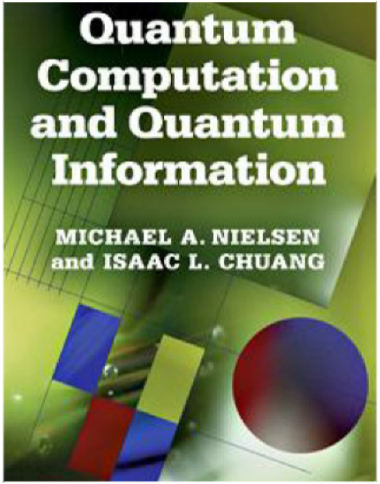

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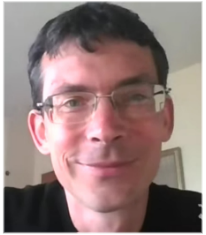

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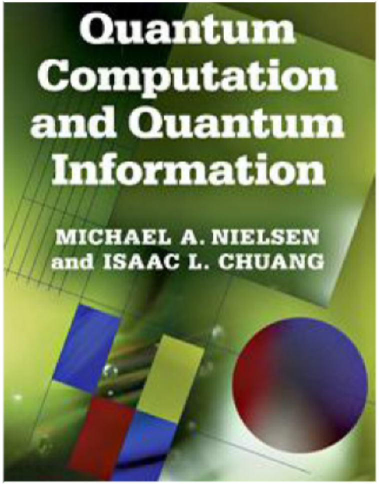
Quantum Computation and Quantum Information
MICHAEL A. NIELSEN and ISAAC L. CHUANG

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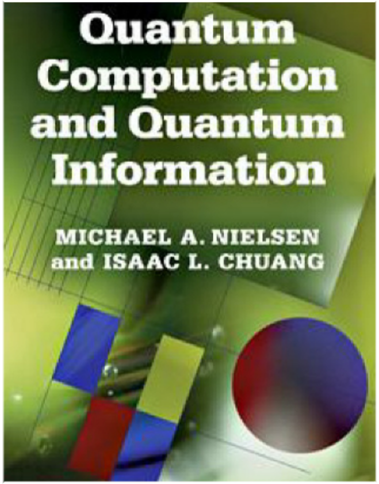
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Contents

2.1 Linear algebra

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2.1 Linear algebra

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2.1 Linear algebra

$$\begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix} \tag{2.1}$$

$$\begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix} + \begin{bmatrix} z_1' \\ \vdots \\ z_n' \end{bmatrix} \equiv \begin{bmatrix} z_1 + z_1' \\ \vdots \\ z_n + z_n' \end{bmatrix}, \tag{2.2}$$

$$z \begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix} \equiv \begin{bmatrix} zz_1 \\ \vdots \\ zz_n \end{bmatrix}, \tag{2.3}$$

2.1 Linear algebra

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Notation	Description
z^*	Complex conjugate of the complex number z. $(1+i)^* = 1-i$
$\|\psi\rangle$	Vector. Also known as a *ket*.
$\langle\psi\|$	Vector dual to $\|\psi\rangle$. Also known as a *bra*.
$\langle\varphi\|\psi\rangle$	Inner product between the vectors $\|\varphi\rangle$ and $\|\psi\rangle$.
$\|\varphi\rangle \otimes \|\psi\rangle$	Tensor product of $\|\varphi\rangle$ and $\|\psi\rangle$.
$\|\varphi\rangle\|\psi\rangle$	Abbreviated notation for tensor product of $\|\varphi\rangle$ and $\|\psi\rangle$.
A^*	Complex conjugate of the A matrix.
A^T	Transpose of the A matrix.
$A^\dagger$	Hermitian conjugate or adjoint of the A matrix, $A^\dagger = (A^T)^*$. $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^\dagger = \begin{bmatrix} a^* & c^* \\ b^* & d^* \end{bmatrix}$.
$\langle\varphi\|A\|\psi\rangle$	Inner product between $\|\varphi\rangle$ and $A\|\psi\rangle$. Equivalently, inner product between $A^\dagger\|\varphi\rangle$ and $\|\psi\rangle$.

