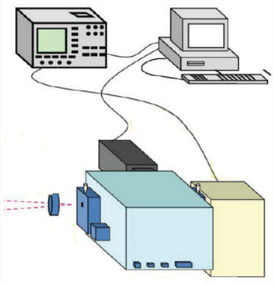
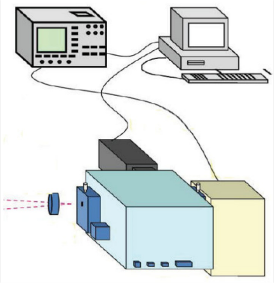
Квантови трансформации

Картина на Шрьодингер: трансформират се състоянията

Квантовата трансформация моделира трансформация на данни (експериментални!) за квантова система

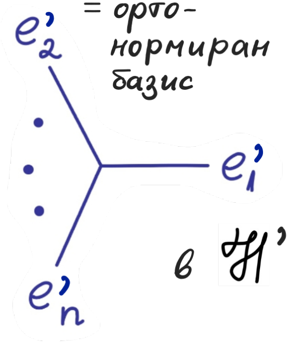
Данни (α') $\longrightarrow$ Данни (α'')

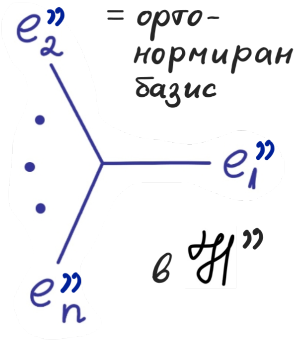




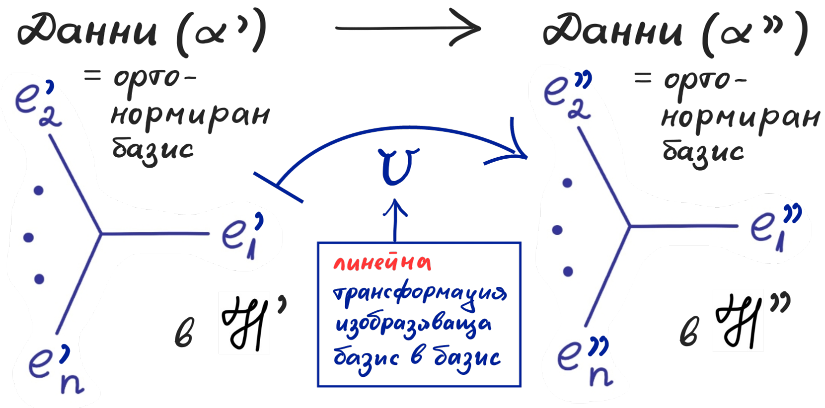
Квантовата трансформация моделира трансформация на данни (експериментални!) за квантова система

Данни (α') $\longrightarrow$ Данни (α'')





Квантовата трансформация моделира трансформация на данни (експериментални!) за квантова система



<u>Теорема</u> (Сведение от линейната алгебра)

а) Ако $U : \mathcal{H}' \longrightarrow \mathcal{H}''$ е линейно изображение, което изобразява един ортонормиран базис отново в ортонормиран базис, то U е изоморфизъм на хилбертови пространства (т.е., линейна биекция, която запазва скаларните произведения).

Теорема (Сведение от линейната алгебра)

б) Ако $U : \mathbb{C}^N \longrightarrow \mathbb{C}^N$ е отново линейно изображение, което изобразява един ортонормиран базис отново в ортонормиран базис,

<u>Теорема</u> (Сведение от линейната алгебра)

б) Ако $U : \mathbb{C}^N \longrightarrow \mathbb{C}^N$ е отново линейно изображение, което изобразява един ортонормиран базис отново в ортонормиран базис, то U се задава от унитарна матрица,

т.е.,
$$U^* = U^{-1}$$

Теорема (Сведение от линейната алгебра)

б) Ако $U : \mathbb{C}^N \longrightarrow \mathbb{C}^N$ е отново линейно изображение, което изобразява един ортонормиран базис отново в ортонормиран базис, то U се задава от унитарна матрица,

т.е.,
$$U^* = U^{-1}$$

Вярно е и твърдението в обратната посока.

Теорема (Сведение от линейната алгебра)

б) Ако $U : \mathbb{C}^N \longrightarrow \mathbb{C}^N$ е отново линейно изображение, което изобразява един ортонормиран базис отново в ортонормиран базис, то U се задава от унитарна матрица,

т.е., $$\boxed{U^* = U^{-1}}$$

Вярно е и твърдението в обратната посока.

