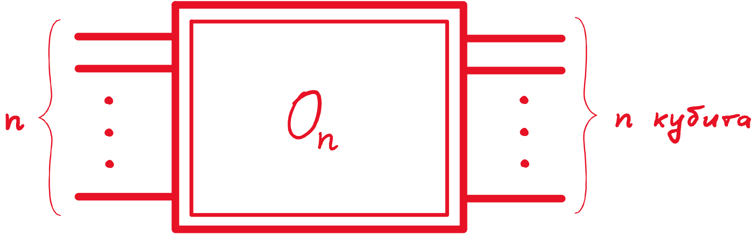
$$\frac{1}{\sqrt{N}}\left(|0\rangle + e^{\frac{2\pi i}{N} j\cdot 1}\cdot|1\rangle + e^{\frac{2\pi i}{N} j\cdot 2}\cdot|2\rangle + \cdots\right)$$

1. Квантови изчисления: начална илюстрация
2. Система от няколко кубити: изчислителен базис
3. Система от няколко кубити: квант. трансформации
4. Дискретната трансформация на Фурие: определение
5. Дискретната трансформация на Фурие: реализация с квантова мрежа

6. Първо понятие за квантов алгоритъм

Квантов алгоритъм е редица от квантови мрежи, която има алгоритмично описание в класически смисъл.

Квантов алгоритъм е редица от квантови мрежи, която има алгоритмично описание в класически смисъл.



Квантов алгоритъм е редица от квантови мрежи, която има алгоритмично описание в класически смисъл. Например: