REVIEW OF SCIENTIFIC INSTRUMENTS **76**, 116104 (2005)

Thermometric characteristics approximation of germanium film temperature microsensors by orthonormal polynomials

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(Received 13 May 2005; accepted 26 September 2005; published online 23 November 2005)

Approximations of thermometric characteristics of germanium film temperature microsensors are presented using a mathematical approach based on their expansion with orthonormal polynomials. A weighted orthonormal polynomial expansion method (OPEM) is applied, involving the experimental errors of calibration test data at every point. The thermometric functions $R(T)$ and $T(R)$ of resistance and temperature are described in the whole temperature range (1.7–300 K) and in three subintervals (1.7–20, 20–150, and 150–300 K). The absolute, relative, and specific sensitivities of the sensor, as well as the main approximation characteristics, are discussed. The OPEM is extended to obtain a mathematical description of $R(T)$ and $T(R)$ functions by usual polynomial coefficients calculated by orthonormal ones. A comparison between maximal relative deviations of $R(T)$ and $T(R)$ three-interval approximations, correspondingly, by usual and orthonormal polynomials, is presented. Numerical results of the approximation parameters of these type of temperature sensors are shown in the figures and tables. © *2005 American Institute of Physics.* [DOI: 10.1063/1.2126814]

I. INTRODUCTION

Different fields of cryogenic engineering and experimental physics as magnetic systems and arrangements for low-temperature thermal property investigations of thin materials, require (i) miniature, small-sized thermometers with minimized magnetic-field effects on them, (ii) extremely fast response times, and (iii) the ability to cover a wide temperature range for operation. Germanium (Ge) film temperature sensors, created in Kiev, Ukraine, appear to be suitable for such kind of application.[1] This work, which summarizes the latest achievements in the field of fabrication of thin-film Ge thermometers, also presents their thermometric characteristics. The results of polynomial $R(T)$ description of these microsensors are also presented at the Fourth European Workshop on Low Temperature Electronics held in the Netherlands in 2000 (Ref. 2).

In the present article, an orthonormal polynomial approximation of $T(\ln R)$ and $R(\ln T)$ functions is presented for Model TTR-1D of these microsensors in the whole temperature range (1.7–300 K) and in the three subintervals (1.7–20, 20–150, and 150–300 K). The R and T three-interval approximations by usual coefficients is also proposed.

II. MATHEMATICAL APPROACH

The first applications of our orthonormal polynomial expansion method (OPEM) in cryogenic thermometry are presented in Refs. 3–5 Methodical and computational aspects of the OPEM concerning its application in cryogenic thermometry at the approximation of thermometric characteristics of different type low-temperature sensors (resistors, diodes, capacitances, and thermocouples) are protected by a patent for an invention.[6] Our OPEM is based on Forsythe's[7] three-term relation for generating orthogonal polynomials over a discrete point set with arbitrary weights in a least-squares method. OPEM is a generalization for calculating derivatives and integrals with a fourth term in the expansion. The principal relation for one-dimensional generation of orthonormal polynomials $\{\Psi_k^{(0)}, k=1,2,\ldots\}$ and their derivatives $\{\Psi_k^{(m)}, m=1,2,\ldots\}$, in the OPEM is

$$\Psi_{k+1}^{(m)}(q) = \gamma_{k+1}[(q-\alpha_{k+1})\Psi_k^{(m)}(q) - (1-\delta_{k0})\beta_k\Psi_{k-1}^{(m)}(q) + m\Psi_k^{(m-1)}(q)]. \quad (1)$$

The polynomials $\{\Psi_k^{(0)}\}$ satisfy the orthogonality relations over the given point set $\{q_i, i=1,2,\ldots M\}$ with weights $w_i=1/\sigma_i^2$, depending on errors σ_i at every point. The approximation values f^{appr} of the function f and its mth derivative $f^{(m)\mathrm{appr}}$ are given by

$$f^{(m)\mathrm{appr}}(q) = \sum_{k=0}^{N} a_k\Psi_k^{(m)}(q) = \sum_{k=0}^{N} c_k q^k. \quad (2)$$

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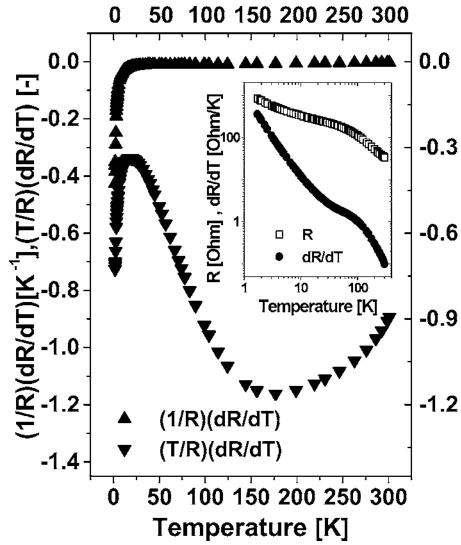