Унитарни матрици и ортонормирани базиси

Унитарни матрици и ортонормирани базиси

$$\underbrace{\begin{pmatrix} \overline{u}_{11} & \overline{u}_{21} & \overline{u}_{31} & \cdots & \overline{u}_{n1} \\ \overline{u}_{12} & \overline{u}_{22} & \overline{u}_{32} & \cdots & \overline{u}_{n2} \\ \overline{u}_{13} & \overline{u}_{23} & \overline{u}_{33} & \cdots & \overline{u}_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \overline{u}_{1n} & \overline{u}_{2n} & \overline{u}_{3n} & \cdots & \overline{u}_{nn} \end{pmatrix}}_{U^*} \underbrace{\begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2n} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & u_{n3} & \cdots & u_{nn} \end{pmatrix}}_{U} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}}_{\hat{1}}$$

Унитарни матрици и ортонормирани базиси

$$\overbrace{\begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2n} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & u_{n3} & \cdots & u_{nn} \end{pmatrix}}^{U} \overbrace{\begin{pmatrix} \overline{u}_{11} & \overline{u}_{21} & \overline{u}_{31} & \cdots & \overline{u}_{n1} \\ \overline{u}_{12} & \overline{u}_{22} & \overline{u}_{32} & \cdots & \overline{u}_{n2} \\ \overline{u}_{13} & \overline{u}_{23} & \overline{u}_{33} & \cdots & \overline{u}_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \overline{u}_{1n} & \overline{u}_{2n} & \overline{u}_{3n} & \cdots & \overline{u}_{nn} \end{pmatrix}}^{U^*} = \overbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}}^{\hat{1}}$$

Унитарни матрици и ортонормирани базиси

$$\underbrace{\begin{pmatrix} \overline{u}_{11} & \overline{u}_{21} & \overline{u}_{31} & \cdots & \overline{u}_{n1} \\ \overline{u}_{12} & \overline{u}_{22} & \overline{u}_{32} & \cdots & \overline{u}_{n2} \\ \overline{u}_{13} & \overline{u}_{23} & \overline{u}_{33} & \cdots & \overline{u}_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \overline{u}_{1n} & \overline{u}_{2n} & \overline{u}_{3n} & \cdots & \overline{u}_{nn} \end{pmatrix}}_{U^*} \underbrace{\begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2n} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & u_{n3} & \cdots & u_{nn} \end{pmatrix}}_{U} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}}_{\hat{1}}$$

Унитарни матрици и ортонормирани базиси

$$\underbrace{\begin{pmatrix} \overline{u}_{11} & \overline{u}_{21} & \overline{u}_{31} & \cdots & \overline{u}_{n1} \\ \overline{u}_{12} & \overline{u}_{22} & \overline{u}_{32} & \cdots & \overline{u}_{n2} \\ \overline{u}_{13} & \overline{u}_{23} & \overline{u}_{33} & \cdots & \overline{u}_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \overline{u}_{1n} & \overline{u}_{2n} & \overline{u}_{3n} & \cdots & \overline{u}_{nn} \end{pmatrix}}_{U^*} \underbrace{\begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2n} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & u_{n3} & \cdots & u_{nn} \end{pmatrix}}_{U} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}}_{\hat{1}}$$

$$\begin{array}{ccccc} \downarrow & \downarrow & \downarrow & \cdots & \downarrow \\ u_{\bullet 1} & u_{\bullet 2} & u_{\bullet 3} & \cdots & u_{\bullet n} \end{array}$$

Унитарни матрици и ортонормирани базиси

$$\underbrace{\begin{pmatrix} \bar{u}_{11} & \bar{u}_{21} & \bar{u}_{31} & \cdots & \bar{u}_{n1} \\ \bar{u}_{12} & \bar{u}_{22} & \bar{u}_{32} & \cdots & \bar{u}_{n2} \\ \bar{u}_{13} & \bar{u}_{23} & \bar{u}_{33} & \cdots & \bar{u}_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{u}_{1n} & \bar{u}_{2n} & \bar{u}_{3n} & \cdots & \bar{u}_{nn} \end{pmatrix}}_{U^*} \underbrace{\begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2n} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & u_{n3} & \cdots & u_{nn} \end{pmatrix}}_{U} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}}_{\hat{1}}$$

$$\begin{array}{ccccc} \downarrow & \downarrow & \downarrow & \cdots & \downarrow \\ u_{\cdot 1} & u_{\cdot 2} & u_{\cdot 3} & \cdots & u_{\cdot n} \end{array}$$