Figure 2.1. Summary of some standard quantum mechanical notation for notions from linear algebra. This style of notation is known as the *Dirac* notation.

$$|v_1\rangle \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix}; \quad |v_2\rangle \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{2.5}$$

since any vector

$$|v\rangle = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \tag{2.6}$$

$$((y_1, \ldots, y_n), (z_1, \ldots, z_n)) \equiv \sum_i y_i^* z_i = [y_1^* \ldots y_n^*] \begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix}. \tag{2.14}$$

2.1 Linear algebra

61 - 79

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http://theo.inrne.bas.bg/~mitov/qi23_yJ4Gmi891z/NMN_QI23_04_WB_notes_v1.pdf стр. 2 - 9 (стр. 4 ↓)

Бра-кет-формализъм. Напомняме: $\mathbb{C}^n$ се състои от вектор-стълбове =: кет-вектори

⟨bra|ket⟩ за бра-вектори $\in (\mathbb{C}^n)^*$

$\langle \Phi | \Psi \rangle = \Phi^* \Psi$ (за $\Phi, \Psi \in \mathbb{C}^n$)

$\langle \Phi | \quad | \Psi \rangle$

кет-вектори = вектор-стълбове

бра-вектори = вектор-редове

Конвенция: $|\Psi\rangle := \Psi$ – операция идентитет,

$\langle \Psi | := \Psi^*$ – операция на ермитово спрягане.

Тоест, поставянето на скоби $|\bullet\rangle, \langle\bullet|$ са едноместни операции.

<u>Примери:</u>

1) $\langle \Phi || \Psi \rangle = \Phi^* \Psi = \langle \Phi | \Psi \rangle$ – число (за $\Phi, \Psi \in \mathbb{C}^n$)

правило 1: две вертикални линии стоящи една до друга могат да се слеят в една, както и обратно, всяка вертикална линия може да се раздвои

2.1 Linear algebra

2.1.6 Adjoints and Hermitian operators

Suppose A is any linear operator on a Hilbert space, V. It turns out that there exists a unique linear operator $A^\dagger$ on V such that for all vectors $|v\rangle, |w\rangle \in V$,

$$(|v\rangle, A|w\rangle) = (A^\dagger|v\rangle, |w\rangle). \tag{2.32}$$

This linear operator is known as the *adjoint* or *Hermitian conjugate* of the operator A. From the definition it is easy to see that $(AB)^\dagger = B^\dagger A^\dagger$. By convention, if $|v\rangle$ is a vector, then we define $|v\rangle^\dagger \equiv \langle v|$. With this definition it is not difficult to see that $(A|v\rangle)^\dagger = \langle v|A^\dagger$.

$$\left(\sum_i a_i A_i\right)^\dagger = \sum_i a_i^* A_i^\dagger. \tag{2.33}$$

$$P \equiv \sum_{i=1}^{k} |i\rangle\langle i| \tag{2.35}$$

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http://theo.inrne.bas.bg/~mitov/qi23_yJ4Gmi891z/NMN_QI23_04_WB_notes_v1.pdf стр. 2 - 9 (стр. 3, 8 ↓)

Ермитово спрягане / Hermitian conjugation : $(m \times n)^* = (n \times m)$

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}^* = \begin{pmatrix} \bar{a}_{11} & \cdots & \bar{a}_{m1} \\ \vdots & & \vdots \\ \bar{a}_{1n} & \cdots & \bar{a}_{mn} \end{pmatrix}, \quad A^* := \bar{A}^T = \overline{A^T}$$

Основен закон : $(AB)^* = B^* A^* \Leftarrow \begin{cases} (AB)^T = B^T A^T \\ \overline{AB} = \bar{A}\bar{B} \end{cases}$

$A^{**} = A$ - инволютивност / involution property

Извод : за всеки ортонормиран базис $f_1, \dots, f_n \in \mathbb{C}^n$ е в сила

(алгебрично) разлагане на $\mathbb{1}$
$\hat{\mathbb{1}} = \vert f_1\rangle\langle f_1\vert + \cdots + \vert f_n\rangle\langle f_n\vert$

2.1 Linear algebra

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2.1.5 Eigenvectors and eigenvalues