FIG. 1. Temperature dependencies of the relative $(1/R)(dR/dT)$ (K^{-1}) and specific $(T/R)(dR/dT)$ (–) sensitivities of the Ge film resistance temperature microsensor. (Inset) Temperature dependencies of the resistance R (Ω) and the absolute sensitivity dR/dT (Ω/K) of the Ge film resistance temperature microsensor.

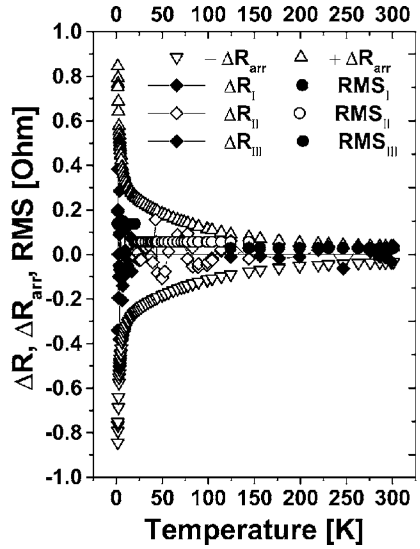

FIG. 2. Temperature dependencies of ΔR, $\Delta R_{\rm arr}$, and RMS^R for three interval approximation of the Ge film resistance temperature microsensor.

The optimal degree N of the approximating polynomials in Eq. (2) is selected by the algorithm, using the following two criteria. First, the fitting curve should lie in the error corridor of the dependent variable $(q_j, f_j^{\exp}\pm\sigma_j, j=1,\ldots,M)$ [Eq. (3)]. Second, the minimum χ^2 should be reached [Eq. (4)]:

$$(f_j^{\rm appr}-f_j^{\rm exp})^2 w_j \leqslant 1, \tag{3}$$

$$\sum_{j=1}^{M} w_j (f_j^{\rm appr}-f_j)^2/(M-n-1) \to \min. \tag{4}$$

When the first criterion is satisfied, the search of the minimum χ^2 stops.

The present version is a development of the OPEM, which proposes an evaluation of usual coefficients c_k from orthogonal ones a_k in accordance with Eq. (2). The mathematical extension of Eq. (2) with the inherited errors in coefficients in usual and orthonormal expansion will be published in a separate paper.

III. APPROXIMATION RESULTS AND ILLUSTRATIONS

A. Orthonormal expansion

The studied miniature thermometers have the advantages of being small and having a high sensitivity. They are based on the vacuum-evaporated Ge film onto semi-insulating GaAs substrate. The Ge film-sensitive elements 0.3 mm wide × 0.3 mm long × 0.2 mm high, are placed into hermetic packs of 1.2 mm diam × 1.0 mm length.

The temperature dependencies of the resistance, R, and the sensitivities, absolute (dR/dT), relative $(1/R)/(dR/dT)$, and specific $(T/R)(dR/dT)$, evaluated at every point, are shown in Fig. 1. It is worth noting that the subintervals are chosen by the temperature behavior of the specific sensitivity $(T/R)(dR/dT)$ of the studied thermometer (see Fig. 1). The $R(T)$ and $T(R)$ functions are described by orthonormal polynomials in the whole temperature range 1.7–300 K and in three subintervals (1.7–20, 20–150, and 150–300 K) using the weights W^R and W^T. The weighting functions W^R and W^T are defined with a variance σ^2 for $R(T)$ and $T(R)$ approximations[3,4] accepted to be, correspondingly, squares of the absolute resistance resolution $\Delta R_{\rm arr}$ and the absolute temperature resolution $\Delta T_{\rm atr}$ of the sensor. Here the accuracy of the resistance experimental data is estimated to be 0.1% of R_i determining $(\Delta R_{\rm arr})_i = 0.001 R_i$ (Ω) within 0.844 Ω and 0.034 Ω. The accuracy of T defined by $(\Delta T_{\rm atr})_i = (\Delta R_{\rm arr})_i / |(dR/dT)_i|$ (K) is evaluated to be between 0.0023 and 0.3365 K, i.e., (0.135–0.112)% of T_i. Then $W_i^R = 1/(\Delta R_{\rm arr}^2)_i = 1.10^6/R_i^2$ (Ω^{-2}), $W_i^T = 1/[(\Delta T_{\rm atr})^2]_i = W_i^R (dR/dT)_i^2$ (K^{-2}). The absolute sensitivity $(dR/dT)_i$ of the described thermometer as well as the deviations ΔR_i and ΔT_i between experimental and approximating values of the resistance and temperature are evaluated at every point. The temperature behavior of the calculated approximation differ-