Унитарни матрици и ортонормирани базиси

$$\overbrace{\begin{pmatrix} \bar{u}_{11} & \bar{u}_{21} & \bar{u}_{31} & \cdots & \bar{u}_{n1} \\ \bar{u}_{12} & \bar{u}_{22} & \bar{u}_{32} & \cdots & \bar{u}_{n2} \\ \bar{u}_{13} & \bar{u}_{23} & \bar{u}_{33} & \cdots & \bar{u}_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{u}_{1n} & \bar{u}_{2n} & \bar{u}_{3n} & \cdots & \bar{u}_{nn} \end{pmatrix}}^{U^*} \overbrace{\begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2n} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & u_{n3} & \cdots & u_{nn} \end{pmatrix}}^{U} = \overbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}}^{\hat{1}}$$

$$\begin{array}{ccccc} \downarrow & \downarrow & \downarrow & \cdots & \downarrow \\ u_{\bullet 1} & u_{\bullet 2} & u_{\bullet 3} & \cdots & u_{\bullet n} \end{array}$$

Унитарни матрици и ортонормирани базиси

$$
\underbrace{\begin{pmatrix} \bar{u}_{11} & \bar{u}_{21} & \bar{u}_{31} & \cdots & \bar{u}_{n1} \\ \bar{u}_{12} & \bar{u}_{22} & \bar{u}_{32} & \cdots & \bar{u}_{n2} \\ \bar{u}_{13} & \bar{u}_{23} & \bar{u}_{33} & \cdots & \bar{u}_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{u}_{1n} & \bar{u}_{2n} & \bar{u}_{3n} & \cdots & \bar{u}_{nn} \end{pmatrix}}_{U^*}
\underbrace{\begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2n} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & u_{n3} & \cdots & u_{nn} \end{pmatrix}}_{U}
=
\underbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}}_{\hat{1}}
$$

$$
\begin{matrix} \downarrow & \downarrow & \downarrow & \cdots & \downarrow \\ u_{\bullet 1} & u_{\bullet 2} & u_{\bullet 3} & \cdots & u_{\bullet n} \end{matrix}
$$

Унитарни матрици и ортонормирани базиси

$$\underbrace{\begin{pmatrix} \overline{u}_{11} & \overline{u}_{21} & \overline{u}_{31} & \cdots & \overline{u}_{n1} \\ \overline{u}_{12} & \overline{u}_{22} & \overline{u}_{32} & \cdots & \overline{u}_{n2} \\ \overline{u}_{13} & \overline{u}_{23} & \overline{u}_{33} & \cdots & \overline{u}_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \overline{u}_{1n} & \overline{u}_{2n} & \overline{u}_{3n} & \cdots & \overline{u}_{nn} \end{pmatrix}}_{U^*} \underbrace{\begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2n} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & u_{n3} & \cdots & u_{nn} \end{pmatrix}}_{U} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}}_{\hat{1}}$$

$$\begin{array}{ccccc} \downarrow & \downarrow & \downarrow & \cdots & \downarrow \\ u_{\bullet 1} & u_{\bullet 2} & u_{\bullet 3} & \cdots & u_{\bullet n} \end{array}$$

Унитарни матрици и ортонормирани базиси

$$\underbrace{\begin{pmatrix} \bar{u}_{11} & \bar{u}_{21} & \bar{u}_{31} & \cdots & \bar{u}_{n1} \\ \bar{u}_{12} & \bar{u}_{22} & \bar{u}_{32} & \cdots & \bar{u}_{n2} \\ \bar{u}_{13} & \bar{u}_{23} & \bar{u}_{33} & \cdots & \bar{u}_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{u}_{1n} & \bar{u}_{2n} & \bar{u}_{3n} & \cdots & \bar{u}_{nn} \end{pmatrix}}_{U^*} \underbrace{\begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2n} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & u_{n3} & \cdots & u_{nn} \end{pmatrix}}_{U} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}}_{\hat{1}}$$

$$\begin{matrix} \downarrow & \downarrow & \downarrow & \cdots & \downarrow \\ u_{\bullet 1} & u_{\bullet 2} & u_{\bullet 3} & \cdots & u_{\bullet n} \end{matrix}$$

Унитарни матрици и ортонормирани базиси

$$
\underbrace{\begin{pmatrix} \overline{u}_{11} & \overline{u}_{21} & \overline{u}_{31} & \cdots & \overline{u}_{n1} \\ \overline{u}_{12} & \overline{u}_{22} & \overline{u}_{32} & \cdots & \overline{u}_{n2} \\ \overline{u}_{13} & \overline{u}_{23} & \overline{u}_{33} & \cdots & \overline{u}_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \overline{u}_{1n} & \overline{u}_{2n} & \overline{u}_{3n} & \cdots & \overline{u}_{nn} \end{pmatrix}}_{U^*}
\underbrace{\begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2n} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & u_{n3} & \cdots & u_{nn} \end{pmatrix}}_{U}
=
\underbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}}_{\hat{1}}
$$

$$
\begin{array}{ccccc} \downarrow & \downarrow & \downarrow & \cdots & \downarrow \\ u_1 & u_2 & u_3 & \cdots & u_n \end{array}
$$

