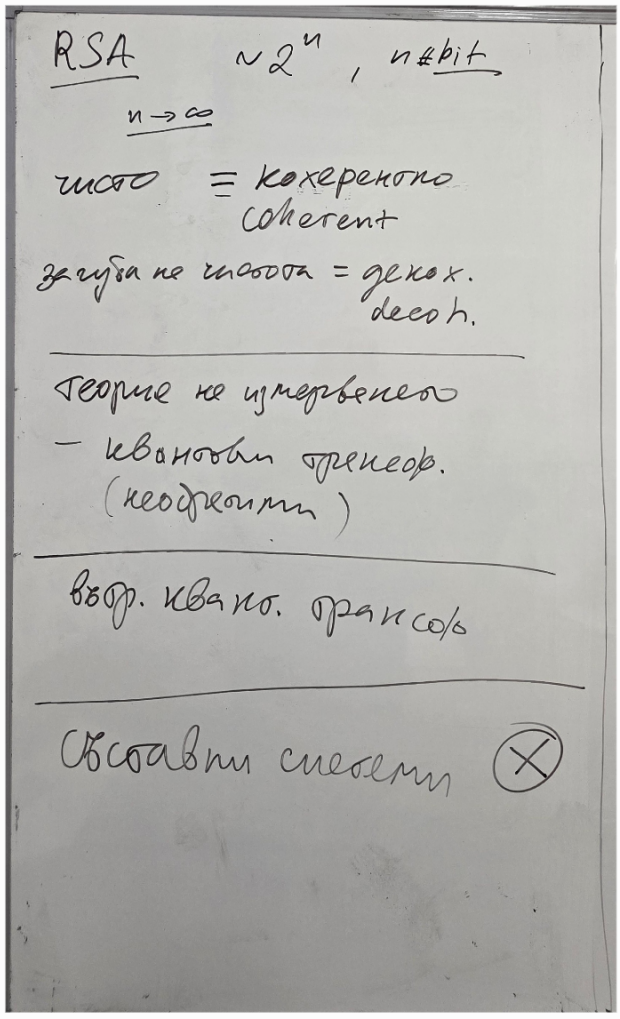
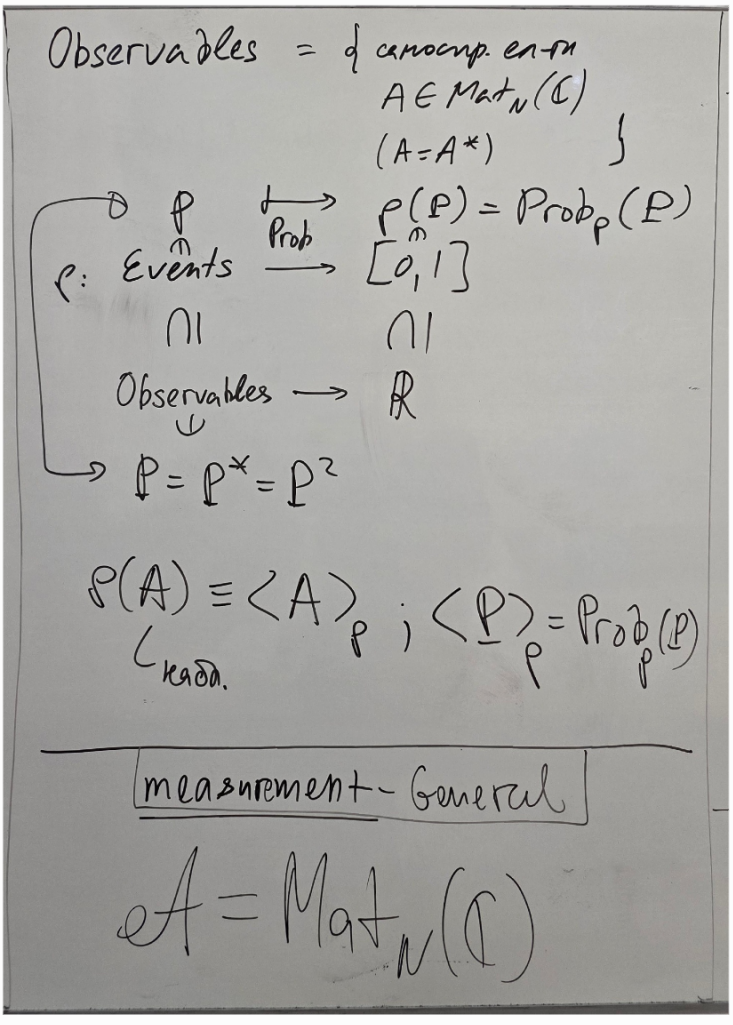
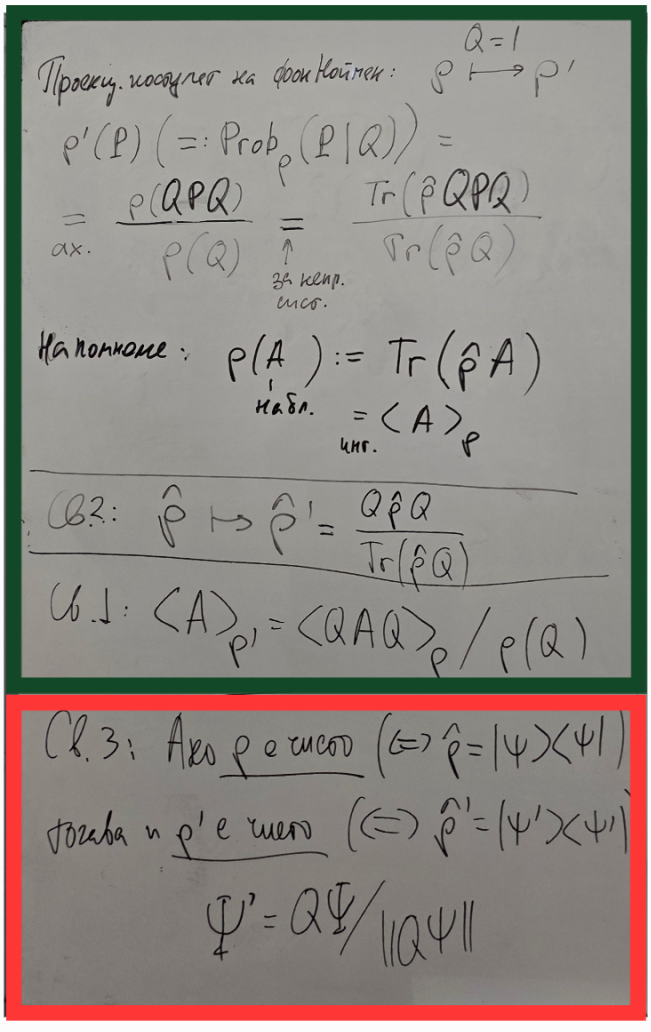