An *eigenvector* of a linear operator A on a vector space is a non-zero vector $|v\rangle$ such that $A|v\rangle = v|v\rangle$, where v is a complex number known as the *eigenvalue* of A corresponding to $|v\rangle$. It will often be convenient to use the notation v both as a label for the eigenvector, and to represent the eigenvalue. We assume that you are familiar with the elementary properties of eigenvalues and eigenvectors – in particular, how to find them, via the characteristic equation. The *characteristic function* is defined to be $c(\lambda) \equiv \det |A - \lambda I|$,

A *diagonal representation* for an operator A on a vector space V is a representation $A = \sum_i \lambda_i |i\rangle\langle i|$, where the vectors $|i\rangle$ form an orthonormal set of eigenvectors for A, with corresponding eigenvalues λ_i. An operator is said to be *diagonalizable* if it has a diagonal representation. In the next section we will find a simple set of necessary and sufficient conditions for an operator on a Hilbert space to be diagonalizable.

2.1 Linear algebra

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$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = |0\rangle\langle 0| - |1\rangle\langle 1|, \tag{2.29}$$

http://theo.inrne.bas.bg/~mitov/qi23_yJ4Gmi891z/NMN-QI23-06_compact-slides_v1.pdf слайд 9

<u>Спектрална теорема</u> Нека $A = A^*$ (самоспрегнат)

$A \in Mat_N(\mathbb{C})$, т.е., A е $N \times N$, ермитова

Съществува ортонормиран базис на $\mathbb{C}^N$: $f_1, \dots, f_N \in \mathbb{C}^N$, от собствени вектори за A

$A f_j = \lambda_j f_j$, $\lambda_j \in \mathbb{R}$, $j = 1, \dots, N$ (реални собствени стойности)

При това:

$A = \lambda_1 |f_1\rangle\langle f_1| + \dots + \lambda_N |f_N\rangle\langle f_N|$

$\hat{I} = |f_1\rangle\langle f_1| + \dots + |f_N\rangle\langle f_N|$

упражнение
– приложете двете страни върху $|f_j\rangle$
и използвайте равенство върху базис

$A \in \mathcal{A}$ – питомна алгебра

Съществуват двойки $(\alpha_1, Q_1), \dots, (\alpha_n, Q_n)$, които са единствени с точност до пренареждане и са такива, че:

- $\alpha_1, \dots, \alpha_n$ са реални и са две по две различни
- $Q_1, \dots, Q_n$ са самоспрегнати идемпотенти $Q_j^* = Q_j = Q_j^2$
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- $\hat{I} = Q_1 + \dots + Q_n$, $Q_j Q_k = \begin{cases} Q_j & \text{при } j = k \\ 0 & \text{при } j \neq k \end{cases}$

разбиване на $\hat{I}$

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http://theo.inrne.bas.bg/~mitov/qi23_yJ4Gmi891z/NMN-QI23-06_compact-slides_v1.pdf слайд 9

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разбиване на $\hat{1}$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = |0\rangle\langle 0| - |1\rangle\langle 1|, \quad (2.29)$$

Задача. Как се прилага теоремата за случая $A := Q$ при $Q^* = Q = Q^2$

самоспрегнат идемпотент

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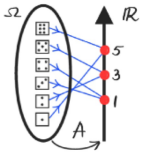
разбиване на $\hat{1}$

Определение: $\mathrm{Spec}\, A := \{\alpha_1, \dots, \alpha_n\}$ – спектър на наблюдаемата A

$\alpha_1, \dots, \alpha_n$ – спектрални стойности, $Q_1, \dots, Q_n$ – съответни спектрални събития

Интерпретация: • в класическа система, ако $A : \Omega \to \mathbb{R}$ (класическа наблюдаема = функция) тогава $\mathrm{Spec}\, A = A(\Omega)$ (стойностите на A като функция) за $\alpha_j \in \mathrm{Spec}\, A$: $Q_j = \chi_{S_j}$, $S_j := A^{-1}(\alpha_j)$