Унитарни матрици и ортонормирани базиси

$$\underbrace{\begin{pmatrix} \overline{u}_{11} & \overline{u}_{21} & \overline{u}_{31} & \cdots & \overline{u}_{n1} \\ \overline{u}_{12} & \overline{u}_{22} & \overline{u}_{32} & \cdots & \overline{u}_{n2} \\ \overline{u}_{13} & \overline{u}_{23} & \overline{u}_{33} & \cdots & \overline{u}_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \overline{u}_{1n} & \overline{u}_{2n} & \overline{u}_{3n} & \cdots & \overline{u}_{nn} \end{pmatrix}}_{U^*} \underbrace{\begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2n} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & u_{n3} & \cdots & u_{nn} \end{pmatrix}}_{U} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}}_{\hat{1}}$$

$$\begin{array}{ccccc} \downarrow & \downarrow & \downarrow & \cdots & \downarrow \\ u_1 & u_2 & u_3 & \cdots & u_n \end{array}$$

$$U^* U = \hat{1}$$

$$\iff \langle u_j | u_k \rangle = \begin{cases} 1 & \text{при } j = k \\ 0 & \text{при } j \neq k \end{cases}$$

Унитарни матрици и ортонормирани базиси

$$
\underbrace{\begin{pmatrix} \bar{u}_{11} & \bar{u}_{21} & \bar{u}_{31} & \cdots & \bar{u}_{n1} \\ \bar{u}_{12} & \bar{u}_{22} & \bar{u}_{32} & \cdots & \bar{u}_{n2} \\ \bar{u}_{13} & \bar{u}_{23} & \bar{u}_{33} & \cdots & \bar{u}_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{u}_{1n} & \bar{u}_{2n} & \bar{u}_{3n} & \cdots & \bar{u}_{nn} \end{pmatrix}}_{U^*}
\underbrace{\begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2n} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & u_{n3} & \cdots & u_{nn} \end{pmatrix}}_{U}
=
\underbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}}_{\hat{1}}
$$

$$
\begin{array}{ccccc} \downarrow & \downarrow & \downarrow & \cdots & \downarrow \\ u_1 & u_2 & u_3 & \cdots & u_n \end{array}
$$

$$U^* U = \hat{1}$$

$$\iff \langle u_j | u_k \rangle = \begin{cases} 1 & \text{при } j = k \\ 0 & \text{при } j \neq k \end{cases}$$

Унитарни матрици и ортонормирани базиси

$$
\underbrace{\begin{pmatrix} \overline{u}_{11} & \overline{u}_{21} & \overline{u}_{31} & \cdots & \overline{u}_{n1} \\ \overline{u}_{12} & \overline{u}_{22} & \overline{u}_{32} & \cdots & \overline{u}_{n2} \\ \overline{u}_{13} & \overline{u}_{23} & \overline{u}_{33} & \cdots & \overline{u}_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \overline{u}_{1n} & \overline{u}_{2n} & \overline{u}_{3n} & \cdots & \overline{u}_{nn} \end{pmatrix}}_{U^*}
\underbrace{\begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2n} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & u_{n3} & \cdots & u_{nn} \end{pmatrix}}_{U}
=
\underbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}}_{\hat{1}}
$$

$$
\begin{array}{ccccc} \downarrow & \downarrow & \downarrow & \cdots & \downarrow \\ u_1 & u_2 & u_3 & \cdots & u_n \end{array}
$$

$$U^* U = \hat{1}$$

$$\iff \langle u_j | u_k \rangle = \begin{cases} 1 & \text{при } j = k \\ 0 & \text{при } j \neq k \end{cases}$$

Унитарни матрици и ортонормирани базиси

$$\overbrace{\begin{pmatrix} \bar u_{11} & \bar u_{21} & \bar u_{31} & \cdots & \bar u_{n1} \\ \bar u_{12} & \bar u_{22} & \bar u_{32} & \cdots & \bar u_{n2} \\ \bar u_{13} & \bar u_{23} & \bar u_{33} & \cdots & \bar u_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar u_{1n} & \bar u_{2n} & \bar u_{3n} & \cdots & \bar u_{nn} \end{pmatrix}}^{U^*} \underset{\begin{matrix} \downarrow & \downarrow & \downarrow & \cdots & \downarrow \\ u_1 & u_2 & u_3 & \cdots & u_n \end{matrix}}{\overbrace{\begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2n} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & u_{n3} & \cdots & u_{nn} \end{pmatrix}}^{U}} = \overbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}}^{\hat{1}}$$

$$U^* U = \hat{1}$$

$$\iff \langle u_j | u_k \rangle = \begin{cases} 1 & \text{при } j = k \\ 0 & \text{при } j \neq k \end{cases}$$