В това поле са снимки на дъските от дясната половина, на които са изписани по-центалните твърдения





Защо ρ' е състояние?

<u>Заб.</u> PQP – не е проектор

$$(PQP)^* = P^* Q^* P^* = PQP$$

P_1, P_2 – дизюнктивни

($\Leftrightarrow$ $P = P_1 + P_2$ е събитие

Th (и тогава $P = P_1 \dot{\vee} P_2$)

$$\rho'(P_1 \dot{\vee} P_2) = \rho'(P_1) + \rho'(P_2)$$

$$\rho'(P_1 + P_2) \overset{?}{=} \rho'(P_1) + \rho'(P_2)$$

<u>Заб.:</u> $\|Q\Psi\|^2 = \langle Q\Psi | Q\Psi \rangle = \langle \Psi | \underbrace{Q^* Q}_{Q} \Psi \rangle$

$= \langle \Psi | Q \Psi \rangle = \rho(Q)$

Проекц. постулат на фон Нойман: $\rho \overset{Q=1}{\longmapsto} \rho'$

$$\rho'(P) \ \left(=: \mathrm{Prob}_\rho(P|Q) \right) = \underset{\text{ax.}}{=} \frac{\rho(QPQ)}{\rho(Q)} \underset{\uparrow}{=} \frac{Tr(\hat{\rho} QPQ)}{Tr(\hat{\rho} Q)}$$

(за конкр. сист.)

