8*. Квантов алгоритъм за "намиране на фазата" по
стр. 221(255)-223 (част 5.1 до 5.2.1 - без) на книгата на Ниелсен и Чуанг,
Nielsen M.A., Chuang I.L., Quantum Computation and Quantum Information, 10th Anniversary Edition,
http://theo.inrne.bas.bg/~mitov/qi21/notes_df34g2tb3/NC2010.pdf

to successive powers of two. The final state of the first register is easily seen to be:

$$\frac{1}{2^{t/2}}\left(|0\rangle + e^{2\pi i 2^{t-1}\varphi}|1\rangle\right)\left(|0\rangle + e^{2\pi i 2^{t-2}\varphi}|1\rangle\right)\ldots\left(|0\rangle + e^{2\pi i 2^{0}\varphi}|1\rangle\right)$$
$$= \frac{1}{2^{t/2}}\sum_{k=0}^{2^t-1} e^{2\pi i \varphi k}|k\rangle\,. \qquad (5.20)$$

We omit the second register from this description, since it stays in the state $|u\rangle$ throughout the computation.

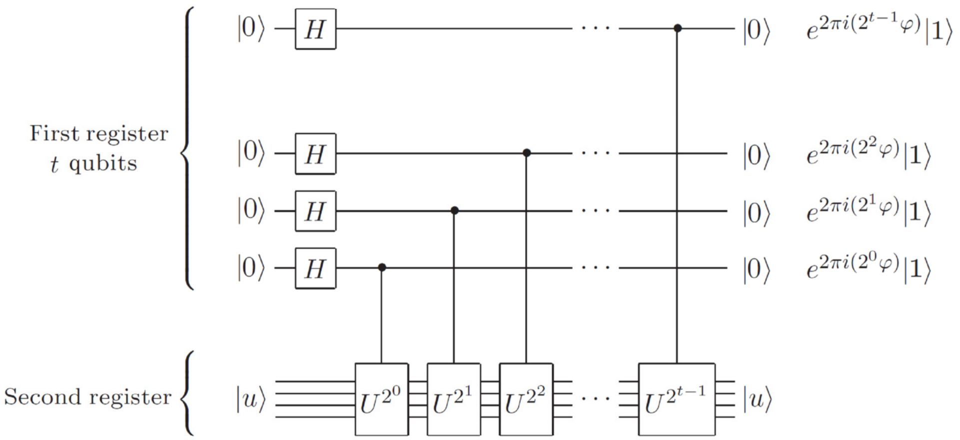

Figure 5.2. The first stage of the phase estimation procedure. Normalization factors of $1/\sqrt{2}$ have been omitted, on the right.

$$\frac{1}{2^{t/2}}\left(|0\rangle + e^{2\pi i 0.\varphi_t}|1\rangle\right)\left(|0\rangle + e^{2\pi i 0.\varphi_{t-1}\varphi_t}|1\rangle\right)\ldots\left(|0\rangle + e^{2\pi i 0.\varphi_1\varphi_2\cdots\varphi_t}|1\rangle\right)\,. \quad (5.21)$$

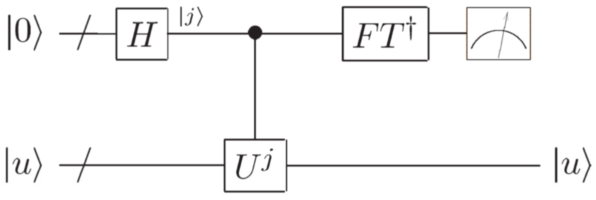

Figure 5.3. Schematic of the overall phase estimation procedure. The top t qubits (the '/' denotes a bundle of wires, as usual) are the first register, and the bottom qubits are the second register, numbering as many as required to perform U. $|u\rangle$ is an eigenstate of U with eigenvalue $e^{2\pi i\varphi}$. The output of the measurement is an approximation to φ accurate to $t - \left\lceil \log\left(2 + \frac{1}{2\epsilon}\right)\right\rceil$ bits, with probability of success at least $1-\epsilon$.