Унитарни матрици и ортонормирани базиси

$$\underbrace{\begin{pmatrix} \overline{u}_{11} & \overline{u}_{21} & \overline{u}_{31} & \cdots & \overline{u}_{n1} \\ \overline{u}_{12} & \overline{u}_{22} & \overline{u}_{32} & \cdots & \overline{u}_{n2} \\ \overline{u}_{13} & \overline{u}_{23} & \overline{u}_{33} & \cdots & \overline{u}_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \overline{u}_{1n} & \overline{u}_{2n} & \overline{u}_{3n} & \cdots & \overline{u}_{nn} \end{pmatrix}}_{U^*} \underbrace{\begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2n} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & u_{n3} & \cdots & u_{nn} \end{pmatrix}}_{U} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}}_{\hat{1}}$$

$$\begin{array}{ccccc} \downarrow & \downarrow & \downarrow & \cdots & \downarrow \\ u_1 & u_2 & u_3 & \cdots & u_n \end{array}$$

$$U^* U = \hat{1}$$

$$\Longleftrightarrow \quad \langle u_j | u_k \rangle = \begin{cases} 1 & \text{при } j = k \\ 0 & \text{при } j \neq k \end{cases}$$

Унитарни матрици и ортонормирани базиси

$$\underbrace{\begin{pmatrix} \bar{u}_{11} & \bar{u}_{21} & \bar{u}_{31} & \cdots & \bar{u}_{n1} \\ \bar{u}_{12} & \bar{u}_{22} & \bar{u}_{32} & \cdots & \bar{u}_{n2} \\ \bar{u}_{13} & \bar{u}_{23} & \bar{u}_{33} & \cdots & \bar{u}_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{u}_{1n} & \bar{u}_{2n} & \bar{u}_{3n} & \cdots & \bar{u}_{nn} \end{pmatrix}}_{U^*} \underbrace{\begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2n} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & u_{n3} & \cdots & u_{nn} \end{pmatrix}}_{U} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}}_{\hat{1}}$$

$$\begin{matrix} \downarrow & \downarrow & \downarrow & \cdots & \downarrow \\ u_1 & u_2 & u_3 & \cdots & u_n \end{matrix}$$

U е унитарна $\iff$ колоните (или редовете) са ортонормиран базис

$$U^* U = \hat{1}$$

$$\iff \langle u_j | u_k \rangle = \begin{cases} 1 & \text{при } j = k \\ 0 & \text{при } j \neq k \end{cases}$$

Унитарни матрици и ортонормирани базиси

$$\underbrace{\begin{pmatrix} \overline{u}_{11} & \overline{u}_{21} & \overline{u}_{31} & \cdots & \overline{u}_{n1} \\ \overline{u}_{12} & \overline{u}_{22} & \overline{u}_{32} & \cdots & \overline{u}_{n2} \\ \overline{u}_{13} & \overline{u}_{23} & \overline{u}_{33} & \cdots & \overline{u}_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \overline{u}_{1n} & \overline{u}_{2n} & \overline{u}_{3n} & \cdots & \overline{u}_{nn} \end{pmatrix}}_{U^*} \underbrace{\begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2n} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & u_{n3} & \cdots & u_{nn} \end{pmatrix}}_{U} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}}_{\hat{1}}$$

$$\begin{matrix} \downarrow & \downarrow & \downarrow & \cdots & \downarrow \\ u_1 & u_2 & u_3 & \cdots & u_n \end{matrix}$$

Унитарни матрици и ортонормирани базиси

$$
\overbrace{\begin{pmatrix}
u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\
u_{21} & u_{22} & u_{23} & \cdots & u_{2n} \\
u_{31} & u_{32} & u_{33} & \cdots & u_{3n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
u_{n1} & u_{n2} & u_{n3} & \cdots & u_{nn}
\end{pmatrix}}^{U}
\overbrace{\begin{pmatrix}
\overline{u}_{11} & \overline{u}_{21} & \overline{u}_{31} & \cdots & \overline{u}_{n1} \\
\overline{u}_{12} & \overline{u}_{22} & \overline{u}_{32} & \cdots & \overline{u}_{n2} \\
\overline{u}_{13} & \overline{u}_{23} & \overline{u}_{33} & \cdots & \overline{u}_{n3} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\overline{u}_{1n} & \overline{u}_{2n} & \overline{u}_{3n} & \cdots & \overline{u}_{nn}
\end{pmatrix}}^{U^*}
=
\overbrace{\begin{pmatrix}
1 & 0 & 0 & \cdots & 0 \\
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1
\end{pmatrix}}^{\hat{1}}
$$

$$
\begin{array}{ccccc}
\downarrow & \downarrow & \downarrow & \cdots & \downarrow \\
u_1 & u_2 & u_3 & \cdots & u_n
\end{array}
$$