Напомняне: $\rho(\underset{\text{набл.}}{A}) := Tr(\hat{\rho} A) \underset{\text{инт.}}{=} \langle A \rangle_\rho$

Св.2: $\hat{\rho} \mapsto \hat{\rho}' = \dfrac{Q \hat{\rho} Q}{Tr(\hat{\rho} Q)}$

Св.1: $\langle A \rangle_{\rho'} = \langle QAQ \rangle_\rho / \rho(Q)$

Св.3: Ако <u>ρ е чисто</u> ($\Leftrightarrow$ $\hat{\rho} = |\psi\rangle\langle\psi|$)

тогава и <u>ρ' е чисто</u> ($\Leftrightarrow$ $\hat{\rho}' = |\psi'\rangle\langle\psi'|$)

$$\Psi' = Q\Psi / \|Q\Psi\|$$

Док. на Св.3)

$\hat{\rho}' \overset{?}{=} |\Psi'\rangle\langle\Psi'|$

за $\Psi' = Q\Psi / \|Q\Psi\|$

<u>По Св.2</u>: $\hat{\rho}' = Q \hat{\rho} Q / \|Q\Psi\|^2$

$$= Q \underbrace{|\Psi\rangle}_{\Psi} \underbrace{\langle\Psi|}_{\Psi^*} Q / \|Q\Psi\|^2$$

$$Q\Psi \qquad \Psi^* Q = (Q^*\Psi)^* = (Q\Psi)^*$$

$$\|Q\Psi\|^{-1} \qquad \|Q\Psi\|^{-1}$$

$$= \Psi' \Psi'^* = |\Psi'\rangle\langle\Psi'|$$

$\square$

Вероятност за преход:

$$\Psi \mapsto \Phi$$

$$|\langle \Psi | \Phi \rangle|^2$$

<u>Св.4</u>: При измерване на елементарно събитие настъпва чисто състояние: именно това, което съотв. на елем. събитие

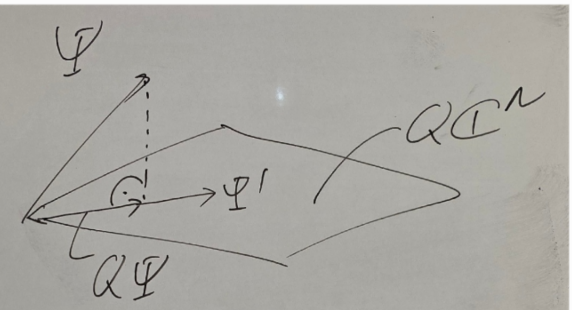
Ком.: $\Psi = |\Psi\rangle$

$\Psi^* = \langle\Psi|$

$\Rightarrow A|\Psi\rangle = |A\Psi\rangle$

Но $\underbrace{\langle\Psi|A}_{\Psi^*A} = \underbrace{\langle A^*\Psi|}_{(A^*\Psi)^*}$

$$\rho \mapsto \rho' = \frac{Q|\Psi\rangle\langle\Psi|Q}{\|Q\Psi\|^2} = \frac{|Q\Psi\rangle\langle Q\Psi|}{\|Q\Psi\|^2} = |\Psi'\rangle\langle\Psi'|$$



Класически случай

Q и P – комутират ($\Longleftrightarrow$ $PQ = QP$)

Тогава $QPQ = QP = PQ$
$= Q \wedge P$

$$\rho'(P) = \frac{\rho(QPQ)}{\rho(Q)} = \mathrm{Prob}_\rho(P|Q) = \frac{\mathrm{Prob}_\rho(Q\wedge P)}{\mathrm{Prob}_\rho(Q)}$$

