Наблюдение – измерване

$A \leftrightarrow$ различни реални числа

$\alpha_1, \dots, \alpha_n \in \mathbb{R}$ – изходи на измерване

$$\updownarrow \qquad \updownarrow$$

$$Q_1 \dot{\vee} \cdots \dot{\vee} Q_n = 1$$

$Q_j^* = Q_j = Q_j^2$ $N \times N$-матрици – взаимно орт. пр.

$$\hat{A} := \alpha_1 Q_1 + \alpha_2 Q_2 + \cdots + \alpha_n Q_n$$

Тогава: $\hat{A} = \hat{A}^*$

Освен: $\mathbb{C}^N = \underbrace{Q_1 \mathbb{C}^N}_{V_1} \oplus \cdots \oplus Q_n \mathbb{C}^N$

$$\hat{A} v_1 = \alpha_1 \underbrace{Q_1 v_1}_{v_1} + \alpha_2 \underbrace{Q_2 v_2}_{0} + \cdots + \alpha_n \underbrace{Q_n v_n}_{0}$$

$\Rightarrow \hat{A} v_1 = \alpha_1 v_1$ – собств. в-р.

аналог.: $v_j \in Q_j \mathbb{C}^N$

тогава $A v_j = \alpha_j v_j$

Обръщане на конструкцията

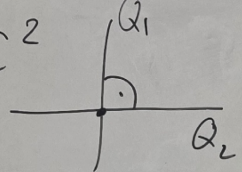
$A = A^*$ – дадено

да конструираме разлагане

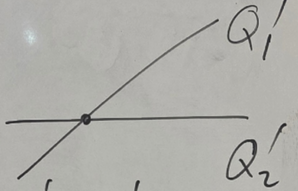
$$A = \alpha_1 Q_1 + \cdots + \alpha_n Q_n$$

за разл. α_j и $Q_1 \dot{\vee} \cdots \dot{\vee} Q_n = 0$

Пример: $\mathbb{C}^2$



Контрапример:



$Q_1 \dot{\vee} Q_2 = 1$ $\qquad$ $Q_1' \vee Q_2' = 1$

$Q_1 \wedge Q_2 = 0$ $\qquad$ $Q_1' \wedge Q' = 0$

$Q_1 \,\mathrm{C}\, Q_2$ $\qquad$ $Q_1' \,\not\mathrm{C}\, Q_2'$

$$\begin{pmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & & \vdots \\ a_{N1} & \cdots & a_{NN} \end{pmatrix}^* = \begin{pmatrix} \bar{a}_{11} & \cdots & \bar{a}_{N1} \\ \vdots & & \vdots \\ \bar{a}_{1N} & \cdots & \bar{a}_{NN} \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & & \vdots \\ a_{N1} & \cdots & a_{NN} \end{pmatrix} = \lambda_1 \begin{pmatrix} f_{11} \\ \vdots \\ f_{1N} \end{pmatrix} (\bar{f}_{11}, \dots, \bar{f}_{1N})$$

$$+ \cdots + \lambda_N \begin{pmatrix} f_{N1} \\ \vdots \\ f_{NN} \end{pmatrix} (\bar{f}_{N1}, \dots, \bar{f}_{NN})$$

$$= \sum_{j=1}^{N} \lambda_j \begin{pmatrix} f_{j1} \bar{f}_{j1} & \cdots & f_{j1} \bar{f}_{jN} \\ \vdots & & \vdots \\ f_{jN} \bar{f}_{j1} & \cdots & f_{jN} \bar{f}_{jN} \end{pmatrix}$$

$$\equiv \lambda_1 \underbrace{|f_1\rangle}_{f_1} \underbrace{\langle f_1|}_{f_1^*} + \cdots + \lambda_N |f_N\rangle\langle f_N|$$

" $\hat{F}_j = \hat{m}\,\hat{a}_j$ | $\hat{U} = \hat{I}\,\hat{R}$ "

$\hat{I}\hat{R} \neq \hat{R}\hat{I}$

Dirac – "q-numbers"

c-numbers

algebra (associative)

1927

$\langle \phi | \psi \rangle = \langle \phi || \psi \rangle$

" $\langle \phi$ "

von Neumann 1930 | Jordan Wigner 1932 | Q-Log. Birchoff 1936

$\lambda_1 \le \lambda_2 \le \lambda_3 \le \cdots \le \lambda_N$

$\underbrace{\lambda_1 = \lambda_2 = \cdots = \lambda_{k_1}}_{\alpha_1} < \underbrace{\lambda_{k_1+1} = \cdots = \lambda_{k_1+k_2}}_{\alpha_2} < \cdots$

$\alpha_1 < \alpha_2$

От лекция 2:

Q е орт. пр. в/у $V \underset{\text{лин.}}{\subseteq} \mathbb{C}^N$

и $f_1, \dots, f_k$ е орт. норм. б. на V

то $Q = |f_1\rangle\langle f_1| + \cdots + |f_k\rangle\langle f_k|$

$\underbrace{(zf_1)\,(zf_1)^*}_{\underbrace{|z|^2}_{1} f_1 f_1^*}$

$\boxed{\text{SpecThm}}$