Унитарни матрици и ортонормирани базиси

$$
\underbrace{\begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2n} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & u_{n3} & \cdots & u_{nn} \end{pmatrix}}_{U}
\underbrace{\begin{pmatrix} \overline{u}_{11} & \overline{u}_{21} & \overline{u}_{31} & \cdots & \overline{u}_{n1} \\ \overline{u}_{12} & \overline{u}_{22} & \overline{u}_{32} & \cdots & \overline{u}_{n2} \\ \overline{u}_{13} & \overline{u}_{23} & \overline{u}_{33} & \cdots & \overline{u}_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \overline{u}_{1n} & \overline{u}_{2n} & \overline{u}_{3n} & \cdots & \overline{u}_{nn} \end{pmatrix}}_{U^*}
=
\underbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}}_{\hat{1}}
$$

$$
\begin{array}{ccccc} \downarrow & \downarrow & \downarrow & \cdots & \downarrow \\ u_1 & u_2 & u_3 & \cdots & u_n \end{array}
$$

$$UU^* = \hat{1}$$

$$\Longleftrightarrow \quad |u_1\rangle\langle u_1| + \cdots + |u_n\rangle\langle u_n| = \hat{1}$$

Унитарни матрици и ортонормирани базиси

$$\underbrace{\begin{pmatrix} \overline{u}_{11} & \overline{u}_{21} & \overline{u}_{31} & \cdots & \overline{u}_{n1} \\ \overline{u}_{12} & \overline{u}_{22} & \overline{u}_{32} & \cdots & \overline{u}_{n2} \\ \overline{u}_{13} & \overline{u}_{23} & \overline{u}_{33} & \cdots & \overline{u}_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \overline{u}_{1n} & \overline{u}_{2n} & \overline{u}_{3n} & \cdots & \overline{u}_{nn} \end{pmatrix}}_{U^*} \underbrace{\begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2n} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & u_{n3} & \cdots & u_{nn} \end{pmatrix}}_{U} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}}_{\hat{1}}$$

$$\begin{matrix} \downarrow & \downarrow & \downarrow & \cdots & \downarrow \\ u_1 & u_2 & u_3 & \cdots & u_n \end{matrix}$$

$$U^* U = \hat{1}$$

$$\Longleftrightarrow \quad \langle u_j | u_k \rangle = \begin{cases} 1 & \text{при } j = k \\ 0 & \text{при } j \neq k \end{cases}$$

Унитарни матрици и ортонормирани базиси

$$\overbrace{\begin{pmatrix} \overline{u}_{11} & \overline{u}_{21} & \overline{u}_{31} & \cdots & \overline{u}_{n1} \\ \overline{u}_{12} & \overline{u}_{22} & \overline{u}_{32} & \cdots & \overline{u}_{n2} \\ \overline{u}_{13} & \overline{u}_{23} & \overline{u}_{33} & \cdots & \overline{u}_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \overline{u}_{1n} & \overline{u}_{2n} & \overline{u}_{3n} & \cdots & \overline{u}_{nn} \end{pmatrix}}^{U^*} \overbrace{\begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2n} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & u_{n3} & \cdots & u_{nn} \end{pmatrix}}^{U} = \overbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}}^{\hat{1}}$$

$$\begin{array}{ccccc} \downarrow & \downarrow & \downarrow & \cdots & \downarrow \\ u_1 & u_2 & u_3 & \cdots & u_n \end{array}$$

$$e_j \overset{U}{\longmapsto} u_j$$

$$U^* U = \hat{1}$$

$$\Longleftrightarrow \quad \langle u_j | u_k \rangle = \begin{cases} 1 & \text{при } j = k \\ 0 & \text{при } j \neq k \end{cases}$$