Проекц. постулат на фон Нойман: $\rho \overset{Q=1}{\longmapsto} \rho'$

$$\rho'(P) \ (=: \mathrm{Prob}_\rho(P|Q)) = \underset{\text{ax.}}{=} \frac{\rho(QPQ)}{\rho(Q)} \underset{\substack{\uparrow \\ \text{за конкр.} \\ \text{сис.}}}{=} \frac{\mathrm{Tr}(\hat\rho QPQ)}{\mathrm{Tr}(\hat\rho Q)}$$

Напомняне: $\underset{\text{набл.}}{\rho(A)} := \mathrm{Tr}(\hat\rho A) \underset{\text{инт.}}{=} \langle A\rangle_\rho$

Св.2: $\hat\rho \mapsto \hat\rho' = \dfrac{Q\hat\rho Q}{\mathrm{Tr}(\hat\rho Q)}$

Св.1: $\langle A\rangle_{\rho'} = \langle QAQ\rangle_\rho / \rho(Q)$

Св.3: Ако ρ е чисто ($\Longleftrightarrow$ $\hat\rho = |\Psi\rangle\langle\Psi|$)

тогава и ρ' е чисто ($\Longleftrightarrow$ $\hat\rho' = |\Psi'\rangle\langle\Psi'|$)

$$\Psi' = Q\Psi / \|Q\Psi\|$$

Вероятност за преход:

$$\Psi \longmapsto \Phi$$

$$|\langle\Psi|\Phi\rangle|^2$$

Св.4: При измерване на елементарно събитие настъпва чисто състояние: именно това, което съотв. на елем. събитие

Вероятност за преход
"transition prob."
между състояния.

чисто съст. $\Longleftrightarrow$ $\hat{\rho} = |\Psi\rangle\langle\Psi|$

ρ

$\updownarrow$ 1-1-съотв.

Елем. съб. P $\Longleftrightarrow$ $P = |\Psi\rangle\langle\Psi|$

$\rho \underset{\text{съотв.}}{\longleftrightarrow} P$

$\Updownarrow$

$\mathrm{Prob}_\rho(P) = 1$

Проекц. постулат на фон Нойман: $\rho \overset{Q=1}{\longmapsto} \rho'$

$$\rho'(P) \; \left(=: \mathrm{Prob}_\rho(P|Q)\right) = \underset{\text{ax.}}{=} \frac{\rho(QPQ)}{\rho(Q)} \underset{\text{за непр. съст.}}{=} \frac{\mathrm{Tr}(\hat{\rho}QPQ)}{\mathrm{Tr}(\hat{\rho}Q)}$$

Напомняне: $\underset{\text{набл.}}{\rho(A)} := \mathrm{Tr}(\hat{\rho}A) \underset{\text{инг.}}{=} \langle A \rangle_\rho$

Св.2: $\hat{\rho} \mapsto \hat{\rho}' = \dfrac{Q\hat{\rho}Q}{\mathrm{Tr}(\hat{\rho}Q)}$

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Нека Ψ, Φ – два век. на съст

$\downarrow$ $\downarrow$

$\hat{\rho}$ Q

$$\mathrm{Prob}_{\hat{\rho}} Q = \mathrm{Tr}\, \hat{\rho} Q =$$

$$= \mathrm{Tr}\, |\Psi\rangle\overbrace{\langle\Psi| \cdot |\Phi\rangle}^{\text{число}}\langle\Phi|$$

$$= \langle\Psi|\Phi\rangle \, \mathrm{Tr}\, |\Psi\rangle\langle\Phi|$$

$\langle\Phi|\Psi\rangle$ число

Вероятност за преход:

$$\Psi \longmapsto \Phi$$

$$|\langle\Psi|\Phi\rangle|^2$$

Св.4: При измерване на елементарно събитие настъпва чисто състояние: именно това, което съотв. на елем. събитие

$$\mathrm{Prob}_{\hat\rho}\, Q = |\langle \Psi | \Phi \rangle|^2$$
$$= \mathrm{Prob}_{\hat\eta}\, P$$
$$\hat\eta \leftrightarrow \Phi, \quad P \leftrightarrow \Psi$$