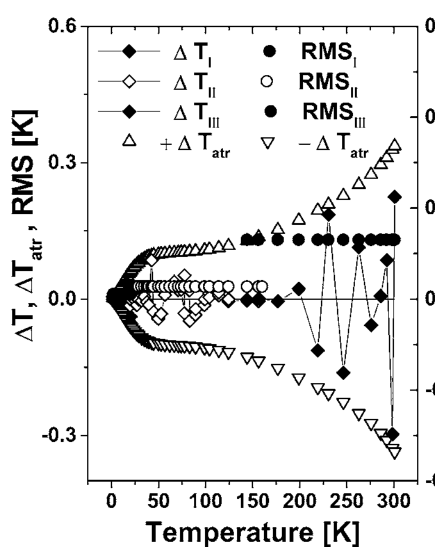

FIG. 3. Temperature dependencies of ΔT, $\Delta T_{\rm atr}$, and RMS^T for three-interval approximation of the Ge film resistance temperature microsensor.

TABLE I. OPEM approximations of $T(R)$ and $R(T)$ for Ge microsensor model TTR-1D.

T (K)	R (Ω)	M	$f(q)$	N	W^R (Ω)	W^T (K)	RMS^R (Ω)	MAD^R (Ω)	RMS^T (K)	MAD^T (K)
1.7–300	844–34	80	$R(T)$	12	1.4–857	⋯	0.083	0.059	⋯	⋯
1.7–300	844–34	80	$T(R)$	15	⋯	$2\times10^5-8.8$	⋯	⋯	0.103	0.062
1.7–20	844–260	35	$R(T)$	5	1.4–15	⋯	0.138	0.104	⋯	⋯
1.7–20	844–260	35	$T(R)$	6		2×10^5-279	⋯	⋯	0.010	0.006
20–150	260–77	35	$R(T)$	5	15–169	⋯	0.057	0.045	⋯	⋯
20–150	260–77	35	$T(R)$	5	⋯	299–58	⋯	⋯	0.027	0.020
150–300	80–34	14	$R(T)$	4	169–857	⋯	0.028	0.024	⋯	⋯
150–300	80–34	14	$T(R)$	5	⋯	74–9.5	⋯	⋯	0.18	0.13

ences ΔR_i and ΔT_i, the root-mean-square deviations (RMS) RMS^R and RMS^T, respectively, for $R(T)$ and $T(R)$ functions, and the absolute resistance and absolute temperature resolutions $\pm\Delta R_{\rm arr}$ and $\pm\Delta T_{\rm atr}$ for studied thermometers in three range approximations, (marked with subscripts I, II, and III, respectively), are shown in Figs. 2 and 3. Parameters of the OPEM whole-interval approximation are presented in Table I together with the ones for three-interval description.

Following the cited criteria in Eqs. (3) and (4) the deviation results are in the error corridor. The common result is that the polynomial degree for the whole temperature range is higher than that for subintervals. For $R=R(\ln T)$ approximation the optimal degree is 12 for the whole temperature range and 5,5,4 for the subintervals. For $T=T(\ln R)$ approximation the optimal degree is 15 for the whole temperature range and 6,5,5 for subintervals. In Table I the main characteristics are given including the overall approximation characteristics: RMS deviation and the mean absolute deviation (MAD). The RMS deviation is given in our paper.[3] Involving $\overline{\Delta T}=1/M\Sigma_i \Delta T_i$, the characteristic MAD is defined as follows:

$$\mathrm{MAD} = \frac{1}{M}\sum_i |\Delta T_i - \overline{\Delta T}|. \qquad (5)$$