Унитарни матрици и ортонормирани базиси

$$\underbrace{\begin{pmatrix} \overline{u}_{11} & \overline{u}_{21} & \overline{u}_{31} & \cdots & \overline{u}_{n1} \\ \overline{u}_{12} & \overline{u}_{22} & \overline{u}_{32} & \cdots & \overline{u}_{n2} \\ \overline{u}_{13} & \overline{u}_{23} & \overline{u}_{33} & \cdots & \overline{u}_{n3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \overline{u}_{1n} & \overline{u}_{2n} & \overline{u}_{3n} & \cdots & \overline{u}_{nn} \end{pmatrix}}_{U^*} \underbrace{\begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2n} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & u_{n3} & \cdots & u_{nn} \end{pmatrix}}_{U} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}}_{\hat{1}}$$

$$\begin{array}{ccccc} \downarrow & \downarrow & \downarrow & \cdots & \downarrow \\ u_1 & u_2 & u_3 & \cdots & u_n \end{array}$$

$$e_j \overset{U}{\longmapsto} u_j$$

U е унитарна $\Longleftrightarrow$ колоните (или редовете) са ортонормиран базис

$$U^* U = \hat{1}$$

$$\Longleftrightarrow \quad \langle u_j | u_k \rangle = \begin{cases} 1 & \text{при } j = k \\ 0 & \text{при } j \neq k \end{cases}$$

$$U U^* = \hat{1}$$

$$\Longleftrightarrow \quad |u_1\rangle\langle u_1| + \cdots + |u_n\rangle\langle u_n| = \hat{1}$$

Унитарни матрици и ортонормирани базиси

Унитарни матрици и ортонормирани базиси

– пример 1.: гейт на Адамар / Hadamard gate

Унитарни матрици и ортонормирани базиси

- пример 1.: гейт на Адамар / Hadamard gate

$$H := \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Унитарни матрици и ортонормирани базиси

- пример 1.: гейт на Адамар / Hadamard gate

$$H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Унитарни матрици и ортонормирани базиси

– пример 1.: гейт на Адамар / Hadamard gate

$$H := \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad e_0 \equiv |0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$$

Унитарни матрици и ортонормирани базиси

– пример 1.: гейт на Адамар / Hadamard gate

$$H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad e_0 \equiv |0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

Унитарни матрици и ортонормирани базиси

- пример 1.: гейт на Адамар / Hadamard gate

$$H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad e_0 \equiv |0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \stackrel{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

Унитарни матрици и ортонормирани базиси

– пример 1.: гейт на Адамар / Hadamard gate

$$H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$e_1 \equiv |1\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

Унитарни матрици и ортонормирани базиси

– пример 1.: гейт на Адамар / Hadamard gate

$$H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad e_1 \equiv |1\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

Унитарни матрици и ортонормирани базиси

– пример 1.: гейт на Адамар / Hadamard gate

$$H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$e_1 \equiv |1\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \xmapsto{H} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

Унитарни матрици и ортонормирани базиси

– пример 1.: гейт на Адамар / Hadamard gate

$$H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad \begin{cases} e_0 \equiv |0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \\ e_1 \equiv |1\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \equiv \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \end{cases}$$

Унитарни матрици и ортонормирани базиси

– пример 1.: гейт на Адамар / Hadamard gate

$$H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\begin{cases} e_0 \equiv |0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \\ e_1 \equiv |1\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \equiv \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \end{cases}$$

Унитарни матрици и ортонормирани базиси

– пример 1.: гейт на Адамар / Hadamard gate

$$H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

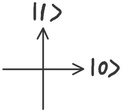
$$\begin{cases} e_0 \equiv |0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \\ e_1 \equiv |1\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \equiv \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \end{cases}$$

Унитарни матрици и ортонормирани базиси

– пример 1.: гейт на Адамар / Hadamard gate

$$H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\begin{cases} e_0 \equiv |0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \\ e_1 \equiv |1\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \equiv \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \end{cases}$$

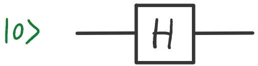


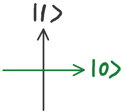
Унитарни матрици и ортонормирани базиси

– пример 1.: гейт на Адамар / Hadamard gate

$$H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\begin{cases} e_0 \equiv |0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \\ e_1 \equiv |1\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \equiv \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \end{cases}$$



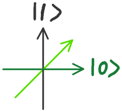


Унитарни матрици и ортонормирани базиси

– пример 1.: гейт на Адамар / Hadamard gate

$$H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad \begin{cases} e_0 \equiv |0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \\ e_1 \equiv |1\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \equiv \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \end{cases}$$

$$|0\rangle \; —\boxed{H}— \; 2^{-1/2}|0\rangle + 2^{-1/2}|1\rangle$$



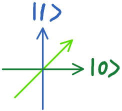
Унитарни матрици и ортонормирани базиси

– пример 1.: гейт на Адамар / Hadamard gate

$$H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\begin{cases} e_0 \equiv |0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \\ e_1 \equiv |1\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \equiv \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \end{cases}$$