А какъв е прехода?

$$\hat\rho \overset{P}{\longmapsto} \hat\rho' = \hat\eta$$
$$\hat\rho = |\psi\rangle\langle\psi|, \quad \hat\rho' = |\psi'\rangle\langle\psi'|$$
$$\psi' = Q\psi / \|Q\Psi\| = \mathrm{const}\cdot\Phi$$

Проекц. постулат на фон Нойман: $\rho \overset{Q=1}{\longmapsto} \rho'$

$$\rho'(P)\ \left(=: \mathrm{Prob}_\rho(P|Q)\right) = \underset{\text{ax.}}{=} \frac{\rho(QPQ)}{\rho(Q)} \underset{\text{за непр. съст.}}{=} \frac{\mathrm{Tr}(\hat\rho QPQ)}{\mathrm{Tr}(\hat\rho Q)}$$

Напомняне: $\rho(\underset{\text{набл.}}{A}) := \mathrm{Tr}(\hat\rho A) \underset{\text{инт.}}{=} \langle A\rangle_\rho$

Св.2: $\hat\rho \mapsto \hat\rho' = \dfrac{Q\hat\rho Q}{\mathrm{Tr}(\hat\rho Q)}$

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Св.3: Ако ρ е чисто ($\Leftrightarrow \hat\rho = |\psi\rangle\langle\psi|$) тогава и ρ' е чисто ($\Leftrightarrow \hat\rho' = |\psi'\rangle\langle\psi'|$)

$$\Psi' = Q\Psi / \|Q\Psi\|$$

Док. на Св.4.

Изх. от Св.2:

$\hat\rho$ – е произв., но $Q = |\Phi\rangle\langle\Phi|$

$$\hat\rho' = \frac{Q\hat\rho Q}{\mathrm{Tr}\,\hat\rho Q} = \frac{|\Phi\rangle\overbrace{\langle\Phi|\hat\rho|\Phi\rangle}^{\text{число}}\langle\Phi|}{\mathrm{Tr}\,\hat\rho|\Phi\rangle\langle\Phi|}$$

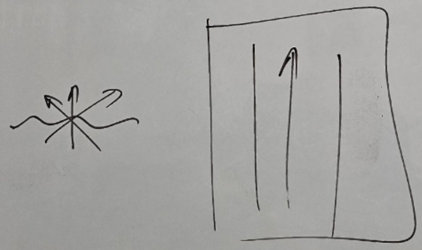
(цикл.)

Вероятност за преход:

$$\Psi \longmapsto \Phi$$
$$|\langle \Psi | \Phi \rangle|^2$$

Св.4: При измерване на елементарно събитие настъпва чисто състояние: именно това, което съотв. на елем. събитие

$$= \frac{\langle \Phi | \hat\rho\, \Phi \rangle}{\langle \Phi | \hat\rho\, \Phi \rangle} |\Phi\rangle\langle\Phi|$$



$$\mathrm{Prob}_{\hat\rho}\, Q = |\langle \Psi | \Phi \rangle|^2$$

$$= \mathrm{Prob}_{\hat\eta}\, P$$

$$\hat\eta \leftrightarrow \Phi, \quad P \leftrightarrow \Psi$$

А какъв е преходът?

$$\hat\rho \overset{P}{\longmapsto} \hat\rho' = \hat\eta$$

$$\hat\rho = |\Psi\rangle\langle\Psi|, \quad \hat\rho' = |\Psi'\rangle\langle\Psi'|$$

$$\Psi' = Q\Psi / \|Q\Psi\| = \mathrm{const} \cdot \Phi$$

Проекц. постулат на фон Нойман: $\rho \overset{Q=1}{\longmapsto} \rho'$

$$\rho'(P) \; (=: \mathrm{Prob}_\rho(P|Q)) = \underset{\text{ax.}}{=} \frac{\rho(QPQ)}{\rho(Q)} \underset{\substack{\uparrow \\ \text{за непр.} \\ \text{сист.}}}{=} \frac{\mathrm{Tr}(\hat\rho QPQ)}{\mathrm{Tr}(\hat\rho Q)}$$