B. Usual expansion obtained by orthonormal one

The numerical experiment is done here showing the method possibilities. The algorithm is extended by the calculation of usual coefficients $\{c_k\}$ from orthogonal ones $\{a_k\}$ with experimental data for calibrating points using Eq. (2). This is carried out for the three input subintervals. The calculations are made for both the $R=R(\ln T)$ and $T=T(\ln R)$ descriptions in two runs: First, in an interval $[-1,1]$, and second, in the input subintervals. Optimal values of usual polynomial degrees are chosen using two criteria. The first criterion is

$$\max|f_i^{\rm exp} - f_i^{{\rm appr},u}|/f_i^{\rm exp} \rightarrow \min, \qquad (6)$$

where $f^{{\rm appr},u}$ is the approximating function defined with the usual expansion. The second criterion is

$$\Delta\{c_k\}_1^N \rightarrow \min, \qquad (7)$$

where $\{\Delta c_k\}$ are inherited errors in the usual coefficients. The usual coefficients c_k for three-interval $R(\ln T)$ and $T(\ln R)$ approximations are presented in Table II. For $R=R(\ln T)$ three-interval approximation the optimal degrees by orthonormal (as cited in Table I) and usual expansion are the same, i.e., 5,5,4, while for $T=T(\ln R)$ three-interval approximation the optimal degrees are (6,5,5) by orthonormal and (4,5,5) by usual expansions.

A comparison between the orthonormal and usual relevant expansion of $R(T)$ and $T(R)$ approximations by the OPEM is presented in Table III with the maximal values of the relative deviations $\Delta f_i^1/f_i^{\rm exp}$ and $\Delta f_i^2/f_i^{\rm exp}$. Here the absolute deviations are given by the expressions $(\Delta f^1)_i=(f_i^{\rm exp} - f_i^{{\rm appr},u})$, $(\Delta f^2)_i=(f_i^{\rm exp}-f_i^{\rm appr})$.

The next step is an interpolation procedure based on the good approximation accuracy shown in the proposed comparison. The set of approximating values $\{f_{l1}^{\rm appr}, l1=1,M_n\}$ is calculated in M_n points chosen between given calibration ones, and having the previously defined coefficients $\{c_k, k=1,N\}$. The interpolation is carried out for every input sub-

TABLE II. Usual coefficients for $R(T)$ and $T(R)$ approximation.

Range $R(\Omega)$	844–260		260–77		77–34	
Range T (K)	1.7–20		20–150		150–300	
Function	$R(\ln T)$	$T(\ln R)$	$R(\ln T)$	$T(\ln R)$	$R(\ln T)$	$T(\ln R)$
c_0	1303.775	23 660.240	642.025	−43 638.870	−3262.111	108 941.200
c_1	−1144.246	−14 451.850	154.141	48 560.520	3164.836	128 349.300
c_2	634.017	3315.664	−350.217	−21 234.100	−1009.652	61 065.620
c_3	−215.181	−338.528	152.042	4600.144	134.221	−14 598.610
c_4	40.623	12.974	−27.613	−495.404	−6.438	1749.987
c_5	−3.275	⋯	1.181	21.238	⋯	84.068

TABLE III. Relative deviations $\Delta R^1(\Delta R^2)/R^{exp}$, $\Delta T^1(\Delta T^2)/T^{exp}$.

T range (K)	1.7–20	20–150	150–300
R range (Ω)	844–260	260–77	77–34
$(\Delta R^1/R^{exp})_{max}$ (%)	−0.0735	0.0687	−0.158
At R (Ω) and T (K)	R=516.63, T=3.69	R=90.43, T=124.4	R=41.7, T=246.3
$(\Delta T^1/T^{exp})_{max}$ (%)	−0.348	−0.293	−0.129
At R (Ω) and T (K)	R=844.60, T=1.70	R=260.20, T=20.46	R=41.67, T=246.29
$(\Delta R^2/R^{exp})_{max}$ (%)	−0.0735	0.0678	−0.152
At R (Ω) and T (K)	R=516.63, T=3.69	R=90.43, T=124.4	R=41.7, T=246.3
$(\Delta T^2/T^{exp})_{max}$ (%)	0.213	0.199	−0.126
At R (Ω) and T (K)	R=282.11, T=16.12	R=197.80, T=42.66	R=41.67, T=246.29

interval with optimal polynomial degrees, presented together with the coefficients c_k in Table II. There are no oscillations in approximating curves in the points lying within given points.

Finally, the developed version of our algorithm offers an orthonormal polynomial approximation of the thermometric functions $R=R(T)$ and $T=T(R)$ of the Ge microsensor, as well as a description of these characteristics applying usual polynomials, obtained by orthonormal ones. The presented approach for description of the above thermometric characteristics proposes possibilities for automation of low-temperature physics experiments, particularly in study of low-temperature properties of thin materials at high magnetic fields.

ACKNOWLEDGMENT

The paper is partially supported by the Bulgarian National Council for Scientific Research, under Grant No. Ph 1001.

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