$|1\rangle$ 

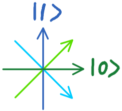


Унитарни матрици и ортонормирани базиси

– пример 1.: гейт на Адамар / Hadamard gate

$$H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad \begin{cases} e_0 \equiv |0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \\ e_1 \equiv |1\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \equiv \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \end{cases}$$

$$|1\rangle \;\text{——}\boxed{H}\text{——}\; 2^{-1/2}|0\rangle + 2^{-1/2}|1\rangle$$

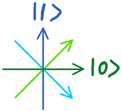


Унитарни матрици и ортонормирани базиси

– пример 1.: гейт на Адамар / Hadamard gate

$$H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad \begin{cases} e_0 \equiv |0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \\ e_1 \equiv |1\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \equiv \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \end{cases}$$

$$\left.\begin{matrix} |0\rangle \\ |1\rangle \end{matrix}\right\} \;—\; \boxed{H} \;—\; \left\{\begin{matrix} 2^{-1/2}|0\rangle + 2^{-1/2}|1\rangle \\ 2^{-1/2}|0\rangle - 2^{-1/2}|1\rangle \end{matrix}\right.$$



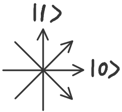
Унитарни матрици и ортонормирани базиси

– пример 1.: гейт на Адамар / Hadamard gate

$$H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad \begin{cases} e_0 \equiv |0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \\ e_1 \equiv |1\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \equiv \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \end{cases}$$

$$\left.\begin{matrix} |0\rangle \\ |1\rangle \end{matrix}\right\}$$

$$\left\{\begin{matrix} 2^{-1/2}|0\rangle + 2^{-1/2}|1\rangle \\ 2^{-1/2}|0\rangle - 2^{-1/2}|1\rangle \end{matrix}\right.$$

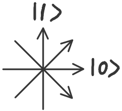


Унитарни матрици и ортонормирани базиси

– пример 1.: гейт на Адамар / Hadamard gate

$$H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad \begin{cases} e_0 \equiv |0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \\ e_1 \equiv |1\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \equiv \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \end{cases}$$

$$\left.\begin{matrix} |0\rangle \\ |1\rangle \end{matrix}\right\} \;—\boxed{H}—\; \left\{\begin{matrix} 2^{-1/2}|0\rangle + 2^{-1/2}|1\rangle \\ 2^{-1/2}|0\rangle - 2^{-1/2}|1\rangle \end{matrix}\right.$$



– пример 2: Ротация на ъгъл α : $\quad R = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$

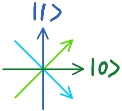
Унитарни матрици и ортонормирани базиси

– пример 1.: гейт на Адамар / Hadamard gate

$$H := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad \begin{cases} e_0 \equiv |0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \\ e_1 \equiv |1\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \overset{H}{\longmapsto} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \equiv \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \end{cases}$$

$$\left.\begin{matrix} |0\rangle \\ |1\rangle \end{matrix}\right\}$$

$$\begin{cases} 2^{-1/2}|0\rangle + 2^{-1/2}|1\rangle \\ 2^{-1/2}|0\rangle - 2^{-1/2}|1\rangle \end{cases}$$



– пример 2: Ротация на ъгъл α :

$$R = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$$

В заключение:

В заключение:

Обръщаме внимание, че

КВАНТОВИТЕ ТРАНСФОРМАЦИИ СА ВИНАГИ ОБРАТИМИ!

В заключение:

Обръщаме внимание, че
КВАНТОВИТЕ ТРАНСФОРМАЦИИ СА ВИНАГИ ОБРАТИМИ!

Няма задоволителна и общоприета теория за прозволни квантови квантови трансформации.

В заключение:

Обръщаме внимание, че
КВАНТОВИТЕ ТРАНСФОРМАЦИИ СА ВИНАГИ ОБРАТИМИ!

Няма задоволителна и общоприета теория за прозволни квантови квантови трансформации.

Обратимостта на квантовите трансформации при квантовите изчисления е аналог на т.нар. ОБРАТИМИ ИЗЧИСЛЕНИЯ.

В заключение:

Обръщаме внимание, че
КВАНТОВИТЕ ТРАНСФОРМАЦИИ СА ВИНАГИ ОБРАТИМИ!

Няма задоволителна и общоприета теория за прозволни квантови квантови трансформации.

Обратимостта на квантовите трансформации при квантовите изчисления е аналог на т.нар. ОБРАТИМИ ИЗЧИСЛЕНИЯ.

en.wikipedia.org/wiki/Reversible_computing