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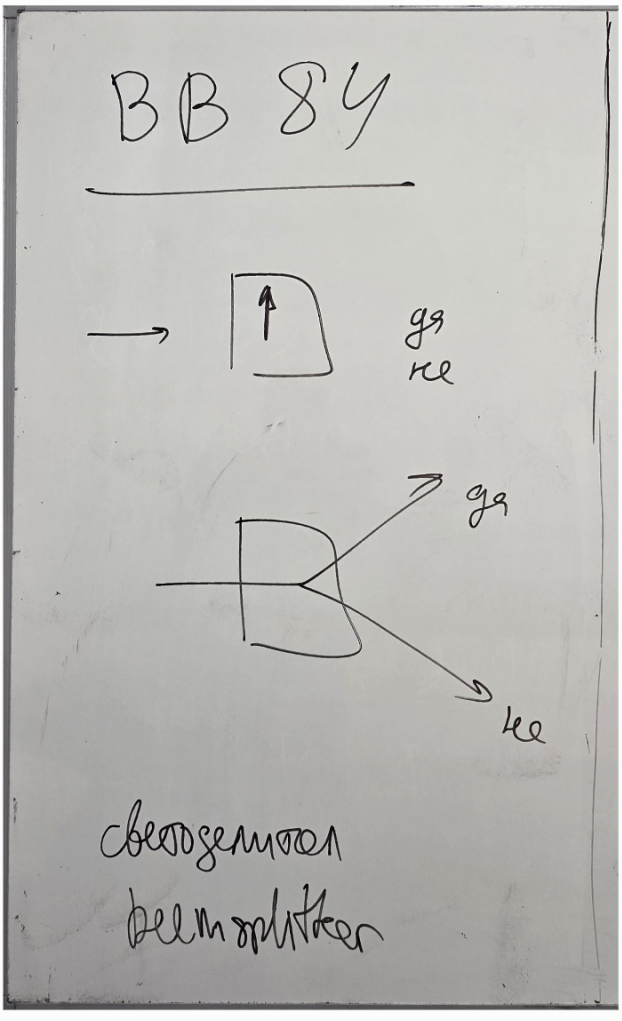
$$\Psi' = Q\Psi / \|Q\Psi\|$$

Вероятност за преход:

$$\Psi \longmapsto \Phi$$

$$|\langle \Psi | \Phi \rangle|^2$$

Св.4: При измерване на елементарно събитие настъпва чисто състояние: именно това, което съотв. на елем. събитие