характеристична функция



$$\chi_S(\omega) = \begin{cases} 1 & \text{при } \omega \in S \\ 0 & \text{при } \omega \notin S \end{cases}$$

$$A = 1 \cdot \chi_{\{3,4\}} + 3 \cdot \chi_{\{2,5\}} + 5 \cdot \chi_{\{1,6\}}$$

За $A = A^*$ $\exists!\, \alpha_1 < \dots < \alpha_n$, $Q_1, \dots, Q_n$

- $Q_j^* = Q_j = Q_j^2$, $j = 1, \dots, n$
- $A = \alpha_1 Q_1 + \dots + \alpha_n Q_n$
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разбиване на $\hat{1}$

Kronecker product

$$A \otimes B \equiv \overbrace{\left[\begin{array}{cccc} A_{11}B & A_{12}B & \dots & A_{1n}B \\ A_{21}B & A_{22}B & \dots & A_{2n}B \\ \vdots & \vdots & \vdots & \vdots \\ A_{m1}B & A_{m2}B & \dots & A_{mn}B \end{array}\right]}^{nq} \Bigg\} mp\,. \qquad (2.50)$$

Exercise 2.28: Show that the transpose, complex conjugation, and adjoint operations distribute over the tensor product,

$$(A \otimes B)^* = A^* \otimes B^*;\ (A \otimes B)^T = A^T \otimes B^T;\ (A \otimes B)^\dagger = A^\dagger \otimes B^\dagger. \qquad (2.53)$$

2.1 Linear algebra

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$$(A_1 \otimes B_1)(A_2 \otimes B_2) = (A_1 A_2) \otimes (B_1 B_2)$$

Kronecker product

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$$(A_1 \otimes B_1)(A_2 \otimes B_2) = (A_1 A_2) \otimes (B_1 B_2)$$

Влаганията : $\iota_1(A) =: A_{(1)}$

$$A \longmapsto A \otimes \hat{I}$$

$$\mathcal{A} \overset{\iota_1}{\hookrightarrow} \mathcal{C} \overset{\iota_2}{\hookleftarrow} \mathcal{B}$$

$$\hat{I} \otimes B \longleftarrow B$$

$$\iota_2(B) =: B_{(2)}$$

$\Rightarrow$ $A \otimes B =$

$$= (A.1)\otimes(1.B) = (A\otimes 1)(1\otimes B)$$

$$= (1.A)\otimes(B.1) = (1\otimes B)(A\otimes 1)$$

$\Rightarrow$

$$\boxed{A_{(1)} B_{(2)} = B_{(2)} A_{(1)} \equiv A \otimes B}$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a_{11}\cdot 1 & a_{11}\cdot 0 & a_{12}\cdot 1 & a_{12}\cdot 0 \\ a_{11}\cdot 0 & a_{11}\cdot 1 & a_{12}\cdot 0 & a_{12}\cdot 1 \\ a_{21}\cdot 1 & a_{21}\cdot 0 & a_{22}\cdot 1 & a_{22}\cdot 0 \\ a_{21}\cdot 0 & a_{21}\cdot 1 & a_{22}\cdot 0 & a_{22}\cdot 1 \end{pmatrix}$$

$$= \begin{pmatrix} a_{11} & 0 & a_{12} & 0 \\ 0 & a_{11} & 0 & a_{12} \\ a_{21} & 0 & a_{22} & 0 \\ 0 & a_{21} & 0 & a_{22} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} 1\cdot b_{11} & 1\cdot b_{12} & 0\cdot b_{11} & 0\cdot b_{12} \\ 1\cdot b_{21} & 1\cdot b_{22} & 0\cdot b_{21} & 0\cdot b_{22} \\ 0\cdot b_{11} & 0\cdot b_{12} & 1\cdot b_{11} & 1\cdot b_{12} \\ 0\cdot b_{21} & 0\cdot b_{22} & 1\cdot b_{21} & 1\cdot b_{22} \end{pmatrix}$$

$$= \begin{pmatrix} b_{11} & b_{12} & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 \\ 0 & 0 & b_{11} & b_{12} \\ 0 & 0 & b_{21} & b_{22} \end{pmatrix}$$