9*. Квантов алгоритъм за търсене по:
стр. 248-253 (част 6.1 до 6.1.4) на книгата на Ниелсен и Чуанг
Nielsen M.A., Chuang I.L., Quantum Computation and Quantum Information, 10th Anniversary Edition,
http://theo.inrne.bas.bg/~mitov/qi21/notes_df34g2tb3/NC2010.pdf

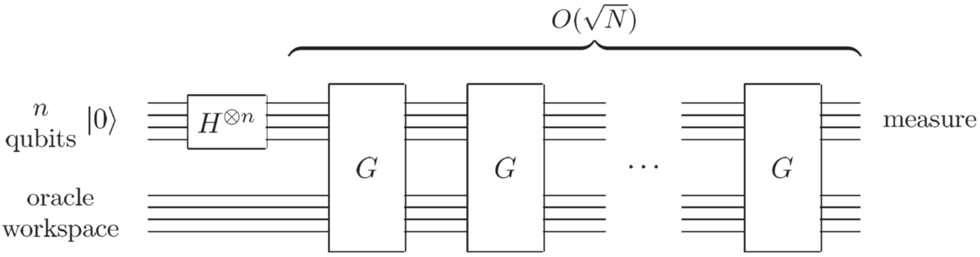

Figure 6.1. Schematic circuit for the quantum search algorithm. The oracle may employ work qubits for its implementation, but the analysis of the quantum search algorithm involves only the n qubit register.

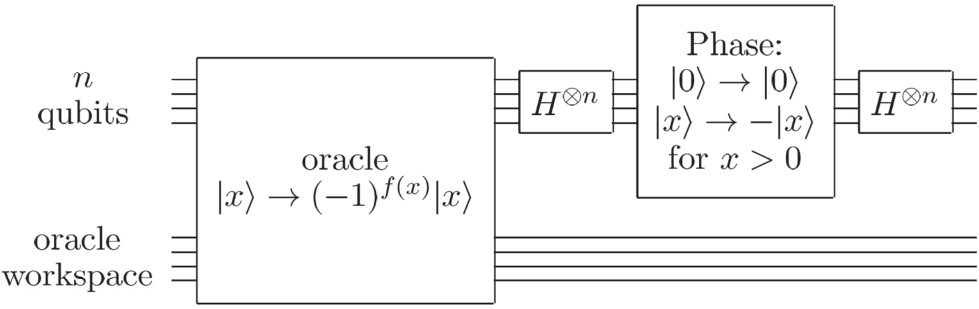

Figure 6.2. Circuit for the Grover iteration, G.

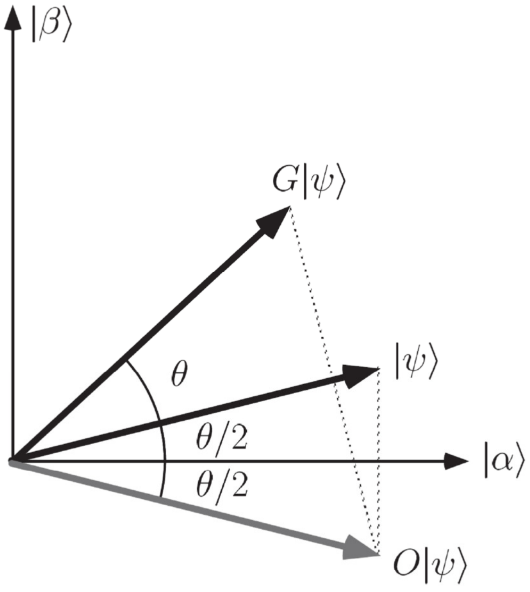

Figure 6.3. The action of a single Grover iteration, G: the state vector is rotated by θ towards the superposition $|\beta\rangle$ of all solutions to the search problem. Initially, it is inclined at angle $\theta/2$ from $|\alpha\rangle$, a state orthogonal to $|\beta\rangle$. An oracle operation O reflects the state about the state $|\alpha\rangle$, then the operation $2|\psi\rangle\langle\psi| - I$ reflects it about $|\psi\rangle$. In the figure $|\alpha\rangle$ and $|\beta\rangle$ are lengthened slightly to reduce clutter (all states should be unit vectors). After repeated Grover iterations, the state vector gets close to $|\beta\rangle$, at which point an observation in the computational basis outputs a solution to the search problem with high probability. The remarkable efficiency of the algorithm occurs because θ behaves like $\Omega(\sqrt{M/N})$, so only $O(\sqrt{N/M})$ applications of G are required to rotate the state vector close to $|\beta\rangle$.