BB 84
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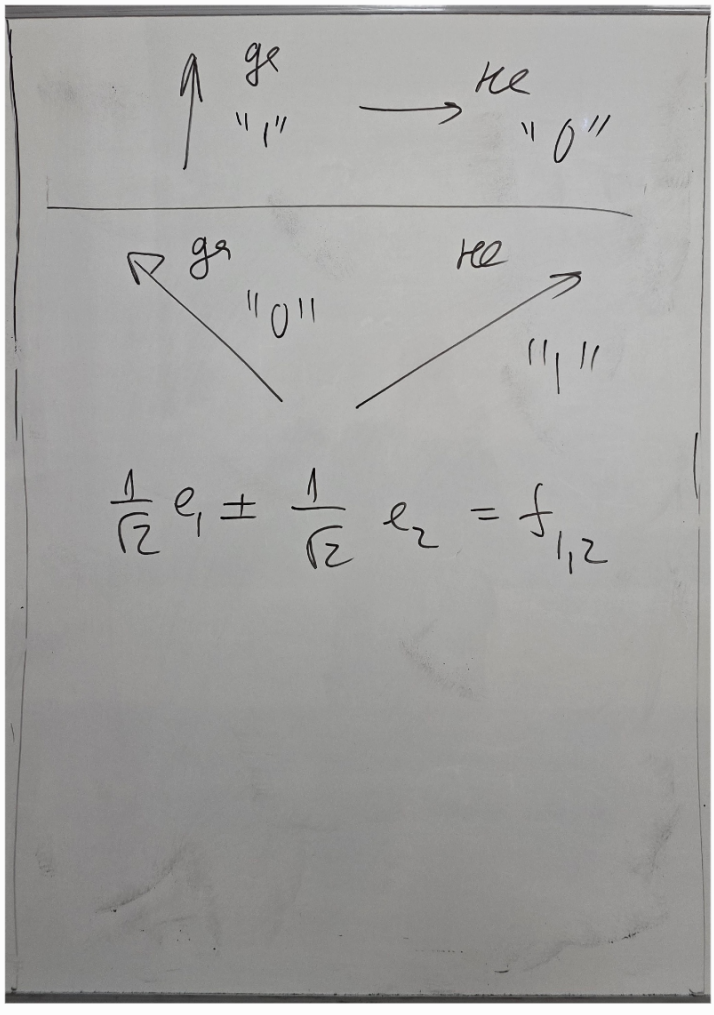
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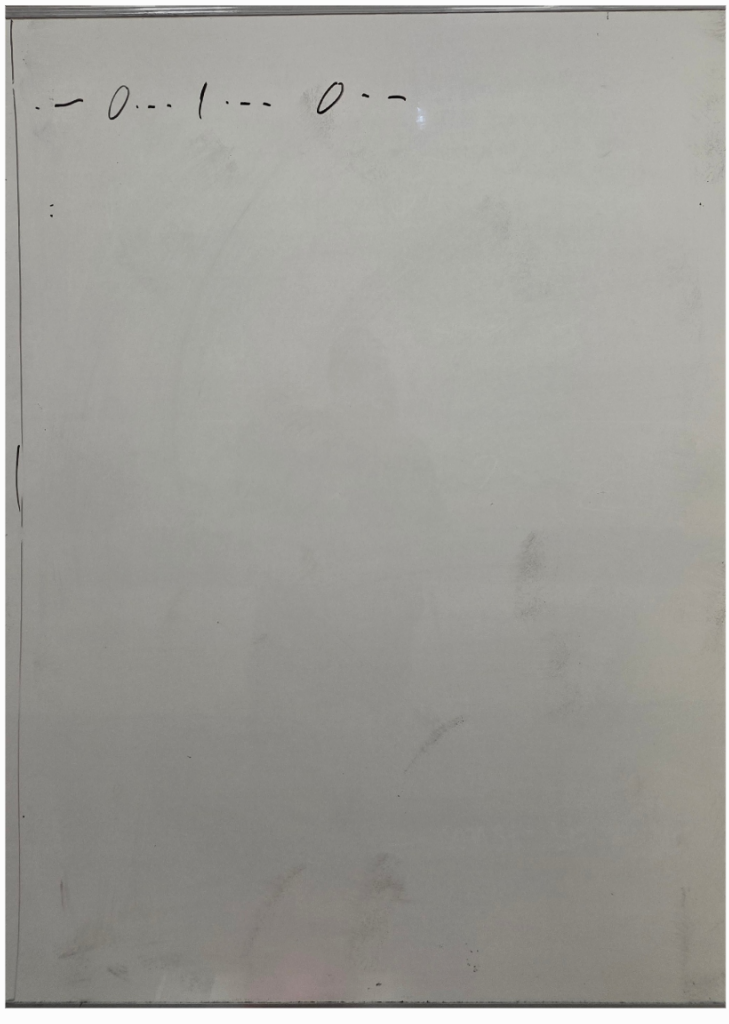
$\frac{1}{\sqrt{2}} e_1 \pm \frac{1}{\sqrt{2}} e_2 = f_{1,2}$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