Kronecker product

$$A \otimes B \equiv \overbrace{\begin{bmatrix} A_{11}B & A_{12}B & \dots & A_{1n}B \\ A_{21}B & A_{22}B & \dots & A_{2n}B \\ \vdots & \vdots & \vdots & \vdots \\ A_{m1}B & A_{m2}B & \dots & A_{mn}B \end{bmatrix}}^{nq} \Bigg\} mp\,. \qquad (2.50)$$

Exercise 2.28: Show that the transpose, complex conjugation, and adjoint operations distribute over the tensor product,

$$(A \otimes B)^* = A^* \otimes B^*;\ (A \otimes B)^T = A^T \otimes B^T;\ (A \otimes B)^\dagger = A^\dagger \otimes B^\dagger. \quad (2.53)$$

$$(A_1 \otimes B_1)(A_2 \otimes B_2) = (A_1 A_2) \otimes (B_1 B_2)$$

Влаганията : $\iota_1(A) =: A_{(1)}$

$$A \longmapsto A \otimes \hat{I}$$

$$\mathcal{A} \overset{\iota_1}{\hookrightarrow} \mathcal{C} \overset{\iota_2}{\hookleftarrow} \mathcal{B}$$

$$\hat{I} \otimes B \longleftarrow B$$

$$\iota_2(B) =: B_{(2)}$$

$$\Rightarrow \quad A \otimes B = (A.1)\otimes(1.B) = (A \otimes 1)(1 \otimes B) = (1.A)\otimes(B.1) = (1 \otimes B)(A \otimes 1)$$

$$\Rightarrow \quad \boxed{A_{(1)} B_{(2)} = B_{(2)} A_{(1)} \equiv A \otimes B}$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a_{11}.1 & a_{11}.0 & a_{12}.1 & a_{12}.0 \\ a_{11}.0 & a_{11}.1 & a_{12}.0 & a_{12}.1 \\ a_{21}.1 & a_{21}.0 & a_{22}.1 & a_{22}.0 \\ a_{21}.0 & a_{21}.1 & a_{22}.0 & a_{22}.1 \end{pmatrix} = \begin{pmatrix} a_{11} & 0 & a_{12} & 0 \\ 0 & a_{11} & 0 & a_{12} \\ a_{21} & 0 & a_{22} & 0 \\ 0 & a_{21} & 0 & a_{22} \end{pmatrix}$$

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http://theo.inrne.bas.bg/~mitov/qi23_yJ4Gmi891z/NMN-QI23-08_compact-slides_v1.pdf слайд 15 ↑

ВНИМАНИЕ: Всички, които се подготвят върху квантови алгоритми, непременно трябва да са запознати и с материала на този слайд както и с слайдиве 45 - 63 на лекция 8.

$$\mathrm{tr}(A) \equiv \sum_i A_{ii}. \tag{2.59}$$

Exercise 2.37: (Cyclic property of the trace) If A and B are two linear operators show that

$$\mathrm{tr}(AB) = \mathrm{tr}(BA). \tag{2.62}$$

Exercise 2.38: (Linearity of the trace) If A and B are two linear operators, show that

$$\mathrm{tr}(A + B) = \mathrm{tr}(A) + \mathrm{tr}(B) \tag{2.63}$$

and if z is an arbitrary complex number show that

$$\mathrm{tr}(zA) = z\mathrm{tr}(A). \tag{2.64}$$

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Следа на $A = \begin{pmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & & \vdots \\ a_{N1} & \cdots & a_{NN} \end{pmatrix}$ ← квадратна

trace

$\mathrm{tr}\, A := a_{11} + a_{22} + \cdots + a_{NN}$

Основни свойства на следата

- $\mathrm{tr}(AB \cdots C) = \mathrm{tr}(B \cdots CA)$ ← цикличност
  квадратна, но $A, B, \ldots, C$ - по-общи
- $\mathrm{tr}\, \hat{1}_N = N$ — $N \times N$

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СЛЕДАТА Е "КВАНТОВАТА БРОЙНА МЯРКА"

2.1 Linear algebra

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2.1 Linear algebra

2.2 The postulates of quantum mechanics

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80 - 96

Postulate 1: Associated to any isolated physical system is a complex vector space with inner product (that is, a Hilbert space) known as the *state space* of the system. The system is completely described by its *state vector*, which is a unit vector in the system's state space.

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Postulate 2: The evolution of a *closed* quantum system is described by a *unitary transformation*. That is, the state $|\psi\rangle$ of the system at time t_1 is related to the state $|\psi'\rangle$ of the system at time t_2 by a unitary operator U which depends only on the times t_1 and t_2,

$$|\psi'\rangle = U|\psi\rangle\,. \tag{2.84}$$

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$$|\psi'\rangle = U|\psi\rangle . \tag{2.84}$$

Postulate 3: Quantum measurements are described by a collection $\{M_m\}$ of *measurement operators*. These are operators acting on the state space of the system being measured. The index m refers to the measurement outcomes that may occur in the experiment. If the state of the quantum system is $|\psi\rangle$ immediately before the measurement then the probability that result m occurs is given by

$$p(m) = \langle\psi|M_m^\dagger M_m|\psi\rangle , \tag{2.92}$$

and the state of the system after the measurement is

$$\frac{M_m|\psi\rangle}{\sqrt{\langle\psi|M_m^\dagger M_m|\psi\rangle}} . \tag{2.93}$$

The measurement operators satisfy the *completeness equation*,

$$\sum_m M_m^\dagger M_m = I . \tag{2.94}$$

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Postulate 4: The state space of a composite physical system is the tensor product of the state spaces of the component physical systems. Moreover, if we have systems numbered 1 through n, and system number i is prepared in the state $|\psi_i\rangle$, then the joint state of the total system is $|\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_n\rangle$.

2.4 The density operator

Postulate 1: Associated to any isolated physical system is a complex vector space with inner product (that is, a Hilbert space) known as the *state space* of the system. The system is completely described by its *density operator*, which is a positive operator ρ with trace one, acting on the state space of the system. If a quantum system is in the state ρ_i with probability p_i, then the density operator fo the system is $\sum_i p_i \rho_i$.

Postulate 2: The evolution of a *closed* quantum system is described by a *unitary transformation*. That is, the state ρ of the system at time t_1 is related to the state ρ' of the system at time t_2 by a unitary operator U which depends only on the times t_1 and t_2,

$$\rho' = U \rho U^\dagger. \tag{2.158}$$

Postulate 3: Quantum measurements are described by a collection $\{M_m\}$ of *measurement operators*. These are operators acting on the state space of the system being measured. The index m refers to the measurement outcomes that may occur in the experiment. If the state of the quantum system is ρ immediately before the measurement then the probability that result m occurs is given by

$$p(m) = \mathrm{tr}(M_m^\dagger M_m \rho), \tag{2.159}$$

and the state of the system after the measurement is

$$\frac{M_m \rho M_m^\dagger}{\mathrm{tr}(M_m^\dagger M_m \rho)}. \tag{2.160}$$

The measurement operators satisfy the *completeness equation*,

$$\sum_m M_m^\dagger M_m = I. \tag{2.161}$$

Postulate 4: The state space of a composite physical system is the tensor product of the state spaces of the component physical systems. Moreover, if we have systems numbered 1 through n, and system number i is prepared in the state ρ_i, then the joint state of the total system is $\rho_1 \otimes \rho_2 \otimes \ldots \rho_n$.

Аксиомите, използвани в този курс целят:

1) Да се постигне аналогия, при която една квантова система да изглежда като класическа вероятностна (статистическа) система, в която не е фиксирано определено вероятностно разпределение, а отнапред е зададено само пространтвото на събитията (т.е., т.нар. "измеримо пространство" в теория на вероятностите). След това допускаме, че може да имаме различни вероятностни разпределения върху събитията и тях ги наричаме "състояния" на разглежданата статистическа система.

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2) Описаната постановка в точка 1), когато се вземе точно за класически вероятностни модели наричаме "класическа статистическа система". Искаме тя да бъде частен случай в нашите аксиоми.

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3) Системите изучавани, в квантовата физика трябва да бъдат друг частен случай на аксиомите на квантовата теория, но заедно с тях искаме да има и "хибридни" модели, които да съчетават и двата гранични случая: напълно класическия и чисто квантовия. Последното е полезно, когато трябва да се прави паралел или преход между класически и квантови понятия.